arihant

2020-21
EDITION

NDA/NA

NATIONAL DEFENCE ACADEMY & NAVAL ACADEMY ENTRANCE EXAMINATION

CHAPTERWISE SECTIONWISE

SOLVED PAPERS

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NAVAL ACADEMY ENTRANCE EXAMINATION**

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Compiled & Edited by
Arihant 'Expert Team'

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ABOUT THE EXAMINATION

The National Defence Academy (NDA) is an iconic institution, a global brand of excellence in the sphere of military education. For recruitment to the Indian Army, Navy and Air force wings of Indian Army, there is prestigious National Defence Academy Entrance Examination. To join National Defence Academy, a candidate must appear in the entrance exam conducted by Union Public Service Commission (UPSC), twice a year. The UPSC is solely responsible for issuing guidelines for selection and the final conduct of the entrance examination.

NATIONALITY

A candidate must be either

(i) Indian citizen, or

(ii) A subject of Bhutan, or

(iii) A subject of Nepal, or

(iv) A Tibetan refugee who came over to India before 1st January, 1962 with the intention of permanently setting in India, or

(v) A person of Indian origin who has migrated from Pakistan, Burma, Sri Lanka and East African countries of Kenya, Uganda, the United Republic of Tanzania, Zambia, Malawi, Zaire and Ethiopia and Vietnam with the intention of permanently setting in India.

Provided that a candidate belonging to categories (ii), (iii), (iv) and (v) above shall be a person in whose favour a certificate of eligibility has been issued by the Government of India. Certificate of eligibility, will not , however, be necessary in the case of candidate who are Gorkha subjects of Nepal.

AGE LIMIT, SEX AND MARITAL STATUS

Only unmarried male candidates whose age is not less than 15 years and not exceeding 18 years on 1st January in accordance with the year of examination are eligible to apply. Candidates must not marry until they complete their full training.

EDUCATIONAL QUALIFICATION

For Indian Army, candidates must have done or appearing class 12th from a recognised board. For Air force and Navy and for 10+2 course at Naval Academy, candidates must have done or appearing class 12th with Physics and Mathematics from a recognised board.

SCHEME OF EXAMINATION

Subject	No. of Questions	Duration	Maximum Marks
Mathematics	120	$2\frac{1}{2}$ hours	300
General Ability Test			
Part-A (English)	50	$2\frac{1}{2}$ hours	200
Part-B (General Studies)	100		400
Total			**900**
SSB Test/Interview			**900**

The papers in all the subjects will consists of objective type questions only. There will be 1/3rd negative marking for wrong answers.

-0.83 for Maths

-1.33 for English

-1.33 for General Ability Test

The SSB procedure consists of two stage-

(a) Stage I comprises of Officer Intelligence Rating (OIR) test, Picture Perception and Description Test (PP and DT).

(b) Stage II comprises of interview, Group Testing Officer tasks, Psychology tests and the Conference.

Subjectwise - Chapterwise

TREND ANALYSIS

PAPER I MATHEMATICS

Aspirants who are willing to crack NDA examinations must acquire excellent clutch on Mathematical abilities as one-third of the total written test marks are determined by this section. The candidates have to attempt 120 questions carrying 300 marks in 2 hours and 30 minutes. Based on the analysis of previous years question papers, it can be concluded that the exam paper generally asks questions on following topics. Here along with the topic name we have also indicated the approximate number of questions, which are generally asked from there topics.

Number Algebra (25-35 Questions)

There are around 5-8 questions which are usually statement based from Sets, Relation and Functions, 8-11 questions are asked from Complex numbers and Quadratic equations, 4-8 questions are there from Arithmetic and Geometric progressions and 8-10 questions from Permutation and Combination Binomial theorem, Binary number and Logarithms.

Matrices And Determinants (3-12 Questions)

The questions from this topic are entirely based on 11th and 12th class syllabus including around 1-4 questions on finding determinant and 2-9 questions on Algebra of matrices.

Trigonometry (15-20 Questions)

Around 10-12 questions are based on simple trigonometric ratios and identities, 2-4 questions are there from Inverse trigonometry and 2-4 questions from Heights and Distances and Properties of Triangle.

Analytical Geometry of Two and Three Dimension (10-20 Questions)

There are around 4-9 questions based on rectangular cartesion system and straight line, 2-3 questions based on Circle, 2-3 question based on Conic Section, 2-5 questions based on Three dimensional geometry.

Differential Calculus (20-25 Questions)

This section is one of the most important section and includes questions from various topics. Around 10-12 questions are asked from functions, Limits and continuity and differentibility, 4-6 questions are based on finding Maxima and minima, 6-8 questions are based on determining increasing and decreasing functions and the remaining question include Fundamentals of differentiation and its applications.

Integral Calculus and Differential Equations (8-12 Questions)

It is comparatively difficult section and around 4-6 questions are there from Definite and Indefinite integrals and around 4-6 Area under the curves and the remaining questions are there from Differential equations.

Vector Algebra (5-8 Questions)

This is one of the most scoring section and includes around 3-4 questions from Scalar and Vector product of vectors and 2-4 questions on finding Two or three dimension vectors.

Statistics And Probability (16-20 Questions)

This section include tricky as well as calculative questions. Around 8-10 questions are there from Probability and around 8-10 remaining from Statistics which involves finding Mean, Median, Mode or other Central tendency of the given data.

PAPER II GENERAL ABILITY TEST

Paper II of the NDA examination is General Ability Test which is of 600 marks. It is divided into two parts. Part 'A' is General English for 200 marks with 50 questions and Part 'B' is General Knowledge for 400 marks with 100 questions. Maximum time for this paper is 2 hour 30 minutes.

GENERAL ENGLISH

General English forms an essential part of 'General Ability Test' of NDA exam which consists of 50 questions. On the basis of previous years' papers, it has been observed that this section covers various topics. Synonyms (5-10 questions), Antonyms (5-10 questions) both are asked usually in word form not in sentence form. Spotting the errors (10-15 questions) deals with the errors in use of Prepositions, Noun, Pronoun, Adjective, Adverb, Verb, Tenses and Conjunctions etc. and candidate has to correct these errors. Sentence improvement (10-12 questions) has asked and questions with the use of phrasal verbs, idioms, sayings, correct form of verb etc. Fillers (6-10 questions) section deals with the most derivable word in the sentence to differentiate between similar words.

Reconstruction of sentence and paragraphs (2-10 questions) is an easy section where candidate has to arrange the sentences in a proper order or sequence.

SCIENCE

Physics (20-25 Questions)

In science section, particularly physics plays an important role in NDA. As we go through the previous years' questions, around 20-25 questions are asked from this section. By perceiveness of concept and segregating the topics on microscopic level, physics can be made easy.

- From Units and Dimension, 1-2 questions are asked.
- As in Kinematics section, around 3-4 questions are asked from Acceleration, Distance and Displacement.
- From Newton's Laws of Motion, around 1-2 questions are asked in this section like Momentum and impulse.
- From Work, Power and Energy, around 1-2 questions based on Kinetic and potential energy, Types of energy are asked.
- From Gravitation and simple harmonic motion, 2-3 questions based on Effect of gravitation, eclipse and weightlessness are asked.
- From Light section, 3-4 questions are asked as from Image formation by mirror and lenses (concave and convex).
- From Properties of Material, 1-2 questions are asked from Mechanical properties of solid and fluid and Thermal properties of matter.
- Apart from all these sections, almost 6-7 questions are asked from one of the major topics i.e. Current electricity like series and parallel connection, Magnet, Thermal effect of electricity, Modern Physics and Nucleus or Radioactivity.

Chemistry (5-8 Questions)

Science section of NDA chemistry comprises around 15-18 questions. This section is classified into three sub-groups based on its properties.

- From some basic concept of chemistry 8-10 questions are asked from physical or chemical changes, element, mixture and compounds, Law of chemical combinations, atomic structure and redox reaction.
- From organic section 6-7 questions are asked from carbon and its compounds, fertilizers, air and water, carbon dioxide and some important chemical compounds.
- From 2-3 questions are asked Acid base and salt, chemical equivalent and classification of element.

Biology (6-8 Questions)

- In NDA exam nearly, 6-8 questions are asked from biology.
  These questions can be categorised into many topics.
- From 'The Living-World', 1-2 questions are asked from Animal and Plant kingdom covering Biodiversity and Taxonomy.
- From Health and Nutrition section, 2-3 questions are asked such as Vitamins and Its deficiency disorders, Diseases and Its diagnosis.
- From 'Genetics and Cell-biology', 1-2 questions are asked such as about all organ cells, blood, DNA and Human organs.
- From Animal and Plant Morphology and Physiology, 1-2 questions are frequently asked. Sometimes, 1-2 questions are asked from Economic zoology and Economic botany.

GENERAL STUDIES

History (20-23 Questions)

After analysing the previous years' question papers of the NDA examination, we have derived that around 22-25 questions are asked from the History section. The section has been splitted into four sub-parts.

- From Ancient History, 3-4 questions are asked from topics like Harappan civilisation, Buddhism and Jainism, Mauryan empire and the Gupta age, South Indian Empire.
- Around 4-5 questions cover the Medieval History of India. It consists of important topics like attacks of various rulers, Delhi sultanate (5 Sultanate dynasties and administration), Mughal empire (its important aspects like Land Revenue System, Art and Architecture) and Bhakti and Sufi saints like Kabir, Nanak, Nizamuddin Auliya, etc.
- From Modern History, 9-10 questions are asked with focus on 1857 Revolt, Socio Religious Reform movement, Formation and Sessions of Congress, Freedom movement, Books and Authors, etc.
- Around 3-4 questions are asked related to events of World History like Renaissance, Revolutions like Russian, American, Germani and French Revolution, World War I and II, etc.

Geography (22-24 Questions)

After due analysis of previous years' question papers, we can conclude that approximately 22-24 questions are asked from Geography. This section can be divided into three broad sections to analyse the pattern of questions asked in the exam.

- Indian Geography has important weightage in the exam. 2-3 questions are asked from topics like origin of Earth, Time zone, Types of rocks, Mountains, etc. Further, 2-3 questions are related to structure and composition of Atmosphere, Oceans, Cyclones and Anticyclones, etc. Apart from this, 7-8 questions cover topics like important industries, mineral and power sources in India, agriculture pattern in India, etc.
- Concepts of World Geography are also a part of this portion. Around 4-5 questions are based on Drainage and Climate pattern of the World, and important Minerals and Energy resources of the world.
- The Environment and Ecology part is also covered in this section. About 4-5 questions are asked from areas like Natural vegetation of India and the world, National parks and Wildlife sanctuaries in India, etc.

Polity (10-11 Questions)

Analysing the previous years' question papers, it can be asserted that polity has its due place with a good number of 8-10 questions in the exam. Various aspects of polity have been asked in the following way:

- Around 2-3 questions are asked from the Constitution of India from the topics like Preamble, Fundamental Rights and Duties, Directive principles of State Policy.
- The Executive and Legislature has been allotted 2-3 questions which cover President and its powers, Sessions of Parliament, difference between Ordinary and Money bill, State Legislature, etc.
- About 1-2 questions are from Judiciary with focus on Supreme Court and High Court, Writ jurisdiction, Lokpal and Lokayukta, etc.
- Around 1-2 questions are also asked from Constitutional and Non-constitutional bodies like Central Vigilance Commission, Election Commission, NITI Ayog, National Human Rights Commission, etc.
- Apart from this, one can find 3-4 questions from topics like Constitutional amendment acts, Schedules and Articles in the Constitution of India, Local Self-Government, etc.

Economy (5-6 Questions)

Around 6-8 questions are asked from the Economy section with focus on the Indian economy. The areas which are given importance in the question paper include Economic Planning in India, concepts like Poverty, Demographic dividend and National Income, Banking Sector in India, etc. Few questions are also asked from various Government policies and schemes and Economic issues of the current importance.

General Knowledge And Current Affairs (6-8 Questions)

The General knowledge and Current affairs has been given weightage to around 6-8 questions. One can find questions related to National and International events, Persons in news, Sports, Places, Awards And Books and Authors.

NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2020 (I & II)

PAPER I : Mathematics

1. If matrix $A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$ where $i = \sqrt{-1}$, then which one of the following is correct?

(a) A is hermitian

(b) A is skew-hermitian

(c) $(\bar{A})^T + A$ is hermitian

(d) $(\bar{A})^T + A$ is skew-hermitian

(**c**) We have

$$A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$$

Now, $\bar{A} = \begin{bmatrix} 1+i & -i \\ i & 1+i \end{bmatrix}$

$$(\bar{A})^T = \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix}$$

Now, consider

$$X = (\bar{A})^T + A = \begin{bmatrix} 1+i & i \\ -i & 1+i \end{bmatrix} + \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2i \\ -2i & 2 \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} 2 & -2i \\ 2i & 2 \end{bmatrix}$$

$$(\bar{X})^T = \begin{bmatrix} 2 & 2i \\ -2i & 2 \end{bmatrix} = X$$

Hence, $X = (\bar{A})^T + A$ is a hermitian matrix.

2. The term independent of x in the binomial expansion of $\left(\frac{2}{x^2} - \sqrt{x}\right)^{10}$ is equal to

(a) 180 (b) 120

(c) 90 (d) 72

(**a**) The general term in the binomial expansion of $\left(\frac{2}{x^2} - \sqrt{x}\right)^{10}$ is

$$T_{r+1} = {}^{10}C_r \left(\frac{2}{x^2}\right)^{10-r} (-\sqrt{x})^r$$

$$= {}^{10}C_r (2)^{10-r} x^{-20+2r+\frac{r}{2}} (-1)^r$$

For independent of x, put

$$-20 + 2r + \frac{r}{2} = 0 \Rightarrow \frac{5r}{2} = 20$$

$$\Rightarrow \quad r = 8$$

$$\therefore \quad T_{8+1} = {}^{10}C_8 (2)^{10-8} (-1)^8$$

$$= \frac{10 \times 9}{2 \times 1} \times 2^2 \times 1$$

$$= 180$$

3. If $(1 + 2x - x^2)^6 = a_0 + a_1 x + a_2 x^2 + \ldots + a_{12} x^{12}$, then what is $a_0 - a_1 + a_2 - a_3 + a_4 - \ldots + a_{12}$ equal to?

(a) 32 (b) 64

(c) 2048 (d) 4096

(**b**) We have,

$$(1 + 2x - x^2)^6 = a_0 + a_1 x + a_2 x^2 + \ldots + a_{12} x^{12}$$

Put $x = -1$ both sides, we get

$$(1 - 2 - 1^2)^6 = a_0 - a_1 + a_2 - \ldots + a_{12}$$

$$\Rightarrow a_0 - a_1 + a_2 - \ldots + a_{12} = (-2)^6 = 64$$

4. If $C(20, n+2) = C(20, n-2)$, then what is n equal to?

(a) 18 (b) 25

(c) 10 (d) 12

(**c**) Given, $C(20, n+2) = C(20, n-2)$

$\Rightarrow C(20, 20 - (n+2)) = C(20, n-2)$ $[\because C(n, r) = C(n, n-r)]$

$$\Rightarrow \quad 20 - (n+2) = n - 2$$

$$\Rightarrow \quad 20 = n + 2 + n - 2$$

$$\Rightarrow \quad 2n = 20$$

$$\Rightarrow \quad n = 10$$

5. For how many values of k, is the matrix $\begin{bmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{bmatrix}$ singular?

(a) Only one (b) Only two

(c) Only four (d) Infinite

(**d**) The condition for singular matrix is

$$\begin{vmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{vmatrix} = 0$$

Expanding along R_1, we get

$$0 - k(k - 5k) + 4(-k^2 + 0) = 0$$

$$\Rightarrow \quad 4k^2 - 4k^2 = 0$$

$$\Rightarrow \quad 0 = 0, \forall k \in R$$

Hence, for infinite values of k, given matrix is singular.

6. The number $(1101101 + 1011011)_2$ can be written in decimal system as

(a) $(198)_{10}$ (b) $(199)_{10}$

(c) $(200)_{10}$ (d) $(201)_{10}$

(**c**) Now, $(1101101)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$$= 64 + 0 + 16 + 8 + 0 + 2 + 1$$

$$= (91)_{10}$$

and

$$(1011011)_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 64 + 32 + 0 + 8 + 4 + 0 + 1 = (109)_{10}$$

$\therefore (1101101 + 1011011)_2$
$= (1101101)_2 + (1011011)_2$
$= (91)_{10} + (109)_{10} = (200)_{10}$

7. What is the value of
$\frac{1}{10}\log_5 1024 - \log_5 10 + \frac{1}{5}\log_5 3125$?
(a) 0 (b) 1 (c) 2 (d) 3

(**a**) $\frac{1}{10}\log_5 1024 - \log_5 10 + \frac{1}{5}\log_5 3125$
$= \frac{1}{10}\log_5 2^{10} - \log_5(5\times 2) + \frac{1}{5}\log_5 5^5$
$= \frac{10}{10}\log_5 2 - [\log_5 5 + \log_5 2] + \frac{5}{5}\log_5 5$
$[\because \log mn = \log m + \log n]$
$= \log_5 2 - [1 + \log_5 2] + 1 \quad [\because \log_m m = 1]$
$= 0$

8. If $x = \log_c(ab)$, $y = \log_a(bc)$, $z = \log_b(ca)$, then which of the following is correct?
(a) $xyz = 1$
(b) $x + y + z = 1$
(c) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1} = 1$
(d) $(1+x)^{-2} + (1+y)^{-2}(1+z)^{-2} = 1$

(**c**) We have, $x = \log_c(ab)$
$y = \log_a(bc)$
$z = \log_b(ca)$
Now, $1 + x = \log_c c + \log_c(ab)$
$= \log_c(abc)$
$1 + y = \log_a(abc)$
and $1 + z = \log_b(abc)$
Now, $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$
$= [\log_c(abc)]^{-1} + [\log_a(abc)]^{-1} + [\log_b(abc)]^{-1}$
$= \frac{1}{\log_c(abc)} + \frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)}$
$= \frac{\log c}{\log(abc)} + \frac{\log a}{\log(abc)} + \frac{\log b}{\log(abc)}$
$\left[\because \log_m n = \frac{\log n}{\log m}\right]$
$= \frac{\log c + \log a + \log b}{\log(abc)} = \frac{\log(abc)}{\log(abc)} = 1$

9. Let $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. If $AB = C$, then what is the value of the determinant of the matrix A?
(a) – 10 (b) – 14
(c) – 24 (d) – 34

(**b**) Given, $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$
$B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$
Also given, $AB = C$
$\Rightarrow \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} 2x+2y-y \\ 4x-x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} 2x+y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$
On equating the corresponding elements, we get
$2x + y = 3$ and $3x + y = 2$
$\Rightarrow x = -1$ and $y = 5$
$\therefore A = \begin{bmatrix} -1+5 & 5 \\ 2\times -1 & -1-5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$
$\therefore$ The determinant of matrix A is
$|A| = \begin{vmatrix} 4 & 5 \\ -2 & -6 \end{vmatrix} = -24 + 10 = -14$

10. If $1.5 \le x \le 4.5$, then which one of the following is correct?
(a) $(2x-3)(2x-9) > 0$
(b) $(2x-3)(2x-9) < 0$
(c) $(2x-3)(2x-9) \ge 0$
(d) $(2x-3)(2x-9) \le 0$

(**d**) We have, $1.5 \le x \le 4.5$
$\Rightarrow \frac{3}{2} \le x \le \frac{9}{2} \Rightarrow 3 \le 2x \le 9$
$\Rightarrow (2x-3)(2x-9) \le 0$

11. Let $S = \{1, 2, 3, \ldots\}$. A relation R on $S \times S$ is defined by xRy if $\log_a x > \log_a y$ when $a = \frac{1}{2}$. Then the relation is
(a) reflexive only
(b) symmetric only
(c) transitive only
(d) both symmetric and transitive

(**c**) We have, $S = \{1, 2, 3, \ldots.\}$
and $\log_a x > \log_a y$
Here, $a = \frac{1}{2} \in (0, 1)$
$\therefore \log_a x > \log_a y \Rightarrow x < y$
Now, $x\,R\,x \Rightarrow x < x$ which is not possible. So it is not reflexive relation.
Now, $x\,R\,y \Rightarrow x < y$
But $y \not< x$, so it is not symmetric relation.
Now, $x\,R\,y$ and $y\,R\,z$
$\Rightarrow x < y$ and $y < z \Rightarrow x < z \Rightarrow x\,R\,z$
Hence, it is transitive relation only.

12. What is the value of the determinant $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix}$ where $i = \sqrt{-1}$?
(a) 0 (b) – 2 (c) $4i$ (d) $-4i$

(**d**) Let $\Delta = \begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix}$
$= \begin{vmatrix} i & -1 & -i \\ 1 & -1 & 1 \\ i & 1 & -i \end{vmatrix}$
$[\because i^2 = -1, i^3 = -i, i^4 = 1]$
$= i(i-1) + 1(-i-i) - i(1+i)$
[Expanding along R_1]
$= i^2 - i - 2i - i - i^2 = -4i$

13. Let $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then what is AB equal to?
(a) $\begin{bmatrix} ax+hy+gz \\ y \\ z \end{bmatrix}$
(b) $\begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ z \end{bmatrix}$
(c) $\begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$
(d) $[ax+hy+gz \quad hx+by+fz \quad gx+fy+cz]$

(**c**) Now,
$AB = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$
$= \begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$

14. What is the number of ways in which the letters of the word 'ABLE' can be arranged so that the vowels occupy even places?
(a) 2 (b) 4 (c) 6 (d) 8

(**b**) In a given word 'ABLE' Vowels are $\{A, E\}$.
$\overline{1} \quad \overline{2} \quad \overline{3} \quad \overline{4}$
As, vowel occupy even places, so two vowels occupy the places 2 and 4.
Therefore, the number of ways of occupying the vowels in even places is 2!.
Now, we have two consonants and these consonants occupy the odd places 1 and 3. Therefore, the number of ways of occupying the consonants in odd places is 2!.
$\therefore$ Total number of ways $= 2! \times 2!$
$= 2 \times 2 = 4$

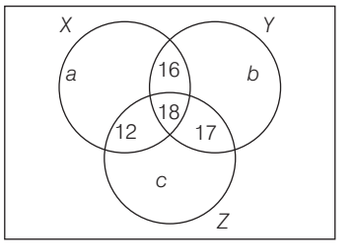
15. What is the maximum number of points of intersection of 5 non-overlapping circles?
(a) 10 (b) 15 (c) 20 (d) 25

(**c**) The maximum number of points of intersection of 5 non-overlapping circles

= Selection of two circles × 2
[∵ Two intersecting circles cut at two points]

$= {}^5C_2 \times 2 = \frac{5 \times 4}{2 \times 1} \times 2 = 20$

Directions (Q. Nos. 16-18)
Consider the following Venn diagram, where X, Y and Z are three sets. Let the number of elements in Z be denoted by $n(Z)$ which is equal to 90.

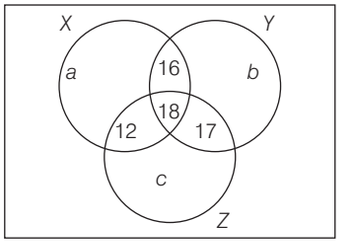


16. If the number of elements in Y and Z are in the ratio 4 : 5, then what is the value of b?
(a) 18 (b) 19 (c) 21 (d) 23

17. What is the value of
$n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$?
(a) $a + b + 43$ (b) $a + b + 63$
(c) $a + b + 96$ (d) $a + b + 106$

18. If the number of elements belonging to neither X, nor Y, nor Z is equal to p, then what is the number of elements in the complement of X?
(a) $p + b + 60$ (b) $p + b + 40$
(c) $p + a + 60$ (d) $p + a + 40$

Solutions (16-18)
Given $n(Z) = 90$
$\Rightarrow 12 + 18 + 17 + c = 90$
$\Rightarrow c = 90 - 47 = 43$



16. (**c**) Also given,
$$\frac{n(Y)}{n(Z)} = \frac{4}{5}$$
$$\Rightarrow \frac{16 + 18 + 17 + b}{90} = \frac{4}{5}$$
$$\Rightarrow 51 + b = 72$$
$$\Rightarrow b = 72 - 51 = 21$$

17.(**d**) Now,
$n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$
$= n(X \cup Y \cup Z)$
$= a + 12 + 18 + 16 + b + 17 + c$
$= a + b + c + 63$
$= a + b + 43 + 63$ [∵ $c = 43$]
$= a + b + 106$

18. (**a**) Complement of X
$= p + b + c + 17$
$= p + b + 43 + 17$ [∵ $c = 43$]
$= p + b + 60$

Directions (Q. Nos. 19 and 20) *Read the following information and answer the two items that follow.*
Let $\frac{\tan 3A}{\tan A} = K$, where $\tan A \neq 0$ and $K \neq \frac{1}{3}$.

19. What is $\tan^2 A$ equal to?
(a) $\frac{K + 3}{3K - 1}$ (b) $\frac{K - 3}{3K - 1}$
(c) $\frac{3K - 3}{K - 3}$ (d) $\frac{K + 3}{3K + 1}$

(**b**) Given, $\frac{\tan 3A}{\tan A} = K$
$$= \frac{3\tan A - \tan^3 A}{(1 - 3\tan^2 A)\tan A} = K$$
$$\Rightarrow \frac{3 - \tan^2 A}{1 - 3\tan^2 A} = K$$
$$\Rightarrow K - 3K\tan^2 A = 3 - \tan^2 A$$
$$\Rightarrow K - 3 = \tan^2 A(3K - 1)$$
$$\Rightarrow \tan^2 A = \frac{K - 3}{3K - 1}$$

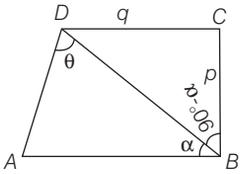
20. For real values of $\tan A$, K cannot lie between
(a) $\frac{1}{3}$ and 3 (b) $\frac{1}{2}$ and 2
(c) $\frac{1}{5}$ and 5 (d) $\frac{1}{7}$ and 7

(**a**) For real values of $\tan A$, K lies when
$$\frac{K - 3}{3K - 1} \geq 0 \text{ and } 3K - 1 \neq 0$$
$$\Rightarrow (K - 3)(3K - 1) \geq 0 \text{ and } K \neq \frac{1}{3}$$
$$\Rightarrow K < \frac{1}{3} \text{ and } K \geq 3$$
Hence, for real values of $\tan A$, K cannot lie between $\left(\frac{1}{3}, 3\right)$.

Directions (Q. Nos. 21 and 22) *Read the following information and answer the two items that follow.*
$ABCD$ is a trapezium such that AB and CD are parallel and BC is perpendicular to them. Let $\angle ADB = \theta$, $\angle ABD = \alpha$, $BC = p$ and $CD = q$.

21. Consider the following
1. $AD \sin\theta = AB \sin\alpha$
2. $BD \sin\theta = AB \sin(\theta + \alpha)$
Which of the above is/are correct?
(a) 1 Only (b) 2 Only
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) We have, $\angle ADB = \theta$, $\angle ABD = \alpha$, $BC = p$ and $CD = q$



1. In ΔABD, use Sine rule,
$$\frac{\sin\theta}{AB} = \frac{\sin\alpha}{AD}$$
$\Rightarrow AD\sin\theta = AB\sin\alpha$, which is correct.
2. In ΔABD, $\angle A = \pi - (\theta + \alpha)$
Use Sine rule in ΔABD,
$$\frac{\sin A}{BD} = \frac{\sin\theta}{AB}$$
$$\Rightarrow \frac{\sin[\pi - (\theta + \alpha)]}{BD} = \frac{\sin\theta}{AB} \quad \ldots(i)$$
$\Rightarrow AB\sin(\theta + \alpha) = BD\sin\theta$, which is correct.
Hence, both statements are correct.

22. What is AB equal to?
(a) $\frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$
(b) $\frac{(p^2 - q^2)\cos\theta}{p\cos\theta + q\sin\theta}$
(c) $\frac{(p^2 + q^2)\sin\theta}{q\cos\theta + p\sin\theta}$
(d) $\frac{(p^2 - q^2)\cos\theta}{q\cos\theta + p\sin\theta}$

(**a**) In right angle, ΔBCD,
$$\angle B = 90° - \alpha$$
$$BD = \sqrt{p^2 + q^2}$$
and $\sin B = \frac{CD}{BD}$
$$\Rightarrow \sin(90° - \alpha) = \frac{q}{\sqrt{p^2 + q^2}}$$
[∵ $\angle B = 90° - \alpha$]

$\Rightarrow \quad \cos\alpha = \dfrac{q}{\sqrt{p^2+q^2}}$

and $\quad \cos B = \dfrac{BC}{BD}$

$\Rightarrow \cos(90° - \alpha) = \dfrac{p}{\sqrt{p^2+q^2}}$

$\Rightarrow \quad \sin\alpha = \dfrac{p}{\sqrt{p^2+q^2}}$

From eq. (i),

$\dfrac{\sin(\pi - (\theta+\alpha))}{BD} = \dfrac{\sin\theta}{AB}$

$\Rightarrow \quad AB = \dfrac{BD\sin\theta}{\sin(\theta+\alpha)}$

$= \dfrac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha}$

$[\because BD = \sqrt{p^2+q^2}]$

$= \dfrac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\dfrac{q}{\sqrt{p^2+q^2}} + \cos\theta\dfrac{p}{\sqrt{p^2+q^2}}}$

$= \dfrac{(p^2+q^2)\sin\theta}{q\sin\theta + p\cos\theta}$

23. If $\tan\theta = \dfrac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$, then what is the value of θ?

(a) 0° (b) 28° (c) 38° (d) 52°

(**b**) We have,

$\tan\theta = \dfrac{\cos17° - \sin17°}{\cos17° + \sin17°}$

$= \dfrac{1 - \tan17°}{1 + \tan17°}$

[Divide numerator and denominator by $\cos17°$]

$\Rightarrow \quad \tan\theta = \tan(45° - 17°)$

$\left[\because \tan(45° - 17°) = \dfrac{\tan45° - \tan17°}{1 + \tan45°\tan17°}\right]$

$\Rightarrow \quad \tan\theta = \tan28°$

$\Rightarrow \quad \theta = 28°$

24. A and B are positive acute angles such that $\cos 2B = 3\sin^2 A$ and $3\sin 2A = 2\sin 2B$. What is the value of $(A + 2B)$?

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

(**d**) We have, $\cos2B = 3\sin^2 A$

and $\quad 3\sin2A = 2\sin2B$

$\therefore \quad \dfrac{2\sin2B}{\cos2B} = \dfrac{3\sin2A}{3\sin^2 A}$

$\Rightarrow \quad 2\dfrac{\sin2B}{\cos2B} = \dfrac{2\times\sin A\cos A}{\sin^2 A}$

$\Rightarrow \quad \tan2B = \cot A$

$\Rightarrow \quad \tan2B = \tan\left(\dfrac{\pi}{2} - A\right)$

$\Rightarrow \quad 2B = \dfrac{\pi}{2} - A$

$\Rightarrow \quad A + 2B = \dfrac{\pi}{2}$

25. What is $\sin 3x + \cos 3x + 4\sin^3 x - 3\sin x + 3\cos x - 4\cos^3 x$ equal to?

(a) 0 (b) 1

(c) $2\sin 2x$ (d) $4\cos 4x$

(**a**) $\sin3x + \cos3x + (4\sin^3 x - 3\sin x) + (3\cos x - 4\cos^3 x)$

$= \sin3x + \cos3x - \sin3x - \cos3x = 0$

26. The value of ordinate of the graph of $y = 2 + \cos x$ lies in the interval

(a) [0, 1 (b) [0, 3] (c) [– 1, 1] (d) [1, 3]

(**d**) We know that,

$-1 \le \cos x \le 1$

$\Rightarrow \quad -1 + 2 \le 2 + \cos x \le 1 + 2$

$\Rightarrow \quad 1 \le y \le 3$

$\therefore \quad y \in [1, 3]$

27. What is the value of $8\cos10°\cdot\cos20°\cdot\cos40°$?

(a) $\tan10°$ (b) $\cot10°$

(c) $\operatorname{cosec}10°$ (d) $\sec10°$

(**b**) $8\cos10°\cos20°\cos40°$

$= 8\cos10°\cos20°\cos40°\times\dfrac{\sin10°}{\sin10°}$

$= \dfrac{4(2\sin10°\cos10°)\cos20°\cos40°}{\sin10°}$

$= \dfrac{4\sin20°\cos20°\cos40°}{\sin10°}$

$\because 2\sin A\cos A = \sin2A$

$= \dfrac{2(2\sin20°\cos20°)\cos40°}{\sin10°}$

$= \dfrac{2\times\sin40°\cos40°}{\sin10°}$

$= \dfrac{\sin80°}{\sin10°} = \dfrac{\sin(90° - 10°)}{\sin10°}$

$= \dfrac{\cos10°}{\sin10°} = \cot10°$

28. What is the value of $\cos 48° - \cos 12°$?

(a) $\dfrac{\sqrt{5}-1}{4}$ (b) $\dfrac{1-\sqrt{5}}{4}$

(c) $\dfrac{\sqrt{5}+1}{2}$ (d) $\dfrac{1-\sqrt{5}}{8}$

(**b**) $\cos48° - \cos12°$

$= -2\sin\left(\dfrac{48°+12°}{2}\right)\sin\left(\dfrac{48°-12°}{2}\right)$

$\left[\because \cos C - \cos D = -2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)\right]$

$= -2\sin30°\sin18°$

$= -2\times\dfrac{1}{2}\times\dfrac{\sqrt{5}-1}{4}$

$= \dfrac{1-\sqrt{5}}{4}$

29. Consider the following statements:

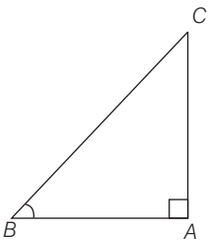
1. If ABC is a right-angled triangle, right-angled at A and if $\sin B = \dfrac{1}{3}$, then $\operatorname{cosec} C = 3$.
2. If $b\cos B = c\cos C$ and if the triangle ABC is not right-angled, then ABC must be isosceles.

Which of the above statements is/are correct?

(a) 1 Only (b) 2 Only

(c) Both 1 and 2 (d) Neither 1 nor 2

(**b**) **1.** We have, $\sin B = \dfrac{1}{3}$



$\Rightarrow \quad \dfrac{AC}{BC} = \dfrac{1}{3}$

$\Rightarrow \quad AC = k$ and $BC = 3k$

Use pythagoras theorem in ΔABC,

$AB = \sqrt{(BC)^2 - (AC)^2}$

$= \sqrt{(3k)^2 - (k)^2}$

$= \sqrt{9k^2 - k^2}$

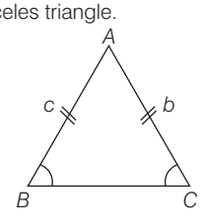
$= \sqrt{8k^2}$

$= 2\sqrt{2}k$

Now, $\operatorname{cosec} C = \dfrac{BC}{AB}$

$= \dfrac{3k}{2\sqrt{2}k} = \dfrac{3}{2\sqrt{2}}$, which is not correct.

2. Suppose we consider ΔABC is an isosceles triangle.



$\therefore \quad \angle B = \angle C$

Also we have, $b\cos B = c\cos C$

$\Rightarrow \quad b\cos C = c\cos C$

[put $B = C$]

$\Rightarrow \quad b = c$, Which is correct.

30. Consider the following statements

1. If in a triangle ABC, $A = 2B$ and $b = c$, then it must be an obtuse-angled triangle.
2. There exists no triangle ABC with $A = 40°$, $B = 65°$ and $\frac{a}{c} = \sin 40° \operatorname{cosec} 15°$.

Which of the above statements is/are correct?

(a) 1 Only (b) 2 Only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*d*) **1.** We have, in ΔABC,

$A = 2B$ and $b = c$

$\Rightarrow$ Angles opposite to equal sides are equal.

$\therefore \quad \angle C = \angle B$

Also, $A = 2B = 2C$

In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 2C + C + C = 180°$

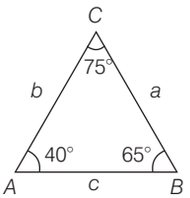
$\Rightarrow \quad C = \frac{180°}{4} = 45°$

$\Rightarrow \quad B = 45°$ and $A = 90°$

Thus, it shows that ΔABC is not an obtuse angle triangle.

Hence, statement 1 is incorrect.

2. We have $A = 40°$, $B = 65°$



In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 40° + 65° + \angle C = 180°$

$\Rightarrow \quad \angle C = 75°$

Use sine rule in ΔABC,

$$\frac{a}{\sin 40°} = \frac{c}{\sin 75°}$$

$$\frac{a}{c} = \sin 40° \operatorname{cosec} 75°,$$

Hence, Statement 2 is incorrect.

Directions (Q. Nos. 31-33) *Read the following information and answer the three items that follow.*

Let $a \sin^2 x + b \cos^2 x = c$,

$b \sin^2 y + a \cos^2 y = d$

and $p \tan x = q \tan y$

31. What is $\tan^2 x$ equal to?

(a) $\frac{c-b}{a-c}$ (b) $\frac{a-c}{c-b}$
(c) $\frac{c-a}{c-b}$ (d) $\frac{c-b}{c-a}$

(*a*) We have,

$a \sin^2 x + b \cos^2 x = c$

On dividing both sides by $\cos^2 x$, we get

$a \tan^2 x + b(1) = c \times \sec^2 x$

$\Rightarrow \quad a \tan^2 x + b = c(1 + \tan^2 x)$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$\Rightarrow \quad \tan^2 x(a - c) = c - b$

$\Rightarrow \quad \tan^2 x = \frac{c-b}{a-c} \quad \ldots(i)$

32. What is $\frac{d-a}{b-d}$ equal to?

(a) $\sin^2 y$ (b) $\cos^2 y$
(c) $\tan^2 y$ (d) $\cot^2 y$

(*c*) We have, $b \sin^2 y + a \cos^2 y = d$

On dividing both sides by $\cos^2 y$, we get

$b \tan^2 y + a(1) = d(\sec^2 y)$

$\Rightarrow \quad b \tan^2 y + a = d(1 + \tan^2 y)$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$\Rightarrow \quad \tan^2 y (b - d) = d - a$

$\Rightarrow \quad \frac{d-a}{b-d} = \tan^2 y \quad \ldots(ii)$

33. What is $\frac{p^2}{q^2}$ equal to?

(a) $\frac{(b-c)(b-d)}{(a-d)(a-c)}$ (b) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$
(c) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$ (d) $\frac{(b-c)(b-d)}{(c-a)(a-d)}$

(*b*) We have,

$p \tan x = q \tan y$

On squaring both sides, we get.

$$\frac{p^2}{q^2} = \frac{\tan^2 y}{\tan^2 x}$$

$$= \frac{(d-a)/(b-d)}{(c-b)/(a-c)}$$

$[\because$ from eq. (i) and (ii)]

$$= \frac{(d-a)(a-c)}{(b-d)(c-b)}$$

$$= \frac{(a-d)(c-a)}{(d-b)(b-c)}$$

Directions (Q. Nos. 34-36) *Read the following information and answer the three items that follow.*

Let $t_n = \sin^n \theta + \cos^n \theta$

34. What is $\frac{t_3 - t_5}{t_5 - t_7}$ equal to?

(a) $\frac{t_1}{t_3}$ (b) $\frac{t_3}{t_5}$
(c) $\frac{t_5}{t_7}$ (d) $\frac{t_1}{t_7}$

(*a*) We have,

$t_n = \sin^n\theta + \cos^n\theta$

$$\text{Now, } \frac{t_3 - t_5}{t_5 - t_7} = \frac{-(\sin^5\theta + \cos^5\theta)}{(\sin^5\theta + \cos^5\theta) - (\sin^7\theta + \cos^7\theta)}$$

$$= \frac{(\sin^3\theta - \sin^5\theta) + (\cos^3\theta - \cos^5\theta)}{(\sin^5\theta - \sin^7\theta) + (\cos^5\theta - \cos^7\theta)}$$

$$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin^5\theta(1 - \sin^2\theta) + \cos^5\theta(1 - \cos^2\theta)}$$

$$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin^5\theta\cos^2\theta + \cos^5\theta\sin^2\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta(\sin\theta + \cos\theta)}{\sin^2\theta\cos^2\theta(\sin^3\theta + \cos^3\theta)}$$

$$= \frac{\sin\theta + \cos\theta}{\sin^3\theta + \cos^3\theta} = \frac{t_1}{t_3}$$

35. What is $t_1^2 - t_2$ equal to?

(a) $\cos 2\theta$ (b) $\sin 2\theta$
(c) $2 \cos \theta$ (d) $2 \sin \theta$

(*b*) $t_1^2 - t_2 = (\sin\theta + \cos\theta)^2 - (\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - (\sin^2\theta + \cos^2\theta)$

$= 2\sin\theta\cos\theta = \sin 2\theta$

36. What is the value of t_{10} where $\theta = 45°$?

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$

(*c*) Now, $t_{10} = \sin^{10}\theta + \cos^{10}\theta$

$= (\sin 45°)^{10} + (\cos 45°)^{10}$ [Put $\theta = 45°$]

$$= \left(\frac{1}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}}\right)^{10}$$

$$= 2\left[\frac{1}{2^5}\right] = \frac{1}{2^4} = \frac{1}{16}$$

Directions (Q. Nos. 37-39) *Read the following information and answer the three items that follow.*

Let $\alpha = \beta = 15°$.

37. What is the value of $\sin\alpha + \cos\beta$?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

(*d*) $\sin\alpha + \cos\beta$

$$= \left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{\sqrt{2}}\cos\beta\right) \times \sqrt{2}$$

$= \sqrt{2}\,(\sin\alpha\cos 45° + \sin 45°\cos\beta)$

$= \sqrt{2}(\sin 15°\cos 45° + \sin 45°\cos 15°)$

$[\because \alpha = \beta = 15°]$

$= \sqrt{2}\sin(15° + 45°)$

$= \sqrt{2}\sin 60° = \sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}}$

38. What is the value of $\sin 7\alpha - \cos 7\beta$?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

(**d**) $\sin 7\alpha - \cos 7\beta$

$= \left(\frac{1}{\sqrt{2}}\sin 7 \times 15° - \frac{1}{\sqrt{2}}\cos 7 \times 15°\right)\sqrt{2}$

$= \sqrt{2}(\sin 105°\cos 45° - \cos 105°\sin 45°)$

$= \sqrt{2}\sin(105° - 45°)$

$= \sqrt{2}\sin 60°$

$= \sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}}$

39. What is $\sin(\alpha + 1°) + \cos(\beta + 1°)$ equal to?

(a) $\sqrt{3}\cos 1° + \sin 1°$

(b) $\sqrt{3}\cos 1° - \frac{1}{2}\sin 1°$

(c) $\frac{1}{\sqrt{2}}(\sqrt{3}\cos 1° + \sin 1°)$

(d) $\frac{1}{2}(\sqrt{3}\cos 1° + \sin 1°)$

(**c**) $\sin(\alpha + 1°) + \cos(\beta + 1°)$

$= \sin(\alpha + 1°) + \cos(\alpha + 1°) \quad [\because \alpha = \beta]$

$= \sin\alpha\cos 1° + \cos\alpha\sin 1° + \cos\alpha\cos 1° - \sin\alpha\sin 1°$

$= \cos 1°(\sin\alpha + \cos\alpha) + \sin 1°(\cos\alpha - \sin\alpha)$

$= \cos 1° \times \frac{\sqrt{3}}{\sqrt{2}} + \sin 1°\left(\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{\sqrt{2}}\sin\alpha\right)\sqrt{2}$

$\left[\because \sin\alpha + \cos\alpha = \sqrt{\frac{3}{2}}\right]$

$= \sqrt{\frac{3}{2}}\cos 1° + \sqrt{2}\sin 1°(\sin 45°\cos\alpha - \cos 45°\sin\alpha)$

$= \sqrt{\frac{3}{2}}\cos 1° + \sqrt{2}\sin 1°\sin(45° - \alpha)$

$= \sqrt{\frac{3}{2}}\cos 1° + \sqrt{2}\sin 1°\sin 30° \quad [\because \alpha = 15°]$

$= \sqrt{\frac{3}{2}}\cos 1° + \frac{\sqrt{2}}{2}\sin 1°$

$= \sqrt{\frac{3}{2}}\cos 1° + \frac{1}{\sqrt{2}}\sin 1°$

$= \frac{1}{\sqrt{2}}(\sqrt{3}\cos 1° + \sin 1°)$

40. If $\sin x + \sin y = \cos y - \cos x$, where $0 < y < x < \frac{\pi}{2}$, then what is $\tan\left(\frac{x - y}{2}\right)$ equal to?

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

(**c**) Given, $\sin x + \sin y = \cos y - \cos x$

$\Rightarrow 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right) = -2\sin\left(\frac{y + x}{2}\right)\sin\left(\frac{y - x}{2}\right)$

$\Rightarrow \cos\left(\frac{x - y}{2}\right) = \sin\left(\frac{x - y}{2}\right)$

$\Rightarrow \tan\left(\frac{x - y}{2}\right) = 1$

41. If A is a matrix of order 3×5 and B is a matrix of order 5×3, then the order of AB and BA will respectively be

(a) 3×3 and 3×3

(b) 3×5 and 5×3

(c) 3×3 and 5×5

(d) 5×3 and 3×5

(**c**) Given, matrix A is of order 3×5 and matrix B is of order 5×3.

$\therefore$ Order of matrix $AB = [A]_{3\times 5}\,[B]_{5\times 3} = [AB]_{3\times 3}$

and order of matrix $BA = [B]_{5\times 3}\,[A]_{3\times 5} = [BA]_{5\times 5}$

42. If p^2, q^2 and r^2 (where $p, q, r > 0$) are in GP, then which of the following is/are correct?

1. p, q and r in GP.

2. $\ln p$, $\ln q$ and $\ln r$ are in AP.

Select the correct answer using the code given below

(a) 1 Only (b) 2 Only

(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) **1.** Given, p^2, q^2 and r^2 in GP.

$\therefore \quad q^2 = \sqrt{p^2 r^2} \Rightarrow q^2 = pr \quad \ldots(i)$

$\Rightarrow p, q$ and r in GP, hence Statement 1 is correct.

2. Now, $\frac{\ln p + \ln r}{2} = \frac{\ln pr}{2} = \frac{\ln q^2}{2}$ [From eq. (i)]

$= 2\frac{\ln q}{2} = \ln q$

$\Rightarrow \ln p$, $\ln q$ and $\ln r$ are in AP, which is correct statement.

Hence, both statements are correct.

43. If $\cot\alpha$ and $\cot\beta$ are the roots of the equation $x^2 - 3x + 2 = 0$, then what is $\cot(\alpha + \beta)$ equal to?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 3

(**b**) Given, $\cot\alpha$ and $\cot\beta$ are the roots of $x^2 - 3x + 2 = 0$.

Now, sum of roots, $\cot\alpha + \cot\beta = 3$

and product of roots, $\cot\alpha\cot\beta = 2$

Now, $\cot(\alpha + \beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

$= \frac{2 - 1}{3} = \frac{1}{3}$

44. The roots α and β of a quadratic equation, satisfy the relations $\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2$. What is the number of such quadratic equations?

(a) 0 (b) 2 (c) 3 (d) 4

(**d**) Given, $\alpha + \beta = \alpha^2 + \beta^2$

and $\alpha\beta = \alpha^2\beta^2$

$\Rightarrow \alpha\beta(1 - \alpha\beta) = 0$

$\Rightarrow \alpha\beta = 0$ and $\alpha\beta = 1$

Consider $\alpha + \beta = \alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

When $\alpha\beta = 0$,

$\alpha + \beta = (\alpha + \beta)^2 - 2 \times 0$

$\Rightarrow (\alpha + \beta) = (\alpha + \beta)^2$

$\Rightarrow (\alpha + \beta)[1 - (\alpha + \beta)] = 0$

$\Rightarrow (\alpha + \beta) = 0$

and $\alpha + \beta = 1$

It implies that when $\alpha\beta = 0$, gives two quadratic equations.

When $\alpha\beta = 1$

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow (\alpha + \beta) = (\alpha + \beta)^2 - 2(1)$

$\Rightarrow (\alpha + \beta)^2 - (\alpha + \beta) - 2 = 0$

$\Rightarrow [(\alpha + \beta) - 2][(\alpha + \beta) + 1] = 0$

$\Rightarrow \alpha + \beta = 2, -1$

Again it implies that, when $\alpha\beta = 1$, gives two quadratic equations.

Hence, there are total four such quadratic equations formed.

45. What is the argument of the complex number $\frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$, where $i = \sqrt{-1}$?

(a) 240° (b) 210° (c) 120° (d) 60°

(**a**) Let $z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$

$= \frac{(1 - i\sqrt{3})^2}{1^2 - (i\sqrt{3})^2}$

$= \frac{1 - 3 - 2i\sqrt{3}}{1 + 3} = \frac{-2 - 2i\sqrt{3}}{4}$

$= \frac{-1 - i\sqrt{3}}{2}$

Now, $\tan\theta = \left|\frac{y}{x}\right| = \left|\frac{-\sqrt{3}/2}{-1/2}\right|$

$= |\sqrt{3}| = 60°$

Since, given complex number lies in IIIrd quadrant.

$\therefore \arg(z) = 180° + 60° = 240°$

46. What is the modulus of the complex number $\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$, where $i = \sqrt{-1}$?

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

(**b**) Let $z = \frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$

$$\Rightarrow \quad |z| = \frac{|\cos\theta + i\sin\theta|}{|\cos\theta - i\sin\theta|} = \frac{\sqrt{\cos^2\theta + \sin^2\theta}}{\sqrt{\cos^2\theta + \sin^2\theta}} = 1$$

47. Consider the proper subsets of {1, 2, 3, 4}. How many of these proper subsets are superset of the set {3}?

(a) 5 (b) 6
(c) 7 (d) 8

(**c**) The proper subset of {1, 2, 3, 4} are
{},{1},{2},{3},{4},{1, 2},{2, 3},{3, 4},
{1, 3},{1, 4},{2, 4},{1, 2, 3},{2, 3, 4},
{1, 2, 4},{1, 3, 4}

The superset of the set {3} in the given proper subset are
{3},{2, 3},{3, 4},{1, 3},
{1, 2, 3}, {2, 3, 4}, {1, 3, 4}

Hence, number of proper subset of the superset of the set {3} is 7.

48. Let p, q and r be three distinct positive real numbers. If $D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$, then which one of the following is correct?

(a) $D < 0$ (b) $D \le 0$
(c) $D > 0$ (d) $D \ge 0$

(**a**) We have,

$$D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$D = \begin{vmatrix} p+q+r & q & r \\ p+q+r & r & p \\ p+q+r & p & q \end{vmatrix} = (p+q+r)\begin{vmatrix} 1 & q & r \\ 1 & r & p \\ 1 & p & q \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$D = (p+q+r)\begin{vmatrix} 1 & q & r \\ 0 & r-q & p-r \\ 0 & p-q & q-r \end{vmatrix}$$

$$= (p+q+r)[1\{(r-q)(q-r) - (p-r)(p-q)\}]$$

$$= (p+q+r)[rq - r^2 - q^2 + qr - \{p^2 - pq - rp + rq\}]$$

$$= (p+q+r)[-p^2 - q^2 - r^2 + pq + pr + rp]$$

$$= \frac{-(p+q+r)}{2}[2p^2 + 2q^2 + 2r^2 - 2pq - 2qr - 2rp]$$

$$= \frac{(p+q+r)}{2}[(p^2 + q^2 - 2pq) + (q^2 + r^2 - 2qr) + (r^2 + p^2 - 2rp)]$$

$$= -\frac{1}{2}(p+q+r)[(p-q)^2 + (q-r)^2 + (r-p)^2]$$

Since, p, q and r are distinct.

$\therefore \quad D < 0$

49. What is the sum of the last five coefficients in the expansion of $(1+x)^9$ when it is expanded in ascending powers of x?

(a) 256 (b) 512 (c) 1024 (d) 2048

(**a**) $(1+x)^9 = {}^9C_0x^9 + {}^9C_1x^8 + \ldots + {}^9C_5x^4 + {}^9C_6x^3 + {}^9C_7x^2 + {}^9C_8x^1 + {}^9C_9$

$\therefore$ The sum of last 5 coefficients in the expansion of $(1+x)^9$ is

$${}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9 = \frac{9\times8\times7\times6}{4\times3\times2\times1} + \frac{9\times8\times7}{3\times2\times1} + \frac{9\times8}{2\times1} + 9 + 1$$

$= 126 + 84 + 36 + 9 + 1 = 256$

50. Consider the following in respect of a non-singular matrix of order 3.

1. $A(\text{adj } A) = (\text{adj } A)A$
2. $|\text{adj } A| = |A|$

Which of the above statements is/are correct?

(a) 1 Only (b) 2 Only
(c) Both 1 and 2 (d) Neither 1 nor 2

(**a**) **1.** By using the property of adjoint of square matrix.

$A(\text{adj } A) = (\text{adj } A)A = |A|I$

Hence, Statement 1 is correct.

2. By using the property of adjoint of square matrix, if matrix A is of order n, then

$|\text{adj } A| = |A|^{n-1}$

Here, order of given matrix A is 3.

$$\therefore \quad |\text{adj } A| = |A|^{3-1} = |A|^2,$$

Which is incorrect statement.
Hence, only Statement 1 is correct.

51. The centre of the circle $(x-2a)(x-2b) + (y-2c)(y-2d) = 0$ is

(a) $(2a, 2c)$ (b) $(2b, 2d)$
(c) $(a+b, c+d)$ (d) $(a-b, c-d)$

(**c**) Given equation $(x-2a)(x-2b) + (y-2c)(y-2d) = 0$ is a circle, whose end points of a diameter are $A(2a, 2c)$ and $B(2b, 2d)$.

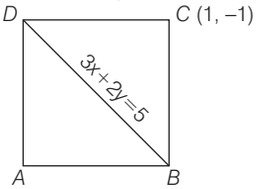
Now, centre of circle = mid point of diameter.

$$= \left(\frac{2a+2b}{2}, \frac{2c+2d}{2}\right) = (a+b, c+d)$$

52. The point $(1, -1)$ is one of the vertices of a square. If $3x + 2y = 5$ is the equation of one diagonal of the square, then what is the equation of the other diagonal?

(a) $3x - 2y = 5$ (b) $2x - 3y = 1$
(c) $2x - 3y = 5$ (d) $2x + 3y = -1$

(**c**) We have point $(1, -1)$ is one of the vertices of a square.



Here, we see that point $(1, -1)$ does not satisfy the diagonal equation $3x + 2y = 5$. Therefore we consider point $(1, -1)$ of the other vertex of diagonal.

We know that, diagonal of a square are perpendicular to each other.

Let perpendicular equation of $3x + 2y = 5$ is $2x - 3y + c = 0 \ldots$(i)

This equation passes through the vertex C $(1, -1)$

$\therefore \quad 2(1) - 3(-1) + c = 0$

$\Rightarrow \quad 2 + 3 + c = 0$

$\Rightarrow \quad c = -5$

Put $c = -5$ in eq. (i), we get

$2x - 3y - 5 = 0 \Rightarrow 2x - 3y = 5$

53. Let $P(x, y)$ be any point on the ellipse $25x^2 + 16y^2 = 400$. If $Q(0, 3)$ and $R(0, -3)$ are two points, then what is $(PQ + PR)$ equal to?

(a) 12 (b) 10 (c) 8 (d) 6

(**b**) Given, equation can be rewritten as

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

Here, $a^2 = 16$, $b^2 = 25$, $b > a$

Eccentricity, $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{16}{25}}$

$= \sqrt{\frac{9}{25}} = \frac{3}{5}$

Foci $= (0, \pm be) = \left(0, \pm 5 \times \frac{3}{5}\right) = (0, \pm 3)$

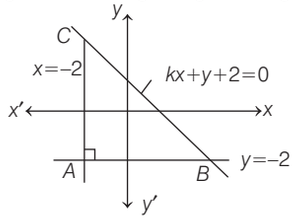
∴ The given points (0, 3) and $R(0, -3)$ are the foci of given ellipse.

$PQ + PR = 2b = 2 \times 5 = 10$

54. If the circumcentre of the triangle formed by the lines $x + 2 = 0$, $y + 2 = 0$ and $kx + y + 2 = 0$ is $(-1, -1)$, then what is the value of k?

(a) -1 (b) -2 (c) 1 (d) 2

(***c***) Here, we see that triangle formed by given lines is right angled triangle.



Therefore, circumcentre of right angled triangle lies on the hypotenuse of triangle.

∴ Point $(-1, -1)$ satisfy the equation of line $kx + y + 2 = 0$

$\Rightarrow \quad k(-1) - 1 + 2 = 0 \Rightarrow k = 1$

55. In the parabola, $y^2 = x$, what is the length of the chord passing through the vertex and inclined to the X-axis at an angle θ?

(a) $\sin\theta \cdot \sec^2\theta$ (b) $\cos\theta \cdot \text{cosec}^2\theta$
(c) $\cot\theta \cdot \sec^2\theta$ (d) $2\tan\theta \cdot \text{cosec}^2\theta$

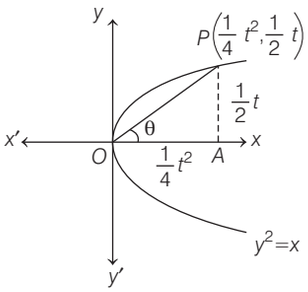
(***b***) We know that, any point on the parabola $y^2 = 4ax$ is $P(at^2, 2at)$

We have equation of parabola is $y^2 = x$

Here, $a = \frac{1}{4}$

∴ Any point on the parabola $y^2 = x$ is

$P\left(\frac{1}{4}t^2, \frac{1}{2}t\right)$



In right angled ΔOAP,

$\sin\theta = \frac{AP}{OP} \Rightarrow \sin\theta = \frac{\frac{1}{2}t}{OP}$

$\Rightarrow \quad t = 2OP\sin\theta \quad \ldots(i)$

and $\cos\theta = \frac{OA}{OP}$

$\Rightarrow \quad \cos\theta = \frac{\frac{1}{4}t^2}{OP}$

$\Rightarrow \quad \cos\theta = \frac{\frac{1}{4}(2 \times OP\sin\theta)^2}{OP}$

[From Eq. (i)]

$\Rightarrow \quad \cos\theta = OP\sin^2\theta$

$\Rightarrow \quad OP = \cos\theta\,\text{cosec}^2\theta$

56. Under which condition, are the points (a, b), (c, d) and $(a - c, b - d)$ collinear?

(a) $ab = cd$ (b) $ac = bd$
(c) $ad = bc$ (d) $abc = d$

(***c***) The condition for the points (a, b), (c, d) and $(a - c, b - d)$ are collinear, is

$$\begin{vmatrix} a & b & 1 \\ c-a & d & 1 \\ -c & b-d & 1 \end{vmatrix} = 0$$

Apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} a & b & 1 \\ c-a & d-b & 0 \\ -c & -d & 0 \end{vmatrix} = 0$$

Expanding along C_3, we get

$1[(c - a)(-d) + c(d - b)] = 0$

$\Rightarrow \quad -cd + ad + cd - cb = 0$

$\Rightarrow \quad ad = cb$

57. Let ABC be a triangle. If D (2, 5) and E (5, 9) are the mid-points of the sides AB and AC respectively, then what is the length of the side BC?

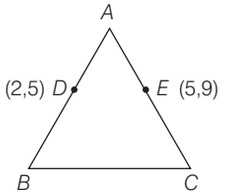
(a) 8 (b) 10 (c) 12 (d) 14

(***b***) Now, length of DE

$= \sqrt{(5-2)^2 + (9-5)^2} = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ sq units.

We know that, if D and E are the mid points of AB and AC, then length of DE is half of BC.



∴ Length of $BC = 2 \times DE$

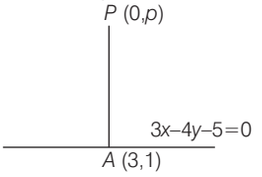
$= 2 \times 5 = 10$ sq units.

58. If the foot of the perpendicular drawn from the point $(0, k)$ to the line $3x - 4y - 5 = 0$ is (3, 1), then what is the value of k?

(a) 3 (b) 4 (c) 5 (d) 6

(***c***) Now equation of line AP, which is perpendicular to the given line $3x - 4y - 5 = 0$ is

$4x + 3y + c = 0 \quad \ldots(i)$



Since, it passes through (3, 1)

∴ $\quad 4 \times 3 + 3 \times 1 + c = 0$

$\Rightarrow \quad c = -15$

Put $c = -15$ in Eq. (i), we get

$4x + 3y - 15 = 0$

Also, this equation passes through the point P $(0, k)$.

∴ $\quad 4(0) + 3(k) - 15 = 0$

$\Rightarrow \quad k = \frac{15}{3} = 5$

59. What is the obtuse angle between the lines whose slopes are $2 - \sqrt{3}$ and $2 + \sqrt{3}$?

(a) 105° (b) 120° (c) 135° (d) 150°

(***b***) Given $m_1 = 2 - \sqrt{3}$ and $m_2 = 2 + \sqrt{3}$

∴ The obtuse angle between the line having slopes m_1 and m_2 is

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$= \frac{2 - \sqrt{3} - (2 + \sqrt{3})}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}$

$= \frac{-2\sqrt{3}}{1 + (4 - 3)} = \frac{-2\sqrt{3}}{2}$

$= -\sqrt{3} = -\tan 60° = \tan(180° - 60°)$

$\theta = 120°$

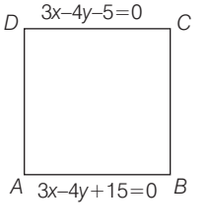
60. If $3x - 4y - 5 = 0$ and $3x - 4y + 15 = 0$ are the equations of a pair of opposite sides of a square, then what is the area of the square?

(a) 4 sq units (b) 9 sq units
(c) 16 sq units (d) 25 sq units

(***c***) Now, distance between two parallel lines $= \frac{15 + 5}{\sqrt{(3)^2 + (-4)^2}} = \frac{20}{\sqrt{9 + 16}}$

[∵ Distance between two parallel lines $= \frac{c_2 - c_1}{\sqrt{a^2 + b^2}}$]

$$= \frac{20}{\sqrt{25}} = \frac{20}{5} = 4$$



It is clear that, side of square = distance between two lines.

$\therefore$ Side = 4

$\therefore$ Area of square = $(\text{Side})^2 = (4)^2$

= 16 sq units.

61. What is the length of the diameter of the sphere whose centre is at $(1, -2, 3)$ and which touches the plane $6x - 3y + 2z - 4 = 0$?

(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units

(**d**) Now, radius of sphere

= Length of perpendicular drawn from centre to the tangent

$$= \frac{|6(1) - 3(-2) + 2(3) - 4|}{\sqrt{(6)^2 + (-3)^2 + (2)^2}}$$

$$= \frac{|6 + 6 + 6 - 4|}{\sqrt{36 + 9 + 4}} = \frac{14}{\sqrt{49}} = \frac{14}{7} = 2$$

$\therefore$ Diameter of sphere

= 2 × radius of sphere

= 2 × 2 = 4 units

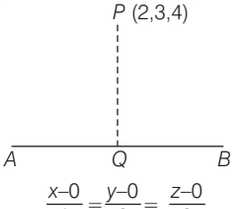
62. What is the perpendicular distance from the point (2, 3, 4) to the line $\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$?

(a) 6 units (b) 5 units
(c) 3 units (d) 2 units

(**b**) Given equation of line is

$$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} = \lambda \text{(say)}$$

Any point on the line is $Q(\lambda, 0, 0)$



$$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$$

Now, Dr's of PQ are $(2 - \lambda, 3, 4)$

Since, PQ is perpendicular to AB.

$\therefore$ $(2 - \lambda)1 + 3 \times 0 + 4 \times 0 = 0$

$[\because a_1 a_2 + b_1 b_2 + c_1 c_2 = 0]$

$\Rightarrow$ $2 - \lambda = 0$

$\Rightarrow$ $\lambda = 2$

$\therefore$ The coordinate of Q is (2, 0, 0).

$\therefore$ The perpendicular distance PQ

$$= \sqrt{(2-2)^2 + (3-0)^2 + (4-0)^2}$$

$$= \sqrt{0 + 9 + 16} = \sqrt{25} = 5 \text{ units}$$

63. If a line has direction ratios $< a + b,\ b + c, c + a >$, then what is the sum of the squares of its direction cosines?

(a) $(a + b + c)^2$ (b) $2(a + b + c)$
(c) 3 (d) 1

(**d**) Given, direction ratio of a given line is $< a + b, b + c, c + a >$

$\therefore$ Direction cosine of given line is

$$< \frac{a+b}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}},$$

$$\frac{b+c}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}},$$

$$\frac{c+a}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}} >$$

Now, sum of the square of direction cosine

$$= \left(\frac{a+b}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}\right)^2 + \left(\frac{b+c}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}\right)^2 + \left(\frac{c+a}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}\right)^2$$

$$= \frac{(a+b)^2 + (b+c)^2 + (c+a)^2}{(a+b)^2 + (b+c)^2 + (c+a)^2}$$

= 1

64. Into how many compartments do the coordinate planes divide the space?

(a) 2 (b) 4
(c) 8 (d) 16

(**c**) The coordinate planes divides the spaces into 8 compartment.

65. What is the equation of the plane which cuts an intercept 5 units on the Z-axis and is parallel to XY-plane?

(a) $x + y = 5$ (b) $z = 5$
(c) $z = 0$ (d) $x + y + z = 5$

(**b**) Plane parallel to xy plane is $z = k$…(i)

Since, it intercept 5 units on Z-axis.

$\therefore$ Point (0, 0, 5) satisfy Eq. (i), we get

$5 = k$

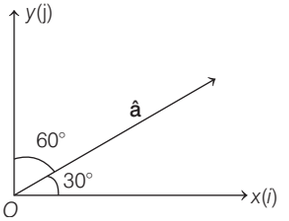
Put $k = 5$ in Eq. (i), we get

$z = 5$

66. If $\hat{a}$ is a unit vector in the xy-plane making an angle 30° with the positive X-axis, then what is $\hat{a}$ equal to?

(a) $\frac{\sqrt{3}\,\hat{i} + \hat{j}}{2}$ (b) $\frac{\sqrt{3}\hat{i} - \hat{j}}{2}$
(c) $\frac{\hat{i} + \sqrt{3}\,\hat{j}}{2}$ (d) $\frac{\hat{i} - \sqrt{3}\,\hat{j}}{2}$

(**a**) Given $\hat{a}$ makes an angle of 30° with X-axis. So, angle between $\hat{a}$ and $\hat{i}$ is 30°.



$\because$ $a\,b = |a||b| \cos\theta$

$\therefore$ $\hat{a} \cdot \hat{i} = |\hat{a}||\hat{i}| \cos 30°$

$\Rightarrow$ $\hat{a}\,\hat{i} = 1 \times 1 \times \frac{\sqrt{3}}{2}$

$[\because \hat{a}$ is a unit vector $|\hat{a}| = 1]$

Similarly, $\hat{a}$ makes an angle of 60° with Y-axis. So, angle between $\hat{a}$ and $\hat{j}$ is 60°.

$\therefore$ $\hat{a}.\hat{j} = |\hat{a}||\hat{j}| \cos 60°$

$$= 1 \times 1 \times \frac{1}{2} = \frac{1}{2}$$

$\therefore$ $\hat{a} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j} = \frac{\sqrt{3}\hat{i} + \hat{j}}{2}$

67. Let A be a point in space such that $|\mathbf{OA}| = 12$, where O is the origin. If $\mathbf{OA}$ is inclined at angles 45° and 60° with X-axis and Y-axis respectively, then what is $\mathbf{OA}$ equal to?

(a) $6\hat{i} + 6\hat{j} \pm \sqrt{2}\hat{k}$

(b) $6\hat{i} + 6\sqrt{2}\,\hat{j} \pm 6\,\hat{k}$

(c) $6\sqrt{2}\,\hat{i} + 6\hat{j} \pm 6\,\hat{k}$

(d) $3\sqrt{2}\,\hat{i} + 3\hat{j} \pm 6\hat{k}$

(**c**) Let $OA = x\hat{i} + y\hat{j} + z\hat{k}$

Where $x = 12\cos 45°$ and $y = 12\cos 60°$

$\therefore OA = 12\cos 45°\,\hat{i} + 12\cos 60°\,\hat{j} + z\hat{k}$

$$= 12 \times \frac{1}{\sqrt{2}}\hat{i} + 12 \times \frac{1}{2}\hat{j} + z\hat{k}$$

$$= \frac{12}{\sqrt{2}}\hat{i} + 6\hat{j} + z\hat{k} \quad \ldots(i)$$

$\Rightarrow$ $|OA| = \sqrt{\frac{144}{2} + 36 + z^2}$

$\Rightarrow$ $12 = \sqrt{108 + z^2}$

On squaring both sides, we get

$144 = 108 + z^2$

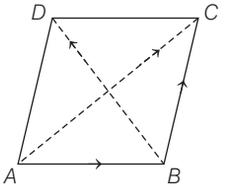
$\Rightarrow$ $z^2 = 36 \Rightarrow z = \pm 6$

Put $z = \pm 6$ in Eq. (i), we get

$OA = 6\sqrt{2}\hat{i} + 6\hat{j} \pm 6\hat{k}$

68. Two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. What is the magnitude of dot product of vectors which represent its diagonals?
(a) 21 (b) 25 (c) 31 (d) 36

(*c*) We have,



$AB = 2\hat{i} - 4\hat{j} + 5\hat{k}$

and $BC = \hat{i} - 2\hat{j} - 3\hat{k}$

Now, $AC = AB + BC$

$= (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\hat{i} - 2\hat{j} - 3\hat{k})$

$= 3\hat{i} - 6\hat{j} + 2\hat{k}$

and $BD = AD - AB = BC - AB$

$= (\hat{i} - 2\hat{j} - 3\hat{k}) - (2\hat{i} - 4\hat{j} + 5\hat{k})$

$= -\hat{i} + 2\hat{j} - 8\hat{k}$

Now, dot product of diagonals,

$AC.BD = (3\hat{i} - 6\hat{j} + 2\hat{k}).(-\hat{i} + 2\hat{j} - 8\hat{k})$

$= -3 - 12 - 16 = -31$

$\therefore$ Magnitude of dot product of diagonals.

$= |AC \cdot BD| = |-31| = 31$

69. If $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 144$ and $|\mathbf{a}| = 4$, then what is $|\mathbf{b}|$ equal to?
(a) 3 (b) 4 (c) 6 (d) 8

(*a*) We have, $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 144$

$\Rightarrow |\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta + |\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta = 144$

$\Rightarrow |\mathbf{a}|^2 |\mathbf{b}|^2 (\sin^2\theta + \cos^2\theta) = 144$

$\Rightarrow (4)^2 \times |\mathbf{b}|^2 (1) = 144$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow |\mathbf{b}|^2 = \frac{144}{16} = 9$

$|\mathbf{b}| = 3$

70. If the vectors $\mathbf{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\mathbf{c} = \hat{j} + p\hat{k}$ are coplanar, then what is the value of p?
(a) 1 (b) – 1 (c) 5 (d) – 5

(*b*) The condition of three vectors to be coplanar is $a.(b \times c) = 0$

$$\Rightarrow \begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -3 \\ 0 & 1 & p \end{vmatrix} = 0$$

$\Rightarrow 2(2p + 3) + 3(p + 0) + 1(1 - 0) = 0$

$\Rightarrow 4p + 6 + 3p + 1 = 0$

$\Rightarrow 7p + 7 = 0$

$p = -1$

71. What is $\lim_{x \to 1} \frac{x + x^2 + x^3 - 3}{x - 1}$ equal to?
(a) 1 (b) 2 (c) 3 (d) 6

(*d*) We have, $\lim_{x \to 1} \frac{x + x^2 + x^3 - 3}{x - 1}$

$\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 1} \frac{1 + 2x + 3x^2}{1}$

[by using L' Hospital's rule]

$= 1 + 2(1) + 3(1)^2$

$= 1 + 2 + 3 = 6$

72. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?
(a) 4.4 cm/sec (b) 8.4 cm/sec
(c) 8.8 cm/sec (d) 15.4 cm/sec

(*a*) We have, $\frac{dr}{dt} = 0.7$ cm/sec

Now, circumference of circle, $C = 2\pi r$

On differentiating w.r.t. t, we get

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$= 2 \times \frac{22}{7} \times 0.7$ cm/sec

$= 4.4$ cm/sec

73. If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, where $k \neq 0$, then what is the value of k?
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) 4

(*c*) $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$

$\Rightarrow 4(1)^{4-1} = \lim_{x \to k} \frac{(x - k)(x^2 + k^2 + kx)}{(x - k)(x + k)}$

$\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$

$\Rightarrow 4 = \lim_{x \to k} \frac{x^2 + k^2 + kx}{(x + k)}$

$\Rightarrow 4 = \frac{k^2 + k^2 + k^2}{k + k}$

$\Rightarrow 4 = \frac{3k^2}{2k} \Rightarrow k = \frac{4 \times 2}{3} = \frac{8}{3}$

74. The order and degree of the differential equation

$$k\frac{dy}{dx} = \int \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{2}{3}} dx \text{ are}$$

respectively
(a) 1 and 1 (b) 2 and 3
(c) 2 and 4 (d) 1 and 4

(*b*) We have,

$$k\frac{dy}{dx} = \int \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{2/3} dx$$

On differentiating both sides w.r.t., x, we get

$$k\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{2/3}$$

On cubing both sides, we get

$$k^3\left(\frac{d^2y}{dx^2}\right)^3 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^2$$

Here, highest order derivative $\frac{d^2y}{dx^2}$ is 2 and whose degree is 3.

75. What is $\lim_{x \to 0} \frac{\sin x \log(1 - x)}{x^2}$ equal to?
(a) – 1 (b) Zero (c) $-e$ (d) $-\frac{1}{e}$

(*a*) $\lim_{x \to 0} \frac{\sin x \log(1 - x)}{x^2}$

$= \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \log \frac{1 - x}{x}$

$= 1 \times (-1) = -1$

$\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \text{ and } \lim_{x \to 0} \log \frac{(1 - x)}{x} = -1\right]$

76. If $f(x) = 3x^2 - 5x + p$ and $f(0)$ and $f(1)$ are opposite in sign, then which of the following is correct?
(a) $-2 < p < 0$ (b) $-2 < p < 2$
(c) $0 < p < 2$ (d) $3 < p < 5$

(*c*) We have,

$f(x) = 3x^2 - 5x + p$

Now, $f(0) = 3(0)^2 - 5(0) + p = p$

and $f(1) = 3(1)^2 - 5(1) + p$

$= 3 - 5 + p = p - 2$

Since, $f(0)$ and $f(1)$ are opposite signs.

$\therefore f(0)\, f(1) < 0$

$\Rightarrow p \times (p - 2) < 0$

$0 < p < 2$

77. If $e^{\theta\phi} = c + 4\theta\phi$, where c is an arbitrary constant and ϕ is a function of θ, then what is $\phi\, d\theta$ equal to?
(a) $\theta\, d\phi$ (b) $-\theta\, d\phi$
(c) $4\theta\, d\phi$ (d) $-4\theta\, d\phi$

(*b*) We have,

$e^{\theta\phi} = c + 4\theta\phi$

On differentiating w.r.t θ, we get

$e^{\theta\phi}\left(\theta\frac{d\phi}{d\theta} + \phi\right) = 0 + 4\left[\theta\frac{d\phi}{d\theta} + \phi\right]$

$\Rightarrow \frac{d\phi}{d\theta}(\theta e^{\theta\phi} - 4\theta) = 4\phi - \phi e^{\theta\phi}$

$$\Rightarrow \quad \frac{d\phi}{d\theta} = \frac{\phi\,(4 - e^{\theta\phi})}{-\theta\,(4 - e^{\theta\phi})}$$

$$\Rightarrow \quad \phi d\theta = -\theta d\phi$$

78. If $p(x) = (4e)^{2x}$, then what is $\int p(x)\,dx$ equal to?

(a) $\frac{p(x)}{1 + 2\ln 2} + C$ (b) $\frac{p(x)}{2\,(1 + 2\ln 2)} + C$

(c) $\frac{2p(x)}{1 + \ln 4} + C$ (d) $\frac{p(x)}{1 + \ln 2} + C$

(**b**) We have,

$$p(x) = (4e)^{2x}$$

$$\therefore \quad \int p(x)dx = \int (4e)^{2x} dx$$

$$= \int 4^{2x}_{\text{I}} . e^{2x}_{\text{II}}\, dx$$

By using integration by parts, we get

$$\int p(x)dx = 4^{2x} \int e^{2x}\, dx - \int \left[\frac{d}{dx}(4^{2x}) \int e^{2x} dx\right] dx$$

$$= \frac{4^{2x} e^{2x}}{2} - \int 4^{2x}\,(\log 4) 2 \times \frac{e^{2x}}{2} dx$$

$$= \frac{4^{2x}.e^{2x}}{2} - \log 4 \int 4^{2x}.e^{2x}\, dx$$

$$\Rightarrow \quad \int p(x)dx = \frac{4^{2x}e^{2x}}{2} - \log 4 \int p(x)dx$$

$$\Rightarrow (1 + \log 4) \int p(x)dx = \frac{4^{2x}.e^{2x}}{2}$$

$$\Rightarrow \quad \int p(x)dx = \frac{4^{2x}.e^{2x}}{2(1 + \log(2)^2)} + C$$

$$= \frac{p(x)}{2(1 + 2\log 2)} + C$$

79. What is the value of $\int_0^{\frac{\pi}{4}} (\tan^3 x + \tan x)\, dx$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

(**b**) Let $I = \int_0^{\pi/4} (\tan^3 x + \tan x)\, dx$

$$= \int_0^{\pi/4} \tan x.\,(\tan^2 x + 1)\, dx$$

$$= \int_0^{\pi/4} \tan x \sec^2 x\, dx$$

$[\because \sec^2 x - \tan^2 x = 1]$

Put $\tan x = t$

$\Rightarrow \quad \sec^2 x dx = dt$

$\therefore$ Upper limit, $t = \tan\frac{\pi}{4} = 1$

Lower limit, $t = \tan 0 = 0$

$$\therefore \quad I = \int_0^1 t\, dt = \left[\frac{t^2}{2}\right]_0^1$$

$$= \frac{1}{2}(1^2 - 0^2) = \frac{1}{2}$$

80. Let $y = 3x^2 + 2$. If x changes from 10 to 10.1, then what is the total change in y?

(a) 4.71 (b) 5.23 (c) 6.03 (d) 8.01

(**c**) Change in y,

$$\delta y = f(10.1) - f(10)$$

$$= 3 \times (10.1)^2 + 2 - [3(10)^2 + 2]$$

$$= 308.03 - 302$$

$$= 6.03$$

81. If $f(x) = \frac{\sin x}{x}$, where $x \in R$, is to be continuous at $x = 0$, then the value of the function at $x = 0$

(a) should be 0
(b) should be 1
(c) should be 2
(d) cannot be determined

(**b**) We have, $f(x) = \frac{\sin x}{x}$

Since, $f(x)$ is continuous at $x = 0$

$$\therefore \quad f(0) = \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{\sin x}{x} = 1$$

82. The solution of the differential equation $dy = (1 + y^2)\, dx$ is

(a) $y = \tan x + c$
(b) $y = \tan (x + c)$
(c) $\tan^{-1} (y + c) = x$
(d) $\tan^{-1} (y + c) = 2x$

(**b**) We have, $dy = (1 + y^2)dx$

$$\Rightarrow \quad \frac{dy}{1 + y^2} = dx$$

On integrating both sides, we get

$$\int \frac{dy}{1 + y^2} = \int dx + c$$

$$\Rightarrow \quad \tan^{-1} y = x + c$$

$$y = \tan(x + c)$$

83. What is $\int (e^{\log x} + \sin x) \cos x\, dx$ equal to?

(a) $\sin x + x \cos x + \frac{\sin^2 x}{2} + c$

(b) $\sin x - x \cos x + \frac{\sin^2 x}{2} + c$

(c) $x \sin x + \cos x + \frac{\sin^2 x}{2} + c$

(d) $x \sin x - x \cos x + \frac{\sin^2 x}{2} + c$

(**c**) Let $I = \int (e^{\log x} + \sin x) \cos x\, dx$

$$= \int x \cos x\, dx + \int \sin x \cos x\, dx$$

$$= I_1 + I_2$$

Where, $I_1 = \int x \cos x dx$

and $I_2 = \int \sin x . \cos x\, dx$

Now, $I_1 = \int x_{\text{I}} \cos_{\text{II}} x dx$

By using integration by parts, we get

$$I_1 = x \int \cos x dx - \int \left[\frac{d}{dx}(x) \int \cos x dx\right] dx$$

$$= x \sin x - \int 1 \times \sin x dx$$

$$= x \sin x + \cos x$$

and $I_2 = \int \sin x \cdot \cos x dx$

Put $\sin x = t \quad \Rightarrow \quad \cos x dx = dt$

$$\therefore \quad I_2 = \int t dt = \frac{t^2}{2} = \frac{\sin^2 x}{2} + c$$

$[\because$ put $t = \sin x]$

$$\therefore \quad I = x \sin x + \cos x + \frac{\sin^2 x}{2} + c$$

84. What is the domain of the function $f(x) = \cos^{-1}(x - 2)$?

(a) $[-1, 1]$ (b) $[1, 3]$ (c) $[0, 5]$ (d) $[-2, 1]$

(**b**) We know domain of $\cos^{-1} x$ is $[-1, 1]$.

$\therefore$ Domain of $\cos^{-1}(x - 2)$ is

$$-1 \le x - 2 \le 1$$

$$\Rightarrow \quad -1 + 2 \le x \le 1 + 2$$

$$\Rightarrow \quad 1 \le x \le 3$$

Hence, domain of the function is $[1, 3]$.

85. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) 2

(**c**) Given curve $y^2 = 2x$ is a parabola having vertex $(0, 0)$ and open right side.

Given straight line $y = x$, which passes through origin.

The point of intersection of line $y = x$ and curve is $(x)^2 = 2x$

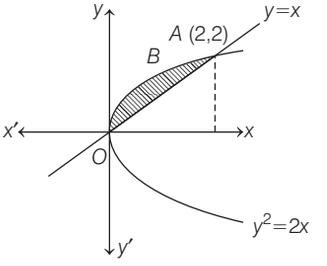
$$\Rightarrow \quad x(x - 2) = 0 \Rightarrow x = 0, 2$$

Put $x = 0$, then $y = 0$

Put $x = 2$, then $y^2 = 2 \times 2$

$$y = \pm 2$$

$\therefore$ The point of intersection of given curve is $O(0, 0)$ and $A(2, 2)$.



Now, area of shaded region $OABO$

$$= \int (y_2 - y_1) dx$$

$$= \int_0^2 (\sqrt{2x} - x)\, dx$$

$$= \left[\sqrt{2}\,\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^2$$

$$= \left[\frac{2\sqrt{2}}{3}x^{3/2} - \frac{x^2}{2}\right]_0^2$$
$$= \left[\frac{2\sqrt{2}}{3}2^{3/2} - \frac{2^2}{2} - 0 - 0\right]$$
$$= \frac{8}{3} - 2 = \frac{2}{3}$$

86. If $f(x) = 2x - x^2$, then what is the value of $f(x+2) + f(x-2)$ when $x = 0$?

(a) – 8 (b) – 4
(c) 8 (d) 4

(a) We have, $f(x) = 2x - x^2$

Now, $f(x+2) + f(x-2) = [2(x+2) - (x+2)^2] + [2(x-2) - (x-2)^2]$
$= [2x + 4 - (x^2 + 4 + 4x)] + [2x - 4 - (x^2 + 4 - 4x)]$
$= [-x^2 - 2x] + [-x^2 + 6x - 8]$
$= -2x^2 + 4x - 8$

At $x = 0$,
$f(x+2) + f(x-2) = -2(0)^2 + 4(0) - 8$
$= -0 + 0 - 8$
$= -8$

87. If $x^m y^n = a^{m+n}$, then what is $\frac{dy}{dx}$ equal to?

(a) $\frac{my}{nx}$ (b) $-\frac{my}{nx}$
(c) $\frac{mx}{ny}$ (d) $-\frac{ny}{mx}$

(b) We have, $x^m y^n = a^{m+n}$

On taking log both sides, we get
$\log x^m + \log y^n = (m+n)\log a$
$m\log x + n\log y = (m+n)\log a$

On differentiating both sides w.r.t. 'x' we get
$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-my}{nx}$$

88. What is $\int \frac{dx}{x(x^n+1)}$ equal to?

(a) $\frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right) + c$

(b) $\ln\left(\frac{x^n+1}{x^n}\right) + c$

(c) $\ln\left(\frac{x^n}{x^n+1}\right) + c$

(d) $\frac{1}{n}\ln\left(\frac{x^n+1}{x^n}\right) + c$

(a) Let $I = \int \frac{dx}{x(x^n+1)}$

$$= \int \frac{x^{n-1}}{x^n(x^n+1)}dx$$

[Multiply numerator and denominator by x^{n-1}]

Put $x^n + 1 = t$

$nx^{n-1}dx = dt$

$$\therefore \quad I = \int \frac{dt}{n(t-1)t}$$
$$= \frac{1}{n}\left[\int \frac{1}{t-1}dt - \int \frac{1}{t}dt\right]$$
$$\frac{1}{n}[\log|t-1| - \log t] + c$$
$$= \frac{1}{n}\log\frac{|t-1|}{t}$$

$\left[\because \log m - \log n = \log\frac{m}{n}\right]$

$$= \frac{1}{n}\log\left(\frac{x^n+1-1}{x^n+1}\right) + c$$

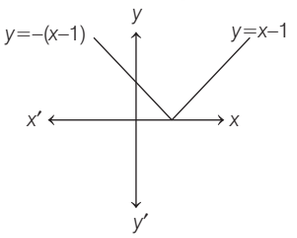
[put $t = x^n + 1$]

$$= \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + c$$

89. What is the minimum value of $|x-1|$, where $x \in R$?

(a) 0 (b) 1
(c) 2 (d) – 1

(a) Let $y = |x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$



The graph of a given function is shown above. From the graph it is clear that the minimum value of given function is 0.

90. What is the value of k such that integration of $\frac{3x^2 + 8 - 4k}{x}$ with respect to x, may be a rational function?

(a) 0 (b) 1 (c) 2 (d) – 2

(c) Now, $I = \int \frac{3x^2 + 8 - 4k}{x}dx$

$$= \int\left(3x + \frac{8}{x} - \frac{4}{x}k\right)dx$$

$$= \frac{3x^2}{2} + 8\log|x| - 4k\log|x| + C$$
$$I = \frac{3x^2}{2} + (8 - 4k)\log|x| + C$$

Integration I become a rational, if
$8 - 4k = 0$
$k = 2$

91. Consider the following statements for $f(x) = e^{-|x|}$:

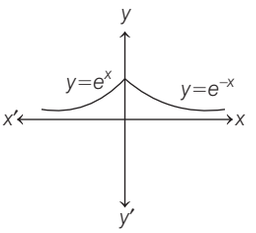
1. The function is continuous at $x = 0$.
2. The function is differentiable at $x = 0$.

Which of the above statements is/are correct?

(a) 1 Only (b) 2 Only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) We have, $f(x) = e^{-|x|}$

$$= \begin{cases} e^{-x}, & x \geq 0 \\ e^{x}, & x < 0 \end{cases}$$



It is clear from the graph that $f(x)$ is continuous everywhere. But $f(x)$ is not differentiable at $x = 0$, due to sharp shape.

Hence, Statement 1 is correct but Statement 2 is incorrect.

92. What is the maximum value of $\sin x \cdot \cos x$?

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $2\sqrt{2}$

(c) Let $f(x) = \sin x \cos x$

$$= \frac{1}{2} \times 2\sin x.\cos x$$

[Multiply numerator and denominator by 2]

$$= \frac{1}{2}\sin 2x$$

We know that maximum value of $\sin 2x$ is 1.

$$\therefore \quad f(x)_{max.} = \frac{1}{2} \times 1 = \frac{1}{2}$$

93. What is $\lim_{x \to 0} \frac{3^x + 3^{-x} - 2}{x}$ equal to?

(a) 0
(b) – 1
(c) 1
(d) Limit does not exist

(a) $\lim_{x \to 0} \frac{3^x + 3^{-x} - 2}{x}\left(\text{form}\frac{0}{0}\right)$

$$\lim_{x \to 0} \frac{3^x\log 3 - 3^{-x}\log 3 - 0}{1}$$

[by using L' Hospital's rule]

$= 3^0\log 3 - 3^0\log 3$

$= \log 3 - \log 3$

$= 0$

94. What is the derivative of $\tan^{-1} x$ with respect to $\cot^{-1} x$?

(a) -1 (b) 1

(c) $\frac{1}{x^2+1}$ (d) $\frac{x}{x^2+1}$

(**a**) Let $f(x) = \tan^{-1} x$ and $g(x) = \cot^{-1} x$

On differentiating w.r.t 'x', we get

$f'(x) = \frac{1}{1+x^2}$ and $g(x) = -\frac{1}{1+x^2}$

$\therefore \quad \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)} = \frac{\frac{1}{1+x^2}}{-\frac{1}{1+x^2}} = -1$

95. The function $u(x, y) = c$ which satisfies the differential equation $x(dx - dy) + y(dy - dx) = 0$, is

(a) $x^2 + y^2 = xy + c$

(b) $x^2 + y^2 = 2xy + c$

(c) $x^2 - y^2 = xy + c$

(d) $x^2 - y^2 = 2xy + c$

(**b**) We have,

$x(dx - dy) + y(dy - dx) = 0$

$\Rightarrow \quad xdx + ydy - (xdy + ydx) = 0$

$\Rightarrow \quad xdx + ydy - d(xy) = 0$

On integrating both sides, we get

$\frac{x^2}{2} + \frac{y^2}{2} - xy = \frac{c}{2}$

$x^2 + y^2 = 2xy + c$

96. What is the minimum value of $3\cos\left(A + \frac{\pi}{3}\right)$ where $A \in R$?

(a) -3 (b) -1

(c) 0 (d) 3

(**a**) Let $f(x) = 3\cos\left(A + \frac{\pi}{3}\right)$

We know that, the minimum value of $\cos\left(A + \frac{\pi}{3}\right)$ is -1.

$\therefore$ Minimum value of $f(x) = 3 \times (-1)$

$= -3$

97. Consider the following statements:

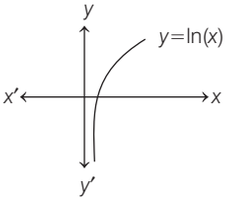
1. The function $f(x) = \ln x$ increases in the interval $(0, \infty)$.
2. The function $f(x) = \tan x$ increases in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Which of the above statements is/are correct?

(a) 1 Only (b) 2 Only

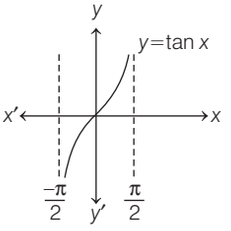
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) **1.** We have $f(x) = \ln(x)$



It is clear from the graph that $f(x)$ is increasing in the interval $(0, \infty)$.

2. We have $f(x) = \tan(x)$



It is clear from the graph that $f(x)$ is increases in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Hence, both statement are correct.

98. Which one of the following is correct in respect of the graph of $y = \frac{1}{x-1}$?

(a) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of reals

(b) The domain is $\{x \in R \mid x \neq 1\}$, the range is $\{y \in R \mid y \neq 0\}$ and the graph intersects y-axis at $(0, -1)$

(c) The domain is the set of reals and the range is the singleton set $\{0\}$

(d) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of points on they y-axis

(**b**) We have, $y = \frac{1}{x-1}$ ...(i)

The domain of given graph is $\{x \in R : x \neq 1\}$

Consider $y = \frac{1}{x-1}$

$\Rightarrow \quad x - 1 = \frac{1}{y}$

$\Rightarrow \quad x = \frac{1}{y} + 1 = \frac{1+y}{y}$

Here, we see that x is not defined for $y = 0$.

$\therefore$ Range of given curve is $\{y \in R : y \neq 0\}$

Also graph intersect Y –axis, put $x = 0$ in Eq. (i), we get

$y = \frac{1}{0-1} = -1$

Hence, graph intersect Y-axis at point $(0, -1)$.

99. What is the solution of the differential equation $\ln\left(\frac{dy}{dx}\right) = x$?

(a) $y = e^x + c$ (b) $y = e^{-x} + c$

(c) $y = \ln x + c$ (d) $y = 2\ln x + c$

(**a**) Given differential equation is

$\ln\left(\frac{dy}{dx}\right) = x \quad \Rightarrow \quad \frac{dy}{dx} = e^x$

$\Rightarrow \quad dy = e^x dx$

On integrating both sides, we get

$y = e^x + c$

100. Let l be the length and b be the breadth of a rectangle such that $l + b = k$. What is the maximum area of the rectangle?

(a) $2k^2$ (b) k^2 (c) $\frac{k^2}{2}$ (d) $\frac{k^2}{4}$

(**d**) We have, $l + b = k$

Now, area of rectangle $A = lb$

$A = l(k - l)$

On differentiating w.r.t, 'l', we get

$\frac{dA}{dl} = (k - l) + l(0 - 1) = k - 2l$

For maximum, put $\frac{dA}{dl} = 0$

$k - 2l = 0, l = \frac{k}{2}$,

Now, $\frac{d^2A}{dl^2} = -2 < 0$

Therefore A is maximum at $l = \frac{k}{2}$

$\therefore$ Value of maximum area,

$A = \frac{k}{2}\left(k - \frac{k}{2}\right) = \frac{k}{2}\left(\frac{k}{2}\right) = \frac{k^2}{4}$

101. The numbers 4 and 9 have frequencies x and $(x - 1)$ respectively. If their arithmetic mean is 6, then what is the value of x?

(a) 2 (b) 3 (c) 4 (d) 5

(**b**) Given, arithmetic mean = 6

$\therefore \quad \frac{4 \times x + 9 \times (x-1)}{x + (x-1)} = 6$

$\Rightarrow 4x + 9x - 9 = 6(2x - 1)$

$\Rightarrow \quad 13x - 9 = 12x - 6$

$\Rightarrow \quad x = 3$

102. If three dice are rolled under the condition that no two dice show the same face, then what is the probability that one of the faces is having the number 6?

(a) $\frac{5}{6}$ (b) $\frac{5}{9}$ (c) $\frac{1}{2}$ (d) $\frac{5}{12}$

(**c**) Let S = Total sample space

$= 6^3 = 216$

E = Event of getting no two face have same number. i.e. all three faces have distinct numbers.

$\therefore \quad n(E) = 6 \times 5 \times 4 = 120$

and F = Event of getting one of the face have 6 number.

$n(F) = (1 \times 5 \times 4) \times 3 = 60$

and $n(E \cap F) = 60$

$$\therefore \quad P\left(\frac{F}{E}\right) = \frac{P(F \cap E)}{P(E)} = \frac{n(F \cap E)/n(S)}{n(E)/n(S)} = \frac{60/216}{120/216} = \frac{60}{120} = \frac{1}{2}$$

103. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\text{not } A) = \frac{1}{2}$, then which one of the following is not correct?

(a) $P(B) = \frac{2}{3}$

(b) $P(A \cap B) = P(A)P(B)$

(c) $P(A \cup B) > P(A) + P(B)$

(d) $P(\text{not } A \text{ and not } B) = P(\text{not } A)\, P(\text{not } B)$

(**c**) We have, $P(A \cup B) = \frac{5}{6}$,

$$P(A \cap B) = \frac{1}{3}$$

and $P(\text{not A}) = \frac{1}{2}$

$$\Rightarrow \quad P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

(a) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3}$$

$$\Rightarrow \quad P(B) = \frac{5}{6} - \frac{1}{2} + \frac{1}{3} = \frac{5 - 3 + 2}{6} = \frac{4}{6} = \frac{2}{3},$$

Which is correct.

(b) $P(A) \cdot P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

$= P(A \cap B)$ which is correct.

(c) Now, $P(A) + P(B) = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

and $P(A \cup B) = \frac{5}{6}$

$\therefore P(A \cup B) \not> P(A) + P(B)$, which is incorrect.

Hence, option (c) is not correct.

104. The sum of deviations of n number of observations measured from 2.5 is 50. The sum of deviations of the same set of observations measured from 3.5 is -50. What is the value of n?

(a) 50 (b) 60 (c) 80 (d) 100

(**d**) We have,

$\Sigma(x_i - 2.5) = 50$

and $\Sigma(x_i - 3.5) = -50$

$\Rightarrow \quad \Sigma x_i - (2.5)n = 50 \quad \ldots(i)$

and $\Sigma x_i - (3.5)n = -50 \quad \ldots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get $n = 100$

105. A data set of n observations has mean $2M$, while another data set of $2n$ observations has mean M. What is the mean of the combined data sets?

(a) M (b) $\frac{3M}{2}$ (c) $\frac{2M}{3}$ (d) $\frac{4M}{3}$

(**d**) Given, $\bar{x}_1 = 2M$

$\bar{x}_2 = M$

and $n_1 = n$ and $n_2 = 2n$

$\therefore$ Mean of the combined data

$$= \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{n \times 2M + 2n \times M}{n + 2n} = \frac{4nM}{3n} = \frac{4}{3}M$$

Directions (Q. Nos. 106-108) *Read the following information and answer the three items that follow.*

Marks	Number of Students	
	Physics	Mathematics
10-20	8	10
20-30	11	21
30-40	30	38
40-50	26	15
50-60	15	10
60-70	10	6

106. The difference between number of students under Physics and Mathematics is largest for the interval

(a) 20-30 (b) 30-40

(c) 40-50 (d) 50-60

(**c**)

Marks	Number of students		Difference
	Physics (P)	Mathematics (M)	$\lvert P - M \rvert$
10-20	8	10	$\lvert 8 - 10 \rvert = 2$
20-30	11	21	$\lvert 11 - 21 \rvert = 10$
30-40	30	38	$\lvert 30 - 38 \rvert = 8$
40-50	26	15	$\lvert 26 - 15 \rvert = 11$
50-60	15	10	$\lvert 15 - 10 \rvert = 5$
60-70	10	6	$\lvert 10 - 6 \rvert = 4$

The largest difference between number of students under Physics and Mathematics is 11, which lies in the interval 40-50.

107. Consider the following statements:

1. Modal value of the marks in Physics lies in the interval 30-40.
2. Median of the marks in Physics is less than of marks in Mathematics.

Which of the above statements is/are correct?

(a) 1 Only

(b) 2 Only

(c) Both 1 and 2

(d) Neither 1 nor 2

(**a**)

Marks	Number of students		Cumulative Frequencies	
	Physics (P)	Mathematics (M)	Physics (cf_1)	Mathematics (cf_2)
10-20	8	10	8	10
20-30	11	21	19	31
30-40	30	38	49	69
40-50	26	15	75	84
50-60	15	10	90	94
60-70	10	6	100	100

1. The maximum frequency in Physics is 30, which lies in the interval 30-40.

Hence, modal value lies in the interval 30-40, which is correct statement.

2. Cumulative frequency of Physics is $N_1 = 100$

Now, $\frac{N_1}{2} = \frac{100}{2} = 50$

Just greater than $\frac{N_1}{2}$ is 75, which lies in the interval 40-50.

Here, $cf_1 = 49$, $f_1 = 26$, $h = 10$, $l = 40$

$\because$ Median of Physics, Md_1

$$= l + \frac{\frac{N_1}{2} - cf_1}{f_1} \times h$$

$$= 40 + \frac{50 - 49}{26} \times 10$$

$$= 40 + \frac{10}{26} = 40 + 0.38$$

$$= 40.38$$

Cumulative frequency of Mathematics is $N_2 = 100$

Now, $\frac{N_2}{2} = \frac{100}{2} = 50$

Just greater than $\frac{N_2}{2}$ is 69, which lies in the interval 30-40.

Here, $cf_2 = 31$, $f_2 = 38$, $h = 10$, $l = 30$

Median of Mathematics (Md_2)

$$= l + \frac{\frac{N_2}{2} - cf_2}{f_2} \times h$$

$$= 30 + \frac{50 - 31}{38} \times 10$$

$$= 30 + \frac{190}{38}$$

$$= 30 + 5 = 35$$

Here we see that median marks of Physics is greater than median marks of Mathematics. Hence, statement 2 is incorrect.

108. What is the mean of marks in Physics?

(a) 38.4 (b) 39.4 (c) 40.9 (d) 41.6

(***c***)

Marks	Mid value	Number of students in Physics (f_i)	$\mu_i = \frac{x_i - 45}{10}$	$\mu_i f_i$
10-20	15	8	–3	–24
20-30	25	11	–2	–22
30-40	35	30	–1	–30
40-50	45	26	0	0
50-60	55	15	1	15
60-70	65	10	2	20
Total		$\Sigma f_i = 100$		$\Sigma f_i \mu_i = -41$

Here, $A = 45$, $h = 10$

$$\therefore \quad \text{Mean} = A + \frac{\Sigma f_i \mu_i}{\Sigma f_i} \times h$$

$$= 45 + \frac{(-41)}{100} \times 10$$

$$= 45 - 4.1 = 40.9$$

109. What is the standard deviation of the observations $-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}$?

(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 4

(***b***) Now, mean of given observations,

$$\bar{x} = \frac{-\sqrt{6} - \sqrt{5} - \sqrt{4} - 1 + 1 + \sqrt{4} + \sqrt{5} + \sqrt{6}}{8}$$

$$= 0$$

Now, standard deviation, σ

$$= \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{(-\sqrt{6} - 0)^2 + (-\sqrt{5} - 0)^2 + (-\sqrt{4} - 0)^2 + (-1 - 0)^2 + (1 - 0)^2 + (\sqrt{4} - 0)^2 + (\sqrt{5} - 0)^2 + (\sqrt{6} - 0)^2}{8}}$$

$$= \sqrt{\frac{6 + 5 + 4 + 1 + 1 + 4 + 5 + 6}{8}}$$

$$= \sqrt{\frac{32}{8}} = \sqrt{4} = 2$$

110. If $\Sigma x_i = 20$, $\Sigma x_i^2 = 200$ and $n = 10$ for an observed variable x, then what is the coefficient of variation?

(a) 80 (b) 100 (c) 150 (d) 200

(***d***) $\because$ Coefficient of variation, $= \frac{\sigma}{\bar{x}} \times 100$

Now, standard deviation,

$$\sigma = \frac{1}{n}\sqrt{n\,\Sigma x_i^2 - (\Sigma x_i)^2}$$

$$= \frac{1}{10}\sqrt{10 \times 200 - (20)^2}$$

$$= \frac{1}{10}\sqrt{2000 - 400}$$

$$= \frac{1}{10}\sqrt{1600}$$

$$= \frac{40}{10} = 4$$

and mean, $\bar{x} = \frac{\Sigma x_i}{n}$

$$= \frac{20}{10} = 2$$

$\therefore$ Coefficient of variation $= \frac{4}{2} \times 100 = 200$

111. What is the probability that February of a leap year selected at random, will have five Sundays?

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$

(c) $\frac{2}{7}$ (d) 1

(***b***) In a leap year, February has 29 days. In out of 29 days, there are four weeks and one day extra. In out of the four weeks, there exist four Sunday. In one extra days, it may be Sunday or Monday or Tuesday or Wednesday or Thursday or Friday or Saturday.

Probability of getting five sundays in the month of February $= \frac{1}{7}$

112. The arithmetic mean of 100 observations is 40. Later, it was found that an observation '53' was wrongly read as '83'. What is the correct arithmetic mean?

(a) 39.8 (b) 39.7 (c) 39.6 (d) 39.5

(***b***) Correct arithmetic mean

$$= \frac{100 \times 40 - 83 + 53}{100}$$

$$= \frac{4000 - 30}{100} = \frac{3970}{100} = 39.7$$

113. A husband and wife appear in an interview for two vacancies for the same post. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. If the events are independent, then the probability of which one of the following is $\frac{11}{35}$?

(a) At least one of them will be selected
(b) Only one of them will be selected
(c) None of them will be selected
(d) Both of them will be selected

(***a***) Now, the probability of husband's selection,

$$P(H) = \frac{1}{7}$$

$$\Rightarrow \quad P(\overline{H}) = 1 - \frac{1}{7} = \frac{6}{7}$$

and the probability of wife's selection,

$$P(W) = \frac{1}{5}$$

$$\Rightarrow \quad P(\overline{W}) = 1 - \frac{1}{5} = \frac{4}{5}$$

Now, probability of none of them is selected

$$= P(\overline{H} \cap \overline{W}) = P(\overline{H})\,P(\overline{W})$$

$$= \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$$

Now, probability that at least one of them is selected

$= 1 - P$(None of them is selected)

$$= 1 - \frac{24}{35} = \frac{11}{35}$$

114. A dealer has a stock of 15 gold coins out of which 6 are counterfeits. A person randomly picks 4 of the 15 gold coins. What is the probability that all the coins picked will be counterfeits?

(a) $\frac{1}{91}$ (b) $\frac{4}{91}$

(c) $\frac{6}{91}$ (d) $\frac{15}{91}$

(***a***) Total number of gold coins = 15
and number of counterfeits coins = 6
∴ Probability of getting all four coins are counterfeits = $\frac{{}^{6}C_4}{{}^{15}C_4}$

$$= \frac{\frac{6\times5}{2\times1}}{\frac{15\times14\times13\times12}{4\times3\times2\times1}} = \frac{6\times5\times4\times3}{15\times14\times13\times12} = \frac{1}{7\times13} = \frac{1}{91}$$

115. A committee of 3 is to be formed from a group of 2 boys and 2 girls. What is the probability that the committee consists of 2 boys and 1 girl?
(a) $\frac{2}{3}$ (b) $\frac{1}{4}$
(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

(***d***) Total number of persons = 2 + 2 = 4
∴ Probability of getting a committee of 3 persons = $\frac{{}^{2}C_2\times{}^{2}C_1}{{}^{4}C_3} = \frac{1\times2}{4} = \frac{1}{2}$

116. In a lottery of 10 tickets numbered 1 to 10, two tickets are drawn simultaneously. What is the probability that both the tickets drawn have prime numbers?
(a) $\frac{1}{15}$ (b) $\frac{1}{2}$ (c) $\frac{2}{15}$ (d) $\frac{1}{5}$

(***c***) Total number of lottery tickets = 10
The prime number from 1 to 10 are {2, 3, 5, 7}
∴ Probability of drawing two prime number tickets

$$= \frac{{}^{4}C_2}{{}^{10}C_2} = \frac{\frac{4\times3}{2\times1}}{\frac{10\times9}{2\times1}} = \frac{4\times3}{10\times9} = \frac{2}{15}$$

117. Let X and Y represent prices (in ₹) of a commodity in Kolkata and Mumbai respectively. It is given that $\bar{X} = 65, \bar{Y} = 67, \sigma_X = 2.5, \sigma_y = 3.5$ and $r(X, Y) = 0.8$. What is the equation of regression of Y on X?
(a) $Y = 0.175X - 5$
(b) $Y = 1.12X - 5.8$
(c) $Y = 1.12X - 5$
(d) $Y = 0.17X + 5.8$

(***b***) We have,
$\bar{X} = 65, \bar{Y} = 67,$
$\sigma_x = 2.5, \sigma_y = 3.5$
and $r(X, Y) = 0.8$
The equation of regression of Y on X is

$$Y - \bar{Y} = r\frac{\sigma_y}{\sigma_x}(X - \bar{X})$$
$$\Rightarrow Y - 67 = 0.8 \times \frac{3.5}{2.5}(X - 65)$$
$$\Rightarrow Y - 67 = 1.12(X - 65)$$
$$\Rightarrow Y = 1.12X - 72.8 + 67$$
$$\Rightarrow Y = 1.12X - 5.8$$

118. Consider a random variable X which follows Binomial distribution with parameters $n = 10$ and $p = \frac{1}{5}$. Then, $Y = 10 - X$ follows Binomial distribution with parameters n and p respectively given by
(a) 5, $\frac{1}{5}$ (b) 5, $\frac{2}{5}$
(c) 10, $\frac{3}{5}$ (d) 10, $\frac{4}{5}$

(***d***) Given, $B(X, n, p) = B(X, 10, \frac{1}{5})$

$$\therefore \quad B(X, n, p) = {}^{n}C_X\, p^X(1-p)^{n-X}$$
$$= {}^{10}C_X\left(\frac{1}{5}\right)^X\left(1-\frac{1}{5}\right)^{10-X} \quad \left[\text{put } p = \frac{1}{5}\right]$$
$$= {}^{10}C_{10-X}\left(\frac{4}{5}\right)^{10-X}\left(\frac{1}{5}\right)^X \quad [\because {}^{n}C_r = {}^{n}C_{n-r}]$$
$$= {}^{10}C_Y\left(\frac{4}{5}\right)^Y\left(\frac{1}{5}\right)^{10-Y} \quad [\because Y = 10 - X]$$
$$= B(Y, n, p)$$

Where, $n = 10$ and $p = \frac{4}{5}$

119. If A and B are two events such that $P(A) = 0.6, P(B) = 0.5$ and $P(A \cap B) = 0.4$, then consider the following statements
1. $P(\bar{A} \cup B) = 0.9$
2. $P(\bar{B} / \bar{A}) = 0.6$

Which of the above statements is/are correct?
(a) 1 Only (b) 2 Only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***d***) We have,
$P(A) = 0.6, P(B) = 0.5$ and $P(A \cap B) = 0.4$

1. $P(\bar{A} \cup B) = 1 - P(A) + P(A \cap B)$
$= 1 - 0.6 + 0.4$
$= 0.8$, which is incorrect.

2. $P(\bar{B} / \bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})}$

$$= \frac{P(\overline{A \cup B})}{P(\bar{A})} = \frac{1 - P(A \cup B)}{1 - P(A)} = \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(A)}$$
$$= \frac{1 - [0.6 + 0.5 - 0.4]}{1 - 0.6} = \frac{1 - [0.7]}{0.4} = \frac{0.3}{0.4}$$
$$= 0.75$$

Which is incorrect.
Hence, both statements are incorrect.

120. Three cooks X, Y and Z bake a special kind of cake and with respective probabilities 0.02, 0.03 and 0.05, it fails to rise. In the restaurant where they work, X bakes 50%, Y bakes 30% and Z bakes 20% of cakes. What is the proportion of failures caused by X?
(a) $\frac{9}{29}$ (b) $\frac{10}{29}$ (c) $\frac{19}{29}$ (d) $\frac{28}{29}$

(***b***) Let A = Bake X-type of cake
B = Bake Y-type of cake
and C = Bake Z-type of cake
Then, $P(A) = 50\% = \frac{50}{100}$,
$P(B) = 30\% = \frac{30}{100}$,
$P(C) = 20\% = \frac{20}{100}$,
$P\left(\frac{X}{A}\right) = 0.02, P\left(\frac{X}{B}\right) = 0.03$
and $P\left(\frac{X}{C}\right) = 0.05$

By using Baye's theorem,

$$P\left(\frac{A}{X}\right) = \frac{P(A)\,P\left(\frac{X}{A}\right)}{P(A)\,P\left(\frac{X}{A}\right) + P(B)\,P\left(\frac{X}{B}\right) + P(C)\,P\left(\frac{X}{C}\right)}$$
$$= \frac{\frac{50}{100}\times0.02}{\frac{50}{100}\times0.02 + \frac{30}{100}\times0.03 + \frac{20}{100}\times0.05}$$
$$= \frac{1.00}{1.00 + 0.90 + 1.00} = \frac{1}{2.9} = \frac{10}{29}$$

PAPER II English Language and General Studies

Part A (English Language)

Directions (Q.Nos. 1-10) *Each item in this section consists of a sentence with an underlined word followed by four words. Select the option that is opposite in meaning to the underlined word and your response accordingly.*

1. He nodded absently throughout the meeting.
(a) capably (b) alertly (c) agitatedly (d) dreamily

(b) The given word means carelessly. Hence, from the given options the correct antonym is alertly which means carefully.

2. I fully believe that the cornerstone of good policy is an electorate that is educated on the national issues.
(a) cerebral (b) enlightened (c) ignorant (d) erudite

(c) The antonym of 'educated' is ignorant.

3. For important medical decisions, even finding a doctor you trust is not enough.
(a) significant (b) trivial (c) basic (d) probable

(b) The antonym of 'important' is trivial meaning 'of little value'.

4. Planets move in their orbits.
(a) push (b) rotate (c) stall (d) flow

(c) The antonym of 'move' is stall meaning 'to stop'.

5. Temperature is a measure of internal energy of an object and is frequently expressed by physicists in units of Kelvin.
(a) found (b) told (c) distributed (d) concealed

(d) The given word 'expressed' means to convey. Hence, its antonym will be concealed meaning 'to hide'.

6. People argue about why Venus is so much warmer than the Earth.
(a) friendlier (b) colder (c) wilder (d) heavier

(b) The antonym of 'warmer' is 'colder'.

7. Scientists are concerned whether the oceans and land biosphere will take up as much carbon in the future as they presently do.
(a) worried (b) indifferent (c) curious (d) puzzled

(b) The antonym of 'concerned' is indifferent meaning 'without any concern'.

8. The biggest debate among scientists today is about cloud feedback.
(a) contend (b) moot (c) wrangle (d) agreement

(d) The given word 'debate' means to argue. Hence, its antonym will be agreement meaning 'to agree to a thought'.

9. The Earth's climate sensitivity is conventionally defined as the equilibrium temperature increase caused by a doubling of carbon dioxide.
(a) imperviousness (b) willingness
(c) responsiveness (d) closeness

(a) The given word 'sensitivity' means responsiveness. Its antonym will be imperviousness meaning 'something that will not be affected or will not respond'.

10. The oceans carry a huge amount of heat from the tropics to the high latitudes.
(a) significant (b) major (c) tiny (d) dormant

(c) The antonym of 'huge' is 'tiny'.

Directions (Q.Nos. 11-20) *Each of the following items in this section consists of a sentence(s), the parts of which have been jumbled. These parts have been labelled as P, Q, R and S. Given below each sentence are four sequences namely (a), (b), (c) and (d). You are required to rearrange the jumbled part of the sentence and mark your response accordingly.*

11. several years ago (P), course on climate change at Texas A & M University (Q) Professor Andrew Dessler created an introductory (R) for freshman and sophomores (S)
(a) P R Q S (b) Q R P S (c) S Q R P (d) P Q R S

(a)

12. I realize that solving the climate change problem (P)
than solving (Q) will be much harder (R)
the ozone depletion problem (S)
(a) P R Q S (b) Q R P S (c) S Q R P (d) P Q R S

(a)

13. although the temperature of this layer of the (P)
when directly comparing the satellite (Q)
measurements of temperature (R)
atmosphere should generally track the surface temperature, we must be careful (S)
(a) P R Q S (b) P S Q R (c) S Q R P (d) P Q R S

(b)

14. the element heats up (P), eventually reaching high temperatures (Q)
glowing like a dark orange that radiates (R) the visible range, (S)
(a) P Q S R (b) Q P S R (c) S Q R P (d) P R S Q

(d)

15. two people argue about why Venus is so much warmer (P)
to the Sun, so it absorbs more solar energy. (Q)
The second argues that it's because Venus has a thick, (R)

greenhouse-gas rich atmosphere than the Earth. The first argues that it's because Venus is closer
S

(a) P S Q R (b) Q R P S (c) S Q R P (d) P Q R S

(a)

16. are now used by more than a thousand firms
P
and are growing in popularity
Q
in the United States and Europe gain sharing-plans
R S

(a) P R Q S (b) Q R P S (c) P R S Q (d) S P R Q

(d)

17. adversity without succumbing
P
to the clouds of doubt and jealousy
Q
between friends which is subjected
R
to both prosperity and the essay 'Of Friendship' by Francis Bacon celebrated the intimacy
S

(a) P R Q S (b) Q R P S (c) S R P Q (d) P Q R S

(c)

18. friends without which the world is
P
make friends and a person wills to want true
Q
it is miserable solitude that compels a person to
R
nothing other than a place of wilderness
S

(a) P R Q S (b) Q R P S (c) R Q P S (d) P Q R S

(c)

19. takes the longest that never started the job to finish
P Q R S

(a) P R Q S (b) R Q P S (c) S Q R P (d) P Q R S

(b)

20. to what you can create control, shift
P
your energy instead of worrying about what you cannot
Q R S

(a) P R S Q (b) Q R S P
(c) S Q R P (d) R S Q P

(d)

Directions (Q.Nos.21-30) *Each item in this section consists of a sentence with an underlined word/words followed by your words. Select the option that is* ***nearest in meaning*** *to the underlined word and mark your response accordingly.*

21. She got the divorce within no time.
(a) detachment (b) breaking down
(c) annulment (d) punishment

(a,c) The given word 'divorce' means annulment or detachment.

22. He was known for his gentle disposition.
(a) harmful (b) amiable (c) cunning (d) adjusting

(b) The given word gentle means kind and friendly. Therefore, from the given options, 'amiable' meaning friendly is its near synonym.

23. The harder we kick, the better the ball bounces back.
(a) recoils (b) deflates (c) inflates (d) ascends

(a) The given phrase 'bounces back' means 'to come back to the original position'. Therefore, from the given options, 'recoil' means the same as bounces back.

24. I plan and execute.
(a) desire (b) debate (c) accomplish (d) discard

(c) The given word 'execute' means to accomplish.

25. He listened to me with equanimity.
(a) carelessly (b) excitedly
(c) patiently (d) half-heartedly

(c) The given phrase 'with equanimity' means carefully and patiently.

26. The whole thing was a futile exercise.
(a) costly (b) pointless (c) indecisive (d) successful

(b) The given word 'futile' means 'useless'. Hence, from the given options, pointless is it's near synonym.

27. He was too cunning for her.
(a) capable (b) fit (c) intelligent (d) dodgy

(d) The given word 'cunning' means deceitful. From the given options, dodgy also means deceitful and hence is its near synonym.

28. This decision is crucial for me.
(a) momentous (b) natural (c) primitive (d) inept

(a) The given word 'crucial' means important. From the given options, 'Momentous' is its near synonym.

29. The media went into a frenzy about the case.
(a) silent (b) creative (c) berserk (d) wrong

(c) The given phrase 'into a frenzy' means 'to go crazy or mad position'. From the given options, 'berserk' means the same as the phrase.

30. He has been yearning to go home.
(a) declining (b) demanding (c) begging (d) hankering

(d) The given word 'yearning' means 'intense longing for something. Therefore, from the given options, 'hankering' means the same as the word.

Directions (Q.Nos. 31-40) *Each item in this section has a sentence with a missing preposition. Select the correct preposition from the given option and mark your response accordingly.*

31. Simulations of the 20th century by climate models that exclude the observed increase ..... greenhouse gases fail to simulate the increase in temperature over the second half of the 20th century.
(a) of (b) in (c) by (d) to

(a) Preposition 'of' is used when we need to refer to a thing.

32. In extremely poor societies, children can be put to work ..... a young age and are therefore a source of income.
(a) in (b) on (c) by (d) at

(d) Preposition 'at' is used when we use a common time phrase.

33. People who are averse ..... hard work, generally do not succeed in life.
(a) to (b) about (c) at (d) on

(a) Preposition 'to' is used when we indicate a receiver of an action.

34. I have known her ..... a long time.
(a) since (b) for
(c) at (d) before

(b) Preposition 'for' is used to indicate the length of time.

35. I accepted the offer ..... certain conditions.
(a) on (b) in
(c) by (d) within

(a) Preposition 'on' is used to indicate a state of something.

36. She is a woman ..... humble origin.
(a) off (b) of
(c) from (d) within

(b) Preposition 'of' is used when we need to refer to a characteristic of a thing/ person.

37. There is no cure ..... the common cold.
(a) for (b) of (c) to (d) on

(a) Preposition 'for' is used to indicate a connection with somebody/something.

38. I ran ..... John yesterday, and it was a pleasant surprise.
(a) by (b) off (c) beside (d) into

(d) Preposition 'into' is used to a point wherein you hit someone or something.

39. My grandmother enjoyed boating ..... the lovely lake.
(a) in (b) on
(c) beside (d) within

(b) Preposition 'on' is used to refer to surfaces such as land or water body.

40. He visits the needy to relieve them ..... their sufferings and poverty.
(a) from
(b) off
(c) of
(d) on

(c) Preposition 'of' is used to indicate a belonging of somebody.

Directions (Q.Nos. 41-50) *Each item in this section has a sentence with three underlined parts labelled as (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part and indicate your response against the corresponding letter i.e., (a) or (b) or (c). If you find no error, your response should be indicated as (d).*

41. This building (a) comprises of six houses, (b) three parking lots and one basement. c No error (d)

(d)

42. I look forward (a) to meet (b) you. (c) No error (d)

(b) For the sentence to be correct 'meet' must be replaced by 'meeting'.

43. I cannot (a) cope up (b) with this presure (c). No error (d)

(b) 'up' must be removed from the sentence to make the sentence grammatically correct.

44. I came (a) to this place (b) by walk (c). No error (d)

(c) 'by walk' must be replaced by 'on foot' to make the sentence grammatically correct.

45. What is (a) the time (b) in you watch? (c) No error (d)

(c) 'in' must be replaced by 'by' to make the sentence grammatically correct.

46. The price of my dress (a) is higher than that of (b) yours. (c) No error (d)

(d)

47. My friend's (a) daughter-in-laws (b) have come. (c) No error (d)

(d)

48. I prefer (a) seafood (b) than Mediterranean food. (c) No error (d)

(c) 'Than' must be replaced by 'to' to make the sentence grammatically correct.

49. We were (a) discussing about (b) the issue. (c) No error (d)

(b) 'About' must be removed to make the sentence grammatically correct.

50. Dispose off (a) the garbage (b) from the shelves. (c) No error (d)

(a) 'Off' must be replaced by 'of' to make the sentence grammatically correct.

Part B (General Studies)

51. New Zealand is considered part of which one of the following island groups?
(a) Micronesia (b) Melanesia
(c) Polynesia
(d) Hawaii Island chain

(c) New Zealand is island country in the South Pacific ocean. It is considered as the South westernmost part of Polynesia. Polynesia is a sub-region of Oceania made up of more than 1000 islands scattered over the Central and southern Pacific ocean. The indigenous people who inhabit the islands of Polynesia are termed Polynesians sharing many similar traits including language family, culture and beliefs.

Melanesia islands (Literal meaning 'Islands of Black People'), a Sulo-region of Oceania extending from New Guinea islands to Arafura sea and to Tonga. Micronesia island is a Sulo region of Oceania, composed of thousands of small islands in the Western Pacific ocean.

Hawaii island chain, formerly known as 'Sandwich islands'', are an archipelago of 137 islands.

52. Which of the following statements with regard to *Pradhan Mantri Krishi Sinchayee Yojana* (PMKSY) is/are correct?
1. PMKSY was launched during 2015-16.

2. The basic aim of PMKSY is to enhance physical access of water on farm.
3. One of the main objectives of PMKSY is to achieve convergence is investments in irrigation at the field level.

Select the correct answer using the codes given below.

(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) All of these

(d) Pradhan Mantri Krishi Sinchayee Yojana (PMKSY) was launched during 2015-16. It will be implemented across the country with an outlay of ₹ 50,000 crore in five years.

The basic aim of PMKSY is to enhance physical access of water on farm by providing assured irrigation, reducing wastage of water and improving water use efficiency.

To achieve its objectives of achieving convergence of investments in irrigation at the field level. It amalgamated ongoing schemes such as Accelerated Irrigation Benefit Programme (AIBP), Integrated Watershed Management Programme (IWMP) and on-farm water management.

Hence, option (d) 'All of these' is the correct answer.

53. According to Article-79 of the Constitution of India, which of the following is/are described as a part of Parliament of India?

1. The House of the People
2. The Council of States
3. The President of India

Select the correct answer using the codes given below.

(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) All of these

(d) As per Article-79 of the Constitution of India, the Council of the Parliament of the Union consists of the President and two Houses known as the Council of States (Rajya Sabha) and the House of the People (Lok Sabha).

54. The Five Year Plan was first launched in

a. China (b) USSR
(c) India (d) Bhutan

(b) Among the given options, Five Year Plan was first launched in erstwhile USSR (1928-1932) by Joseph Stalin (Soviet Ruler), under 'New Economic Policy'.

- The First Five Year Plan in China was started in 1953 and ended in 1957.
- The First Five Year plan in Bhutan was started in 1961 and ended in 1966, and also it was fully funded by India.
- In India, the First Five Year Plan was introduced in 1951, which was based on Harrod-Domar model and focused primarily on the development of primary sector, especially agriculture and irrigation. On, 1st January, 2015 planning commission (which was responsible for formulating five year plan) was replaced by NITI Aayog (National Institution for Transforming India). The twelfth Five Year Plan (2012-17) would be called the last five year plan of India.

55. The call for *Garibi Hatao* was incorporated in

(a) Fourth Five Year Plan
(b) Fifth Five Year Plan
(c) Sixth Five Year Plan
(d) Seventh Five Year Plan

(b) Garibi Hatao (Remove poverty) was the theme and slogan of Indira Gandhi's 1971 election campaign.The slogan and the proposed anti-poverty programme that came with it were designed to give Gandhi an independent national support, based on rural and urban poor.

It was part of the Fifth Five-Year Plan (1975-79). The Fifth Five-Year Plan laid stress on employment, poverty alleviation (Garibi Hatao) and justice. The plan also focused on self-reliance in agricultural production and defence.

In 1978, the newly elected Morarji Desai government rejected the plan.

The objectives of Fourth Five Year Plan (1969-74) was growth with stability and progressive achievement of self-reliance.

The basic objective of Sixth Five Year Plan (1980-85) was economic liberalisation by eradicating poverty and achieving technological self-reliance.

The objectives of Seventh Five Year Plan (1985-90) includes the establishment of self-sufficient economy, opportunities for productive employment and upgradation of technology.

56. Match List I with List II and select the correct answer using the code given below the lists:

	List I (Major Objective)		**List II** (Five Year Plan)
A.	Faster and more inclusive growth	1.	First
B.	Faster, more inclusive and sustainable growth	2.	Fifth
C.	Correction of disequalibrium caused by Second World War	3.	Eleventh
D.	Attaining self-reliance	4.	Twelfth

Codes

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	4	1	3
(d)	2	1	4	3

(b) The term economic planning is used to describe the long-term plans of the Government of India to develop and co-ordinate the economy with efficient utilisation of resources.

First Five Year Plan (1951 to 1956) Its objective was Rehabilitation of refugees, rapid agricultural development to achieve food self-sufficiency in the shortest possible time and control of inflation.

Fifth Five Year Plan (1956 to 1961) Its objective was removal of poverty and attainment of self-reliance.

Eleventh Five Year Plan (2007 to 2012) Its objectives were Rapid and inclusive growth; Empowerment through education and skill development; Reduction of gender inequality and Environmental sustainability.

Twelfth Five Year Plan (2012 to 2017) Its objectives were "faster, sustainable and more inclusive growth". It proposed a growth target of 8%; Raised agriculture output to 4%; Manufacturing sector growth to 10% and services to 9% respectively.

57. Which one of the following Directive Principles was inserted by the Constitution (42nd Amendment) Act, 1976?

(a) The State shall minimise inequalities in income
(b) Equal justice and free legal aid
(c) Promotion of co-operative societies
(d) Provision for early childhood care

(b) To promote equal justice and to provide free legal aid to the poor (Article-39A) was inserted by the Constitution 42nd Amendment Act, 1976.

The 42nd Amendment Act of 1976 added four new Directive Principles to the original list.

Others are

- To secure opportunities for healthy development of children (Article-39).
- To take steps to secure the participation of workers in the management of industries (Article-43A).

- To protect and improve the environment and to safeguard forests and wildlife (Article-48 A).
- Article 38(2) of Directive Principles of State policies says that State shall strive to minimise the inequaliting of income, status, facilities, opportunities etc.
- Article 43-B (inserted by 97th Amendment, 2011) deals with the promotion of co-operative societies.
- Article 45 of DPSPs says that state shall endeavour to provide early childhood care and education for all children until they complete the age of six year.
- Directive principles of State Policies (DPSPs), enshrined in Part IV (Article 36 to 51) of Constitution, with the basic idea that the 'State' should keep these principles while framing laws, policies, ordinances etc.

58. A common High Court for two or more States can be established by
(a) a law passed by the Parliament
(b) an order of the Supreme Court of India
(c) an order of the President of India
(d) an amendment to the Constitution of India

(a) A common High Court for two or more state can be established by a law passed by the Parliament. As per the Constitution of India, Articles-214-231 deals with the provisions of the High Courts in India. At present, we have 25 High Courts in the country, which includes 7 common High Courts.

These are Bombay High Court, Calcutta High Court, Madras High Court, Gauhati High Court, Kerala High Court, Punjab and Haryana High Court and Jammu and Kashmir High Court,

Article-217 deals with the appointment of judges.

Article 231 of our Constitution states that Parliament by law establish common high courts for two or more states or for two or more states and a Union Territory. This provision was brought by Seventh Amendment Act, 1956.

59. In the year 1928, a committee of Congress leaders drafted a Constitution for India. The Committee was headed by
(a) Mahatma Gandhi
(b) TB Sapru
(c) Motilal Nehru
(d) Jawaharlal Nehru

(c) In the 1928, a committee of Congress leaders drafted a Constitution for India. It was headed by Motilal Nehru. It was prepared by a committee of All Parties Conference chaired by Motilal Nehru with Jawaharlal Nehru as its Secretary. There were a total of 9 members in this committee.

This committee was created when Lord Birkenhead, Secretary of State of India asked the Indian leaders to draft a Constitution for the country. The report, which demanded a Dominion Status for India was considered by the Congress. The main idea of the Nehru report was that India would be given Dominion status. The Nehru draft report also defined the citizenship and Fundamental Rights.

Tej Bahadur Sapru (TB Sapru) was an Indian freedom fighter, lawyer, politician and the leader of the liberal party in British ruled India.

60. Which one of the following statements about Jaipal Singh is not correct?
(a) He was a member of the Constituent Assembly
(b) He founded the Adivasi Maha Sabha
(c) He was the captain of the first Indian National Hockey Team
(d) He campaigned for a separate State of Chhattisgarh

(d) Jaipal Singh Munda (1903-1970) was a politician, prolific writer and sportsman. He was the member of the Constituent Assembly which debated on the new Constitution of the Indian Union and who spoke for the first time in the Assembly (on 19th December, 1946) and owned up proudly to his 'tribal heritage'. He captained the Indian field hockey team to clinch gold in the 1928 Summer Olympics in Amsterdam. Singh became President of Adivasi Mahasabha in 1939.

61. What is 'Tikki Mausi' in the context of Malnutrition?
(a) A specially packaged food item
(b) A mascot
(c) Name of a scheme
(d) Name given to the healthcare providers

(b) 'Tikki Mausi' is the mascot unveiled by the Odisha government and UNICEF.

It aims to spread awareness about nutrition of children and women. Through state level communication and awareness generation, this mascot is expected to bring out changes in the behaviour of common people and make them aware about child and women's nutrition and development.

62. Which one of the following was the theme of the World Tourism Day, 2019?
(a) Sustainable tourism
(b) Tourism and the digital transformation
(c) Tourism : responding to the challenge of climate change
(d) Tourism and jobs : a better future for all

(d) The theme of World Tourism Day' 2019 was 'Tourism and Jobs: A Better Future For All'.

The World Tourism Day which observed globally on 27th September is hosted by a different country every year. India has been selected as the host country for celebration of World Tourism Day 2019 by United Nations World Tourism Organisation (UNWTO). This is for the very first time that India hosted the day in its National Capital, New Delhi.

- 'Sustainable Tourism' was the theme for World Tourism Day, 2017
- 'Tourism and the Digital Transformation' was the theme for World Tourism Day, 2017.

63. Which one of the following countries has hosted the Army Exercise TSENTR, 2019?
(a) Russia (b) China
(c) Kazakhstan (d) Kyrgyzstan

(a) Armies of India, Pakistan took part in the Army Exercise TSENTR, 2019 that was conducted by Russia from 9th to 23rd September, 2019. It was the first time that forces from India and Pakistan were participated in multilateral exercise together since airstrikes conducted by Indian force, deep in Pakistan's Balakot district on February 2019. Earlier armies of India and Pakistan had met last year as part of military exercise conducted by Shanghai Cooperation Organisation (SCO) also held in Russia.

64. Which one of the following countries is called the 'country of winds'?
(a) India
(b) China
(c) Denmark
(d) Germany

(c) Denmark is called 'Country of Winds'. Denmark ranks first in the world to produce electricity from wind.

65. Which one of the following is the oldest scientific department of Government of India?
(a) Department of Biotechnology
(b) Survey of India
(c) India Meteorological Department
(d) DRDO

(b) Survey of India, The National Survey and Mapping Organisation of the country under the Department of Science and Technology, is the Oldest Scientific Department of the Government of India. It was set-up in 1767 and has evolved rich traditions over the years.

The Department of Biotechnology is an Indian Government department, under the Ministry of Science and Technology responsible for administrating development and commercialisation in the field of modern biology and biotechnology in India. It was set-up in 1986.

The India Meteorological Department is an agency of the Ministry of Earth Sciences of the Government of India.

It is the principal agency responsible for meteorological observations, weather forecasting and seismology. It was founded in 1875.

The Defence Research and Development Organisation is an agency under the Department of Defence Research and Development in Ministry of Defence of the Government of India, charged with the military's research and development, headquartered in Delhi. It was founded in 1958.

66. 'Naseem-Al-Bahr' is a bilateral naval exercise between India and
(a) United Arab Emirates
(b) Iran
(c) Saudi Arabia
(d) Oman

(d) Naseem-Al-Bahr naval exercise is a bilateral exercise between India and Oman.

India and Oman conducted 12th edition of bilateral maritime exercise Naseem-Al-Bahr in Mormugao Port, Goa in January 2020.

Naseem-Al-Bahr (or sea breeze) is a naval exercise between the Indian Navy and the Royal Navy of Oman (RNOV), being conducted since 1993.

'AL Nagan' is a joint military exercise between Indian and Royal Army of Oman.

'Ex Eastern Bridge' is another bilateral exercise between Indian Airforce and Royal Air Force of Oman.

67. Koneru Humpy excels in which one of the following sports?
(a) Boxing
(b) Table Tennis
(c) Chess
(d) Billiards

(c) Koneru Humpy is an Indian chess player who is also reigning world rapid champion. In 2002, she became the youngest woman ever to achieve the title of Grandmaster at the age of 15 years, 1 month, 27 days, beating Judit Polgar's previous record by 3 months.

68. Which one of the following was the official mascot of Khelo India Youth Games, 2020?
(a) Vijaya (b) Yaya
(c) Rongmon (d) Ammu

(a) Jaya the black buck and Vijay the tiger are the mascots for Khelo India Youth Games.The third Khelo India Youth Games was held from 10th January, 2020 and 22nd January, 2020 in Guwahati (Assam) India. The event witnessed 20 national level multidisciplinary grassroots games played among the age groups of under-17 and under-21 categories. Maharashtra defended its title as it topped.

69. In January, 2020, a passenger aircraft crashed in Iran soon after taking off from Tehran's Imam Khomeini airport killing about 170 people onboard. The airplane belongs to
(a) Qatar Airways
(b) Ukraine International Airlines
(c) Singapore Airlines
(d) Cathay Pacific

(b) A Ukraine International Airlines (UIA) flight 752 (PS752) crashed shortly after taking off from the Iranian capital Tehran's Imam Khomeini airport on 8th January, killing all 176 passengers and crew members on board.

This flight was shot by the Iranian Islamic Revolutionary Guards Corp. This shoot down occurred, five days after US President Donald Trump launched a drone strike that killed Iranian General Qusem Soleimani.

Qatar airways is the state owned flag carrier airline of Qatar.

Cathy Pacific airways limited is the flag carrier airline of Hongkong.

Singapore airlines is the flag carrier airline of Singapore.

70. MILAN, a multilateral naval exercise, 2020 was hosted by which one of the following cities?
(a) Port Blair (b) Kochi
(c) Visakhapatnam (d) Panaji

(c) MILAN 2020-a multilateral naval exercise scheduled to be conducted by the Indian Navy from 18th to 28th March, 2020, at Visakhapatnam, has been postponed taking into consideration the safety of all participants and travel restrictions imposed by the spread of COVID-19'. MILAN was first held in 1995.

Apart from the Indian Navy, the navies of Indonesia, Singapore, Sri Lanka and Thailand participated in the inaugural edition.

71. Chalk and marble are different forms of
(a) Calcium hydrogen carbonate
(b) Calcium carbonate
(c) Calcium acetate
(d) Sodium carbonate

(b) Chalk and marble are different forms of calcium carbonate.

The chemical formula of calcium carbonate is $CaCO_3$. In Earth it is produced by the sedimentation of shells of fossilised fishes, shells one million of years.

It is a precursor of quick lime and slaked lime.

72. The number of maximum electrons in N Shell is
(a) 2
(b) 8
(c) 18
(d) 32

(d) The maximum number of electrons in N shell is 32. Total electrons in other shells are as follows

K shell	–	2	L shell	–	8
M shell	–	18	N shell	–	32

73. Vinegar is also known as
(a) Ethanoic acid (b) Nitric acid
(c) Sulphuric acid (d) Tartaric acid

(a) Vinegar is also known as Ethanoic acid. It's chemical formula is CH_3COOH. It contains 5-8% acetic acid by volume.

74. A liquid is kept in a glass beaker. Which one of the following statements is correct regarding the pressure exerted by the liquid column at the base of the beaker?
(a) The pressure depends on the area of the base of the beaker
(b) The pressure depends on the height of liquid column
(c) The pressure does not depend on the density of the liquid
(d) The pressure neither depends on the area of the base of the beaker nor on the height of liquid column

(b) Statement (b) is correct regarding the pressure exerted by the liquid column at the base of the beaker the

pressure depends upon the density (d), height (h) of liquid column and also on acceleration due to gravity (g).

$$p = hdg$$

75. Which of the following statements is not correct regarding the travel of sound waves?
(a) Sound waves can travel through water
(b) Sound waves can travel through air
(c) Sound waves can travel through steel
(d) Sound waves can travel through vacuum

(d) Sound waves need a medium to travel. In vacuum, there is no medium, hence, sound waves cannot travel through vacuum.

76. Deendayal Port is located at
(a) Kerala (b) Gujarat
(c) Maharashtra (d) Goa

(b) Deendayal Port Trust (Kandla) is a seaport in Kutch District of Gujarat. Kandla port was renamed as Deendayal Port Trust under the India Port Act of 1908. It is the largest port of India by volume of cargo handled. Kandla Port Trustis, India's busiest major port in recent years.
- Kochi Port is in Kerala.
- Mumbai port and Jawaharlal Nehru port are the two major ports of Maharashtra.
- Marmugao is the major port of Goa.
- At present there are 13 major ports in India and nearly 187 notified Minor and Intermediate ports.

77. Which one of the following cities is associated with Biosafety Protocol to the Convention on Biological Diversity (2000)?
(a) Geneva (b) Nairobi
(c) Cartagena (d) Rio de Janeiro

(c) Cartagena (Colombia) is associated with Biosafety Protocol to the Convention on Biological Diversity. The Cartagena Protocol on Biosafety to the Convention on Biological Diversity is an international agreement which aims to ensure the safe handling, transport and use of Living Modified Organisms (LMOs) resulting from modern biotechnology that may have adverse effects on biological diversity, taking also into account risks to human health.

It was adopted on 29th January, 2000 and entered into force on 11th September, 2003.
- Nairobi (Kenya) is the headquarter of 'United Nations Environment Programme (UNEP) which is responsible for coordinating UN's Environmental activities.
- Geneva (Switzerland) is famous for protocol which prohibits the use of poisonous gases and biological warfare.
- Rio de Janerio (Brazil) is famous for United Nations Conference on Environment and Development (UNCED), also known as Earth Summit or Rio Summit in 1992.

78. Krishna Raja Sagara Dam/Reservoir is developed on
(a) river Krishna
(b) river Tungabhadra
(c) river Godavari
(d) river Kavery

(d) The Krishna Raja Sagar Dam was built across river Kavery for the Mysore and Mandya districts in Karnataka in 1932.The dam is named for the then ruler of the Mysore Kingdom, Krishnaraja Wodeyar IV.
- The dam is the creation one of the greatest engineers that India had produced, Sir M. Vishweshwaraiah. His birthday, 15th September is celebrated as Engineers Day.
- The reservoir is also the main source of drinking water for all of Mysore city and almost the whole of Bangalore.
- The water released from this dam is further used as an important source of water in the State of Tamil Nadu.
- Tungabhadra dam is located at Tungabhadra river, a tributary of Krishna river.
- Nagarjuna dam, Srisailam are some of the major dams of Krishna river.
- Gangapur dam, Jayakwadi dam , Ghatghor dam are some of the famous dams across Godavari river.

79. Which one of the following Indian states does not share international border with two or more countries?
(a) Arunachal Pradesh
(b) Assam
(c) Mizoram
(d) Tripura

(d) Tripura does not share international border with two or more countries.

Tripura is bordered by the country of Bangladesh to the West, North and South; and the Indian States of Assam to the North-East; and Mizoram to the East.

Arunachal Pradesh shares international borders with Bhutan in the West, Myanmar in the East, and a disputed border with China in the North at the McMohan Line.

Assam is bordered by Bhutan (in North) and Bangladesh (in South).

Mizoram shares international border with Bangladesh and Myanmar.

80. According to Census of India, 2011, which one among the following is the least populated State in India?
(a) Maharashtra (b) Madhya Pradesh
(c) Odisha (d) Punjab

(d) Among the given options, Punjab (27,704,236) is the least populated state, according to 2011 census. It is the 16th most populated state of India.
- Maharashtra (112,372,972) is the second most populous state of India.
- Madhya Pradesh (72,597,565) is the fifth most populous state of India.
- Odisha (41,947,358) is the eleventh largest state of India in terms of population.
- Uttar Pradesh (199,581,477) is the most populated state of India whereas Sikkim (607,688) is the least populated state of India. According to census, 2011, the total population of India is 1,210,193,422.

81. The speakers of major Indian languages belong to how many language families?
(a) Two
(b) Three
(c) Four
(d) Six

(c) The Speakers of major Indian language belong to four language families.

Most Indians speak a language belonging to the families of the Indo-Aryan branch of Indo-European (c. 77%), the Dravidian (c. 20.61%), the Austroasiatic (Munda) (c. 1.2%) or the Sino-Tibetan (c. 0.8%).

Article-343 of the Indian Constitution stated that the official language of the Union is Hindi in Devanagari script instead of the extant English. Later, a constitutional amendment, the Official Languages Act, 1963, allowed for the continuation of English alongside Hindi in the Indian Government indefinitely until legislation decides to change it.

The Eighth Schedule of the Indian Constitution lists 22 languages.

82. Where did Gandhiji initially forge the techniques of *Satyagraha*?
(a) England (b) South Africa
(c) North Africa (d) India

(b) Gandhiji first conceived Satyagraha in 1906 in response to a law discriminating against Asians that was

passed by the British colonial Government of the Transvaal in South Africa.

In 1917, the first Satyagraha campaign in India was mounted in the indigo-growing district of Champaran.

Satyagraha (holding onto truth or truth force), a term coined and developed by Gandhiji, is a particular form of non-violent resistance or civil resistance.

83. Who was the Viceroy of India at the time of Gandhiji's Dandi March?
(a) Lord Irwin (b) Lord Linlithgow
(c) Lord Reading (d) Lord Willingdon

(a) Lord Irwin was the Viceroy of India at the time of Gandhiji's Dandi March. He served as Viceroy from 1926 to 1931.

In his tenure the First Round Table Conference was organised by Gandhiji started Salt March on 12th March, 1930 to break the oppressive salt taxation levied on Indian. It culminated into begining of Civil- Disobedience movement.

Lord Linlithgow was viceroy from 1936 to 1944. Cripps Mission and launch of Quit India Movement were significant event of his tenure.

Lord Reading was viceory from 1921 to 1926. In his tenurship, Swaraj Party was formed in India.

Lord Willingdon was viceroy from 1931 to 1936. Signing of Poona Pact and communal award was significant event of his tenure.

84. Which one among the following American news magazines was highly sceptical of Gandhiji's Dandi March initially but within a week completely changed its opinion and saluted him as a Saint and Statesman?
(a) Saturday Evening Post
(b) Readers Digest
(c) Time
(d) Life

(c) Time was the American news magazine which was highly sceptical of Gandhiji's Dandi March initially, but within a week completely changed its opinion and saluted him as a Saint and Statesman. It named him its 1930 'Man of the Year'. The wide coverage of Salt March in International media helped in unravelling the true nature of British colonialism in India. It also helped in making Mahatma Gandhi a popular world leader.

Saturday Evening Post (founded on 4th August, 1821) is an American Magazine, currently published 6 times a year.

Readers Digest (founded in 1922) is an American magazine published 10 times a year.

Life an American magazine was founded in 1883.

85. In which of the following years were passenger trains introduced in England?
(a) 1823 (b) 1825
(c) 1848 (d) 1861

(b) In 1825, passenger trains were introduced in England. It used steam locomotive built by George Stephenson. The Liverpool and Manchester Railway, opened in 1830, was the first modern railroad. It was a public carrier of both passengers and freight.

86. The correct relation between the radius of curvature R and focal length f of a spherical mirror is
(a) $R = f$ (b) $R = 2f$
(c) $R = 3f$ (d) $R = 4f$

(b) For a spherical mirror, radius of curvature = 2 × focal length
i.e., $R = 2f$

87. A lemon kept in water in a glass tumbler appears to be larger than its actual size. It is because of
(a) reflection of light
(b) scattering of light
(c) refraction of light
(d) polarization of light

(c) A lemon kept in water in a glass tumbler appears to be larger than its actual size due to refraction of light.

Light gets refracted as it passes from water into air. In this case, the tumbler with water acts as a convex lens which is curved outwards in the middle and can focus light rays to magnify objects.

88. Light enters the eye through a thin membrane called
(a) Retina (b) Cornea
(c) Pupil (d) Iris

(b) Light enters the eye through a thin membrane called 'cornea'. Cornea is the front part of the eye. It forms a thin membrane and lets light enter into the eye.

89. Name the scientist who first used a glass prism to obtain the spectrum of sunlight.
(a) CV Raman (b) Lord Rayleigh
(c) Isaac Newton (d) S. Chandrasekhar

(c) Isaac Newton was the first Scientist, who used a glass prism to obtain spectrum of sunlight.

90. The cost of energy to operate an industrial refrigerator that consumes 5 kW power working 10 h per day for 30 days will be (Given that the charge per kWh of energy = ₹ 4)
(a) ₹ 600 (b) ₹ 6000
(c) ₹ 1200 (d) ₹ 1500

(b) Given, power $P = 5$ kW
Consumption time per day, $t = 10$ h
Energy consumption per day
$= P \times t = 5 \text{ kW} \times 10 \text{ h} = 50 \text{ kWh}$
Cost per kWh = ₹ 4
Daily cost = 50×4 = ₹ 200
Cost for 30 days = 30×200 = ₹ 6000

91. Which one of the following statements regarding magnetic field is not correct?
(a) Magnetic field is a quantity that has direction and magnitude
(b) Magnetic field lines are closed curves
(c) Magnetic field lines are open curves
(d) No two magnetic field lines are found to cross each other

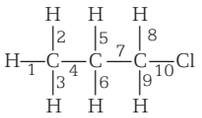
(c) Magnetic field lines are always closed curves because magnetic monopoles do not exist.

92. Which one of the following statements is not correct?
(a) Buckminsterfullerence is an allotrope of carbon
(b) Diamond is a good conductor of electricity
(c) Graphite is a good conductor of electricity
(d) In graphite, each carbon atom is linked to three other carbon atoms

(b) Statement (b) is not correct whereas statement (a), (c) and (d) are correct. Diamond is a poor conductor of electricity because here each carbon atom is bonded of four other carbon atoms forming rigid three dimensional structure.

93. How many covalent bonds are present in a Chloropropane molecule having molecular formula, C_3H_2CI?
(a) 6 (b) 8
(c) 9 (d) 10

(d) Ten covalent bonds are present in Chloropropane having molecular formula C_3H_7Cl



94. Which one of the following is the most fundamental characteristic of an element?
(a) Melting point (b) Atomic number
(c) Colour (d) Atomic weight

(b) Atomic number is the most fundamental characteristic of an element. It is defined as the number of protons found in the nucleus of every atom of that element.

95. Neutrons were discovered by
(a) James Chadwick
(b) Ernest Rutherford
(c) JJ Thomson
(d) John Dalton

(a) Neutrons were discovered by James Chadwick. Neutrons are uncharged particles found within all atomic nucleus.

96. Atomic mass of an element is equal to the sum of number of
(a) electrons and protons only
(b) protons and neutrons only
(c) electrons and neutrons only
(d) electrons, protons and neutrons

(b) Atomic mass of an element is equal to the sum of number of protons and neutrons only. e.g. carbon atom,

proton = 6 nuetron = 6

Atomic mass = 6 + 6 = 12

97. Which one of the following element's isotope is used in the treatment of cancer?
(a) Uranium (b) Cobalt
(c) Sodium (d) Iodine

(b) Isotope of Cobalt (Co) is used in the treatment of cancer. Cobalt therepy is the medical use of gamma rays from the radioisotope cobalt-60 to treatment of cancer.

98. Which one of the following cell organelles may play a role in expelling excess water and wastes in case of unicellular organisms?
(a) Lysosome
(b) Vacuole
(c) Golgi body
(d) Endoplasmic reticulum

(b) In some unicellular eukaryotic organisms (e.g *Amoeba*), cellular nitrogenous waste such as ammonia and excess water are excreted by exosmosis *via* contractile vacuole. These are special types of vacuoles, which regulates osmotic pressure of cell through osmosis and control water movement across the cell membrane. When these vacuoles merge with the cell membrane they expel wastes into the external aquatic environment.

99. Which one of the following terms describes the practice of growing two or more crops simultaneously on the same piece of land?
(a) Crop rotation (b) Mixed cropping
(c) Intercropping (d) Mixed farming

(b) Mixed cropping means the practice of growing two or more crops simultaneously on the same piece of land. In this system, pulses and some oilseeds are grown with maize, sorghum and pearl millet. It prevents complete loss of crop due to inadequate rain, pests invasion and pathogen infection on a particular crop.

Crop rotation involves growing different crops on the same land in the preplanned succession.

Intercropping is the practice of growing more than one crop on the same field at the same time in different row pattern.

Mixed farming involves the growing of crops as well as the raising of the livestock. On the same piece of land.

100. Which one of the following statements is correct about effects of antibiotics on viruses?
(a) Viruses are 'non-living' entities but it can interact with antibiotics
(b) Taking antibiotics cures viral infections
(c) Viruses do not possess metabolic pathways on which antibiotics can function, whereas bacteria have such pathways
(d) Viruses are resistant to antibiotics

(c) Virus is an acellular structure which is a non-living particle outside of the body of its host. Thus, its lacks protoplasm which is the site of most of the metabolic activities, antibiotics which are effective against bacteria (living prokaryotic cell), are ineffective on viruses.

101. The Panama Canal opened in 1914, links
(a) Red sea and Mediterranean sea
(b) Atlantic ocean and Pacific ocean
(c) Indian ocean and Pacific ocean
(d) Adriatic sea and Black sea

(b) The Panama Canal opened in 1914, links Atlantic ocean and Pacific ocean. This canal (82 km), cuts across Isthmus of Panama and is a conduit for maritime trade. Construction of this canal started under French in 1881 and completed under the United States in 1914.

The administrative control of this canal is under the Republic of Panama.

Suez canal joins Red sea and Mediterranean sea.

Strait of Malacca joins Pacific ocean and Indian ocean.

102. The United Nations Charter was signed by 51 original members of the United Nations in 1945 at the
(a) Hague Conference
(b) London Conference
(c) San Francisco Conference
(d) Berlin Conference

(c) The United Nations Charter was signed by SI original members of the United Nations in 1945 at the San Franciso conference. 850 delegates from 51 nations deliberated, discussed and finalised the charter which initially divided into 4 sections. It was passed unanimously on 26th June, 1945.

London conference (officially named as Declaration of St. James' palace) was signed on 12th June, 1941. This conference led to the idea of formation of United Nation.

The Hague Conference, 1954 is related to protection of cultural property in the Event of Armed Conflict.

Berlin conference of 1884-1885 regulated European Colonisation and trade in Africa.

103. Land Revenue Records maintained in India have categorised land-use into
(a) 6 categories (b) 9 categories
(c) 15 categories (d) 21 categories

(b) Land Revenue Records maintained in India have categorised land-use into 9 categories. The following are the land-use categories in India
(i) Forests
(ii) Land put to non-agricultural use
(iii) Barren and wasteland
(iv) Area under permanent pasture and grazing land
(v) Area under miscellaneous tree crops and groves
(vi) Culturable wasteland
(vii) Current fallow
(viii) Fallow other than current fallow
(ix) Net sown area

104. Dry land farming in India is largely confined to areas with rainfall less than
(a) 100 cm (b) 85 cm (c) 80 cm (d) 75 cm

(d) Dry land farming is largely confined to the areas with rainfall less than 75 cm. These regions grow hardy and drought resistant crops such as ragi, bajra, moong, gram and guar. Farmers in this areas practise various measures of soil moisture conservation and rain water harvesting.

105. For an area to be excluded from the drought-prone category, what percentage of its gross cropped area should be under irrigation?

(a) 10% or more (b) 20% or more
(c) 25% or more (d) 30% or more

(d) For an area to be excluded from the drought-prone category, 30% or more of its gross cropped area should be under irrigation. The irrigation commission considers both rainfall and irrigation as factors in identifying a drought. It recommended that areas with more than 30% coverage of irrigation should be excluded from being listed as drought prone areas. According to the vulnerability profile of NDMA, around 68% of India's cultivable area is vulnerable to droughts.

106. Which one of the following is not a current of Pacific ocean?

(a) Oyashio current (b) Alaska current
(c) Agulhas current (d) California current

(c) Agulhas current is not a current of Pacific ocean. It flows South along the East coast of Africa in India ocean. The source of this current is East Madagascar current and Mozambique current. It acts as an oceanic convergence zone which enhances its primary productivity as compared to surrounding waters.

Oyashio current is a cold current which circulates counter clockwise in the western North Pacific ocean.

Alaska current is current of Pacific ocean, which circulate alongside the West coast of the North American continent.

California current is cold water current of Pacific ocean which moves Southward along the Western coast of North America.

107. The duration of monsoon in India extends for an average period of

(a) 80–140 days (b) 100–120 days
(c) 90–130 days (d) 100–140 days

(a) The duration of monsoon in India extends for an average period of 80-140 days. The rainfall received from the South-West monsoon is seasonal in character, which occurs between June and September.

Generally, the monsoon arrives at the southern tip of Indian Peninsula by the first week of June. Subsequently, it divided into two branches, *viz* the Arabian sea branch and the Bay of Bengal branch. The retreat of monsoon which is characterised by clear skies and rise in temperature starts in early September.

108. Which one of the following is the natural vegetation of South-East China?

(a) Subtropical broadleaf evergreen forest
(b) Tropical broadleaf evergreen forest
(c) Tropical deciduous forest
(d) Temperate evergreen forest

(a) The natural vegetation of South-East China is subtropical broadleaf evergreen forests. The South-East China experiences temperate monsoon type of climate which is characterised by warm moist summer and a cool and dry winter. It receives moderate rainfall between 25 inches to 60 inches, which is evenly distributed throughout the year. The well distributed rainfall all the year round make the regions look green at all times. The flora of this region includes oak, camphor, camelia and magnolia.

Tropical Deciduous forest, also known as Monsoon forests are found in the belt along the equator between Tropic of Cancer and Tropic of Capricorn.

Temperate Evergreen forests are found in temperate areas with warmer summers and cool winter. There are mostly located in the coastal region with mild winter and drier climates and are dominated by large trees.

109. Which one of the following is a cold local wind?

(a) Santa Ana (b) Chinook
(c) Mistral (d) Loo

(c) Mistral is a cold local wind. It blows around Alps mountain in France. It brings good health, sunshine and dryness to the adjoining regions.

Santa Ana is a hot local wind which blows in South California.

Chinook is a hot and dry local wind which blows in the Rocky Mountains of USA and Canada.

Loo is a hot and dry local wind which blows from West in summer season of Northern India.

110. What was the consequence of Permanent Settlement on rural society in Bengal?

(a) The zamindars invested capital and enterprise to improve agriculture along lines of British yeoman farmers
(b) A group of rich peasants known as Jotedars succeeded in consolidating their position in the villages
(c) The ryots prospered as a result of fixed revenue levy imposed on them
(d) The system of Collectorate introduced by the Company for exercising supervisory control on zamindars failed to take off

(b) The main consequences of Permanent settlement on rural society in Bengal was the rise of group of rich peasants known as Jotedars. They succeeded in consolidating their positions in the villages. They lived in villages, advanced loans to peasants and resisted efforts by Zamindars to increase the Jama of the village. They also purchased the estates of Zamindars when they failed to pay timely revenue.

Permanent settlement was introduced by Lord Cornwallis in 1793 in Eastern parts of the country. It fixed the land revenue and recognised Zamindar as owners of land.

111. What was *Damin-i Koh* in Rajmahal area?

(a) A large area of land demarcated and declared to be the land of the Santhals
(b) The land of the Paharias cultivated exclusively for paddy
(c) The British territory marked for their military camp
(d) The land earmarked for locating settled agriculturists

(a) *Damin-i Koh* in Rajmahal area was large area of land demarcated and declared to be the land of the Santhals. It was declared in 1832 to persuade Santhals to do settled agriculture in previously forested area. This region was enclosed by boundary pillars and was separated from the world of settled agriculturists of plains and Pahariyas of the hills.

112. Which one of the following statements about the Revolt of 1857 is correct?

(a) It was a Revolt carefully organised and planned by the Rajas, Nawabs and Taluqdars
(b) Rumours and prophecies did not play any role in its outbreak and spread
(c) The rebel proclamations in 1857 repeatedly appealed to all sections of the population irrespective of their caste and creed.
(d) The British succeeded in quickly and easily controlling the rebels

(c) The Rebel proclamations in 1857 repeatedly appealed to all sections of the population irrespective of their caste and creed. During the course of revolt, many proclamations and notifications were issued by the rebels. Many of this proclamations were issued by Muslim princes or in their names but they took care of Sentiments of every communities of the area. Famous proclamations were Delhi proclamation and Azamgarh proclamations.

This revolt was poorly organised and planned by the Rajas, Nawabs and Taluqdars which was greatest weakness of Revolt of 1857.

Rumours and prophecies played huge role in inciting the sentiments of masses against the Britishers.

Britishers found very difficult to supress the revolt due to huge participation of masses alongside rebels in Awadh area.

113. Which one among the following was not a *Panch sheel* principle?

(a) Peaceful Co-existence
(b) Mutual respect for territorial integrity
(c) Nuclear deterrence
(d) Non-interference in internal affairs

(c) Nuclear deterrence was not a *Panch Sheel* principle. It is a theory which provides that a nuclear power nation however weak, it can successfully protect itself against much powerful nation state.

Panch Sheel principles were agreed between India and China under the leadership of Jawaharlal Nehru and Zhou Enlai respectively in 1954 to establish close relations between the two countries. *They are*

- Mutual respect for each other's territorial integrity and sovereignty.
- Mutual non-aggression
- Mutual non-interference in each other's internal affairs.
- Equality and mutual benefit
- Peaceful co-existence

114. Who among the following had organised, in 1904, a secret society of revolutionaries named *Abhinav Bharat*?

(a) Khudiram Bose
(b) Shyamji Krishna Verma
(c) Har Dayal
(d) VD Savarkar

(d) VD Savarkar organised in 1904, a secret society of revolutionaries named Abhinav Bharat. Based in Pune, it prepared youths for revolutionary activities and promoted the cause of national independence.

Khudiram Bose was revolutionary freedom fighter who attempted to assassinate a British Judge, Douglas Kingsford in Muzaffarpur.

Shyamji Krishna Verma was revolutionary freedom fighter who organised India House in London.

He also published a monthly magazine Indian sociologist.

Har Dayal was revolutionary freedom fighter who founded Gadar Party in 1913.

115. The principle that the framing of the new Constitution for independent India should be primarily (though not solely) the responsibility of Indians themselves, was for the first time conceded in the

(a) Government of India Act, 1935
(b) August Offer of Viceroy Linlithgow
(c) Cripps Proposals
(d) Cabinet Mission

(b) The principle that the framing of new Constitution for Independent India should be primarily (though not solely) the responsibility of Indians themselves was for the first time conceded in the August offer of Viceroy Linlithgow.

Made on 8th August, 1940, it also offered the expansion of Viceroy's executive council besides establishment of an advisory war council. However, Congress Party rejected the offer as it provided dominion status.

Government of India Act, 1935 provided for establishment of All India federation, division of subjects into three lists and provincial autonomy.

Cripps proposals were made in March, 1942 it promised dominion status, Constituent Assembly besides control of defence under the British.

Cabinet Mission, 1946 proposed a very loose confederation with centre controlling merely defence, foreign affairs and communication.

116. Which one of the following is a major environmental issue in Eastern Canada ?

(a) Acid precipitation
(b) Groundwater depletion
(c) Land degradation
(d) Desertification

(a) Acid precipitation or rain is a major environmental issue in eastern Canada. Acid rain occurs when gaseous emissions of combustion of coal and other fossil fuels like Sulphur dioxide and nitrogen oxide transform into their Secondary pollutants by combining with atmospheric moisture (H_2O). These secondary pollutants such as sulphuric acid and nitric acid fall on land with precipitation.

The Eastern region of Canada and it's neighbour USA is highly industrialised zone. This alongwith other causes such as Vehicular emissions are the main reason for Acid rain in Eastern Canada.

Groundwater depletion is the depletion of ground water level caused by sustained ground water pumping.

- Land degradation is the decline in the productive capacity of land for some time or permanently.
- Desertification is the persistent degradation of dryland ecosystems by climate variation and human activities.

117. Which one of the following mountains lies in between Caspian sea and Black sea ?

(a) Caucasus (b) Carpathians
(c) Apennine (d) Elburz

(a) Caucasus mountain lies in between Caspian sea and Black sea. This mountain ranges adjoins Vast Eurasian landmass stretching from Russia to Turkey. Mount Elburz the highest peak in Europe lies in this mountain range.

Carpathians is a mountain system extending South-eastwards from Southern Poland and Slovakia into Romania.

Apennine is a mountain range extending in North-South direction in Italian Peninsula.

Mount Elburz is the highest peak of Europe. It is a type of dormant volcano which lies in territory of Russia.

118. Bagalihar, Dulhasti and Salal hydro power projects have been developed on which of the following rivers?

(a) Chenab and Jhelum
(b) Chenab and Indus
(c) Ravi
(d) Chenab only

(d) Bagalihar, Dulhasti and Salal hydro power projects have been developed on Chenab.

Bagalihar hydro power project with installed capacity of 900 MW is situated in the Ramban district of UT of Jammu and Kashmir.

Dulhasti is a 390 MW hydroelectric power plant in Kishtwar district of UT of

Jammu and Kashmir. It is a run off river type hydro project build on Chandra river a tributary of Chenab.

Salal hydroelectric power station is a run off the river project on Chenab rivers in the Reasi district of UT of Jammu and Kashmir.

119. Which of the following statements with regard to Biosphere Reserve is/are correct?

1. The idea of Biosphere Reserve was initiated by UNESCO in 1973-74.
2. There are 18 designated Biosphere Reserves in India.
3. All Biosphere Reserves in India have been included in world network of Biosphere Reserves of UNESCO.

Select the correct answer using the codes given below.

(a) Only 1 (b) Only 2
(c) 1 and 2 (d) 1, 2 and 3

(b) The concept of Biosphere Reserve was initiated by UNESCO in 1971, when it launched Man and Biosphere Programme (MAB). It aims to establish a Scientific basis for the improvement of relationship between people and their environments.

In India, there is 18 Biosphere reserves recognised by the government to protect man and environment. Nilgiri Biosphere Reserve in Tamil Nadu and Kerala, was the first to be included in this list.

In India, 11 out of 18 Biosphere Reserves have been included in world network of Biosphere Reserves of UNESCO.

120. Which one of the following countries does not have direct access to the sea/ocean ?

(a) Syria (b) Jordan
(c) Azerbaijan (d) Armenia

(d) Armenia does not have direct access to the sea/ocean. It is landlocked country in the western Asia. It is bordered by Turkey to the West, Georgia to the North, Artsakh and Azerbaijan to the East and Iran to the South. Yerevan is the capital of Armenia.

Syria is a country in Western Asia with its capital at Damascus. It is bordered by the Mediterranean sea to the West.

Jordan is a country in Western Asia with its capital at Amman. It has 26 km coastline on the Red sea in its extreme South-West.

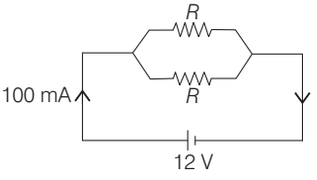
Azerbaijan is a country in the Caucasus region of Eurasia. It is bounded by the caspian sea to the East.

121. In an incandescent electric bulb, the filament of the bulb is made up of which metal?

(a) Aluminium (b) Copper
(c) Tungsten (d) Silver

(c) In an incandescent electric bulb, the filament of the bulb is made up of tungsten because it has very high melting point (3400°C).

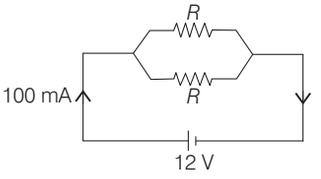
122. Two equal resistors R are connected in parallel, and a battery of 12 V is connected across this combination. A dc current of 100 mA flows through the circuit as shown below:



The value of R is

(a) 120 Ω (b) 240 Ω
(c) 60 Ω (d) 100 Ω

(b)



In the above circuit, both resistance of RR are connected in parallel combination.

Hence, Equivalent resistance of circuit is,

$$R_{eq} = \frac{R.R}{R + R}$$

$$R_{eq} = \frac{R}{2}$$

Here, $I = 100\ \text{mA} = 100 \times 10^{-3}\ \text{A} = 0.1\text{A}$

$$V = 12\ \text{V}$$

Using ohm's law

$$V = I\,R_{eq}$$

$$\Rightarrow \quad R_{eq} = \frac{V}{I}$$

$$\Rightarrow \quad \frac{R}{2} = \frac{12}{0.1}$$

$$\Rightarrow \quad R = 240\ \Omega$$

123. Which one of the following is not the unit of energy?

(a) Joule (b) Watt-hr
(c) Newton-metre (d) kg-metre/sec^2

(d) Amongst the given options, kg-metre/sec^2 is not the unit of energy. kg-(metre)^2/sec^2 is the unit of energy. All other options are the units of energy.

124. Which one of the following statements is not correct?

(a) Biomass is a renewable source of energy
(b) Gobar gas is produced - when cowdung, crop residues, vegetable waste and sewage are allowed to decompose in the absence of oxygen
(c) Biogas generation reduces soil and water pollution
(d) Heating capacity of biogas is very low

(d) The heating capacity or calorific value of biogas is high due to which it is used for lighting.

125. In prokaryotic organisms, nuclear region is not surrounded by a membrane. This undefined nuclear region is known as

(a) Nucleic acid (b) Nucleoid
(c) Nucleolus (d) Nucleosome

(b) Prokaryotic organisms such as bacteria lack true nucleus, as their nuclear region is not bound by outer nuclear membrane. It is known as Nucleoid. It has a circular DNA without histone proteins, which is the main genetic material of bacteria.

126. Which one of the following plant tissues has dead cells ?

(a) Epidermis (b) Parenchyma
(c) Collenchyma (d) Sclerenchyma

(d) Sclerenchyma is a dead permanent tissue. Its cells These have heavily thickened secondary walls containing lignin. The cells are rigid, non-stretchable and lack protoplasma. They provide protection and mechanical support to plant. They are usually found in non-growing regions of plant bodies such as the bark protective layering of seeds and fruits or mature stems. Rest epithelium, paranchyma and collenchyma have living tissues.

127. Cartilage is not found in

(a) larynx (b) nose
(c) ear (d) urinary bladder

(d) Cartilage is not found urinary bladder. Urinary bladder consists of epithelium on the lumen surrounded by a collagen rich connective tissue and muscle layer. Hyaline Cartilage is the type of cartilage found in areas such as nose, ears and larynx of the human body.

128. Two planets orbit the Sun in circular orbits, with their radius of orbit as $R_1 = R$ orbit as $R_2 = 4$ R. Ratio of their periods (T_1 / T_2) around the Sun will be
(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/2

(b) According to Kepler's 3rd law,
$T^2 \propto R^3 \Rightarrow T \propto R^{3/2}$

$\therefore$ As per question,

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{R}{4R}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{1}{64}\right)^{1/2} = \frac{1}{8}$$

129. A metallic wire having resistance of 20 Ω is cut into two equal parts in length. These parts are then connected in parallel. The resistance of this parallel combination is equal to
(a) 20 Ω (b) 10 Ω (c) 5 Ω (d) 15 Ω

(c) As, Resistance ∝ length
$\therefore$ When wire of resistance 20 Ω is cut into two equal parts in length, the resistance of each part will be

$$\frac{R}{2} = \frac{20}{2} = 10\,\Omega$$

Equivalent resistance of parallel combination.

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} \Rightarrow R_{eq} = 5\Omega$$

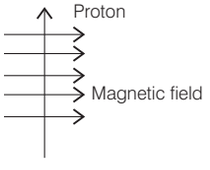
130. Light of uniform intensity impinges perpendicularly on a totally reflecting surface. If the area of the surface is halved, the radiation force on it will become
(a) double (b) half
(c) four times (d) one fourth

(b) As, Radiation force ∝ Area of surface so, if area of the surface is halved, radiation force will also get halved.

131. The part of the human eye on which the image is formed is
(a) pupil (b) cornea
(c) retina (d) iris

(c) Retina is the light sensitive surface of eye on which the image is formed.

132. Consider the following image:



A proton enters a magnetic field at right angles to it, as shown above. The direction of force acting on the proton will be
(a) to the right (b) to the left
(c) out of the page (d) into the page

(c) According to Fleming's left hand rule, the direction of force on conductor is perpendicular to the direction of magnetic field and current. The direction of current is taken same as the direction of motion of positive charge (proton), therefore, the force on proton is directed out of the page.

133. Which one of the following statements about sound is not correct?
(a) Sound travels at a speed slower than the speed of light
(b) Sound waves are transverse waves
(c) Sound waves are longitudinal waves
(d) Sound travels faster in moist air than in dry air

(b) Sound waves are longitudinal waves, but they are not transverse waves. In sound waves, the individual particles of the medium move in a direction parallel to the direction of propagation of the disturbance.

134. When the short circuit condition occurs, the current in the circuit
(a) becomes zero
(b) remains constant
(c) increases substantially
(d) keeps on changing randomly

(c) In short circuit condition the resistance of the circuit is zero.
So, as per ohm's law

$$I_{short} = \frac{V}{R} = \frac{V}{0} = \infty$$

So, current increases substantially in short circuit condition.

135. Which one of the following is not a component of human male reproductive system ?
(a) Cervix
(b) Urethra
(c) Seminal vesicle (d) Vas deferens

(a) Cervix is a part of female reproductive system. Rest Urethra, seminal vesicle and Vas deferens are parts of male reproductive system. It is the lower part of uterus having marrow tube-like structure forming vegina.

136. Which one of the following is not a reason of decrease in biodiversity?
(a) Large scale deforestation
(b) Exploitation of forest produce
(c) Maintaining sacred groves
(d) Encroachment in forest areas

(c) Maintaining sacred groves is not a reason of decreasing biodiversity. It is an *in-situ* conservation method which provide protection to the organisms in some area in its natural habitat. Rest three are the causes of decrease of biodiversity.

137. Which one of the following is not a cause of depletion in groundwater?
(a) Afforestation
(b) Loss of forests
(c) Excessive pumping of groundwater
(d) Construction of large scale concrete buildings

(a) Groundwater will not be depleted in case afforestation. Afforestation refers to planting of various types of trees over a particular area.
This allows rain and surface water to percolate with in the soil and prevents surface run off of water. It also helps in recharging or refilling the groundwater level. Hence, the water table rises up and clear uncontaminated water is available.
But in all other options, groundwater will be depleted without getting recharge.

138. Which one of the following types of radiations has the smallest wavelength ?
(a) Microwaves
(b) Infra-red
(c) Visible light
(d) X-rays

(d) X-rays have the smallest wavelength amongst the other three options.
The range of wavelengths for these waves are :
Microwaves : 10^{-3} to 3×10^{-1} m
Infra-red : 8×10^{-7} to 5×10^{-2} m
Visible light : 4×10^{-7} to 7×10^{-7}m
X-rays : 10^{-13} to 3×10^{-8} m

139. The instrument used for detecting the presence of electric current in a circuit is
(a) Refractometer (b) Galvanometer
(c) Viscometer (d) Diffractometer

(b) Galvanometer is an instrument used for detecting the electric current in a circuit. Galvanometer is used to measured small amount of current in the circuit.

140. Which one of the following is the largest composition in biogas?
(a) Carbon dioxide
(b) Methane
(c) Hydrogen
(d) Hydrogen sulphide

(b) Biogas is generally composed of methane (55-65%), carbondioxide (35-45%), nitrogen (0-3%), hydrogen (0-1%) and hydrogen sulfide (0-1%). Thus, methane has the largest composition in biogas.

141. The Sun appears reddish during sunrise and sunset. The phenomenon in optics which is responsible for this appearance of the Sun is
(a) Reflection
(b) Total internal reflection
(c) Scattering
(d) Interference

(c) The Sun appears reddish during sunrise and sunset due to scattering of light.

142. A lens has a power of +2·0 Dioptre. Which one of the following statements about the lens is true?
(a) The lens is concave and has a focal length of 0.5 m
(b) The lens is convex and has a focal length of 2.0 m
(c) The lens is convex and has a focal length of 0.5 m
(d) The lens is concave and has a focal length of 2.0 m

(c) Power, P = + 2 D

Focal length, $f = \frac{1}{p} = \frac{1}{2} = 0.5$ m

The converging (or convex) lens has positive focal length.

So, the lens is convex with focal length of 0.5 m.

143. At nearly 70°C, sodium bicarbonate shows the property of gradually decomposing, which makes it usable in bakery products. The product of decomposition responsible for this use of sodium bicarbonate is
(a) Carbon dioxide
(b) Hydrogen
(c) Water vapour
(d) Oxygen

(a) Sodium bicarbonate ($NaHCO_3$) at 70°C gradually decomposes to give sodium carbonate (Na_2CO_3), water (H_2O) and carbon dioxide (CO_2). Complete reaction is as follows

$$2NaHCO_3 \xrightarrow{Heat} Na_2CO_3 + H_2O + CO_2$$

144. Number of molecules of water of crystallisation in copper sulphate, sodium carbonate and gypsum are
(a) 5, 10 and 2 respectively
(b) 10, 2 and 5 respectively
(c) 5, 2 and 10 respectively
(d) 2. 5 and 10 respectively

(a) Number of molecules of water of crystallisation of copper sulphate, sodium carbonate and Gypsum are 5, 10 and 2 respectively.

Complete chemical formula are as follows

Copper sulphate — $CuSO_4 . 5H_2O$
Sodium Carbonate — $Na_2CO_3 . 10H_2O$
Gypsum — $CaSO_4 . 2H_2O$

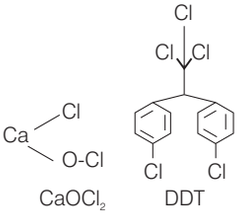
145. Which one of the following is the correct sequence of change in colours when a turmeric stain on white clothes is scrubbed by soap and then washed with water?
(a) Yellow-pink-blue
(b) Yellow-reddish brown-yellow
(c) Yellow-reddish brown-blue
(d) Yellow-blue-pink

(b) When a turmeric stain appear on white clothes it becomes reddish brown when it come in contact with soap, which is basic in nature it turns yellow again when the cloth is washed with plenty of water.

146. Which one of the following statements regarding bleaching powder and DDT is correct?
(a) Both are inorganic compounds
(b) Both are organic compounds
(c) Both contain chlorine
(d) Both contain calcium

(c) Both bleaching powder ($CaOCl_2$) and Dichloro Diphenyl Tricholoroethane (DDT) contain chlorine. Bleaching powder is inorganic compound, but DDT is an organic halogen compound.

structures are as follows



$CaOCl_2$ DDT

147. Which one of the following is the best example of desiccant?
(a) Silica gel
(b) Polystyrene
(c) Sodium chloride
(d) Sodium carbonate

(a) Silica gel is the best example of desiccant. It is hygroscopic in nature. It absorbs water from the surrounding.

148. Which one of the following was the first mineral acid discovered?
(a) Sulphuric acid (b) Hydrochloric acid
(c) Nitric acid (d) Phosphoric acid

(a) Sulphuric acid (H_2SO_4) was the first mineral acid discovered.

149. The refractive index of fused quartz is 1·46 and that of sapphire is 1·77. If V_q is the speed of light in quartz and v_s is the speed of light in sapphire, then which one of the following relations is correct?
(a) $v_q > v_s$ (b) $v_s > v_q$
(c) $v_s = v_q$ (d) $v_s = \frac{v_q}{2}$

(a) As, Refractive index

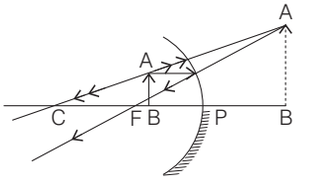
$$= \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

i.e., $\mu = \frac{c}{v} \Rightarrow \mu \propto \frac{1}{v}$

As, $\mu_{quartz} < \mu_{sapphire} \Rightarrow v_q > v_s$

150. In case of a concave mirror, if an object is kept between principal focus F and pole P of the mirror, then which one of the following statements about the image is not correct?
(a) The image will be virtual
(b) The image will be enlarged or magnified
(c) The image will be formed at infinity
(d) The image will be erect

(c) If the object is placed between F and P of concave mirror, the image is formed behind the mirror. The image formed is virtual, erect and magnified.



MATHEMATICS

1

SETS, RELATIONS AND FUNCTIONS

2019 (II)

1. Let $S = \{2, 4, 6, 8, \ldots\ldots, 20\}$.

What are the maximum number of subsets of S?

(a) 10 (b) 20
(c) 512 (d) 1024

(**d**) $S = \{2, 4, 6, 8, \ldots\ldots, 20\}$

Here, number of elements of set
$S = 10\ (n)$

∴ Maximum number of subsets of set
$S = 2^n = 2^{10}$
$= 1024$

2. Let $A \cup B = \{x \mid (x-a)(x-b) > 0$, where $a < b\}$. What are A and B equal to?

(a) $A = \{x \mid x > a\}$ and $B = \{x \mid x > b\}$
(b) $A = \{x \mid x < a\}$ and $B = \{x \mid x > b\}$
(c) $A = \{x \mid x < a\}$ and $B = \{x \mid x < b\}$
(d) $A = \{x \mid x > a\}$ and $B = \{x \mid x < b\}$

(**c**) Let $A \cup B = \{x : (x-a)(x-b) > 0$, where $a < b\}$.

It is possible if $x - a < 0$ and $x - b < 0$
or $x < a$ and $x < b$
∴ $A = \{x : x < a\}$ and $B = \{x : x < b\}$

3. If $A = \{x : 0 \le x \le 2\}$ and $B = \{y : y$ is a prime number$\}$, then what is $A \cap B$ equal to?

(a) ϕ (b) $\{1\}$
(c) $\{2\}$ (d) $\{1, 2\}$

(**c**) $A = \{x : 0 \le x \le 2\} = \{0, 1, 2\}$

and $B = \{y : y$ is a prime number$\}$
$= \{2, 3, 5, 7, 11, \ldots\ldots\}$
∴ $A \cap B = \{0, 1, 2\} \cap \{2, 3, 5, 7, 11, \ldots\ldots\}$
$= \{2\}$

4. In a school, 50% students play cricket and 40% play football. If 10% of students play both thze games, then what per cent of students play neither cricket nor football?

(a) 10% (b) 15%
(c) 20% (d) 25%

(**c**) Students, who play cricket = 50%
Students, who play football = 40%
Students who play both games = 10%
Students who play only cricket
$= 50 - 10 = 40\%$
Students who play only football
$= 40 - 10 = 30\%$
∴ Total students who play any game
$= 40 + 30 + 10 = 80\%$
∴ Students who play neither cricket nor football = 100 − 80 = 20%

5. If a set A contains 3 elements and another set B contains 6 elements, then what is the minimum number of elements that $(A \cup B)$ can have?

(a) 3 (b) 6
(c) 8 (d) 9

(**b**) $n(A) = 3$, $n(B) = 6$

∴ The minimum number of elements in
$A \cup B = 6$
i.e $n(A \cup B) = 6$
(because max $n(A \cap B) = 3$

6. If A, B and C are subsets of a given set, then which one of the following relations is not correct?

(a) $A \cup (A \cap B) = A \cup B$
(b) $A \cap (A \cup B) = A$
(c) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
(d) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

(**a**) Let U be the set and A, B and C are the subset of U.
We know that, $A \cup (A \cap B) = A$,
So option (a) is not correct.
$A \cap (A \cup B) = A$, so option (b) is correct.
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$,
so option (c) is correct.
and $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$,
so option (d) is correct.

2019 (I)

7. Suppose $X = \{1, 2, 3, 4\}$ and R is a relation on X. If $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$, then which one of the following is correct?

(a) R is reflexive and symmetric, but not transitive
(b) R is symmetric and transitive, but not reflexive
(c) R is reflexive and transitive, but not symmetric
(d) R is neither reflexive nor transitive, but symmetric

(**d**) We have, $X = \{1, 2, 3, 4\}$

$R = \{(1,1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

Since, $(4, 4) \notin R$,

Hence, R is not reflexive.

Since, $(1, 2) \in R$, $(2, 3) \in R$ but $(1, 3) \notin R$, R is not transitive.

$(1, 2), (2, 3) \in R$
and also $(2, 1), (3, 2) \in R$
∴ R is symmetric.

Hence, R is neither reflexive nor transitive but symmetric.

8. A relation R is defined on the set N of natural numbers as $xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$. Then, which one of the following is correct?

(a) R is reflexive and symmetric, but not transitive
(b) R is reflexive and transitive, but not symmetric
(c) R is reflexive, symmetric and transitive
(d) R is reflexive, but neither symmetric nor transitive

(**d**) Given, $xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$

For reflexive

$xRx \Rightarrow x^2 - 4x^2 + 3x^2 = 0$

So, $(x, x) \in R, \forall x \in N$

Hence, R is reflexive.

For symmetric

$xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$

$\therefore \quad yRx \Rightarrow y^2 - 4xy + 3x^2$

It is not clear, that $y^2 - 4xy + 3x^2$ is equal to zero or not.

i.e. $(x, y) \in R$ but $(y, x) \notin R \cdot \forall x, y \in N$

Hence, R is not symmetric.

For transitive

$xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$
$yRz \Rightarrow y^2 - 4yz + 3z^2 = 0$ (let)
$xRz \Rightarrow x^2 - 4xz + 3z^2$
It is not clear, that $x^2 - 4xz + 3z^2$ is equal to zero or not.
So, $(x, y) \in R, (y, z) \in R$
$\Rightarrow (x, z) \notin R \ \forall \ x, y, z \in N$
Hence, R is not transitive.

9. If $A = \{x \in Z : x^3 - 1 = 0\}$ and $B = \{x \in Z : x^2 + x + 1 = 0\}$, where, Z is set of complex numbers, then what is $A \cap B$ equal to?

(a) Null set
(b) $\left[\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right]$
(c) $\left[\frac{-1+\sqrt{3}i}{4}, \frac{-1-\sqrt{3}i}{4}\right]$
(d) $\left[\frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}\right]$

(**b**) We have,

$A = \{x \in Z : x^3 - 1 = 0\}$

and $B = \{x \in Z : x^2 + x + 1 = 0\}$

$A = \left\{1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$

$B = \left\{\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$

$A \cap B = \left\{\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$

10. Consider the following statements for the two non-empty sets A and B.

1. $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup B$

2. $(A \cup (\overline{A} \cap \overline{B})) = A \cup B$

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**a**) We have,

1. $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup B$

$\text{LHS} \equiv (A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B)$
$= \{A \cap (B \cup \overline{B})\} \cup (\overline{A} \cap B)$
[by distributive property]
$= (A \cap U) \cup (\overline{A} \cap B)$
$[\because B \cup \overline{B} = U]$
$= A \cup (\overline{A} \cap B)$
$= (A \cup \overline{A}) \cap (A \cup B)$
$= U \cap (A \cup B)$
$= A \cup B = \text{RHS}$

Hence, 1 is correct.

2. $A \cup (\overline{A} \cap \overline{B}) = A \cup B$

$\text{LHS} \equiv A \cup (\overline{A} \cap \overline{B})$
$= (A \cup \overline{A}) \cap (A \cup \overline{B})$
$= U \cap (A \cup \overline{B})$
$= A \cup \overline{B} \neq A \cup B$

Hence, 2 is false.

$\therefore$ Only 1 is correct.

11. Let X be a non-empty set and let A, B, C be subsets of X. Consider the following statements.

1. $A \subset C \Rightarrow (A \cap B) \subset (C \cap B)$, $(A \cup B) \subset (C \cup B)$
2. $(A \cap B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$
3. $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$

Which of the above statements are correct?

(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) 1, 2 and 3

(**d**) Let $X = \{1,2,3,4\}$

$A = \{1, 2\}, B = \{2, 3, 4\}, C = \{1, 2, 3\}$

$A \subset C$

$A \cap B = \{2\}, C \cap B = \{2, 3\}$

Clearly, $(A \cap B) \subset (C \cap B)$

$A \cup B = \{1, 2, 3, 4\}, (C \cup B) = \{1, 2, 3, 4\}$

$(A \cup B) \subset (C \cap B)$

Hence, Statement 1 is correct.

2. $(A \cap B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$

Hence, Statement 2 is also correct.

3. $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$

Hence, Statement 3 is also correct.

12. If $A = \{\lambda, \{\lambda, \mu\}\}$, then the power set of A is

(a) $\{\phi, \{\phi\}, \{\lambda\}, \{\lambda,\mu\}\}$
(b) $\{\phi,\{\lambda\},\{\lambda,\mu\},\{\lambda,\{\lambda,\mu\}\}\}$
(c) $\{\phi,\{\lambda\},\{\lambda,\mu\},\{\lambda,\{\lambda,\mu\}\}\}$
(d) $\{\{\lambda\},\{\lambda,\mu\},\{\lambda,\{\lambda,\mu\}\}\}$

(**b**) We have, $A = \{\lambda, \{\lambda, \mu\}\}$

$P(A) = \{\phi, \{\lambda\}, \{\{\lambda, \mu\}\}, \{\lambda, \{\lambda, \mu\}\}\}$

Directions (Q. Nos. 13 and 14) *Read the information carefully and answer the given questions.*

In a school, all the students play atleast one of three indoor games– chess, carrom and table tennis. 60 play chess, 50 play table tennis, 48 play carrom, 12 play chess and carrom, 15 play carrom and table tennis, 20 play table tennis and chess.

13. What can be the minimum number of students in the school?

(a) 123 (b) 111 (c) 95 (d) 63

(**b**) Let

A = Student play chess
B = Student play table tennis
C = Student play carrom

Given, $n(A) = 60, n(B) = 50, n(C) = 48$
$n(A \cap B) = 20, n(B \cap C) = 15$
$n(A \cap C) = 12$

For minimum number of students in school

$n(A \cap B \cap C)$ must be zero.

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$= 60 + 50 + 48 - 20 - 15 - 12 + 0$
$= 111$

14. What can be the maximum number of students in the school?

(a) 111 (b) 123 (c) 125 (d) 135

(**b**) For maximum number of students in school $n(A \cap B \cap C)$ must be 12.

$\therefore n(A \cup B \cup C)$
$= 60 + 50 + 48 - 20 - 15 - 12 + 12$
$= 123$

2018 (II)

15. If A, B and C are subsets of a universal set, then which one of the following is not correct?

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A' \cup (A \cup B) = (B' \cap A)' \cup A$
(c) $A' \cup (B \cup C) = (C' \cap B)' \cap A'$
(d) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

where, A' is the complement of A

(c) Let A, B and C are subsets of a universal set.

Let $A = \{1\}, B = \{2\}, C = \{3\}$

$U = \{1, 2, 3\}$, $A' = \{2, 3\}$, $B' = \{1, 3\}$, $C' = \{1, 2\}$

by checking options, we get

LHS $= A' \cup (B \cup C)$

$= \{2, 3\} \cup \{2, 3\}$

$= \{2, 3\}$

RHS $= (C' \cap B)' \cap A'$

$= (\{1, 2\} \cap \{2\}) \cap \{2, 3\}$

$= (\{2\})' \cap \{2, 3\}$

$= \{1, 3\} \cap \{2, 3\}$

$= \{3\}$

LHS ≠ RHS

So, option (c) is wrong.

Directions (Q. Nos. 16 and 17)

Consider the information given below and answer the two items that follow.

A survey was conducted among 300 students. It was found that 125 students like to play cricket, 145 students like to play football and 90 students like to play tennis. 32 students like to play exactly two games out of the three games.

16. How many stdudents like to play all the three games ?

(a) 14 (b) 21
(c) 28 (d) 35

(a) Let, A be the set of students like to play cricket B be the set of students like to play football. C be the set of students like to play tennis.

We have, $n(A \cup B \cup C) = 300$

$n(A) = 125$, $n(B) = 145$, $n(C) = 90$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(B \cap C) + n(C \cap A)] + n(A \cap B \cap C)$

$\Rightarrow 300 = 125 + 145 + 90 - [n(A \cap B) + n(B \cap C) + n(C \cap A)] + n(A \cap B \cap C)$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 60 + n(A \cap B \cap C)$ ...(i)

Again,

$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C) = 32$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 32 + 3n(A \cap B \cap C)$ ...(ii)

From Eqs. (i) and (ii), we get

$60 + n(A \cap B \cap C) = 32 + 3n(A \cap B \cap C)$

$\Rightarrow 2n(A \cap B \cap C) = 28$

$\Rightarrow n(A \cap B \cap C) = 14$

17. How many students like to play exactly only one game?

(a) 196 (b) 228 (c) 254 (d) 268

(c) Number of students like to play exactly one

game $= n(A) + n(B) + n(C) - 2[n(A \cap B) + n(B \cap C) + n(C \cap A)] + 3n(A \cap B \cap C)$

$= 125 + 145 + 90 - 2[32 + 3 \times 14] + 3 \times 14$

$= 360 - 106 = 254$

18. Let $A = \{x \in R : -1 \le x \le 1\}$,

$B = \{y \in R : -1 \le y \le 1\}$ and S be the subset of $A \times B$, defined by

$S = \{(x, y) \in A \times B : x^2 + y^2 = 1\}$.

Which one of the following is correct ?

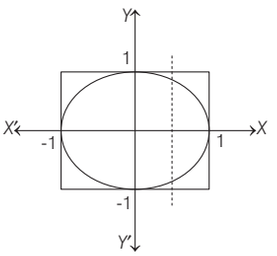
(a) S is a one-one function from A into B
(b) S is a many-one function from A into B
(c) S is a bijective mapping from A into B
(d) S is not a function

(d) Given that,

$A = \{x \in R : -1 \le x \le 1\}$,

$B = \{y \in R : -1 \le y < 1\}$

and $S = \{(x, y) \in A \times B : x^2 + y^2 = 1\}$



By vertical line test. when we draw a vertical line, then line cuts the circle in two points. Hence, S is not a function.

2018 (I)

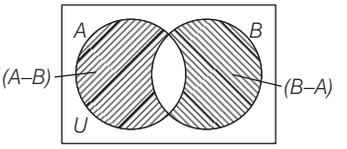
19. Let A and B be subsets of X and $C = (A \cap B') \cup (A' \cap B)$, where A' and B' are complements of A and B respectively in X. What is C equal to ?

(a) $(A \cup B') - (A \cap B')$
(b) $(A' \cup B) - (A' \cap B)$
(c) $(A \cup B) - (A \cap B)$
(d) $(A' \cup B') - (A' \cap B')$

(c) We have, $C = (A \cap B') \cup (A' \cap B)$

$= (A - B) \cup (B - A)$ [$\because X \cap Y' = X - Y$]

$= (A \cup B) - (A \cap B)$ [from Venn diagram]



Directions (Q. Nos. 20 and 21)

Consider the information given below and answer the two items that follow.

In a class, 54 students are good in Hindi only, 63 students are good in Mathematics only and 41 students are good in English only. There are 18 students who are good in both Hindi and Mathematics. 10 students are good in all three subjects.

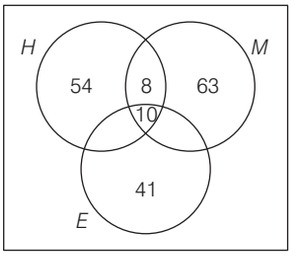
20. What is the number of students who are good in either Hindi or Mathematics but not English?

(a) 99 (b) 107
(c) 125 (d) 130

21. What is the number of students who are good in Hindi and Mathematics but not in English?

(a) 18 (b) 12 (c) 10 (d) 8

Solutions (Q. Nos. 20 and 21)



20. (c) From given Venn-diagram

$n(H \cup M \cup E') = 54 + 8 + 63 = 125$

21. (d) From given Venn-diagram

$n(H \cap M \cap E') = 18 - 10 = 8$

22. A survey of 850 students in a University yields that 680 students like music and 215 like dance. What is the least number of students who like both music and dance?

(a) 40 (b) 45 (c) 50 (d) 55

(b) Let A be the set of students who like music and B be the set of students who like dance.

$\therefore n(A) = 680$, $n(B) = 215$ and $n(U) = 850$

We know that,

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow n(A \cap B)_{min} = n(A) + n(B) - n(A \cup B)_{max}$

$\Rightarrow n(A \cap B)_{min} = 680 + 215 - 850$ [$\because n(A \cup B)_{max} = n(U)$]

$= 45$

2017 (II)

23. If E is the universal set and $A = B \cup C$, then the set $E-(E-(E-(E-(E-A))))$ is same as the set

(a) $B' \cup C'$ (b) $B \cup C$
(c) $B' \cap C'$ (d) $B \cap C$

(c) Given, E is the universal set and

$A = B \cup C$

$\therefore E-(E-(E-(E-(E-A))))$

$= E-(E-(E-(E-A')))$

$[\because E$ is universal set $\therefore E - A = A']$

$= E-(E-(E-A))$ $[\because E - A' = A]$

$= E-(E-A')$

$= E - A = A' = (B \cup C)'$ $[\because A = B \cup C]$

$= B' \cap C'$ [by De-morgan's theorem]

24. If $A = \{x : x$ is a multiple of $2\}$, $B = \{x : x$ is a multiple of $5\}$ and $C = \{x : x$ is a multiple of $10\}$, then $A \cap (B \cap C)$ is equal to

(a) A (b) B
(c) C
(d) $\{x : x$ is a multiple of 100$\}$

(c) From questions,

$A = \{2, 4, 6, 8, \ldots\ldots\}$

$B = \{5, 10, 15, 20, \ldots\ldots\}$

$C = \{10, 20, 30, 40, \ldots\ldots\}$

Here, C is subset of A. i.e $C \subset A$

and C is subset of B.

i.e. $C \subset B$

Now, $(A \cap (B \cap C) = A \cap C = C$

25. If we define a relation R on the set $N \times N$ as $(a,b)\ R\ (c,d) \Leftrightarrow a + d = b + c$ for all $(a,b),(c,d) \in N \times N$, then the relation is

(a) symmetric only
(b) symmetric and transitive only
(c) equivalence relation
(d) reflexive only

(c) $(a, b)R(a, b)\forall(a, b) \in N \times N$

Since, $a + b = b + a$,

$\therefore$ R is reflexive.

for R is to symmetric, $(a, b)\ R\ (c, d)$

$\Rightarrow$ $a + d = b + c$

$\Rightarrow$ $d + a = c + b$

$\Rightarrow$ $c + b = d + a$

$\Rightarrow$ $(c, d)\ R\ (a, b)$

$\therefore$ R is symmetric.

$(a, b)R\ (c, d) \Leftrightarrow a + d = b + c$ ...(i)

Let $(c, d)R(e, f) \Leftrightarrow c + f = d + e$ ...(ii)

$\forall (e,f) \in N \times N$

Adding Eqs. (i) and (ii), we get

$a + d + c + f = b + c + d + e$

$\Rightarrow$ $a + f = b + e$

$\Rightarrow$ $(a, b)\ R\ (e, f)$

Now, $(a,b)R(c,d)$ and $(c,d)\ R\ (c,f)$

$\Rightarrow$ $(a,b)\ R\ (e,f)$

$\therefore R$ is transitive.

Since, R is reflexive, symmetric and transitive

$\therefore R$ is an equivalance relation.

2017 (I)

26. Let S be the set of all persons living in Delhi. We say that x, y in S are related if they were born in Delhi on the same day. Which one of the following is correct?

(a) The relation is an equivalent relation
(b) The relation is not reflexive but it is symmetric and transitive
(c) The relation is not symmetric but it is reflexive and transitive
(d) The relation is not transitive but it is reflexive and symmetric

(a) Given, set S is the set of all persons living in Delhi

and x, y in S are related such that they were born in Delhi on the same day.

Let R be a relation on S defined as x and y were born in Delhi on same day.

For reflexive Here, $(x, x) \in R$ as x and x was born on the same day.

$\therefore R$ is reflexive.

For symmetric

Let $(x, y) \in R$

$\Rightarrow x$ and y were born in Delhi on same day

$\Rightarrow y$ and x were born in Delhi on same day $\Rightarrow (y, x) \in R$

$\therefore R$ is symmetric.

For Transitive Now, let $(x, y) \in R$ and $(y, z) \in R$, $\forall z \in S$

$\Rightarrow x$ and y were born in Delhi on same day and y and z were born in Delhi on same day.

$\Rightarrow x$ and z were born in Delhi on same day $\Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

Thus, R is reflexive symmetric and transitive.

$\therefore R$ is an equivalence relation.

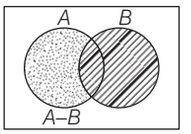
27. Consider the following in respect of sets A and B.

1. $(A - B) \cup B = A$
2. $(A - B) \cup A = A$
3. $(A - B) \cap B = \phi$
4. $A \subseteq B \Rightarrow A \cup B = B$

Which of the above are correct?

(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 3 and 4 (d) 1, 2 and 4

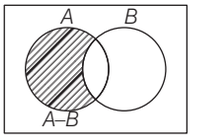
(b) 1.



Here, $(A - B) \cup B = A \cup B$

So, 1 is incorrect.

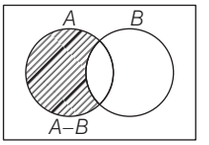
2.



Here, $(A - B) \cup A = A$

So, 2 is correct.

3.



Here, $(A - B) \cap B = \phi$

So, 3 is correct.

4. $A \subseteq B \Rightarrow A \cup B = B$

So, 4 is also correct.

$\therefore$ Option (b) is correct.

2016 (II)

28. In an examination, 70% students passed in Physics, 80% students passed in Chemistry, 75% students passed in Mathematics and 85% students passed in Biology, and x% students failed in all the four subjects. What is the minimum value of x?

(a) 10 (b) 12
(c) 15 (d) None of these

(c) We have,

70% passed in Physics

80% passed in Chemistry

75% passed in Mathematics

85% passed in Biology

and 30% failed in Physics

20% failed in Chemistry

25% failed in Mathematics

15% failed in Biology

Since, all students failed in all four subjects.

$\therefore$ 15% must be minimum failed in all the four subject.

Hence, x = 15%

29. Let S be a set of all distinct numbers of the form $\frac{p}{q}$, where $p, q \in \{1,2,3,4,5,6\}$. What is the cardinality of the set S?

(a) 21 (b) 23 (c) 32 (d) 36

(b) We have,

S be a set of all distinct numbers of the form p/q,

where, $p, q \in \{1, 2, 3, 4, 5, 6\}$

when, $p = 1, q \in \{1, 2, 3, 4, 5, 6\}$

$p = 2, q \in \{1, 3, 5\}$

$p = 3, q \in \{1, 2, 4, 5\}$

$p = 4, q \in \{1, 3, 5\}$

$p = 5, q \in \{1, 2, 3, 4, 6\}$

$p = 6, q \in \{1, 5\}$

$\therefore$ Total number of elements

$= 6 + 3 + 4 + 3 + 5 + 2 = 23$

30. If $A = \{x \in R : x^2 + 6x - 7 < 0\}$ and $B = \{x \in R : x^2 + 9x + 14 > 0\}$, then which of the following is/are correct?

I. $A \cap B = \{x \in R : -2 < x < 1\}$

II. $A/B = \{x \in R : -7 < x < -2\}$

Select the correct answer using the code given below.

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(c) Given,

$A = \{x \in R : x^2 + 6x - 7 < 0\}$

and $B = \{x \in R : x^2 + 9x + 14 > 0\}$

$\therefore \quad A = \{x \in R : (x + 7)(x - 1) < 0\}$

$\Rightarrow \quad A = -7 < x < 1$

and $B = \{x \in R : (x + 2)(x + 7) > 0\}$

$B = x < -7$ or $x > -2$

$\therefore \quad A \cap B = -2 < x < 1$

$= \{x \in R : -2 < x < 1\}$

$A/B = -7 < x < -2$

$= \{x \in R : -7 < x < -2\}$

Hence, both I and II are correct.

31. Let R be a relation from $A = \{1,2,3,4\}$ to $B = \{1,3,5\}$ such that $R = \{(a, b) : a < b$, where $a \in A$ and $b \in B\}$.

What is RoR^{-1} equal to?

(a) {(1, 3), (1, 5), (2, 3), (2, 5) (3, 5), (4, 5)}
(b) {(3, 1), (5, 1), (3, 2), (5,2), (5, 3), (5, 4)}
(c) {(3, 3), (3, 5), (5, 3), (5, 5)}
(d) {(3, 3), (3, 4), (4, 5)}

(c) Given,

$A = \{1, 2, 3, 4,\}$

and $B = \{1, 3, 5\}$

Such that, $R = \{(a,b) : a < b$ where, $a \in A$ and $b \in B\}$

$\therefore \quad R = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$

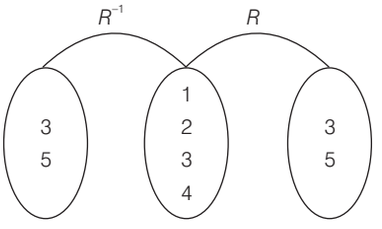
$\Rightarrow \quad R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

Here, domain $(R) = \{1, 2, 3, 4\}$

Range $(R) = \{3, 5\}$

domain $(R^{-1}) = \{3, 5\}$

Range $(R^{-1}) = \{1, 2, 3, 4\}$



$\therefore RoR^{-1} = \{(3, 3), (3, 5), (5, 3), (5, 5)\}$

2016 (I)

32. What is the number of natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25?

(a) 860 (b) 854
(c) 840 (d) 824

(b) Let A be the set of natural numbers ($\leq$ 1000) which are divisible by 10,

B be the set of natural numbers ($\leq$1000) which are divisible by 15 and

C be the set of natural numbers ($\leq$1000) which are divisible by 25.

Then, $n(A) = \left[\frac{1000}{10}\right] = 100;$

$n(B) = \left[\frac{1000}{15}\right] = 66$

and $n(C) = \left[\frac{1000}{25}\right] = 40,$

where $[\cdot]$ denotes the greatest integer function.

Now, $n(A \cap B) = n$ (set of numbers which are divisible by both 10 and 15)

$= n$ (set of numbers divisible by 30)

$= \left[\frac{1000}{30}\right] = 33$

Similarly, $n(A \cap C) = n$ (set of numbers divisible by 50)

$= \left[\frac{1000}{50}\right] = 20$

$n(B \cap C) = n$ (set of numbers divisible by 75)

$= \left[\frac{1000}{75}\right] = 13$

and $n(A \cap B \cap C) = n$ (set of numbers divisible by 10, 15 and 25)

$= n$ (set of numbers divisible by 150)

$= \left[\frac{1000}{150}\right] = 6$

$\therefore n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 100 + 66 + 40 - 33 - 13 - 20 + 6$

$= 146$

Hence, required numbers

$= n(U) - n(A \cup B \cup C)$

$= 1000 - 146 = 854$

33. Suppose, there is a relation * between the positive numbers x and y given by $x * y$ if and only if $x \leq y^2$. Then, which one of the following is correct?

(a) * is reflexive but not transitive and symmetric
(b) * is transitive but not reflexive and symmetric
(c) * is symmetric and reflexive but not transitive
(d) * is symmetric but not reflexive and transitive

(b) Let I be the set of positive integers and $x, y \in I$

(i) **Reflexive**

Given, $xRy \Rightarrow x$ is less than equal to y^2.

$\therefore xRx \Rightarrow x$ is less than equal to x^2.

Which is not true.

Hence, R is not reflexive.

(ii) **Symmetric**

$xRy \Rightarrow x$ is less than equal to y^2.

$yRx \Rightarrow y$ is less than equal to x^2 is not true.

For example

$1R2 \Rightarrow 1$ is less than 2^2.

$2R1 \Rightarrow 2$ is less than 1^2.

It is not symmetric.

(iii) **Transitive**

$\because xRy \Rightarrow x$ is less than equal to y^2.

Let $yRz \Rightarrow y$ is less than equal to z^2, $\forall z \in I$

Then, $xRz \Rightarrow x$ is less than equal to z^2

So, R is transitive.

Hence, option (b) is correct.

34. Let R be a relation on the set N of natural numbers defined by '$nRm \Leftrightarrow n$ is a factor of m'. Then, which one of the following is correct?

(a) R is reflexive, symmetric but not transitive
(b) R is transitive, symmetric but not reflexive
(c) R is reflexive, transitive but not symmetric
(d) R is an equivalence relation

(c) Given, R is a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m.

Reflexive Since, n is a factor of n for every $n \in N$, therefore nRn, $\forall\, n \in N$, i.e. R is reflexive.

Symmetric since, n is a factor of m for every $n \in N$, but m is not a factor of n because m is a multiple of n. Therefore, nRm but $m \not R n$ is not true. Since, 2 is a factor of 4 but 4 is not a factor of 2, i.e. $2R4$ but $4 \not R 2$

Thus, R is not symmetric.

2015 (II)

35. Let X be the set of all persons living in Delhi. The persons a and b in X are said to be related, if the difference in their ages is atmost 5 yr. The relation is

(a) an equivalence relation
(b) reflexive and transitive but not symmetric
(c) symmetric and transitive but not reflexive
(d) reflexive and symmetric but not transitive

(d) Given, $R = \{(a, b) : |a - b| \le 5\}$

$\forall\, a, b \in x$

For reflexive

$|a - a| = 0,\ 0 \le 5$

$\Rightarrow \quad (a, a) \in R$

Hence, R is reflexive.

For symmetric

Let $a, b \in x$, then

$$|a - b| \le 5 = |b - a| \le 5$$

$\therefore \quad (a, b) \in R \ \Rightarrow (b, a) \in R$

Hence, R is symmetric.

For transitive Let a, b, c are in x.

$$= |a - b| \le 5 \Rightarrow (a, b) \in R$$
$$= |b - c| \le 5 \Rightarrow (b, c) \in R$$

Then, $|a - c| \le 10 \Rightarrow (a, c) \notin R$.

Hence, R is not transitive.

$\therefore$ R is reflexive and symmetric but not transitive.

36. If $A = \{x \in R : x^2 + 6x - 7 < 0\}$ and $B = \{x \in R : x^2 + 9x + 14 > 0\}$, then which of the following is/are correct?

1. $(A \cap B) = (-2, 1)$
2. $(A - B) = (-7, -2)$

Select the correct answer using the code given below

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) We have,

$$A = \{x \in R : x^2 + 6x - 7 < 0\}$$
and $B = \{x \in R : x^2 + 9x + 14 > 0\}$
$\therefore \quad A = \{x \in R : (x - 1)(x + 7) < 0\}$
and $B = \{x \in R : (x + 2)(x + 7) > 0\}$
$\Rightarrow \quad A = (-7, 1)$
and $B = (-\infty, -7) \cup (-2, \infty)$
$A \cap B = (-2, 1)$
$A - B = (-7, -2)$

Hence, both 1 and 2 are correct.

37. A, B, C and D are four sets such that $A \cap B = C \cap D = \phi$. Consider the following

1. $A \cup C$ and $B \cup D$ are always disjoint.
2. $A \cap C$ and $B \cap D$ are always disjoint.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) Given, $A \cap B = C \cap D = \phi$

$\therefore \quad (A \cup C) \cap (B \cup D)$
$= [(A \cup C) \cap B] \cup [(A \cup C) \cap D]$
$= [(A \cap B) \cup (B \cap C)] \cup [(D \cap A) \cup (D \cap C)]$
$= [\phi \cup (B \cap C)] \cup [(D \cap A) \cup \phi]$
$= (B \cap C) \cup (D \cap A)$

Hence, $(A \cup C) \cap (B \cup D)$ is not disjoint set.

and $(A \cap C) \cap (B \cap D)$
$= [(A \cap C) \cap B] \cap [(A \cap C) \cap D]$
$= [(A \cap B) \cap C] \cap [A \cap (C \cap D)]$
$= [\phi \cap C] \cap [A \cap \phi]$
$= \phi \cap \phi = \phi$

$\therefore$ $(A \cap C) \cap (B \cap D)$ is disjoint set.

2015 (I)

38. Let X be the set of all persons living in a city. Persons x, y in X are said to be related as $x < y$, if y is atleast 5 yr older than x. Which one of the following is correct?

(a) The relation is an equivalence relation on X
(b) The relation is transitive but neither reflexive nor symmetric
(c) The relation is reflexive but neither transitive nor symmetric
(d) The relation is symmetric but neither transitive nor reflexive

(b) We have, X = Set of all persons living in a city

Let R be a relation on X, define as $x < y$, if y is atleast 5 yr older than x.

Clearly, $x \ne x$, so R is not reflexive.

Now, let xRy, then $x < y$ i.e. y is atleast 5 yr older than x.

Thus, x is smaller than y, so y is not smaller than x.

i.e. $(x, y) \in R \Rightarrow (y, x) \notin R$

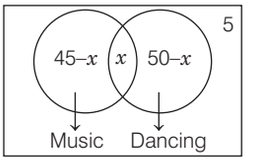
Hence, R is not symmetric.

Again, let xRy and yRz, then $x < y$ and $y < z$, clearly $x < z$. Hence, R is transitive.

39. In a class of 60 students, 45 students like music, 50 students like dancing, 5 students like neither. Then, the number of students in the class who like both music and dancing, is

(a) 35 (b) 40
(c) 50 (d) 55

(b) Let the number of students in the class be x, who like both music and dancing.



$\therefore (45 - x) + x + (50 - x) + 5 = 60$
$\Rightarrow \quad 100 - x = 60$
$\Rightarrow \quad x = 100 - 60$
$= 40$

40. Let Z be the set of integers and aRb, where $a, b \in Z$ if and only if $(a - b)$ is divisible by 5.

Consider the following statements

I. The relation R partitions Z into five equivalent classes.
II. Any two equivalent classes are either equal or disjoint.

Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(**c**) We have, aRb, where $a, b \in Z$, if $(a - b)$ is divisible by 5.
Thus, $(a - b) = 0, 5, 10, 15, \ldots$
So, Statement I is correct, since the relation R partitions Z into five equivalent classes, which are 0, 1, 2, 3, 4.
Also, any two equivalent classes are disjoint, but not equal.
Hence, both the statements are correct.

41. If $A = \{x : x$ is a multiple of $3\}$ and $B = \{x : x$ is a multiple of $4\}$ and $C = \{x : x$ is a multiple of $12\}$, then which one of the following is a null set?
(a) $(A / B) \cup C$ (b) $(A / B)/C$
(c) $(A \cap B) \cap C$ (d) $(A \cap B)/C$

(*d*) Here, $A = \{3, 6, 9, 12, 15, \ldots\}$
$B = \{4, 8, 12, 16, 20, \ldots\}$
$C = \{12, 24, 36, 48, \ldots\}$
$\therefore \quad A \cap B = \{12, 24, 36, 48, \ldots\}$
$\Rightarrow \quad \frac{(A \cap B)}{C} = (A \cap B) - C$
$= \{12, 24, 36, \ldots\} - \{12, 24, 36, \ldots.\}$
which is a null set.

42. Let $A = \{x, y, z\}$ and $B = \{p, q, r, s\}$, what is the number of distinct relations from B to A?
(a) 4096 (b) 4094
(c) 128 (d) 126

(*a*) Number of distinct relations from B to $A = 2^{mn} = 2^{4 \times 3} = 2^{12} = 4096$

2014 (II)

43. A and B are two sets having 3 elements in common. If $n(A) = 5$ and $n(B) = 4$, then what is $n(A \times B)$ equal to?
(a) 0 (b) 9
(c) 15 (d) 20

(*d*) Here, $n(A) = 5$ and $n(B) = 4$
$\therefore \quad n(A \times B) = 5 \times 4 = 20$
$[\because n(A) = m, n(B) = n \Rightarrow n(A \times B) = mn]$

44. The relation S is defined on the set of integers Z as xSy, if integer x divides integer y. Then,
(a) S is an equivalence relation
(b) S is only reflexive and symmetric
(c) S is only reflexive and transitive
(d) S is only symmetric and transitive

(*c*) The relation S is defined on the set of integers Z and xSy, if integer x divides integer y.

Reflexive Since, every integer divides itself.
$\therefore$ Integer x divides integer x.
$\Rightarrow \quad xSx$
Hence, S is reflexive.

Symmetric Let $x, y \in Z$ such that xSy
i.e., integer x divides integer y.
Now, this does not implies that integer y divides integer x.
e.g., Take $x = 2$ and $y = 4$. Then, 2 divides 4 but 4 does not divides 2.
Thus, S is not symmetric.

Transitive Let $x, y, z \in Z$ such that xSy and ySz.
$\Rightarrow$ Integer x divides integer y and integer y divides integer z.
$\Rightarrow$ Integer x divides integer z.
$\Rightarrow \quad xSz$
Hence, S is transitive.

2014 (I)

Directions (Q. Nos. 45-47) *Read the following information carefully to answer the questions that follow.*

In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects.

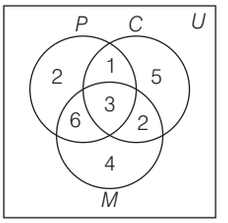
45. The number of students who had taken only Physics, is
(a) 2 (b) 3 (c) 5 (d) 6

46. The number of students who had taken only two subjects, is
(a) 7 (b) 8 (c) 9 (d) 10

47. Consider the following statements
I. The number of students who had taken only one subject is equal to the number of students who had taken only two subjects.
II. The number of students who had taken atleast two subjects is four times the number of students who had taken all the three subjects.

Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Solutions (Q. Nos. 45-47)



Given that,
Total number of surveyed students = 25
Number of students who had taken all three subjects = 3
Number of students who had taken Physics and Chemistry = 4
Number of students who had taken Mathematics and Physics = 9
Number of students who had taken Mathematics and Chemistry = 5
Number of students who had taken Chemistry = 11
Number of students who had taken Physics = 12
Number of students who had taken Mathematics = 15

45. (*a*) The number of students who had taken only Physics
$= 12 - (1 + 3 + 6) = 12 - 10 = 2$

46. (*c*) The number of students who had taken only two subjects = $6 + 2 + 1 = 9$

47. (*b*) I. The number of students who had taken only one subject
$= 2 + 5 + 4 = 11$
and the number of students who had taken only two subjects
$= 6 + 2 + 1 = 9 \Rightarrow 11 \neq 9$
II. The number of students who had taken atleast two subjects
$= 1 + 2 + 6 + 3 = 12$
The number of students who had taken all three subjects = 3
$\Rightarrow \quad 12 = 4 \times 3 = 12$
Hence, number of students taken atleast two subject
$12 = 4 \times$ (number of student taken all these subjects)

48. Let X be the set of all citizens of India. Elements x, y in X are said to be related, if the difference of their age is 5 yr. Which one of the following is correct?
(a) The relation is an equivalence relation on X
(b) The relation is symmetric but neither reflexive nor transitive
(c) The relation is reflexive but neither symmetric nor transitive
(d) None of the above

› *(**b**)* Given that,

$X = \{\text{Set of all citizens of India}\}$

and $R = \{(x, y) : x, y \in X, |x - y| = 5\}$

(i) **Reflexive** $|x - x| = 0 \neq 5$

$\therefore \quad x R x \notin R$

So, R is not reflexive.

(ii) **Symmetric** Again,

$xRy \Rightarrow |x - y| = 5$

$\Rightarrow \quad |y - x| = 5$

$\Rightarrow yRx$

So, R is symmetric.

(iii) **Transitive** Let $x, y, z \in X$,

then $x R y \Rightarrow |x - y| = 5$

and $y R z \Rightarrow |y - z| = 5$

But $|x - z| \neq 5$

So, R is not transitive.

Hence, the relation is symmetric but neither reflexive nor transitive.

49. Consider the following relations from A to B, where $A = \{u, v, w, x, y, z\}$ and $B = \{p, q, r, s\}$.

I. $\{(u, p), (v, p), (w, p), (x, q), (y, q), (z, q)\}$

II. $\{(u, p), (v, q), (w, r), (z, s)\}$

III. $\{(u, s), (v, r), (w, q), (u, p), (v, q), (z, q)\}$

IV. $\{(u, q), (v, p), (w, s), (x, r), (y, q), (z, s)\}$

Which of the above relations are not functions?

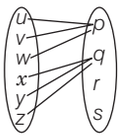
(a) I and II
(b) I and IV
(c) II and III
(d) III and IV

› (**c**) Given that, $A = \{u, v, w, x, y, z\}$

and $B = \{p, q, r, s\}$

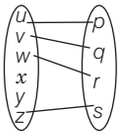
We know that a mapping $f : X \to Y$ is said to be a function, if each element in the set X has its image in set Y. It is also possible that there are few elements in set Y which are not the image of any element in set X. Every element in set X should have one and only one image.

(i)



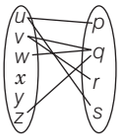
(Function)

(ii)



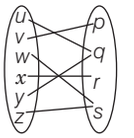
(Not function)

(iii)



(Not Function)

(iv)



(Function)

50. Let S denotes the set of all integers. Define a relation R on S as 'aRb if $ab \geq 0$, where $a, b \in S$.' Then, R is

(a) reflexive but neither symmetric nor transitive relation
(b) reflexive, symmetric but not transitive relation
(c) an equivalence relation
(d) symmetric but neither reflexive nor transitive relation

› (**c**) Given that, S = Set of all integers

and $R = \{(a, b), a, b \in S \text{ and } ab \geq 0\}$

(i) **Reflexive** $a R a \Rightarrow a \cdot a = a^2 \geq 0, \forall a \in S$

So, R is reflexive.

(ii) **Symmetric** $a R b \Rightarrow ab \geq 0, \forall a, b \in S$

$\Rightarrow \quad b \cdot a \geq 0$

$\Rightarrow \quad b R a$

So, R is symmetric.

(iii) **Transitive** If $a R b \Rightarrow ab \geq 0$

and $b R c \Rightarrow bc \geq 0, \forall a, b, c \in S$,

then $ac \geq 0 \Rightarrow a R c$

So, R is also transitive.

Hence, R is an equivalence relation.

2

COMPLEX NUMBERS

2019 (II)

1. If $x = 1 + i$, then what is the value of $x^6 + x^4 + x^2 + 1$?

(a) $6i - 3$ (b) $-6i + 3$
(c) $-6i - 3$ (d) $6i + 3$

(c) Given, $x = 1 + i$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Now, $x^6 + x^4 + x^2 + 1$

$= x^4(x^2 + 1) + 1(x^2 + 1)$

$= (x^2 + 1)(x^4 + 1)$

$$= \left[(\sqrt{2})^2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^2 + 1\right]\left[(\sqrt{2})^4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^4 + 1\right]$$

$$= \left[2\left(\cos\frac{2\pi}{4} + i\sin\frac{2\pi}{4}\right) + 1\right]\left[4\left(\cos\frac{4\pi}{4} + i\sin\frac{4\pi}{4}\right) + 1\right]$$

$[\because (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta]$

$$= \left[2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) + 1\right]$$

$[4(\cos\pi + i\sin\pi) + 1]$

$= [2(0 + i) + 1][4(-1 + 0) + 1]$

$= (2i + 1)(-4 + 1) = -6i - 3$

2. What is the value of

$$\left[\frac{i+\sqrt{3}}{2}\right]^{2019} + \left[\frac{i-\sqrt{3}}{2}\right]^{2019} ?$$

(a) 1 (b) -1
(c) $2i$ (d) $-2i$

(c) $\left[\frac{i+\sqrt{3}}{2}\right]^{2019} + \left[\frac{i-\sqrt{3}}{2}\right]^{2019}$

$$= \left[\frac{\sqrt{3}}{2} + \frac{1}{2}i\right]^{2019} - \left[\frac{\sqrt{3}}{2} - \frac{1}{2}i\right]^{2019}$$

$$= \left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]^{2019} - \left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]^{2019}$$

$$= \cos\frac{2019\pi}{6} + i\sin\frac{2019\pi}{6} - \cos\frac{2019\pi}{6} + i\sin\frac{2019\pi}{6}$$

[De-moivre's theorem

$(\cos\theta \pm i\sin\theta)^n = \cos n\theta \pm i\sin n\theta]$

$$= 2i\sin\frac{2019\pi}{6}$$

$$= 2i\sin\left(168 \times 2\pi + \frac{3\pi}{6}\right)$$

$$= 2i\sin\frac{3\pi}{6}$$

$[\because \sin(2n\pi + \theta) = \sin\theta, n \text{ is an integer}]$

$$= 2i\sin\frac{\pi}{2} = 2i$$

2019 (I)

3. The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{2017} + z^{2018} + 1 = 0$ are

(a) $-1, \omega$ (b) $1, \omega^2$
(c) $-1, \omega^2$ (d) ω, ω^2

(d) We have, $z^3 + 2z^2 + 2z + 1 = 0$

$(z + 1)(z^2 + z + 1) = 0$

$\Rightarrow$ $z + 1 = 0$ or $z^2 + z + 1 = 0$

$z = -1$

or $z = \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2} = \omega, \omega^2$

Now, $z^{2017} + z^{2018} + 1 = 0$

Put $z = -1$,

LHS $= (-1)^{2017} + (-1)^{2018} + 1$

$= -1 + 1 + 1$

$= 1 \neq 0$ (RHS)

$\therefore$ $z = -1$ is not a root of equation.

Put $z = \omega$,

LHS $= (\omega)^{2017} + (\omega)^{2018} + 1$

$= (\omega^3)^{672}.\omega + (\omega^3)^{672}.\omega^2 + 1$

$= \omega + \omega^2 + 1$ $[\because \omega^3 = 1]$

$[\because 1 + \omega + \omega^2 = 0]$

$= 0 =$ RHS

$\therefore$ $z = \omega$ is a root of equation.

put $z = \omega^2$,

LHS $= (\omega^2)^{2017} + (\omega^2)^{2018} + 1$

$= \omega^{4034} + \omega^{4036} + 1$

$= (\omega^3)^{1344}.\omega^2 + (\omega^3)^{1345}.\omega + 1$

$= \omega^2 + \omega + 1 = 0$ RHS

$\therefore$ $z = \omega^2$ is a root of equation.

Hence, ω, ω^2 are the common roots of these equations.

Directions (Q. Nos. 4 and 5) *Read the information carefully and answer the given questions.*

A complex number is given by

$$z = \frac{1 + 2i}{1 - (1 - i)^2}.$$

4. What is the modulus of z?

(a) 4 (b) 2
(c) 1 (d) $\frac{1}{2}$

(c) We have, $z = \frac{1 + 2i}{1 - (1 - i)^2}$

$$z = \frac{1 + 2i}{1 - (1 - 1 - 2i)}$$

$$= \frac{1 + 2i}{1 + 2i} = 1$$

$\therefore$ $|z| = 1$

5. What is the principal argument of z?

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

(***a***) $\arg(z) = \tan^{-1}\left(\frac{\text{Im}(z)}{\text{Re}(z)}\right)$

$= \tan^{-1}\left(\frac{0}{1}\right) = \tan^{-1} 0 = 0$

2018 (II)

6. What is the value of

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3n},$$

where, $i = \sqrt{-1}$?

(a) 3 (b) 2
(c) 1 (d) 0

(b) We have,

$\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}$

$= (\omega)^{3n} + (\omega^2)^{3n}$

$\left[\because \omega = \frac{-1+i\sqrt{3}}{2}, \omega^2 = \left(\frac{-1-i\sqrt{3}}{2}\right)\right]$

$= (\omega^3)^n + (\omega^3)^{2n} = (1)^n + (1)^{2n} \quad [\because \omega^3 = 1]$

$= 1 + 1 = 2$

7. Which one of the following is correct in respect of the cube roots of unity?

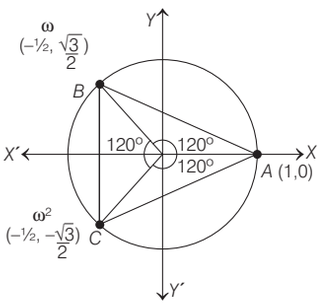
(a) They are collinear
(b) They lie on a circle of radius $\sqrt{3}$
(c) They form an equilateral triangle
(d) None of the above

(***c***) We know that, cube roots of unity is $1, \omega, \omega^2$,

where, $\omega = \frac{-1+i\sqrt{3}}{2}$

and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

Draw the value of roots on argand plane.



From figure,

They are not collinear. They lie on a circle of radius1.

They form an equilateral triangle.

Because $AB = BC = CA$.

2018 (I)

8. What is the principal argument of $(-1-i)$, where $i = \sqrt{-1}$?

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $-\frac{3\pi}{4}$ (d) $\frac{3\pi}{4}$

(***c***) Let $z = -1 - i$

Now, $\tan\alpha = \frac{|b|}{|a|} = \frac{|-1|}{|-1|}$

$[\because a = -1, b = -1]$

$\therefore \quad \alpha = \tan^{-1}(1) = \frac{\pi}{4}$

Since a, b both are negative,

$\therefore \arg(z) = \alpha - \pi = \frac{\pi}{4} - \pi = \frac{-3\pi}{4}$

9. The number of non-zero integral solution of the equation $|1-2i|^x = 5^x$ is

(a) Zero (no solution) (b) One
(c) Two (d) Three

(***a***) We have,

$|1-2i|^x = 5^x$

$\Rightarrow \quad (\sqrt{(1)^2 + (-2)^2})^x = 5^x$

$[\because |a+ib| = \sqrt{a^2+b^2}]$

$\Rightarrow \quad (\sqrt{1+4})^x = 5^x$

$\Rightarrow \quad (\sqrt{5})^x = 5^x \Rightarrow 5^{x/2} = 5^x$

$\Rightarrow \quad \frac{x}{2} = x$

$[\because a^m = a^n \Rightarrow m = n]$

$\Rightarrow \quad x - \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = 0$

$\Rightarrow \quad x = 0$

But, x is non-zero integral.

$\therefore$ Given equation has no solution.

10. If α and β are different complex numbers with $|\alpha| = 1$, then what is $\left|\frac{\alpha-\beta}{1-\alpha\bar{\beta}}\right|$ equal to ?

(a) $|\beta|$ (b) 2 (c) 1 (d) 0

(c) We have,

$\left|\frac{\alpha-\beta}{1-\alpha\bar{\beta}}\right| = \left|\frac{\alpha-\beta}{\alpha\bar{\alpha}-\alpha\bar{\beta}}\right|$

$[\because |\alpha| = 1 \Rightarrow |\alpha|^2 = 1 \Rightarrow \alpha \cdot \bar{\alpha} = 1]$

$= \left|\frac{\alpha-\beta}{\alpha(\bar{\alpha}-\bar{\beta})}\right| = \frac{1}{|\alpha|}\left|\frac{\alpha-\beta}{\bar{\alpha}-\bar{\beta}}\right|$

$= \frac{|\alpha-\beta|}{|\alpha||\overline{\alpha-\beta}|} = \frac{|\alpha-\beta|}{|\alpha||\alpha-\beta|}$

$[\because |\bar{z}| = |z|]$

$= \frac{1}{|\alpha|} = 1 \qquad [\because |\alpha| = 1]$

11. What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal to (where, $i = \sqrt{-1}$) ?

(a) 0 (b) i (c) $-i$ (d) 1

(a) We have,

$i^{1000} + i^{1001} + i^{1002} + i^{1003}$

$= i^{1000}[1 + i + i^2 + i^3]$

$= i^{1000}[1 + i - 1 - i]$

$[\because i^2 = -1, i^3 = -i]$

$= 0$

12. The modulus-amplitude form of $\sqrt{3} + i$, where $i = \sqrt{-1}$ is

(a) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$
(b) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
(c) $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$
(d) $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(b) Let $z = \sqrt{3} + i$

$\therefore \quad |z| = \sqrt{(\sqrt{3})^2 + (1)^2}$

$[\because z = a + ib \Rightarrow |z| = \sqrt{a^2+b^2}]$

$= \sqrt{3+1} = \sqrt{4} = 2$

Now, $\text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} \quad \left[\because \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}\right]$

$\therefore$ modulus-amplitude form of $z = r(\cos\theta + i\sin\theta)$

$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$[\because r = |z| = 2 \text{ and } \theta = \text{amp}(z) = \frac{\pi}{6}]$

13. What is the value of the sum $\sum_{n=2}^{11}(i^n + i^{n+1})$, where $i = \sqrt{-1}$?

(a) i (b) $2i$
(c) $-2i$ (d) $1+i$

(c) We have,

$\sum_{n=2}^{11}(i^n + i^{n+1}) = \sum_{n=2}^{11} i^n(1+i)$

$= (1+i)\sum_{n=2}^{11} i^n$

$= (1+i)[i^2 + i^3 + i^4 + \ldots + i^{11}]$

$= (1+i)\,i^2\left[\frac{i^{10}-1}{i-1}\right]$

$\left[\because a + ar + ar^2 + \ldots + ar^{n-1} = a\left[\frac{r^n-1}{r-1}\right]\right]$

$= \frac{(1+i)i^2(i^{2\times 4+2}-1)}{(i-1)}$

$= \frac{-(1+i)(i^2-1)}{(i-1)} \qquad [\because i^2 = -1]$

$$= \frac{-(1+i)(-1-1)}{(i-1)}$$
$$= \frac{2(1+i)}{(i-1)}$$
$$= \frac{2(1+i)}{(i-1)} \times \frac{(i+1)}{(i+1)}$$
$$= \frac{2(i+1+i^2+i)}{i^2-1}$$
$$= \frac{2(i+1-1+i)}{-1-1}$$
$$= -2i$$

2017 (II)

14. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is

(a) 1 (b) 4
(c) 8 (d) 16

(b) $\left(\frac{1+i}{1-i}\right)^n = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n$

$$= \left[\frac{(1+i)^2}{(1)^2-(i)^2}\right]^n$$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$$= \left[\frac{1+i^2+2i}{1-(-1)}\right]^n [\because i^2 = -1]$$

$$= \left[\frac{1-1+2i}{2}\right]^n = (i)^n$$

$\therefore n = 4$ is the smallest value for which $(i)^n = 1$.

15. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to

(a) $1+\sqrt{3}$ (b) $1+\sqrt{5}$
(c) $1-\sqrt{5}$ (d) $\sqrt{5}-1$

(b) We have,

$$|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right| \le \left|z - \frac{4}{z}\right| + \left|\frac{4}{z}\right|$$

[using triangle inequality]

$$\Rightarrow \quad |z| \le 2 + \frac{4}{|z|} \quad \left[\because \left|z - \frac{4}{z}\right| = 2\right]$$

$$\Rightarrow \quad |z|^2 - 2|z| - 4 \le 0$$
$$\Rightarrow \quad (|z| - 1 + \sqrt{5})(|z| - 1 - \sqrt{5}) \le 0$$
$$\Rightarrow \quad 1 - \sqrt{5} \le |z| \le 1 + \sqrt{5}$$

Thus, the maximum value of $|z|$ is $1+\sqrt{5}$

16. Geometrically Re $(z^2 - i) = 2$, where, $i = \sqrt{-1}$ and Re is the real part, represents

(a) circle
(b) ellipse
(c) rectangular hyperbola
(d) parabola

(c) Let $z = x + iy$

$$\therefore \quad z^2 - i = (x+iy)^2 - i$$
$$= x^2 + 2xyi + i^2y^2 - i$$
$$= x^2 + 2xyi - y^2 - i$$
$$= x^2 - y^2 + (2xy-1)i$$

Since, $\text{Re}(z^2 - i) = 2 \Rightarrow x^2 - y^2 = 2$

Which represents a rectangular hyperbola.

2017 (I)

17. The value of $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3}$, where $i = \sqrt{-1}$, is

(a) 0 (b) 1 (c) i (d) $-i$

(a) $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3}$

$$= i^{2n}(1 + i + i^2 + i^3)$$
$$= i^{2n}(1 + i - 1 - i) = i^{2n} \times 0 = 0$$

18. The value of

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n,$$

where n is not a multiple of 3 and $i = \sqrt{-1}$, is

(a) 1 (b) –1 (c) i (d) $-i$

(b) Let $\frac{-1+i\sqrt{3}}{2} = \omega$

then $\frac{-1-i\sqrt{3}}{2} = \omega^2$

Now, given expression is

$$(\omega)^n + (\omega^2)^n = \omega^n + \omega^{2n} \quad \ldots (i)$$

Since, n is not multiple of 3 ·

Let $n = 3r + 1$ (where r is an integer) then

$$\omega^n = \omega^{3r+1} = \omega \quad [\because \omega^3 = 1]$$

and $\omega^{2n} = \omega^{6r+2} = \omega^2$

Now, from Eq. (i), we get

$$\omega^n + \omega^{2n} = \omega + \omega^2 = -1$$

$[\because 1 + \omega + \omega^2 = 0]$

19. If $1, \omega, \omega^2$ are the cube roots of unity, then $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2)$ is equal to

(a) –2 (b) –1 (c) 0 (d) 2

(c) $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2)$

$= 0 \quad [\because 1 + \omega + \omega^2 = 0]$

20. The modulus and principal argument of the complex number $\frac{1+2i}{1-(1-i)^2}$ are respectively.

(a) 1, 0 (b) 1, 1
(c) 2, 0 (d) 2, 1

(a) Let $z = \frac{1+2i}{1-(1-i)^2}$

$$= \frac{1+2i}{1-1-i^2+2i} = \frac{1+2i}{1+2i}$$

$$z = 1 = 1 + 0i$$

Modulus of z is

$$|z| = \sqrt{(1)^2 + (0)^2}$$
$$= \sqrt{1+0} = 1$$

and argument,

$$\theta = \tan^{-1}\left|\frac{\text{imaginary part}}{\text{real part}}\right|$$
$$= \tan^{-1}\left|\frac{0}{1}\right| = \tan^{-1} 0$$

$$\Rightarrow \quad \theta = \tan^{-1}\tan 0°$$
$$\therefore \quad \theta = 0°$$

21. If $|z+4| \le 3$, then the maximum value of $|z+1|$ is

(a) 0 (b) 4
(c) 6 (d) 10

(c) $|z+1| = |z+4-3|$

$$\le |z+4| + |-3|$$

(by triangle inequality)

$$\Rightarrow \quad |z+1| \le 3 + 3$$
$$\Rightarrow \quad |z+1| \le 6$$

22. The number of roots of the equation $z^2 = 2\bar{z}$ is

(a) 2 (b) 3
(c) 4 (d) zero

(c) We have, $z = x + iy$, then

$$z^2 = 2\bar{z}$$
$$\Rightarrow \quad (x+iy)^2 = 2(x-iy)$$
$$\Rightarrow \quad x^2 - y^2 + 2xy.i = 2x - 2iy$$
$$\Rightarrow \quad x^2 - y^2 - 2x + i(2xy + 2y) = 0$$

On comparing real and imaginary parts, we have

$$\Rightarrow \quad x^2 - y^2 - 2x = 0 \quad \ldots(i)$$

and $\quad 2xy + 2y = 0 \quad \ldots(ii)$

From Eq. (ii) $\quad 2xy + 2y = 0$

$$\Rightarrow \quad xy + y = 0$$
$$\Rightarrow \quad y(x+1) = 0$$
$$\Rightarrow \quad y = 0$$

or $\quad x = -1$

When, $y = 0$

Then, from Eq. (i)

$$\Rightarrow \quad x^2 - 2x = 0$$
$$\Rightarrow \quad x(x-2) = 0$$
$$\Rightarrow \quad x = 0, 2$$

When $x = -1$, then from Eq. (i)

$$\Rightarrow \quad (-1)^2 - y^2 - 2(-1) = 0$$
$$\Rightarrow \quad 1 - y^2 + 2 = 0$$
$$\Rightarrow \quad y^2 = 3$$
$$\Rightarrow \quad y = \pm\sqrt{3}$$

Therefore, four roots exist.

2016 (II)

23. What is $\sqrt{\frac{1+\omega^2}{1+\omega}}$ equal to, where ω is the cube root of unity?

(a) 1
(b) ω
(c) ω^2
(d) $i\omega$, where $i = \sqrt{-1}$

(b) We have,

$$\sqrt{\frac{1+\omega^2}{1+\omega}} = \sqrt{\frac{-\omega}{-\omega^2}} \quad [\because 1+\omega+\omega^2 = 0]$$

$$= \sqrt{\frac{1}{\omega} \times \frac{\omega^2}{\omega^2}}$$

$$= \sqrt{\frac{\omega^2}{\omega^3}} = \omega \quad [\because \omega^3 = 1]$$

24. What is $\omega^{100} + \omega^{200} + \omega^{300}$ equal to, where ω is the cube root of unity?

(a) 1 (b) 3ω
(c) $3\omega^2$ (d) 0

(d) We have, $\omega^{100} + \omega^{200} + \omega^{300}$

$$= (\omega^3)^{33} \cdot \omega + (\omega^3)^{66} \cdot \omega^2 + (\omega^3)^{100}$$

$$= \omega + \omega^2 + 1 \quad [\because \omega^3 = 1]$$

$$= 0$$

25. If $\text{Re}\left(\frac{z-1}{z+1}\right) = 0$, where $z = x + iy$ is a complex number, then which one of the following is correct?

(a) $z = 1 + i$ (b) $|z| = 2$
(c) $z = 1 - i$ (d) $|z| = 1$

(d) We have, $z = x + iy$

Then, $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy}$

$$= \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)]-iy]}$$

$$= \frac{x^2-1+y^2+i(xy+y-xy+y)}{(x+1)^2+y^2}$$

$$\Rightarrow \text{Re}\left(\frac{z-1}{z+1}\right) = \frac{x^2+y^2-1}{(x+1)^2+y^2}$$

It is given that,

$$\text{Re}\left(\frac{z-1}{z+1}\right) = 0$$

$$\therefore \quad \frac{x^2+y^2-1}{(x+1)^2+y^2} = 0$$

$$\Rightarrow \quad x^2 + y^2 = 1$$

$$\Rightarrow \quad |z| = 1$$

26. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{107}$, then what is the imaginary part of z equal to?

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

(a) Given,

$$z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{107}$$

$$= (\cos 30° + i \sin 30°)^{107} + (\cos 30° - i \sin 30°)^{107}$$

By De-Moivre's theorem,

$$z = [\cos(30° \times 107) + i \sin(30° \times 107)] + [\cos(30° \times 107) - i \sin(30° \times 107)]$$

$$= 2\cos(30° \times 107) = 2\cos 3210°$$

$\therefore$ Imaginary part of $z = 0$

27. What is the number of distinct solutions of the equation $z^2 + |z| = 0$ (where, z is a complex number)?

(a) One (b) Two (c) Three (d) Five

(c) Given, $z^2 + |z| = 0$ ... (i)

$$\Rightarrow z^2 = -|z| \Rightarrow |z^2| = |-|z|| = |z|$$

$$\Rightarrow \quad |z|^2 - |z| = 0 \quad \Rightarrow |z|(|z|-1) = 0$$

$$\Rightarrow \quad |z| = 0 \text{ or } |z| = 1$$

On putting $|z| = 0$ in Eq. (i), we get

$$z^2 + 0 = 0 \Rightarrow z = 0$$

On putting $|z| = 1$ in Eq. (i), we get

$z^2 + 1 = 0$, is a quadratic equation.

So,, it has two distinct solutions.

Hence, given equation has three distinct solutions.

2016 (I)

Directions (Q. Nos. 28 and 29) *Read the following information carefully and answer the questions given below.*

Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2 = i\bar{z}$, where $i = \sqrt{-1}$.

28. What is $z_1 + z_2 + z_3$ equal to?

(a) i (b) $-i$ (c) 0 (d) 1

29. Consider the following statements

1. $z_1 z_2 z_3$ is purely imaginary.
2. $z_1 z_2 + z_2 z_3 + z_3 z_1$ is purely real.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

Solutions (Q. Nos. 28 and 29)

We have, $z^2 = i\bar{z}$ ...(i)

$$\Rightarrow \quad |z|^2 = |i\bar{z}|$$

$$\Rightarrow \quad |z|^2 = |z| \quad [\because |i\bar{z}| = |z|]$$

$$\Rightarrow \quad |z|^2 - |z| = 0$$

$$\Rightarrow \quad |z|(|z| - 1) = 0$$

As, z is non-zero complex number.

$$\therefore \quad |z| = 1$$

$$\Rightarrow \quad |z|^2 = 1$$

$$\Rightarrow \quad z\bar{z} = 1 \quad \Rightarrow \quad \bar{z} = \frac{1}{z}$$

From Eq. (i), we have

$$z^2 = i\bar{z} \quad \Rightarrow \quad z^2 = i\frac{1}{z}$$

$$\Rightarrow \quad z^3 = i \quad \Rightarrow \quad z^3 - i = 0 \quad ...(ii)$$

If z_1, z_2 and z_3 satisfying the Eq. (ii), then

$$z_1 + z_2 + z_3 = 0$$

$$z_1z_2 + z_2z_3 + z_1z_3 = 0 \text{ and } z_1z_2z_3 = i$$

28. (c) We have, $z_1 + z_2 + z_3 = 0$

29. (c) We have, $z_1z_2z_3 = i$

$\Rightarrow z_1z_2z_3$ is purely imaginary.

$\therefore$ Statement 1 is correct and

$z_1z_2 + z_2z_3 + z_3z_1 = 0$

$\Rightarrow z_1z_2 + z_2z_3 + z_3z_1$ is purely real.

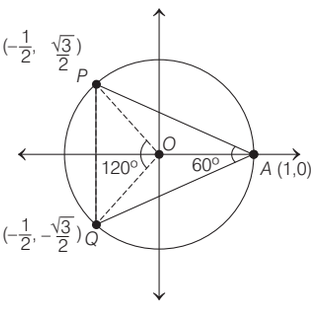
$\therefore$ Statement 2 is correct.

30. Suppose, ω is a cube root of unity with $\omega \neq 1$. Suppose, P and Q are the points on the complex plane defined by ω and ω^2. If O is the origin, then what is the angle between OP and OQ?

(a) 60° (b) 90°
(c) 120° (d) 150°

(c) We know that, nth roots of unity represent vertex of a regular polygon in argand plane.

$\therefore$ Cube roots of unity 1, ω and ω^2 represent the vertex of an equilateral triangle.



$\therefore$ Required angle ($\angle POQ$) = 120°

31. Suppose, ω_1 and ω_2 are two distinct cube roots of unity different from 1. Then, what is $(\omega_1 - \omega_2)^2$ equal to?

(a) 3 (b) 1 (c) –1 (d) –3

(d) Given, $(\omega_1 - \omega_2)^2 = \omega_1^2 + \omega_2^2 - 2\omega_1\omega_2$

Let $\omega_1 = \omega^2$

and $\omega_2 = \omega$

$\therefore (\omega_1 - \omega_2)^2 = (\omega^2)^2 + \omega^2 - 2\omega^2 \cdot \omega$

$= \omega^4 + \omega^2 - 2\omega^3$

$= \omega^3 \cdot \omega + \omega^2 - 2\omega^3$

$= \omega + \omega^2 - 2 \quad [\because \omega^3 = 1]$

$= -1 - 2 \quad [\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow \quad (\omega_1 - \omega_2)^2 = -3$

2015 (II)

32. If z_1 and z_2 are complex numbers with $|z_1| = |z_2|$, then which of the following is/are correct?

1. $z_1 = z_2$
2. Real part of z_1 = Real part of z_2
3. Imaginary part of z_1 = Imaginary part of z_2

Select the correct answer using the code given below.

(a) Only 1 (b) Only 2
(c) Only 3 (d) None of these

(d) We have, $|z_1| = |z_2|$

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$\therefore \quad x_1^2 + y_1^2 = x_2^2 + y_2^2$

$\Rightarrow \quad (x_1^2 - x_2^2) + (y_1^2 - y_2^2) = 0$

$\Rightarrow x_1^2 - x_2^2 = 0$ or $y_1^2 - y_2^2 = 0$

$\Rightarrow \quad x_1 = \pm x_2$ or $y_1 = \pm y_2$

e.g. Let $z_1 = 1 + i$ and $z_2 = -1 - i$

Then, $|z_1| = \sqrt{2}$ and $|z_2| = \sqrt{2}$

$\therefore$ Re $(z_1) \neq$ Re (z_2) and Im $(z_1) \neq$ Im (z_2) and $z_1 \neq z_2$

33. If the point $z_1 = 1 + i$, where $i = \sqrt{-1}$ is the reflection of a point $z_2 = x + iy$ in the line $i\bar{z} - iz = 5$, then the point z_2 is

(a) $1 + 4i$ (b) $4 + i$ (c) $1 - i$ (d) $-1 - i$

(a) Given equation of line is $i\bar{z} - iz = 5$.

Let, $z = x + iy$, then $\bar{z} = x - iy$

$\therefore \quad i(x - iy) - i(x + iy) = 5 \Rightarrow 2y = 5$

$\Rightarrow \quad 2y - 5 = 0 \quad \ldots(i)$

Since, it is given that reflection of point $x + iy$, i.e. (x, y) about the line (i) is $(1 + i)$, i.e. (1, 1).

$\therefore \quad \frac{1-x}{0} = \frac{1-y}{2} = \frac{-2(2y-5)}{4}$

$\Rightarrow \quad \frac{1-x}{0} = \frac{-(2y-5)}{2}$

and $\frac{1-y}{2} = -\frac{(2y-5)}{2}$

$\Rightarrow \quad x = 1$ and $1 - y = -2y + 5$

$\Rightarrow \quad x = 1$ and $y = 4$

$\therefore$ Required point is $1 + 4i$.

34. $z\bar{z} + (3 - i)z + (3 + i)\bar{z} + 1 = 0$ represents a circle with

(a) centre (– 3, – 1) and radius 3
(b) centre (– 3, 1) and radius 3
(c) centre (– 3, – 1) and radius 4
(d) centre (– 3, 1) and radius 4

(a) Given,

$z\bar{z} + (3 - i)z + (3 + i)\bar{z} + 1 = 0$

Put, $z = x + iy$ and $\bar{z} = x - iy$, we get

$(x + iy)(x - iy) + (3 - i)$

$(x + iy) + (3 + i)(x - iy) + 1 = 0$

$\Rightarrow \quad x^2 + y^2 + 3x + 3iy - ix + y + 3x - 3iy + ix + y + 1 = 0$

$\Rightarrow \quad x^2 + y^2 + 6x + 2y + 1 = 0$

Comparing equation by

$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $= (-g, -f) = (-3, -1)$

Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{9 + 1 - 1}$

$= \sqrt{9} = 3$

2015 (I)

35. If $z = \frac{-2(1 + 2i)}{(3 + i)}$, where $i = \sqrt{-1}$, then the argument $\theta\,(-\pi < \theta \leq \pi)$ of z is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$
(c) $\frac{5\pi}{6}$ (d) $-\frac{3\pi}{4}$

(d) We have, $z = \frac{-2(1 + 2i)}{3 + i}$

$= \frac{-2(1 + 2i)(3 - i)}{(3 + i)(3 - i)}$

$= \frac{-2(3 - i + 6i - 2i^2)}{9 - i^2}$

$= \frac{-2}{10}(3 + 5i + 2)$

$= \frac{-10}{10}(i + 1) = -i - 1$

$\therefore$ Argument $(\theta) = -\pi + \theta$

$= -\pi + \tan^{-1}\left(\frac{-1}{1}\right)$

$= -\pi + \frac{\pi}{4}$

$= \frac{-4\pi + \pi}{4}$

$= \frac{-3\pi}{4}$

36. If 1, ω and ω^2 are the cube roots of unity, then the value of $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$ is

(a) –1 (b) 0 (c) 1 (d) 2

(c) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$

$= (1 + \omega + \omega^2 + \omega^3)(1 + \omega^3 \cdot \omega)[1 + (\omega^3)^2 \cdot \omega^2]$

$= 1 \cdot (1 + \omega)(1 + \omega^2)$

$[\because 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1]$

$= (1 + \omega + \omega^2 + \omega^3) = 1$

37. What is the square root of i, where $i = \sqrt{-1}$?

(a) $\frac{1+i}{2}$ (b) $\frac{1-i}{2}$
(c) $\frac{1+i}{\sqrt{2}}$ (d) None of these

(c) Given, $i = \sqrt{-1}$

$i = (-1)^{1/2} = (\cos\pi + i\sin\pi)^{1/2}$

$= e^{\frac{i\pi}{2}}$

$\therefore \quad \sqrt{i} = \sqrt{e^{i\cdot\frac{\pi}{2}}} = e^{i\cdot\frac{\pi}{4}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = \frac{1+i}{\sqrt{2}}$

38. $(x^3 - 1)$ can be factorised as

(a) $(x - 1)(x - \omega)(x + \omega^2)$
(b) $(x - 1)(x - \omega)(x - \omega^2)$
(c) $(x - 1)(x + \omega)(x + \omega^2)$
(d) $(x - 1)(x + \omega)(x - \omega^2)$

where, ω is one of the cube roots of unity.

(b) $(x^3 - 1) = (x - 1)(x^2 + 1 + x)$

$= (x - 1)(x^2 + x - \omega - \omega^2)$

$[\because 1 + \omega + \omega^2 = 0]$

$= (x - 1)(x^2 - \omega^2 + x - \omega)$

$= (x - 1)[(x + \omega)(x - \omega)(x - \omega)]$

$= (x - 1)(x - \omega)(x + \omega + 1)$

$= (x - 1)(x - \omega)(x - \omega^2)$

39. What is $\left[\frac{\sin\frac{\pi}{6} + i\left(1 - \cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6} - i\left(1 - \cos\frac{\pi}{6}\right)}\right]^3$, where, $i = \sqrt{-1}$, equal to?

(a) 1 (b) –1
(c) i (d) $-i$

(c) Let $z = \left[\frac{\sin\frac{\pi}{6} + i\left(1 - \cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6} - i\left(1 - \cos\frac{\pi}{6}\right)}\right]^3$

$= \left[\frac{2\sin\frac{\pi}{12}\cos\frac{\pi}{12} + i \cdot 2\sin^2\frac{\pi}{12}}{2\sin\frac{\pi}{12}\cos\frac{\pi}{12} - i \cdot 2\sin^2\frac{\pi}{12}}\right]^3$

$$= \left[\frac{\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}}{\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}}\right]^3$$

$$= \left[\frac{\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}{\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}\right]^3$$

[by rationalisation]

$$= \left[\frac{\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^2}{\cos^2\frac{\pi}{12} + \sin^2\frac{\pi}{12}}\right]^3$$

$$= \left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^6$$

$$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$$

$[\because (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta]$

40. What is the real part of $(\sin x + i\cos x)^3$, where $i = \sqrt{-1}$?

(a) $-\cos 3x$ (b) $-\sin 3x$
(c) $\sin 3x$ (d) $\cos 3x$

(b) $(\sin x + i\cos x)^3$

$$= \left[\cos\left(\frac{\pi}{2} - x\right) + i\left(\sin\frac{\pi}{2} - x\right)\right]^3$$

$$= \left[e^{i\left(\frac{\pi}{2} - x\right)}\right]^3 \quad [\because e^{i\theta} = \cos\theta + i\sin\theta]$$

$$= e^{3i\left(\frac{\pi}{2} - x\right)}$$

$$= \cos 3\left(\frac{\pi}{2} - x\right) + i\sin 3\left(\frac{\pi}{2} - x\right)$$

$$= \cos\left(\frac{3\pi}{2} - 3x\right) + i\sin\left(\frac{3\pi}{2} - 3x\right)$$

$$= (-\sin 3x - i\cos 3x)$$

Hence, the real part is $-\sin 3x$.

2014 (II)

41. What is $\frac{(1+i)^{4n+5}}{(1-i)^{4n+3}}$ equal to, where, n is a natural number and $i = \sqrt{-1}$?

(a) 2 (b) $2i$ (c) $-2i$ (d) i

(a) $\frac{(1+i)^{4n+5}}{(1-i)^{4n+3}} = \frac{(1+i)^{4n+3}\cdot(1+i)^2}{(1-i)^{4n+3}}$

$[\because a^m \cdot a^n = a^{m+n}]$

$$= \left(\frac{1+i}{1-i}\right)^{4n+3}\cdot(1+i)^2$$

$$= \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^{4n+3}\cdot(1+i^2+2i)$$

$$= \left[\frac{1+i^2+2i}{1+1}\right]^{4n+3}\cdot 2i$$

$$= (i)^{4n+3}\cdot 2i = 2(i)^{4n+4} = 2$$

42. What is $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6$ equal to, where $i = \sqrt{-1}$?

(a) 1 (b) 1/6 (c) 6 (d) 2

(a) $\frac{\sqrt{3}+i}{\sqrt{3}-i} = \frac{\sqrt{3}+i}{\sqrt{3}-i}\times\frac{\sqrt{3}+i}{\sqrt{3}+i}$

$$= \frac{3 + i^2 + 2\sqrt{3}i}{3 - i^2} = \frac{3 - 1 + 2\sqrt{3}i}{3+1}$$

$$= \frac{2(1+\sqrt{3}i)}{4} = \frac{1}{2} + i\,\frac{\sqrt{3}}{2}$$

$$= \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = e^{i\frac{\pi}{3}}$$

$$\therefore \left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6 = (e^{i\frac{\pi}{3}})^6 = e^{i2\pi}$$

$$= \cos 2\pi + i\sin 2\pi = 1 + 0\cdot i = 1$$

43. If z be a complex number such that $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$, then what is z equal to, where $i = \sqrt{-1}$?

(a) $2\sqrt{3} + 2i$ (b) $2\sqrt{3} - 2i$
(c) $-2\sqrt{3} + 2i$ (d) $-\sqrt{3} + i$

(d) Given, $\arg(z) = \frac{5\pi}{6}$

So, z lies in second quadrant.

Let $z = a + ib$, then $|z| = a^2 + b^2 = 4$

Also, $\theta = \left(\pi - \frac{5\pi}{6}\right) = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$

$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \left[\because \tan\theta = \left(\frac{b}{a}\right)\right]$

$\therefore \quad z = -\sqrt{3} + i \quad [\because a < 0 \text{ and } b > 0]$

2014 (I)

44. If P and Q are two complex numbers, then the modulus of the quotient of P and Q is

(a) greater than the quotient of their moduli
(b) less than the quotient of their moduli
(c) less than or equal to the quotient of their moduli
(d) equal to the quotient of their moduli

(d) Let $P = x + iy$ and $Q = \alpha + i\beta$ be two complex numbers.

Then, its quotient $= \frac{P}{Q} = \frac{x+iy}{\alpha+i\beta}$

Now, $\left|\frac{P}{Q}\right| = \left|\frac{x+iy}{\alpha+i\beta}\right| = \frac{|x+iy|}{|\alpha+i\beta|}$

$$= \frac{\sqrt{x^2+y^2}}{\sqrt{\alpha^2+\beta^2}} = \sqrt{\frac{x^2+y^2}{\alpha^2+\beta^2}}$$

Hence, the modulus of the quotient of P and Q is equal to the quotient of their moduli *i.e.* $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$.

45. Let $z = x + iy$, where x, y are real variables and $i = \sqrt{-1}$. If $|2z - 1| = |z - 2|$, then the point z describes

(a) a circle (b) an ellipse
(c) a hyperbola (d) a parabola

(a) Given that, $z = x + iy$ and x, y are real vaeriables

We have, $|2z - 1| = |z - 2|$

$\Rightarrow \quad |2(x + iy) - 1| = |x + iy - 2|$

$\Rightarrow \quad |(2x - 1) + 2iy| = |(x - 2) + iy|$

$\Rightarrow \quad |(2x - 1) + 2iy|^2 = |(x - 2) + iy|^2$

$\Rightarrow \quad (2x - 1)^2 + (2y)^2 = (x - 2)^2 + y^2$

$\Rightarrow \quad 4x^2 + 1 - 4x + 4y^2 = x^2 + 4 - 4x + y^2$

$\Rightarrow 3x^2 + 3y^2 - 3 = 0 \Rightarrow x^2 + y^2 = 1$

which represents a circle.

Hence, point z describes a circle.

46. If $|z + \bar{z}| = |z - \bar{z}|$, then the locus of z is

(a) a pair of straight lines
(b) a line
(c) a set of four straight lines
(d) a circle

(b) Given, $|z + \bar{z}| = |z - \bar{z}|$

Let $z = x + iy$, then $\bar{z} = x - iy$

$\therefore \quad |x + iy + x - iy| = |x + iy - x + iy|$

$\Rightarrow \quad |2x| = |2iy|$

$\Rightarrow \quad 2x = 2y \; [\because |a + ib| = \sqrt{a^2 + b^2}]$

$\therefore \quad x = y$

Which represents a line passing through the origin.

Hence, the locus of z is a line.

47. What is the argument of the complex number $\frac{(1+i)(2+i)}{3-i}$, where $i = \sqrt{-1}$?

(a) 0 (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(d) Let $z = \frac{(1+i)(2+i)}{3-i}$

$$= \frac{2 + 3i + i^2}{3 - i} = \frac{2 + 3i - 1}{3 - i} \quad [\because i^2 = -1]$$

$$= \frac{1 + 3i}{3 - i}\times\frac{3 + i}{3 + i} = \frac{3 + 10i + 3i^2}{9 - i^2}$$

$$= \frac{3 + 10i - 3}{9 + 1} = \frac{10i}{10} = i \Rightarrow z = 0 + i\cdot 1$$

So, $\arg(z) = \tan^{-1}\left(\frac{1}{0}\right)$

$$= \tan^{-1}(\infty) = \frac{\pi}{2}$$

3

BINARY NUMBERS

2019 (II)

1. A binary number is represented by $(cdccddcccddd)_2$, where $c > d$. What is its decimal equivalent?

(a) 1848 (b) 2048 (c) 2842 (d) 2872

(d) Binary number = $(cdccddcccddd)_2$ where, $c > d$. We know that only two bit (digits) 0 and 1 be any binary number.

$\therefore$ Given binary number

$= (101100111000)_2$

$= (1\times2^{11} + 0\times2^{10} + 1\times2^9 + 1\times2^8 +$
$+ 0\times2^7 + 0\times2^6 + 1\times2^5 + 1\times2^4$

$+ 1\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0)_{10}$

$= (2048 + 512 + 256 + 32 + 16 + 8)_{10}$

$= (2872)_{10}$

2018 (II)

2. The sum of the binary numbers $(11011)_2$, $(10110110)_2$ and $(10011x0y)_2$ is the binary numbers $(101101101)_2$. What are the values of x and y ?

(a) $x = 1, y = 1$
(b) $x = 1, y = 0$
(c) $x = 0, y = 1$
(d) $x = 0, y = 0$

(b) Sum of the binary number $(11011)_2$, $(10110110)_2$ and $(10011x0y)_2$ is $(101101101)_2$

i.e. $(11011)_2 + (10110110)_2$
$+ (10011x0y)_2 = (101101101)_2$

$\Rightarrow$ $(10011x0y)_2 = (101101101)_2$
$- (11011)_2 - (10110110)_2$

So, 101101101
− 10110110
10110111
− 11011
10011100

Compare with $(10011x0y)_2$

We get, $x = 1$

and $y = 0$

2018 (I)

3. The binary number expression of the decimal number 31 is

(a) 1111 (b) 10111
(c) 11011 (d) 11111

(d)

2	31	Remainder
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$\therefore (31)_{10} = (11111)_2$

2017 (II)

4. The remainder and the quotient of the binary division $(101110)_2 \div (110)_2$ are respectively.

(a) $(111)_2$ and $(100)_2$
(b) $(100)_2$ and $(111)_2$
(c) $(101)_2$ and $(101)_2$
(d) $(100)_2$ and $(100)_2$

(b)

1 0 1 1 1 0

$0 \times 2^0 = 0$
$1 \times 2^1 = 2$
$1 \times 2^2 = 4$
$1 \times 2^3 = 8$
$0 \times 2^4 = 0$
$1 \times 2^5 = 32$
Total 46

1 1 0

$0 \times 2^0 = 0$
$1 \times 2^1 = 2$
$1 \times 2^2 = 4$
Total 6

$\therefore (101110)_2 = (46)_{10}$ $\therefore (110)_2 = (6)_{10}$

Now, 6) 46 (7
42
4

$\therefore$ Quotient = 7,

2	7	
2	3	1
2	1	1
	0	1

$\therefore$ $(7)_{10} = (111)_2$

$\therefore$ Quotient = $(111)_2$ Remainder = 4

2	4	
2	2	0
2	1	0
	0	1

$\therefore$ $(4)_{10} = (100)_2$

$\therefore$ Remainder = $(100)_2$

2016 (II)

5. What is the binary equivalent of the decimal number 0.3125?

(a) 0.0111 (b) 0.1010
(c) 0.0101 (d) 0.1101

(c) $0.3125 \times 2 = 0.625$, carry = 0

$0.625 \times 2 = 1.25$, carry = 1

$0.25 \times 2 = 0.50$, carry = 0

$0.50 \times 2 = 1.00$, carry = 1

$(0.3125)_{10} = (0.0101)_2$

2015 (II)

6. What is $(100000001)_2 - (0.0101)_2$ equal to?

(a) $(512.6775)_{10}$
(b) $(512.6875)_{10}$
(c) $(512.6975)_{10}$
(d) $(512.0909)_{10}$

(b) We have, $(1000000001)_2 - (0.0101)_2$

$= (2^9 + 2^0) - (1 \times 2^{-2} + 1 \times 2^{-4})$

$= (512 + 1) - \left(\frac{1}{4} + \frac{1}{16}\right)$

$= 513 - \frac{5}{16} = 513 - 0.3125$

$= (512.6875)_{10}$

2015 (I)

7. The decimal number $(127.25)_{10}$, when converted to binary number, takes the form

(a) $(1111111.11)_2$ (b) $(1111110.01)_2$
(c) $(1110111.11)_2$ (d) $(1111111.01)_2$

(d) $(127.25)_{10} = (?)_2$

2	127	
2	63	1
2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

Here, $0.25 \times 2 = 0.50$ carry 0

and $0.50 \times 2 = 1.00$ carry 1

$\therefore \quad (127.25)_{10} = (1111111.01)_2$

8. If $(1110101)_2$ is converted to decimal system, then the resulting number is

(a) 235 (b) 175
(c) 160 (d) 126

(a) $(11101011)_2$

$= [1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0]_{10}$

$= [128 + 64 + 32 + 0 + 8 + 0 + 2 + 1]_{10}$

$= (235)_{10}$

2014 (II)

9. What is $(1001)_2$ equal to?

(a) $(5)_{10}$ (b) $(9)_{10}$ (c) $(17)_{10}$ (d) $(11)_{10}$

(b) $(1001)_2$

$= (2^3 \times 1 + 2^2 \times 0 + 2^1 \times 0 + 2^0 \times 1)_{10}$

$= (8 + 1)_{10} = (9)_{10}$

2014 (I)

10. What is the sum of the two numbers $(11110)_2$ and $(1010)_2$?

(a) $(101000)_2$ (b) $(110000)_2$
(c) $(100100)_2$ (d) $(101100)_2$

(a) Given binary numbers,

$(11110)_2 = 2^4 \times 1 + 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 0$

$= 16 + 8 + 4 + 2 + 0 = (30)_{10}$

and $(1010)_2 = 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 0$

$= 8 + 0 + 2 + 0 = (10)_{10}$

$\therefore$ Required sum $= (11110)_2 + (1010)_2$

$= (30 + 10)_{10} = (40)_{10}$

2	40	
2	20	0
2	10	0
2	5	0
2	2	1
2	1	0
	0	1

$\therefore \quad (40)_{10} = (101000)_2$

11. The number 251 in decimal system is expressed in binary system by

(a) 11110111 (b) 11111011
(c) 11111101 (d) 11111110

(b)

2	251	
2	125	1
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$\therefore \quad (251)_{10} = (11111011)_2$

which is the required binary system.

4

SEQUENCES AND SERIES

2019 (II)

1. If 3rd, 8th and 13th terms of a GP are p, q and r respectively, then which one of the following is correct?

(a) $q^2 = pr$ (b) $r^2 = pq$
(c) $pqr = 1$ (d) $2q = p + r$

(a) Let first term and common ratio of a GP be a and R.

$\therefore \quad T_3 = aR^2 = p \quad \ldots(i)$

$T_8 = aR^7 = q \quad \ldots(ii)$

$T_{13} = aR^{12} = r \quad \ldots(iii)$

Multiplying of Eqs. (i) and (iii)

$(aR^2)(aR^{12}) = pr$

$\Rightarrow \quad a^2R^{14} = pr \Rightarrow (aR^7)^2 = pr$

$\Rightarrow \quad q^2 = pr$ [from Eq. (ii)]

2. Let S_n be the sum of the first n terms of an AP. If $S_{2n} = 3n + 14n^2$, then what is the common difference?

(a) 5 (b) 6 (c) 7 (d) 9

(c) $S_{2n} = 3n + 14n^2$ (S_n be the sum of first n terms of an AP)

$\Rightarrow \quad S_{2n} = \frac{3}{2}.(2n) + \frac{7}{2}(2n)^2$

Put $2n = n$

we get, $S_n = \frac{3n}{2} + \frac{7n^2}{2}$

$\therefore T_n = S_n - S_{n-1}$

$= \frac{3}{2}n + \frac{7n^2}{2} - \frac{3}{2}(n-1) - \frac{7}{2}(n-1)^2$

$= \frac{3}{2}n + \frac{7}{2}n^2 - \frac{3}{2}n + \frac{3}{2} - \frac{7}{2}n^2 - \frac{7}{2} + \frac{7}{2}2n$

$T_n = 7n - 2$

Put $n = 1, 2, \ldots.$

$T_1 = 7(1) - 2 = 5$

$T_2 = 7(2) - 2 = 12$

$\therefore \quad d = T_2 - T_1 = 12 - 5 = 7$

3. How many two-digit numbers are divisible by 4?

(a) 21 (b) 22
(c) 24 (d) 25

(b) Series of two-digit number that divisible by 4 is

12, 16, 20, ..., 96

This series is an AP

Here, $a = 12, d = 4, l = 96$

Let total number of terms be n.

$\therefore \quad l = a + (n-1)d$

$\Rightarrow \quad 96 = 12 + (n-1)4$

$\Rightarrow \quad 84 = (n-1)4$

$\Rightarrow \quad n - 1 = 21$

$\Rightarrow \quad n = 21 + 1 = 22$

4. If the sum of first n terms of a series is $(n + 12)$, then what is its third term?

(a) 1 (b) 2 (c) 3 (d) 4

(a) Sum of first n term of a series $= n + 12$

$\Rightarrow a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_n = n + 12$

Put $n = 1$,

$a_1 = 1 + 12 = 13$

Put $n = 2$,

$a_1 + a_2 = 2 + 12 \Rightarrow a_1 + a_2 = 14$

$\Rightarrow 13 + a_2 = 14 \Rightarrow a_2 = 1$

Put $n = 3$

$a_1 + a_2 + a_3 = 3 + 12$

$\Rightarrow \quad 13 + 1 + a_3 = 15$

$\Rightarrow \quad a_3 = 15 - 14 = 1$

5. Let a, b, c be in AP and $k \neq 0$ be a real number. Which of the following are correct?

1. ka, kb, kc are in AP
2. $k - a, k - b, k - c$ are in AP
3. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP

Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(d) a, b, c are in AP.

We know that equal number addition, subtraction and multiply, divide, by equal number of each term of an AP, the resultent, series be an AP.

$\therefore ka, kb, kc$ are in AP (multiplying by k).

$k - a, k - b, k - c$ are in AP (subtraction from k)

and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP (divide by k)

Hence, option (d) is correct answer.

6. Let m and $n (m < n)$ be the roots of the equation $x^2 - 16x + 39 = 0$. If four terms p, q, r and s are inserted between m and n to form an AP, then what is the value of $p + q + r + s$?

(a) 29 (b) 30 (c) 32 (d) 35

(c) m and n be the roots of the equation $x^2 - 16x + 39 = 0$ $(m < n)$.

$\therefore \quad m + n = 16 \quad \ldots(i)$

and $\quad mn = 39 \quad \ldots(ii)$

We know that, $n - m = \sqrt{(m+n)^2 - 4mn}$ $(\because m < n)$

$= \sqrt{256 - 156}$

$= \sqrt{100}$

$n - m = 10 \quad \ldots(iii)$

Solving the Eqs. (ii) and (iii), $n = 13, m = 3$

Four terms p, q, r and s are inserted between m and n to form an AP.

$\therefore$ AP is 3, p, q, r, s, 13

Here, $a = 3, l = 13, n = 6$

$\therefore \quad l = a + (n-1)d$

$13 = 3 + (6-1)d$

$\Rightarrow \quad d = 2$

$\therefore$ $p = a + d = 3 + 2 = 5$,
$q = a + 2d = 3 + 4 = 7$
$r = a + 3d = 3 + 6 = 9$,
$d = a + 4d = 3 + 8 = 11$
Now, $p + q + r + s = 5 + 7 + 9 + 11$
$= 32$

7. A geometric progression (GP) consists of 200 terms. If the sum of odd terms of the GP is m, and the sum of even terms of the GP is n, then what is its common ratio?
(a) m / n (b) n / m
(c) $m + (n / m)$ (d) $n + (m / n)$

(***b***) Let a, ar, ar^2..... 200 terms be a geometric progression.
Where, a is the first terms and r be the common ratio.
GP of odd terms a, ar^2, ar^4, ...100 terms.
GP of even terms ar, ar^3, ar^5, ... 100 terms.
$\therefore$ Sum of odd terms of the GP = m

$$\Rightarrow \quad \frac{a\{r^{200} - 1\}}{r - 1} = m \qquad \ldots(i)$$

Sum of even terms of the GP = n

$$\Rightarrow \quad \frac{ar(r^{200} - 1\}}{r - 1} = n \qquad \ldots(ii)$$

Dividing of Eq. (i) by Eq. (ii),

$$\Rightarrow \quad \frac{1}{r} = \frac{m}{n} \Rightarrow r = \frac{n}{m}$$

Hence, the common ratio of the GP is $\frac{n}{m}$.

8. What is the value of $1 - 2 + 3 - 4 + 5 - \ldots\ldots + 101$?
(a) 51 (b) 55
(c) 110 (d) 111

(***a***) Given series,
$= 1 - 2 + 3 - 4 + 5 - \ldots\ldots + 101$
$= (1 + 3 + 5 + \ldots\ldots + 101)$
$- (2 + 4 + 6 + \ldots\ldots + 100)$
$= (1 + 3 + 5 + \ldots\ldots 51 \text{ terms})$
$- (2 + 4 + 6 + \ldots\ldots 50 \text{ terms})$
$= \frac{51}{2}[2 + (51 - 1) \times 2]$
$- \frac{50}{2}[4 + (50 - 1) \times 2]$
[$\because$ both series are AP and

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$= \frac{51}{2} \times 102 - \frac{50}{2} \times 102$
$= 2601 - 2550 = 51$

2019 (I)

9. What is the nth term of the sequence $25, -125, 625, -3125, \ldots$?
(a) $(-5)^{2n-1}$ (b) $(-1)^{2n}5^{n+1}$
(c) $(-1)^{2n-1}5^{n+1}$ (d) $(-1)^{n-1}\ 5^{n+1}$

(***d***) Given, sequence 25, $-125, 625, -3125$
Here, $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots\ldots\ldots$
So, this sequence in GP whose common ratio is -5.
then $a = 25$, $r = -5$
$\therefore n$ th term of sequence $= ar^{n-1}$
$= 25(-5)^{n-1}$
$= (-1)^{n-1}\ 5^2 \times 5^{n-1} = (-1)^{n-1}\ 5^{n+1}$

10. The numbers 1, 5 and 25 can be three terms (not necessarily consecutive) of
(a) only one AP
(b) more than one but finite numbers of APs
(c) infinite number of APs
(d) finite number of GPs

(***d***) We have, 1, 5, 25 be three terms.
Clearly, 1, 5, 25 are finite number of GPs.

11. The sum of $(p + q)$th and $(p - q)$th terms of an AP is equal to
(a) $(2p)$ th term (b) $(2q)$ th term
(c) twice the p th term
(d) twice the q th term

(***c***) Let a is first term and d is common difference of AP.

$$a_{p+q} = a + (p + q - 1)d$$

and $\quad a_{p-q} = a + (p - q - 1)d$
Sum of $(p + q)$th and $(p - q)$th terms
$= a_{p+q} + a_{p-q} = 2a + (2p - 2)d$
$= 2(a + (p - 1)d) = 2a_p$
= twice of p th term

12. What is the fourth term of an AP of n terms whose sum is $n(n + 1)$?
(a) 6 (b) 8 (c) 12 (d) 20

(***b***) Given,
Sum of n terms of an AP
i.e. $S_n = n(n + 1)$
$a_4 = S_4 - S_3$ [$\because a_n = S_n - S_{n-1}$]
$a_4 = 4(4 + 1) - 3(3 + 1)$
$a_4 = 20 - 12$
$= 8$
$\therefore$ Fourth term of an AP = 8

2018 (II)

13. If an infinite GP has the first term x and the sum 5, then which one of the following is correct?
(a) $x < -10$ (b) $-10 < x < 0$
(c) $0 < x < 10$ (d) $x > 10$

(c) Given that, first term of an infinite GP is x and sum = 5

$$\therefore \quad \frac{x}{1 - r} = 5$$

[$\because$ sum of infinite GP $= \frac{a}{1 - r}$]

$$\Rightarrow \quad \frac{x}{5} = 1 - r$$

$$\Rightarrow \quad r = 1 - \frac{x}{5}$$

Where, $|r| < 1$

$$\Rightarrow \quad -1 < 1 - \frac{x}{5} < 1$$

$$\Rightarrow \quad -2 < -\frac{x}{5} < 0$$

$$\Rightarrow \quad -10 < -x < 0$$

$$\Rightarrow \quad 10 > x > 0$$

14. The sum of the series $3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots.$ is equal to
(a) $\frac{20}{9}$ (b) $\frac{9}{20}$
(c) $\frac{9}{4}$ (d) $\frac{4}{9}$

(c) Given series,
$3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots..$ are in GP

$$\therefore \quad r = \frac{-1}{3}$$

$$S_n = \frac{3}{1 - \left(-\frac{1}{3}\right)} \qquad \left[\because S_n = \frac{a}{1 - r}\right]$$

$$= \frac{3}{\frac{4}{3}} = \frac{9}{4}$$

15. Let T_r be the r^{th} term of an AP for $r = 1, 2, 3, \ldots$. If for some distinct positive integers m and n we have $T_m = 1 / n$ and $T_n = 1 / m$, then what is T_{mn} equal to ?
(a) $(mn)^{-1}$ (b) $m^{-1} + n^{-1}$
(c) 1 (d) 0

(c) Let first term of an AP is a and common difference is d
Given that, $\quad T_m = \frac{1}{n}$

$$a + (m-1)d = \frac{1}{n} \quad \ldots(i)$$

and $T_n = \frac{1}{m}a + (n-1)d = \frac{1}{m}$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$(m-1)d - (n-1)d = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow \quad (m-n)d = \frac{m-n}{mn}$$

$$\Rightarrow \quad d = \frac{1}{mn}$$

Put in Eq. (i),

$$a + (m-1)\frac{1}{mn} = \frac{1}{n}$$

$$\Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$$

Now, $T_{mn} = a + (mn-1)d$

$$= \frac{1}{mn} + (mn-1)\frac{1}{mn}$$

$$= \frac{1}{mn} + 1 - \frac{1}{mn}$$

$$T_{mn} = 1$$

16. If the second term of a GP is 2 and the sum of its infinite terms is 8, then the GP is

(a) $8, 2, \frac{1}{2}, \frac{1}{8}, \ldots$ (b) $10, 2, \frac{2}{5}, \frac{2}{25}, \ldots$

(c) $4, 2, 1, \frac{1}{2}, \frac{1}{2^2} \ldots$ (d) $6, 3, \frac{3}{2}, \frac{3}{4}, \ldots$

(c) Let first term of a GP is a and common ratio is r

$$\therefore \quad a_2 = 2$$

$$\Rightarrow \quad ar = 2 \quad \ldots(i)$$

and $S_\infty = 8 \Rightarrow \frac{a}{1-r} = 8$

From Eq. (i)

$$\Rightarrow \quad \frac{2}{r(1-r)} = 8 \Rightarrow 8r - 8r^2 = 2$$

$$\Rightarrow \quad 4r^2 - 4r + 1 = 0$$

$$\Rightarrow 4r^2 - 2r - 2r + 1 = 0$$

$$\Rightarrow \quad 2r(2r-1) - 1(2r-1) = 0$$

$$\Rightarrow \quad (2r-1)(2r-1) = 0 \Rightarrow r = \frac{1}{2}$$

$\therefore$ Putting the value of r in Eq. (i), we get,

$$a\left(\frac{1}{2}\right) = 2 \Rightarrow a = 4$$

$\therefore$ GP is $4, 2, 1, \frac{1}{2} \ldots$

17. If a, b, c are in AP or GP or HP, then $\frac{a-b}{b-c}$ is equal to

(a) $\frac{b}{a}$ or 1 or $\frac{b}{c}$ (b) $\frac{c}{a}$ or $\frac{c}{b}$ or 1

(c) 1 or $\frac{a}{b}$ or $\frac{a}{c}$ (d) 1 or $\frac{a}{b}$ or $\frac{c}{a}$

(c) a, b, c are in AP, then

$$\therefore \quad a - b = b - c$$

$$\Rightarrow \quad \frac{a-b}{b-c} = 1$$

If a, b, c are in GP, then

$$\therefore \quad \frac{a}{b} = \frac{b}{c} \quad \ldots(i)$$

$$\Rightarrow \quad \frac{a-b}{b} = \frac{b-c}{c}$$

(by componando dividendo rule)

$$\Rightarrow \quad \frac{a-b}{b-c} = \frac{b}{c}$$

$$\Rightarrow \quad \frac{a-b}{b-c} = \frac{a}{b} = \frac{b}{c} \quad \text{[from Eq. (i)]}$$

If a, b, c are in HP, then

$$\therefore \quad b = \frac{2ac}{a+c} \Rightarrow ab + bc = 2ac$$

$$\Rightarrow \quad ab - ac = ac - bc$$

$$\Rightarrow \quad a(b-c) = c(a-b) \Rightarrow \frac{a-b}{b-c} = \frac{a}{c}$$

Hence, the value of $\frac{a-b}{b-c}$ is 1 or $\frac{a}{b}$ or $\frac{a}{c}$.

2018 (I)

18. If the ratio of AM and GM of two positive numbers a and b is $5:3$, then $a:b$ is equal to

(a) 3 : 5 (b) 2 : 9

(c) 9 : 1 (d) 5 : 3

(c) Let a and b be two numbers.

According to the question,

$$\frac{\frac{a+b}{2}}{\frac{\sqrt{ab}}{}} = \frac{5}{3}$$

$$\left[\because A : G = 5:3,\ A = \frac{a+b}{2},\ G = \sqrt{ab}\right]$$

$$\Rightarrow \quad \frac{a+b}{\sqrt{ab}} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{(a+b)^2}{ab} = \left(\frac{10}{3}\right)^2$$

$$\Rightarrow \quad \frac{a^2 + b^2 + 2ab}{ab} = \frac{100}{9}$$

$$\Rightarrow \quad \frac{a}{b} + \frac{b}{a} + 2 = \frac{100}{9}$$

$$\Rightarrow \quad t + \frac{1}{t} + 2 = \frac{100}{9} \qquad \left[\because \frac{a}{b} = t\right]$$

$$\Rightarrow \quad \frac{t^2 + 1 + 2t}{t} = \frac{100}{9}$$

$$\Rightarrow \quad 9t^2 - 82t + 9 = 0$$

$$\Rightarrow \quad (t-9)(9t-1) = 0 \Rightarrow t = 9, \frac{1}{9}$$

$$\therefore \quad \frac{a}{b} = 9 \text{ or } \frac{a}{b} = \frac{1}{9} \qquad \left[\because t = \frac{a}{b}\right]$$

$$\Rightarrow \quad a : b = 9:1 \text{ or } 1:9$$

19. If $x = 1 - y + y^2 - y^3 \ldots$ up to infinite terms, where $|y| < 1$, then which one of the following is correct?

(a) $x = \frac{1}{1+y}$ (b) $x = \frac{1}{1-y}$

(c) $x = \frac{y}{1+y}$ (d) $x = \frac{y}{1-y}$

(a) We have,

$$x = 1 - y + y^2 - y^3 + \ldots \infty, |y| < 1$$

$$= \frac{1}{1-(-y)}$$

$$\left[\because a + ar + ar^2 + \ldots \infty = \frac{a}{1-r},\ r < 1\right]$$

$$= \frac{1}{1+y}$$

20. What is the sum of all two-digit numbers, which when divided by 3 leave 2 as the remainder ?

(a) 1565 (b) 1585

(c) 1635 (d) 1655

(c) Required numbers are 11, 14, 17, ... 98 which is an AP.

Here, $a = 11, a_n = 98$

We know that, $a_n = a + (n-1)d$

$$\Rightarrow \quad 98 = 11 + (n-1)(3)$$

$$\Rightarrow \quad 98 = 11 + 3n - 3 \Rightarrow 98 = 3n + 8$$

$$\Rightarrow \quad 90 = 3n \Rightarrow n = 30$$

$\therefore$ Required sum

$$= 11 + 14 + 17 + \ldots + 98$$

$$= \frac{30}{2}[11 + 98] \qquad \left[\because S_n = \frac{n}{2}(a+l)\right]$$

$$= 15 \times 109 = 1635$$

21. The third term of a GP is 3. What is the product of the first five terms?

(a) 216 (b) 226

(c) 243

(d) Cannot be determined due to insufficient data

(c) Let a and r be the first term and common ratio of of the GP.

According to the question,

$$a_3 = 3$$

$$\Rightarrow \quad ar^2 = 3 \qquad [\because a_n = ar^{n-1}] \ \ldots(i)$$

Required product $= a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5$

$$= (a)(ar)(ar^2)(ar^3)(ar^4)$$

$$= a^5 r^{10} = (ar^2)^5$$

$$= (3)^5 \qquad \text{[from Eq. (i)]}$$

$$= 243$$

22. If $x, \frac{3}{2}, z$ are in AP, $x, 3, z$ are in GP, then which one of the following will be in HP?

(a) $x, 6, z$ (b) $x, 4, z$

(c) $x, 2, z$ (d) $x, 1, z$

(a) We have, $x, \frac{3}{2}, z$ are in AP.

$$\Rightarrow \quad \frac{x+z}{2} = \frac{3}{2}$$

$$\Rightarrow \quad x + z = 3 \quad \ldots(i)$$

Also, x, 3, z are in GP

$\Rightarrow \quad xz = 3^2$

$\Rightarrow \quad xz = 9$ … (ii)

Now, from Eqs. (i) and (ii), we have

$$\frac{2xz}{x+y} = \frac{2\times 9}{3} \Rightarrow \frac{2xz}{x+z} = 6$$

$\therefore$ x, 6, z are in HP.

2017 (II)

23. The value of the product $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times \ldots$ up to infinite terms is

(a) 6 (b) 36
(c) 216 (d) 512

(a) $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times \ldots\ldots$ up infinite terms

$$= 6^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots\infty} = 6^{\frac{1/2}{1-\frac{1}{2}}}$$

[$\because$ we know that, sum of an infinite GP with first terms a and common ratio r is

$S_\infty = \frac{a}{1-r}$, where $-1 < r < 1$] $= 6$

24. A person is to count 4500 notes. Let a_n denote the number of notes he counts in the n^{th} minute.

If $a_1 = a_2 = a_3 = \ldots = a_{10} = 150$, and $a_{10}, a_{11}, a_{12}, \ldots$ are in AP with the common difference -2, then the time taken by him to count all the notes is

(a) 24 min (b) 34 min
(c) 125 min (d) 135 min

(b) Given,

$a_1 = a_2 = a_3 = \ldots\ldots = a_{10} = 150$

$\therefore$ From question

Number of notes counted by the person in 9 min $= 9 \times 150 = 1350$

Let total minutes to count all the 4500 notes by the person $= t$

$\therefore$ Remaining minutes after 9 min

$= t - 9$

We know, sum of n terms of an AP whose first term 'a' and common difference d is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now, from question,

$$1350 + \frac{t-9}{2}$$

$$[2 \times 150 + \{(t-9) - 1\}(-2)] = 4500$$

[$\because$ total number of notes = 4500 and common difference = -2]

$$1350 + \frac{t-9}{2}[300 - 2t + 18] = 4500$$

$t^2 - 169t + 4590 = 0$, $(t-135)(t-34) = 0$

$\therefore \quad t = 135$ or $t = 34$

Here, $t = 135$ [impossible]

$\therefore \quad t = 34$ min

25. If $y = x + x^2 + x^3 + \ldots$ up to infinite terms where $x < 1$, then which one of the following is correct?

(a) $x = \frac{y}{1+y}$ (b) $x = \frac{y}{1-y}$
(c) $x = \frac{1+y}{y}$ (d) $x = \frac{1-y}{y}$

(a) Given, $y = x + x^2 + x^3 + \ldots\ldots$ upto infinite terms

where, $x < 1$

Which is sum of infinite term of GP whose first term is x and common ratio is x when $x < 1$.

$\therefore \quad y = \frac{x}{1-x}$

[$\because$ sum of infinite terms of GP whose first term is a and common ratio $r = \frac{a}{1-r}$ where, $r < 1$]

$\Rightarrow \quad x = \frac{y}{1+y}$

26. The value of $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots$up to infinite terms is

(a) $\log_e 9$ (b) 0 (c) 1 (d) $\log_e 3$

(a) Let

$$y = \frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots\infty$$

$$= \frac{1}{\log_3 e} + \frac{1}{2\log_3 e} + \frac{1}{4\log_3 e} + \ldots\infty$$

$$= \frac{1}{\log_3 e}\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots.\infty\right]$$

$$= \frac{1}{\log_3 e}\left[\frac{1}{1-\frac{1}{2}}\right] = \frac{1}{\log_3 e} \cdot 2$$

$= 2\log_e 3 = \log_e 3^2 = \log_e 9$

27. If $1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \ldots + n\cdot 3^n = \frac{(2n-1)3^a + b}{4}$, then a and b are respectively.

(a) $n, 2$ (b) $n, 3$
(c) $n+1, 2$ (d) $n+1, 3$

(d) Given, series is

$1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \ldots\ldots + n\cdot 3^n$

The above series is AGP

Let, $S_n = 1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \ldots\ldots + n\cdot 3^n$ …(i)

$\Rightarrow \quad 3S_n = 1\cdot 3^2 + 2\cdot 3^2 + \ldots\ldots + (n-1)3^n + n\cdot 3^{n+1}$ …(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$-2S_n = 1\cdot 3 + [3^2 + 3^3 + \ldots\ldots + 3^n] - n\cdot 3^{n+1}$

$\Rightarrow -2S_n = (3 + 3^2 + 3^3 + \ldots\ldots + 3^n) - n\cdot 3^{n+1}$

$\Rightarrow \quad -2S_n = (3 + 3^2 + 3^3 + \ldots\ldots + 3^n) - n\cdot 3^{n+1}$

$\Rightarrow \quad -2S_n = 3\left[\frac{3^n - 1}{3-1}\right] - n\cdot 3^{n+1}$

$\Rightarrow \quad S_n = \frac{n}{2}\cdot 3^{n+1} - \frac{3}{4}(3^n - 1)$

$\Rightarrow \quad S_n = \frac{2n\cdot 3^{n+1} - 3^{n+1} + 3}{4}$

$\Rightarrow \quad S_n = \frac{(2n-1)3^{n+1} + 3}{4}$

On comparing by RHS

$a = n + 1$, $b = 3$

28. If $S_n = nP + \frac{n(n-1)Q}{2}$, where, S_n denotes the sum of the first n terms of an AP, then the common difference is

(a) $P + Q$ (b) $2P + 3Q$
(c) $2Q$ (d) Q

(d) We have,

$$S_n = nP + \frac{n(n-1)Q}{2} \quad \ldots(i)$$

We know, sum of n terms of AP

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = na + \frac{n(n-1)}{2}d \quad \ldots(ii)$$

[Here, a = first term and d = common difference]

By comparing Eqs. (i) and (ii), we get

common difference, $d = Q$

2017 (I)

29. What is the sum of the series $0.3 + 0.33 + 0.333 + \ldots n$ terms?

(a) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

(b) $\frac{1}{3}\left[n - \frac{2}{9}\left(1 - \frac{1}{10^n}\right)\right]$

(c) $\frac{1}{3}\left[n - \frac{1}{3}\left(1 - \frac{1}{10^n}\right)\right]$

(d) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 + \frac{1}{10^n}\right)\right]$

(a) Let $S = 0.3 + 0.33 + 0.333 + \ldots$ n terms

$= 3(0.1 + 0.11 + 0.111 + \ldots + n \text{ terms})$

$= \frac{3}{9}[0.9 + 0.99 + 0.999 + \ldots n \text{ terms}]$

$= \frac{1}{3}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots + n \text{ terms}\right]$

$= \frac{1}{3}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{100}\right) + \ldots + n \text{ terms}\right]$

$= \frac{1}{3}\left[(1 + 1 + 1 + \ldots n \text{ times}) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots n \text{ terms}\right)\right]$

$= \frac{1}{3}\left[n - \frac{\frac{1}{10}\left\{1 - \left(\frac{1}{10}\right)^n\right\}}{1 - 1/10}\right]$

$[\because \text{sum of GP} = \frac{a(1 - r^n)}{1 - r}, r < 1]$

$= \frac{1}{3}\left[n - \frac{1}{9}\left\{1 - \left(\frac{1}{10}\right)^n\right\}\right]$

$= \frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

30. If the sum of m terms of an AP is n and the sum of n terms is m, then the sum of $(m + n)$ terms is

(a) mn (b) $m + n$
(c) $2(m + n)$ (d) $-(m + n)$

(d) Let a and d be the first term and common difference of the given AP respectively.

According to the question, we have

$$\frac{m}{2}[2a + (m - 1)d] = n \quad \ldots(i)$$

and $$\frac{n}{2}[2a + (n - 1)d] = m \quad \ldots(ii)$$

On subtracting, Eq. (ii) from Eq. (i), we get

$2a(m - n) + [(m^2 - n^2) - (m - n)]d = 2(n - m)$

$\Rightarrow 2a(m - n) + (m - n)(m + n - 1)d = 2(n - m)$

$\Rightarrow 2a(m - n) + (m - n)(m + n - 1)d = -2(m - n)$

$\Rightarrow 2a + (m + n - 1)d = -2$

$\therefore S_{m+n} = \frac{m + n}{2}[2a + (m + n - 1)d]$

$= \frac{(m + n)}{2} \times (-2) = -(m + n)$

31. The fifth term of an AP of n terms, whose sum is $n^2 - 2n$, is

(a) 5 (b) 7 (c) 8 (d) 15

(b) Given, $S_n = n^2 - 2n$

On replacing n by $(n - 1)$, we get

$S_{n-1} = (n - 1)^2 - 2(n - 1)$

$= n^2 + 1 - 2n - 2n + 2$

$= n^2 - 4n + 3$

Now, $T_n = S_n - S_{n-1}$

$= (n^2 - 2n) - (n^2 - 4n + 3)$

$= n^2 - 2n - n^2 + 4n - 3$

$= 2n - 3 \quad \ldots(i)$

On putting $n = 5$ in Eq. (i), we get

$T_5 = 2 \times 5 - 3 = 10 - 3 = 7$

32. The sum of all the two-digit odd numbers is

(a) 2475 (b) 2530 (c) 4905 (d) 5049

(a) Two-digit odd numbers are

11, 13, 15, ... , 99

Here $a = 11, d = 2, l = 99$

$\therefore l = a + (n - 1)d$

$\Rightarrow 99 = 11 + (n - 1)(2)$

$\Rightarrow 88 = (n - 1)(2)$

$\Rightarrow n - 1 = 44 \Rightarrow n = 45$

$\therefore$ Sum of all the two-digit odd numbers

$S_{45} = \frac{45}{2}(11 + 99) \quad \left[\because S_n = \frac{n}{2}(a + l)\right]$

$= \frac{45}{2} \times 110 = 45 \times 55 = 2475$

33. The sum of the first n terms of the series

$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$ is equal to

(a) $2^n - n - 1$ (b) $1 - 2^{-n}$
(c) $2^{-n} + n - 1$ (d) $2^n - 1$

(c) $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$

$\Rightarrow \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \ldots n \text{ terms}$

$= n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots n \text{ terms}\right)$

$= n - \frac{1}{2}\left(\frac{1 - (1/2)^n}{1 - 1/2}\right) = n + 2^{-n} - 1$

2016 (II)

34. What is the greatest value of the positive integer n satisfying the condition?

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^{n-1}} < 2 - \frac{1}{1000}$

(a) 8 (b) 9 (c) 10 (d) 11

(c) We have,

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^{n-1}} < 2 - \frac{1}{1000}$

$\Rightarrow \frac{1 - \frac{1}{2^n}}{1 - \frac{1}{2}} < 2 - \frac{1}{1000}$

$\Rightarrow 2\left(\frac{2^n - 1}{2^n}\right) < 2 - \frac{1}{1000}$

$\Rightarrow \frac{2^n - 1}{2^{n-1}} < 2 - \frac{1}{1000}$

$\Rightarrow 2 - \frac{1}{2^{n-1}} < 2 - \frac{1}{1000} \Rightarrow 2^{n-1} < 1000$

$\therefore n \leq 10$

35. How many geometric progressions is/are possible containing 27, 8 and 12 as three of its/their terms?

(a) One (b) Two
(c) Four (d) Infinitely many

(d) Let a be the first term and r be the common ratio.

Let 27, 8, 12 be the p^{th}, q^{th}, t^{th} terms of the GP.

$\therefore 27 = ar^{p-1}$,

$8 = ar^{q-1}$ and $12 = ar^{t-1}$

We know, $27 \times 8^2 = 12^3$

$\therefore ar^{p-1}(ar^{q-1})^2 = (ar^{t-1})^3$

$\Rightarrow r^{p-1}(r^{q-1})^2 = (r^{t-1})^3$

$\Rightarrow p - 1 + 2q - 2 = 3t - 3$

$\Rightarrow p + 2q - 3t = 0$

There are infinite solutions for the equation $p + 2q - 3t = 0$.

$\therefore$ There are infinite number of geometric progressions possible.

Directions (Q. Nos. 36-38) *Read the following information carefully and answer the questions given below.*

Let a, x, y, z, b be in AP, where $x + y + z = 15$. Let a, p, q, r, b be in HP, where $p^{-1} + q^{-1} + r^{-1} = 5/3$.

36. What is the value of ab?

(a) 10 (b) 9 (c) 8 (d) 6

(b) Given, a, x, y, z, b are in AP.

$\Rightarrow x, y, z$ are three arithmetic means between a and b.

$\Rightarrow x + y + z = 3\left(\frac{a + b}{2}\right)$

But $x + y + z = 15$ [given]

$\therefore 3\left(\frac{a + b}{2}\right) = 15$

$\Rightarrow a + b = 10 \quad \ldots(i)$

Also, a, p, q, r, b are in HP.

$\Rightarrow \frac{1}{a}, \frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{b}$ are in AP.

$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{3}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

$\Rightarrow \frac{5}{3} = \frac{3}{2}\left(\frac{a + b}{ab}\right) \quad \left[\because \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{5}{3}\right]$

$\Rightarrow \frac{5}{3} = \frac{3}{2}\left(\frac{10}{ab}\right) \quad [\because a + b = 10]$

$\Rightarrow ab = 9 \quad \ldots(ii)$

We know, $a - b = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{(10)^2 - 4 \times 9}$

$= \sqrt{100 - 36} = \sqrt{64}$

$\Rightarrow \quad a - b = 8 \quad \ldots(iii)$

On solving Eqs. (i) and (ii), we get

$a = 9, b = 1$

37. What is the value of xyz?

(a) 120
(b) 105
(c) 90
(d) Cannot be determined

(b) We have, 9, x, y, z, 1 are in AP.

Common difference,

$d = \frac{1-9}{4}$ [$\because l = a + (n-1)d$] $= -2$

$\therefore \quad x = 9 + d = 9 - 2 = 7$

$y = 9 + 2d = 9 - 4 = 5$

and $z = 9 + 3d$

$= 9 - 6 = 3$

$\therefore \quad xyz = 7 \times 5 \times 3 = 105$

38. What is the value of pqr?

(a) 35 / 243
(b) 81 / 35
(c) 243/35
(d) Cannot be determined

(c) Since, 9, p, q, r, 1 are in HP.

[$\because a = 9, b = 1$ from Q. 28]

$\therefore \quad \frac{1}{9}, \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$, 1 are in AP.

Common difference

$(d_1) = \frac{1 - 1/9}{4}$ [$\because l = a + (n-1)d$] $= \frac{2}{9}$

$\therefore \quad \frac{1}{p} = \frac{1}{9} + \frac{2}{9} = \frac{1}{3} \Rightarrow p = 3$

$\frac{1}{q} = \frac{1}{9} + \frac{4}{9} = \frac{5}{9}$

$\Rightarrow \quad q = \frac{9}{5}$

and $\frac{1}{r} = \frac{1}{9} + \frac{6}{9} = \frac{7}{9}$

$\Rightarrow \quad r = \frac{9}{7}$

$\therefore \quad pqr = 3 \times \frac{9}{5} \times \frac{9}{7} = \frac{243}{35}$

Directions (Q.Nos. 39 and 40) *Read the following information carefully and answer the questions given below.*

The sixth term of an *AP* is 2 *and its common difference is greater than* 1.

39. What is the common difference of the AP, so that the product of the first, fourth and fifth terms is greatest?

(a) 8/5 (b) 9/5 (c) 2 (d) 11/5

(a) Given, $a + 5d = 2$ and $d > 1$.

$\therefore$ Product of first, fourth and fifth terms

$= a \times (a + 3d) \times (a + 4d)$

$= (2 - 5d)(2 - 5d + 3d)(2 - 5d + 4d)$

$= (2 - 5d)(2 - 2d)(2 - d)$

$P(d) = 8 - 32d + 34d^2 - 10d^3$

$P'(d) = -32 + 68d - 30d^2$

For greatest value of $P'(d)$, $P - (d) = 0$

$\Rightarrow \quad -32 + 68d - 30d^2 = 0$

$\Rightarrow \quad 15d^2 - 34d + 16 = 0$

$\Rightarrow \quad (3d - 2)(5d - 8) = 0$

$\Rightarrow \quad d = 2/3, 8/5$

Now, $P'(d) = 68 - 60d$

For $d = 2/3$,

$P'(d) = 68 - 60 \times 2/3 = 28 > 0$

For $d = 8/5$,

$P'(d) = 68 - 60 \times 8/5 = -28 < 0$

Hence, for $d = 8/5$, $P(d)$ has greatest value.

40. What is the first term of the AP, so that the product of the first, fourth and fifth terms is greatest?

(a) –4 (b) –6
(c) –8 (d) –10

(b) When, $d = \frac{8}{5}$, then product of first, fourth and fifth terms is maximum,

and we have $a + 5d = 2$

$\Rightarrow \quad a + 5 \times \frac{8}{5} = 2$

$\therefore \quad a = 2 - 8 = -6$

Directions (Q.Nos. 41 and 42) *Read the following information carefully and answer the questions given below.*

The interior angles of a polygon of n sides are in *AP*. The smallest angle is 120° and the common difference is 5°.

41. How many possible values can n have?

(a) One (b) Two
(c) Three (d) Infinitely many

(a) $\frac{n}{2}[2a + (n-1)d] = (n-2) \times 180$

$\Rightarrow \frac{n}{2}[2 \times 120 + (n-1)(5)] = (n-2) \times 180$

$\Rightarrow \quad \frac{n}{2}[2 \times 24 + (n-1)] = (n-2) \times 36$

$\Rightarrow \quad n(48 + n - 1) = (n-2) \times 72$

$\Rightarrow \quad n(n + 47) = 72n - 144$

$\Rightarrow \quad n^2 + 47n - 72n + 144 = 0$

$\Rightarrow \quad n^2 - 25n + 144 = 0$

$\Rightarrow \quad (n - 16)(n - 9) = 0$

$\Rightarrow \quad n = 9, 16$

Here, only $n = 9$ is the required number sides.

Since, if $n = 16$,

then $a_n = 195 > 180°$,

which is not possible.

42. What is the largest interior angle of the polygon?

(a) 160° only
(b) 195° only
(c) Either 160° or 195°
(d) Neither 160° nor 195°

(**a**) Largest angle $= a_9 = a + 8d$

$= 120° + 8 \times 5° = 120° + 40° = 160°$

43. If $x^{\ln\left(\frac{y}{z}\right)} \cdot y^{\ln(xz)^2} \cdot z^{\ln\left(\frac{x}{y}\right)} = y^{4 \ln y}$ for any $x > 1$, $y > 1$ and $z > 1$, then which one of the following is correct?

(a) ln y is the GM of ln x, ln x, ln x and ln z
(b) ln y is the AM of ln x, ln x, ln x and ln z
(c) ln y is the HM of ln x, ln x, ln x and ln z
(d) ln y is the AM of ln x, ln x, ln z and ln z

(b) Given, $x^{\ln\left(\frac{y}{z}\right)} \cdot y^{\ln(xz)^2} \cdot z^{\ln\left(\frac{x}{y}\right)} = y^{4 \ln y}$

On taking natural log on both sides, we get

$\ln\left(x^{\ln\left(\frac{y}{z}\right)} \cdot y^{\ln(xz)^2} \cdot z^{\ln\left(\frac{x}{y}\right)}\right) = \ln(y^{4 \ln y})$

$\Rightarrow \ln\left(\frac{y}{z}\right)\ln x + 2\ln(xz) \cdot \ln y + \ln\left(\frac{x}{y}\right)\ln z$

$= 4 \ln y \cdot \ln y$

$\Rightarrow \ln y \cdot \ln x - \ln z \cdot \ln x + 2$

$[\ln x \cdot \ln y + \ln z \cdot \ln y]$

$+ \ln x \cdot \ln z - \ln y \cdot \ln z = 4(\ln y)^2$

$\Rightarrow \quad 3 \ln x \cdot \ln y + \ln y \cdot \ln z = 4(\ln y)^2$

$\Rightarrow \ln y[3 \ln x + \ln z - 4 \ln y] = 0$

$\Rightarrow \ln y = 0$ (as $y > 1$, so $\ln y = 0$) is not possible.

$\therefore \quad 3 \ln x + \ln z - 4 \ln y = 0$

$\therefore \quad \ln y = \frac{\ln x + \ln x + \ln x + \ln z}{4}$

2016 (I)

Directions (Q. Nos. 44 and 45) *Read the following information carefully and answer the questions given below.*

Given that $\log_x y$, $\log_z x$, $\log_y z$ are in GP, $xyz = 64$ and x^3, y^3, z^3 are in AP.

44. Which one of the following is correct?
x, y and z are
(a) in AP only
(b) in GP only
(c) in both AP and GP
(d) Neither in AP nor in GP

45. Which one of the following is correct?
xy, yz and zx are
(a) in AP only
(b) in GP only
(c) in both AP and GP
(d) Neither in AP nor in GP

(Solutions Q. Nos. 44 and 45)
Given, $\log_x y, \log_z x, \log_y z$ are in G P.
$\Rightarrow (\log_z x)^2 = \log_x y \times \log_y z = \log_x z$
$= \dfrac{1}{\log_z x}$
$\Rightarrow (\log_z x)^3 = 1 \quad \Rightarrow \log_z x = 1$
$\Rightarrow x = z$
Now, x^3, y^3, z^3 are in AP.
$\therefore \quad 2y^3 = x^3 + z^3$
$\Rightarrow 2y^3 = z^3 + z^3 \quad [\because x = z]$
$\Rightarrow y^3 = z^3$
$\therefore \quad y = z$
$\therefore \quad x = y = z$
Also, $xyz = 64$
$\Rightarrow (x)^3 = 4^3 \Rightarrow x = y = z = 4$

44. (**c**) x, y and z are in both AP and GP.
45. (**c**) xy, yz and zx are in both AP and GP.

2015 (II)

46. The value of the infinite product
$6^{\frac{1}{2}} \times 6^{\frac{1}{2}} \times 6^{\frac{3}{8}} \times 6^{\frac{1}{4}} \times \ldots$ is
(a) 6 (b) 36 (c) 216 (d) ∞

(***b***) We have,
$6^{1/2} \times 6^{1/2} \times 6^{3/8} \times 6^{1/4} \times \ldots$
$= 6^{\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} \ldots}$
$= 6^{\frac{1}{2}\left(1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \frac{5}{2^4} \ldots\right)}$
Let $\quad S = 1 + \dfrac{2}{2} + \dfrac{3}{2^2} + \dfrac{4}{2^3} \ldots$
$\Rightarrow \quad \dfrac{1}{2} S = \dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} \ldots$
$\Rightarrow S - \dfrac{1}{2} S = 1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \dfrac{1}{2^3} \ldots$
$\Rightarrow \quad \dfrac{S}{2} = \dfrac{1}{1 - \frac{1}{2}} \quad \Rightarrow \quad S = 4$
$\therefore \quad 6^{\frac{1}{2} \times 4} = 6^2 = 36$

47. If the nth term of an AP is $\dfrac{3+n}{4}$, then the sum of first 105 terms is
(a) 270 (b) 735
(c) 1409 (d) 1470

(***d***) We have, $t_n = \dfrac{3+n}{4}$
then first term = 1
and common difference = $\dfrac{1}{4}$
$S_{105} = \dfrac{105}{2}\left[2 + 104 \times \dfrac{1}{4}\right]$
$= \dfrac{105}{2} \times 28$
$= 105 \times 14 = 1470$

48. If p, q, r are in one geometric progression and a, b, c are in another geometric progression, then ap, bq, cr are in
(a) AP (b) GP
(c) HP (d) None of these

(***b***) Clearly, ap, bq, cr are in GP because on multiplying corresponding terms of two GP's and the resulting series is also a GP.
e.g. $S_1 = 2, \sqrt{2}, 1, \dfrac{1}{\sqrt{2}}, \ldots$
$S_2 = 3, \sqrt{3}, 1, \dfrac{1}{\sqrt{3}}, \ldots$
$S_3 = S_1 \times S_2 = 6, \sqrt{6}, \dfrac{1}{\sqrt{6}}, \ldots$
$a = 6, r = \dfrac{\sqrt{6}}{6} = \dfrac{1}{\sqrt{6}}$

49. What is the sum of n terms of the series
$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$?
(a) $\dfrac{n(n-1)}{\sqrt{2}}$ (b) $\sqrt{2}n\,(n+1)$
(c) $\dfrac{n(n+1)}{\sqrt{2}}$ (d) $\dfrac{n(n-1)}{2}$

(***c***) We have, the series
$\sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots$
which is an AP.
$\because \quad a = \sqrt{2}$ and $d = \sqrt{2}$
$\because \quad S_n = \dfrac{n}{2}[2a + (n-1)d]$
$\therefore \quad S_n = \dfrac{n}{2}[2\sqrt{2} + (n-1)\sqrt{2}]$
$= \dfrac{n}{2}[2\sqrt{2} + \sqrt{2}n - \sqrt{2}]$
$= \dfrac{n}{2}[\sqrt{2}n + \sqrt{2}]$
$= \dfrac{n}{2}\sqrt{2}\,(n+1)$
$= \dfrac{n}{\sqrt{2}}(n+1)$

2015 (I)

50. What is the sum of the series $0.5 + 0.55 + 0.555 + \ldots + n$ terms?
(a) $\dfrac{5}{9}\left[n - \dfrac{2}{9}\left(1 - \dfrac{1}{10^n}\right)\right]$
(b) $\dfrac{1}{9}\left[5 - \dfrac{2}{9}\left(1 - \dfrac{1}{10^n}\right)\right]$
(c) $\dfrac{1}{9}\left[n - \dfrac{5}{9}\left(1 - \dfrac{1}{10^n}\right)\right]$
(d) $\dfrac{5}{9}\left[n - \dfrac{1}{9}\left(1 - \dfrac{1}{10^n}\right)\right]$

(***d***) $0.5 + 0.55 + 0.555 + \ldots + n$ terms
$= \dfrac{5}{10} + \dfrac{55}{100} + \dfrac{555}{1000} + \ldots + n$ terms
$= \dfrac{5}{10}\left[1 + \dfrac{11}{10} + \dfrac{111}{100} + \ldots + n \text{ terms}\right]$
$= \dfrac{5}{10} \times \dfrac{1}{9}\left[9 + \dfrac{99}{10} + \dfrac{999}{100} + \ldots + n \text{ terms}\right]$
$= \dfrac{5}{90}\left[(10-1) + \dfrac{(10^2-1)}{10} + \dfrac{(10^3-1)}{10^2} + \ldots + n \text{ terms}\right]$
$= \dfrac{5}{90}\left[10 + \dfrac{10^2}{10} + \dfrac{10^3}{10^2} + \ldots + n \text{ terms}\right]$
$+ \dfrac{5}{90}\left[-1 - \dfrac{1}{10} - \dfrac{1}{10^2} - \dfrac{1}{10^3} - \ldots + n \text{ terms}\right]$
$= \dfrac{5}{90}[10 + 10 + 10 + \ldots + n \text{ terms}]$
$+ \dfrac{5}{90}(-1)\left[1 + \dfrac{1}{10} + \dfrac{1}{10^2} + \dfrac{1}{10^3} + \ldots + n \text{ terms}\right]$
$= \dfrac{5}{90} \times 10n - \dfrac{5}{90}\left[\dfrac{1 - \left(\frac{1}{10}\right)^n}{1 - \frac{1}{10}}\right]$
$= \dfrac{5n}{9} - \dfrac{5}{90} \times \dfrac{10}{9}\left[\dfrac{10^n - 1}{10^n}\right]$
$= \dfrac{5}{9}\left[n - \dfrac{1}{9}\left(1 - \dfrac{1}{10^n}\right)\right]$

51. If $\log_{10} 2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are three consecutive terms of an AP, then the value of x is
(a) 1 (b) $\log_5 2$
(c) $\log_2 5$ (d) $\log_{10} 5$

(***c***) If a, b and c are in AP, then $2b = a + c$
$\therefore 2\log_{10}(2^x - 1) = \log_{10}(2^x + 3) + \log_{10} 2$
$\Rightarrow \log_{10}(2^x - 1)^2 = \log_{10}[(2^x + 3) \times 2]$
$\Rightarrow \quad (2^x - 1)^2 = 2^{x+1} + 6$

$\Rightarrow \quad (2^x)^2 + 1 - 2 \cdot 2^x = 2^{x+1} + 6$

$\Rightarrow \quad 2^{2x} - 2^{x+1} - 2^{x+1} = 6 - 1$

$\Rightarrow \quad 2^{2x} - 2 \cdot 2^{x+1} - 5 = 0$

Let $2^x = y$, then

$y^2 - 4y - 5 = 0$

$\Rightarrow \quad y^2 - 5y + y - 5 = 0$

$\Rightarrow \quad y(y-5) + 1(y-5) = 0$

$\Rightarrow \quad (y+1)(y-5) = 0$

$\Rightarrow \quad y = -1 \quad$ or $\quad y = 5$

$\Rightarrow \quad 2^x = -1 \quad$ [not possible]

or $\quad 2^x = 5$

$\Rightarrow \quad \log 2^x = \log 5$

$\Rightarrow \quad x \log 2 = \log 5$

$\therefore \quad x = \dfrac{\log 5}{\log 2} = \log_2 5$

2014 (II)

Directions (Q. Nos. 52 and 53) *Read the following information carefully and answer the questions given below.*

Consider the given information

Let S_n denotes the sum of first n terms of an AP and $3S_n = S_{2n}$.

52. What is $S_{3n} : S_n$ equal to?

(a) 4 : 1 (b) 6 : 1
(c) 8 : 1 (d) 10 : 1

(b) We have, S_n = Sum of first n terms of an AP

$\therefore \quad S_n = \dfrac{n}{2}[2a + (n-1)d]$

Similarly,

$S_{2n} = \dfrac{2n}{2}[2a + (2n-1)d]$

and $\quad S_{3n} = \dfrac{3n}{2}[2a + (3n-1)d]$

Now, $3S_n = S_{2n}$

$\Rightarrow 3\left(\dfrac{n}{2}\right)[2a + (n-1)d]$

$= 2\left(\dfrac{n}{2}\right)[2a + (2n-1)d]$

$\Rightarrow \quad 2a = d(n+1)$

$\therefore \quad S_n = \dfrac{n}{2}[d(n+1) + d(n-1)]$

$= \dfrac{n}{2}(2nd) = n^2 d \quad \ldots(i)$

Now,

$S_{2n} = n[d(n+1+2n-1)] = 3n^2 d \quad \ldots(ii)$

and $\quad S_{3n} = \dfrac{3n}{2}[d(n+1+3n-1)]$

$= 6n^2 d \quad \ldots(iii)$

$\therefore \quad \dfrac{S_{3n}}{S_n} = \dfrac{6n^2 d}{n^2 d} = 6 : 1$

53. What is $S_{3n} : S_{2n}$ equal to?

(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 5 : 1

(a) $\quad \dfrac{S_{3n}}{S_{2n}} = \dfrac{6n^2 d}{3n^2 d}$ [from Eqs. (ii) and (iii)]

$= \dfrac{6}{3} = 2 : 1$

2014 (I)

54. The sum of an infinite GP is x and the common ratio r is such that $|r| < 1$. If the first term of the GP is 2, then which one of the following is correct?

(a) $-1 < x < 1$ (b) $-\infty < x < 1$
(c) $1 < x < \infty$ (d) None of these

(c) Given that, the sum of an infinite GP = x

$\Rightarrow \quad \dfrac{a}{1-r} = x$

where, a = first term and r = common ratio

$\Rightarrow \quad \dfrac{2}{1-r} = x \quad \ldots(i)$

[$\because$ given that, $a = 2$ and $|r| < 1$]

$\because \quad |r| < 1$

$\Rightarrow \quad -1 < r < 1 \Rightarrow \quad 1 > -r > -1$

$\Rightarrow \quad 1 + 1 > 1 - r > 1 - 1$

$\Rightarrow \quad 0 < 1 - r < 2$

$\Rightarrow \quad (1-r) < 2$

$\Rightarrow \quad \dfrac{1}{1-r} > \dfrac{1}{2} \Rightarrow \dfrac{2}{1-r} > 1$

$\Rightarrow \quad x > 1 \quad$ [from Eq. (i)]

Hence, $x \in (1, \infty)$ i.e. $1 < x < \infty$.

55. The sum of the series formed by the sequence $3, \sqrt{3}, 1, \ldots$ upto infinity is

(a) $\dfrac{3\sqrt{3}(\sqrt{3}+1)}{2}$ (b) $\dfrac{3\sqrt{3}(\sqrt{3}-1)}{2}$
(c) $\dfrac{3(\sqrt{3}+1)}{2}$ (d) $\dfrac{3(\sqrt{3}-1)}{2}$

(a) Given series is $3, \sqrt{3}, 1, \ldots, \infty$

and the series form an infinite GP.

whose first term (a) = 3

and common ratio (r) = $\dfrac{\sqrt{3}}{3} = \dfrac{1}{\sqrt{3}}$

$\therefore$ Sum of infinite GP,

$S_\infty = \dfrac{a}{1-r} = \dfrac{3}{1 - \frac{1}{\sqrt{3}}} = \dfrac{3\sqrt{3}}{\sqrt{3}-1} \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{3\sqrt{3}(\sqrt{3}+1)}{3-1} = \dfrac{3\sqrt{3}(\sqrt{3}+1)}{2}$

5

QUADRATIC EQUATIONS AND INEQUATIONS

2019 (II)

1. If p and q are the roots of the equation $x^2 - 30x + 221 = 0$, what is the value of $p^3 + q^3$?

(a) 7010 (b) 7110 (c) 7210 (d) 7240

(**b**) Since, p and q are the roots of the equation

$$x^2 - 30x + 221 = 0$$

$\therefore\ p + q = 30$ and $pq = 221$

Now, $p^3 + q^3 = (p + q)(p^2 + q^2 - pq)$

$= 30[p^2 + q^2 + 2pq - 3pq]$

$= 30[(p + q)^2 - 3pq]$

$= 30[(30)^2 - 663]$

$= 30[900 - 663] = 30 \times 237 = 7110$

2. Consider the following statements in respect of the quadratic equation

$4(x - p)(x - q) - r^2 = 0$,

where p, q and r are real numbers.

1. The roots are real.
2. The roots are equal, if $p = q$ and $r = 0$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) Given quadratic equation,

$$4(x - p)(x - q) - r^2 = 0$$

$\Rightarrow\ 4x^2 - (4q + 4p)x + 4pq - r^2 = 0$

Comparing it Eq. by $ax^2 + bx + c = 0$

$a = 4,\ b = -4(p + q),\ c = 4pq - r^2$

$b^2 - 4ac = 16(p + q)^2 - 4 \times 4(4pq - r^2)$

$= 16p^2 + 16q^2 + 32pq - 64pq + 16r^2$

$= 16p^2 + 16q^2 - 32pq + 16r^2$

$= 16(p - q)^2 + 16r^2$

$\because\ b^2 - 4ac$ will be positive.

So, the roots are real.

If $p = q$ and $r = 0$, then $b^2 - 4ac = 0$

So, the roots are equal.

Hence, the statements both 1 and 2 are correct.

3. How many real roots does the equation $x^2 + 3|x| + 2 = 0$ have?

(a) Zero (b) One (c) Two (d) Four

(**a**) Given equation, $x^2 + 3|x| + 2 = 0$

Case I $x^2 + 3x + 2 = 0$ (when $x > 0$)

$\Rightarrow\ x^2 + x + 2x + 2 = 0$

$\Rightarrow\ x(x + 1) + 2(x + 1) = 0$

$\Rightarrow\ (x + 1)(x + 2) = 0$

$\therefore\ x = -1, -2$

Hence, no real roots because $x > 0$.

Case II $x^2 - 3x + 2 = 0$ (when $x < 0$)

$\Rightarrow\ x^2 - 2x - x + 2 = 0$

$\Rightarrow\ x(x - 2) - 1(x - 2) = 0$

$\Rightarrow\ (x - 2)(x - 1) = 0$

$\therefore\ x = 1, 2$

Hence, no real roots because $x < 0$.

$\therefore$ The number of real roots of given equation is zero.

4. What is the solution of $x \le 4,\ y \ge 0$ and $x \le -4,\ y \le 0$?

(a) $x \ge -4,\ y \le 0$ (b) $x \le 4,\ y \ge 0$
(c) $x \le -4,\ y = 0$ (d) $x \ge -4,\ y = 0$

(**c**) Given inequalities

$$x \le 4,\ y \ge 0 \quad \ldots(i)$$

and $x \le -4,\ y \le 0 \quad \ldots(ii)$

Possible value of x and y.

$x = \{4, 3, 2, 1, 0, -1, -2, -3, -4, -5, \ldots\}$

$y = \{0, 1, 2, 3, 4 \ldots\ldots\} \quad \ldots(i)$

and $x = \{-4, -5, -6, -7, \ldots\}$,

$y = \{0, -1, -2, -3, -4 \ldots\} \quad \ldots(ii)$

Take combine (i) and (ii),

$x = \{-4, -5, -6, -7 \ldots\}, y = 0$

or $x \le -4,\ y = 0$.

5. If α and β are the roots of $x^2 + x + 1 = 0$, then what is $\sum_{j=0}^{3}(\alpha^j + \beta^j)$ equal to?

(a) 8 (b) 6
(c) 4 (d) 2

(**d**) α and β are the roots of the equation

$$x^2 + x + 1 = 0$$

$\therefore\ \alpha + \beta = -1$

and $\alpha\beta = 1$

Now, $\sum_{j=0}^{3}(\alpha^j + \beta^j) = (\alpha^0 + \beta^0)$

$+ (\alpha^1 + \beta^1) + (\alpha^2 + \beta^2) + (\alpha^3 + \beta^3)$

$= (1 + 1) + (-1) + \{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta\}$

$+ (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= 2 - 1 + \{(\alpha + \beta)^2 - 2\alpha\beta\} + (-1)$

$\{\alpha^2 + \beta^2 + 2\alpha\beta - 3\alpha\beta\}$

$= 1 + \{(-1)^2 - 2(1)\} - \{(\alpha + \beta)^2 - 3(1)\}$

$= 1 - 1 - \{(-1)^2 - 3\}$

$= -(1 - 3) = 2$

6. Under which one of the following conditions will the quadratic equation $x^2 + mx + 2 = 0$ always have real roots?

(a) $2\sqrt{3} \le m^2 < 8$ (b) $\sqrt{3} \le m^2 < 4$
(c) $m^2 \ge 8$ (d) $m^2 \le \sqrt{3}$

(**c**) The quadratic equation

$$x^2 + mx + 2 = 0,$$

have real roots.

$\therefore\ m^2 - 4(1)(2) \ge 0$

[quadratic equation $ax^2 + bx + c = 0$ have real roots if $b^2 - 4ac \ge 0$]

$\Rightarrow\ m^2 - 8 \ge 0$

$\Rightarrow\ m^2 \ge 8$

7. If both p and q belong to the set $\{1,2,3,4\}$, then how many equations of the form $px^2 + qx + 1 = 0$ will have real roots?

(a) 12 (b) 10 (c) 7 (d) 6

(**d**) Equation $px^2 + qx + 1 = 0$, has real roots, where p and q belong to the set $\{1, 2, 3, 4\}$.

$\therefore \quad q^2 - 4p \geq 0$

[$\because$ for real roots of a quadratic equation $b^2 - 4ac \geq 0$]

It is possible if value of
$(p, q) = (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)$ and $(3, 4)$

Hence, the number of equations are 6.

8. What is the value of k for which the sum of the squares of the roots of $2x^2 - 2(k-2)x - (k+1) = 0$ is minimum?

(a) – 1 (b) 1 (c) $\frac{3}{2}$ (d) 2

(**c**) Let α, β be the roots of equation.

$2x^2 - 2(k-2)x - (k+1) = 0$

$\because \alpha + \beta = \frac{2(k-2)}{2} = k - 2,$

$\alpha\beta = \frac{-(k+1)}{2}$

We know that

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
$$= (k-2)^2 + 2 \times \frac{k+1}{2}$$
$$= k^2 + 4 - 4k + k + 1$$
$$= k^2 - 3k + 5$$
$$= k^2 - 3k + \frac{9}{4} - \frac{9}{4} + 5$$
$$= \left(k - \frac{3}{2}\right)^2 + \frac{11}{4}$$

$\alpha^2 + \beta^2$ is minimum, if $\left(k - \frac{3}{2}\right) = 0$

$\Rightarrow \quad k = \frac{3}{2}$

9. If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, then which one of the following is correct?

(a) a, b and c are in AP
(b) a, b and c are in GP
(c) a, b and c are in HP
(d) a, b and c do not follow any regular pattern

(**c**) The roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal.

$\therefore b^2(c-a)^2 - 4a(b-c).c(a-b) = 0$

[$\because ax^2 + bx + c = 0$ of roots are real if $b^2 - 4ac \geq 0$

$\Rightarrow b^2(c^2 + a^2 - 2ca) - 4ac\,(ab - b^2 - ac + bc) = 0$

$\Rightarrow b^2c^2 + a^2b^2 - 2ab^2c - 4a^2bc + 4ab^2c + 4a^2c^2 - 4abc^2 = 0$

$\Rightarrow b^2c^2 + a^2b^2 + 2ab^2c - 4a^2bc - 4abc^2 + 4a^2c^2 = 0$

$\Rightarrow b^2(c^2 + a^2 + 2ac) - 4abc\,(a + c) + 4a^2c^2 = 0$

$\Rightarrow b^2(c+a)^2 - 4abc\,(a+c) + (2ac)^2 = 0$

$\Rightarrow [b(c+a) - 2ac]^2 = 0$

$\Rightarrow b(c+a) - 2ac = 0$

$\Rightarrow b(c+a) = 2ac \Rightarrow b = \frac{2ac}{c+a}$

So, a, b and c are is HP.

10. If $|x^2 - 3x + 2| > x^2 - 3x + 2$, then which one of the following is correct?

(a) $x \leq 1$ or $x \geq 2$ (b) $1 \leq x \leq 2$
(c) $1 < x < 2$
(d) x is any real value except 3 and 4

(**c**) $|x^2 - 3x + 2| > x^2 - 3x + 2$

$\Rightarrow -(x^2 - 3x + 2) > x^2 - 3x + 2$

[if $x^2 - 3x + 2 < 0$, and $x^2 - 3x + 2 > 0$ not possible]

$\Rightarrow \quad -2(x^2 - 3x + 2) > 0$

$\Rightarrow \quad x^2 - 3x + 2 > 0$

$\Rightarrow \quad x^2 - 2x - x + 2 > 0$

$\Rightarrow \quad (x-2)(x-1) > 0$

$\therefore 1 < x < 2$ is correct.

2019 (I)

11. What are the roots of the equation $|x^2 - x - 6| = x + 2$?

(a) –2, 1, 4 (b) 0, 2, 4
(c) 0, 1, 4 (d) –2, 2, 4

(**d**) We have,

$|x^2 - x - 6| = x + 2$

$\Rightarrow \quad |(x-3)(x-2)| = x + 2$

Case I $x < 2$

$x^2 - x - 6 = x + 2$

$x^2 - 2x - 8 = 0$

$\left[\begin{array}{l} x^2 - 4x + 2x - 8 = 0 \\ x(x-4) + 2(x-4) = 0 \end{array}\right]$

$(x-4)(x+2) = 0$

$x = -2$ but $x \neq 4 \quad [\because x < 2]$

Case II $2 \leq x < 3$

$x^2 - x - 6 = -(x+2)$

$x^2 - x - 6 + x + 2 = 0$

$x^2 - 4 = 0$

$x = \pm 2$

$x = 2$ but $x \neq -2$ [$\because x \in (2, 3)$]

Case III $x \geq 3$

$x^2 - x - 6 = x + 2$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = 4$ but $x \neq -2 \quad [\because x \geq 3]$

$\therefore \quad x = -2, 2, 4$

12. The equation $px^2 + qx + r = 0$ (where p, q, r, all are positive) has distinct real roots a and b. Which one of the following is correct?

(a) $a > 0, b > 0$ (b) $a < 0, b < 0$
(c) $a > 0, b < 0$ (d) $a < 0, b > 0$

(**b**) Given, $px^2 + qx + r = 0$, where $p, q, r > 0$ and a and b are distinct roots.

$\therefore \quad a + b = \frac{-q}{p}$ and $ab = r$

Now, $r > 0$

$\therefore \quad ab > 0 \Rightarrow a > 0, b > 0 \quad ...(i)$

or $a < 0, b < 0 \quad ...(ii)$

Now, $\frac{-q}{p} < 0$ $q, p > 0$ $\therefore a + b < 0$

$a < 0, b < 0 \quad ...(iii)$

From Eqs. (i), (ii) and (iii), we get

$\therefore \quad a < 0$ and $b < 0$

13. The number of real roots for the equation $x^2 + 9|x| + 20 = 0$ is

(a) zero (b) one
(c) two (d) three

(**a**) Given, $x^2 + 9|x| + 20 = 0$

$\Rightarrow \quad x^2 + 9x + 20 = 0$

or $\quad x^2 - 9x + 20 = 0$

$\Rightarrow \quad x^2 + 4x + 5x + 20 = 0$

or $\quad x^2 - 4x - 5x + 20 = 0$

$\Rightarrow \quad x(x+4) + 5(x+4) = 0$

or $\quad x(x-4) - 5(x-4) = 0$

$\Rightarrow \quad (x+4)(x+5) = 0$

or $\quad (x-4)(x-5) = 0$

$\Rightarrow \quad x = -4, -5,$ or $4, 5$

But these values of x does not satisfy the given equation.

Hence, number of real roots of the given equation is zero.

2018 (II)

14. If α and β $(\neq 0)$ are the roots of the quadratic equation $x^2 + \alpha x - \beta = 0$, then the quadratic expression $-x^2 + \alpha x + \beta$, where $x \in R$ has

(a) least value $-\frac{1}{4}$ (b) least value $-\frac{9}{4}$
(c) greatest value $\frac{1}{4}$ (d) greatest value $\frac{9}{4}$

(d) α and β are the roots of quadratic equation $x^2 + \alpha x - \beta = 0$

So, $(\alpha\beta = -\beta) \Rightarrow \alpha\beta + \beta = 0$

$\Rightarrow \beta(\alpha + 1) = 0$

$\alpha = -1$ $[\because \beta \neq 0]$

$\alpha + \beta = -\alpha \Rightarrow 2\alpha + \beta = 0 \Rightarrow \beta = 2$

$\therefore \quad -x^2 + \alpha x + \beta \quad [\because \alpha = -1, \beta = 2]$

$= -x^2 - x + 2$

Greatest value $= 2 - \left[\frac{(-1)^2}{4(-1)}\right]$

$[\because$ greatest value $= c - \left(\frac{b^2}{4a}\right)$

$= 2 - \left(\frac{1}{-4}\right) = 2 + \frac{1}{4} = \frac{9}{4}$

2018 (I)

15. Let α and β be real number and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct non-real roots with $\text{Re}(z) = 1$, then it is necessary that

(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$
(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

(c) Let $z = x + iy$

Now, we have $z^2 + \alpha z + \beta = 0$

$\Rightarrow (x + iy)^2 + \alpha(x + iy) + \beta = 0$

$\Rightarrow x^2 - y^2 + 2ixy + \alpha x + i\alpha y + \beta = 0$

$[\because i^2 = -1]$

$\Rightarrow (x^2 - y^2 + \alpha x + \beta) + (2xy + \alpha y)\, i = 0$

On comparing,

$x^2 - y^2 + \alpha x + \beta = 0$ and $2xy + \alpha y = 0$

$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0$ and $(2x + \alpha)y = 0$

$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0$

and $2x + \alpha = 0$ $[\because y \neq 0]$

$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0$ and $\alpha = -2x$

$\Rightarrow x^2 - y^2 + \alpha x + \beta = 0$ and $\alpha = -2$

$[\because \text{Re}(z) = 1 = x]$

$\Rightarrow 1 - y^2 - 2 + \beta = 0 \quad [\because x = 1, \alpha = -2]$

$\Rightarrow \beta = y^2 + 1 \Rightarrow \beta \in (1, \infty)$

$[\because y^2 \geq 0 \Rightarrow y^2 + 1 \geq 1]$

16. The equation $|1 - x| + x^2 = 5$ has

(a) a rational root and an irrational root
(b) two rational roots
(c) two irrational roots
(d) no real roots

(a) We have, $|1 - x| + x^2 = 5$

Case I When $x < 1$,

$1 - x + x^2 = 5$

$[\because x < 1 \Rightarrow |1 - x| = 1 - x]$

$\Rightarrow x^2 - x - 4 = 0$

$\Rightarrow \quad x = \frac{1 \pm \sqrt{1 - 4(1)(-4)}}{2} = \frac{1 \pm \sqrt{17}}{2}$

$\Rightarrow \quad x = \frac{1 - \sqrt{17}}{2}$ $[\because x < 1]$

Case II When $x \geq 1$

$-(1 - x) + x^2 = 5$

$[\because x \geq 1 \Rightarrow |1 - x| = -(1 - x)]$

$\Rightarrow -1 + x + x^2 = 5 \Rightarrow x^2 + x - 6 = 0$

$\Rightarrow (x + 3)(x - 2) = 0 \Rightarrow x = -3, 2$

$\Rightarrow \quad x = 2$ $[\because x \geq 1]$

$\therefore$ Given equation has a rational root and an irrational root.

17. Let $[x]$ denote the greatest integer function. What is the number of solutions of the equation $x^2 - 4x + [x] = 0$ in the interval $[0, 2]$?

(a) Zero (no solution) (b) One
(c) Two (d) Three

(b) We have,

$x^2 - 4x + [x] = 0$

Case I $x \in [0, 1)$

$\therefore \quad x^2 - 4x + 0 = 0$

$[\because x \in [0, 1) \Rightarrow [x] = 0]$

$\Rightarrow \quad x^2 - 4x = 0$

$\Rightarrow \quad x(x - 4) = 0$

$\Rightarrow \quad x = 0, 4$

$\Rightarrow \quad x = 0$

$[\because x \in [0, 1)]$

Case II $x \in [1, 2)$

$\therefore \quad x^2 - 4x + 1 = 0$

$[\because x \in [1, 2) \Rightarrow [x] = 1]$

$\Rightarrow \quad x = \frac{4 \pm \sqrt{16 - 4}}{2} \Rightarrow x = \frac{4 \pm 2\sqrt{3}}{2}$

$\Rightarrow \quad x = 2 \pm \sqrt{3}$

$\Rightarrow \quad x = 0.268, 3.732$

No solution

$[\because x \in [1,2)]$

Case III $[x] = 2$

$\therefore \quad x^2 - 4x + 2 = 0 \quad [\because x = 2]$

$x = 2 \pm \sqrt{2}$

No solution

$\therefore$ Given equation has only one solution i.e. $x = 0$.

2017 (II)

18. The roots of the equation $(q - r)x^2 + (r - p)x + (p - q) = 0$ are

(a) $\frac{(r - p)}{(q - r)}, \frac{1}{2}$ (b) $\frac{(p - q)}{(q - r)}, 1$
(c) $\frac{(q - r)}{(p - q)}, 1$ (d) $\frac{(r - p)}{(p - q)}, \frac{1}{2}$

(b) We have,

$(q - r)x^2 + (r - p)x + (p - q) = 0$

Here, sum of coefficients

$(q - r) + (r - p) + (p - q) = 0$

$\therefore$ One roots of given equation must be 1.

Here, Product of roots $= \frac{p - q}{q - r}$

$[\because$ In quadratic equation

$ax^2 + bx + c = 0$, product of roots $= \frac{c}{a}]$

$\therefore \quad$ Another root $\times 1 = \frac{p - q}{q - r}$

$\therefore$ Roots of the given equation are $\frac{p - q}{q - r}, 1$

19. The sum of all real roots of the equation $|x - 3|^2 + |x - 3| - 2 = 0$ is

(a) 2 (b) 3 (c) 4 (d) 6

(d) We have,

$|x - 3|^2 + |x - 3| - 2 = 0$

Let $|x - 3| = t$

$\therefore \quad t^2 + t - 2 = 0$

$\Rightarrow \quad t^2 + 2t - t - 2 = 0$

$\Rightarrow \quad t(t + 2) - 1(t + 2) = 0$

$\therefore \quad t = 1$ or $t = -2$

[here, $t = -2$ is not possible because $t = |x - 3|$ is always +ve]

$\therefore \quad t = 1$

$\therefore \quad |x - 3| = 1$

$\Rightarrow x - 3 = 1$ or $x - 3 = -1$

$x = 4$ or $x = 2$

$\therefore$ Sum of roots of given equation

$= 4 + 2 = 6$

20. It is given that the roots of the equation $x^2 - 4x - \log_3 P = 0$ are real. For this the minimum value of P is

(a) $\frac{1}{27}$ (b) $\frac{1}{64}$ (c) $\frac{1}{81}$ (d) 1

(c) We have,

$x^2 - 4x - \log_3 P = 0$ ...(i)

It is given that the roots of the given equation are real

$\therefore$ Discriminant, $D \geq 0$

$\therefore \quad (-4)^2 - 4(1)(-\log_3 P) \geq 0$

$[\because D = b^2 - 4ac]$

$\Rightarrow \quad 16 + 4\log_3 P \geq 0$

$\Rightarrow \quad 4\log_3 P \geq -16$

$\Rightarrow \quad \log_3 P \geq -\frac{16}{4}$

$\Rightarrow \log_3 P \geq -4 \Rightarrow P \geq 3^{-4} \Rightarrow P \geq \frac{1}{81}$

$\therefore$ Minimum value of $P = \frac{1}{81}$

21. If α and β are the roots of the equation $3x^2 + 2x + 1 = 0$, then the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$ is

(a) $3x^2 + 8x + 16 = 0$
(b) $3x^2 - 8x - 16 = 0$
(c) $3x^2 + 8x - 16 = 0$
(d) $x^2 + 8x + 16 = 0$

(**a**) Given, α and β are the roots of equations

$$3x^2 + 2x + 1 = 0$$

$$\therefore \quad \alpha + \beta = -\frac{2}{3} \qquad \ldots(i)$$

and $$\alpha\beta = \frac{1}{3} \qquad \ldots(ii)$$

We have to find the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$.

$\therefore$ Sum of roots of required equation

$= (\alpha + \beta^{-1}) + (\beta + \alpha^{-1})$

$= \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{-2}{3} + \frac{-2/3}{1/3}$ [from Eqs. (i) and (ii)]

$= -\frac{8}{3}$

and product of roots of required equation

$= (\alpha + \beta^{-1})(\beta + \alpha^{-1})$

$= \alpha\beta + \alpha \cdot \alpha^{-1} + \beta^{-1} \cdot \beta + \beta^{-1}\alpha^{-1}$

$= \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta}$

$= \frac{1}{3} + 2 + \frac{1}{\frac{1}{3}} = \frac{16}{3}$ [from Eq. (ii)]

$\therefore$ Required equation will be

$x^2 -$ (sum of roots) $x +$ product of roots $= 0$

$$\therefore \quad x^2 - \left(-\frac{8}{3}\right)x + \frac{16}{3} = 0$$

$$\Rightarrow \quad 3x^2 + 8x + 16 = 0$$

22. In ΔPQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

(a) $a = b + c$ (b) $b = c + a$
(c) $c = a + b$ (d) $b = c$

(**c**) We have, ΔPQR in which $\angle R = \frac{\pi}{2}$.

$$\therefore \quad \angle P + \angle Q = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{1}{2}\angle P + \frac{1}{2}\angle Q = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2}\tan\frac{Q}{2}} = 1$$

$$\Rightarrow \tan\frac{P}{2} + \tan\frac{Q}{2} = 1 - \tan\frac{P}{2}\tan\frac{Q}{2} \qquad \ldots(i)$$

Since, $\tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are the roots of the equation $ax^2 + bx + c = 0$

$$\therefore \quad \tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a} \qquad \ldots(ii)$$

and $$\tan\frac{P}{2} \cdot \tan\frac{Q}{2} = \frac{c}{a} \qquad \ldots(iii)$$

$\therefore$ From Eqs. (i), (ii) and (iii), we have

$$-\frac{b}{a} = 1 - \frac{c}{a}$$

$$\Rightarrow \quad -b = a - c$$

$$\Rightarrow \quad c = a + b$$

2017 (I)

23. If the difference between the roots of the equation $x^2 + kx + 1 = 0$ is strictly less than $\sqrt{5}$, where $|k| \geq 2$, then k can be any element of the interval

(a) $(-3, -2] \cup [2, 3)$
(b) $(-3, 3)$
(c) $[-3, -2] \cup [2, 3]$
(d) None of the above

(**a**) Given equation

$$x^2 + kx + 1 = 0$$

Let roots of equation be α and β.

According to the question, we get

$$|\alpha - \beta| < \sqrt{5} \Rightarrow |\alpha - \beta|^2 < 5 \qquad \ldots(i)$$

[squaring on both sides]

Now, $\alpha + \beta = -k$

and $\alpha\beta = 1$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$\Rightarrow \quad (\alpha - \beta)^2 = k^2 - 4 \cdot 1$

$\Rightarrow \quad (\alpha - \beta)^2 = k^2 - 4$

On substituting $(\alpha - \beta)^2 = k^2 - 4$ in Eq. (i), we get

$$k^2 - 4 < 5 \Rightarrow k^2 < 9$$

$$\Rightarrow \quad -3 < k < 3$$

Given $|K| \geq 2$

$\therefore$ k $(-3, -2] \cup [2, 3)$

24. If the graph of a quadratic polynomial lies entirely above X-axis, then which one of the following is correct?

(a) Both the roots are real
(b) One root is real and the other is complex
(c) Both the roots are complex
(d) Cannot say

(**c**) If the graph of a quadratic polynomial lies entirely above the X-axis, then real roots does not exists because for the existence of real roots the graph of polynomial should cut X-axis at atleast one point.

25. If $\cot\alpha$ and $\cot\beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$, then the value of $\cot(\alpha + \beta)$ is

(a) $\frac{c - 1}{b}$ (b) $\frac{1 - c}{b}$ (c) $\frac{b}{c - 1}$ (d) $\frac{b}{1 - c}$

(**b**) Here,

$\cot\alpha + \cot\beta = -b/1 = -b$

and $\cot\alpha \cdot \cot\beta = c/1 = c$

$$\therefore \quad \cot(\alpha + \beta) = \frac{\cot\alpha \cdot \cot\beta - 1}{\cot\alpha + \cot\beta}$$

$$= \frac{c - 1}{-b} = \frac{1 - c}{b}$$

26. The sum of the roots of the equation $x^2 + bx + c = 0$ (where, b and c are non-zero) is equal to the sum of the reciprocals of their squares. Then, $\frac{1}{c}, b, \frac{c}{b}$ are in

(a) AP (b) GP
(c) HP (d) None of these

(**c**) Let α and β be roots of given equation $x^2 + bx + c = 0$

$\therefore \alpha + \beta = -b$ and $\alpha\beta = c$

According to the question,

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \quad \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\Rightarrow \quad -b = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \quad -b = \frac{(-b)^2 - 2(c)}{(c)^2}$$

$$\Rightarrow \quad b^2 - 2c = -bc^2$$

$\Rightarrow b^2 + bc^2 - 2c = 0$

$\Rightarrow b(b + c^2) - 2c = 0$

$$\Rightarrow \quad b = \frac{2c}{b + c^2}$$

$$\Rightarrow \quad b = \frac{2}{b/c + c}$$

$$\frac{1}{b} = \frac{\frac{b}{c} + c}{2}$$

$\Rightarrow c, 1/b, b/c$ are in AP

$\Rightarrow \frac{1}{c}, b, \frac{c}{b}$ are in HP.

27. The sum of the roots of the equation $ax^2 + x + c = 0$ (where, a and c are non-zero) is equal to the sum of the reciprocals of their squares. Then, a, ca^2, c^2 are in

(a) AP (b) GP
(c) HP (d) None of these

(***a***) Let α and β are roots of the equation

$$ax^2 + x + c = 0$$

then $\alpha + \beta = -1/a$

and $\alpha\beta = c/a$

According to the question,

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\Rightarrow -\frac{1}{a} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow -\frac{1}{a} = \frac{\left(-\frac{1}{a}\right)^2 - 2(c/a)}{(c/a)^2}$$

$$\Rightarrow -\frac{1}{a} = \frac{(1/a^2) - (2c/a)}{c^2/a^2}$$

$$\Rightarrow -\frac{1}{a} = \frac{(1-2ac)/a^2}{c^2/a^2}$$

$$\Rightarrow -\frac{1}{a} = \frac{(1-2ac)}{c^2} \Rightarrow -c^2 = a(1-2ac)$$

$$\Rightarrow -c^2 = a - 2a^2c$$

$$\Rightarrow 2a^2c = a + c^2 \Rightarrow a^2c = \frac{a + c^2}{2}$$

$\Rightarrow a, ca^2, c^2$ are in AP

28. If $S = \{x : x^2 + 1 = 0, x \text{ is real}\}$, then S is

(a) $\{-1\}$ (b) $\{0\}$
(c) $\{1\}$ (d) an empty set

(***d***) $x^2 + 1 = 0$

$\Rightarrow x^2 = -1 \Rightarrow x = \pm\sqrt{-1} = \pm i$

Since, x is real

Then, $S = \phi$ = an empty set

2016 (II)

Directions (Q. Nos. 29 and 30) *Read the following information carefully and answer the questions given below.*

Let α and β be the roots of the equation $x^2 - (1-2a^2)x + (1-2a^2) = 0$.

29. Under what condition does the above equation have real roots?

(a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$
(c) $a^2 \le \frac{1}{2}$ (d) $a^2 \ge \frac{1}{2}$

(***d***) We have,

$$x^2 - (1-2a^2)x + (1-2a^2) = 0$$

For real roots,

$$D \ge 0$$

$$\therefore (1-2a^2)^2 - 4(1-2a^2) \ge 0$$

$$\Rightarrow (1-2a^2)(1-2a^2-4) \ge 0$$

$$\Rightarrow (1-2a^2)(2a^2+3) \le 0$$

$$\Rightarrow a^2 \ge \frac{1}{2} \quad [\because 2a^2 + 3 > 0]$$

30. Under what condition is $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1$?

(a) $a^2 < \frac{1}{2}$

(b) $a^2 > \frac{1}{2}$

(c) $a^2 > 1$

(d) $a^2 \in \left(\frac{1}{3}, \frac{1}{2}\right)$ only

(***a***) We have,

α, β are the roots of the equation,

$$x^2 - (1-2a^2)x + (1-2a^2) = 0$$

$$\therefore \alpha + \beta = 1 - 2a^2$$

$$\alpha\beta = 1 - 2a^2$$

Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2}{(\alpha\beta)^2} - \frac{2}{\alpha\beta}$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(1-2a^2)^2}{(1-2a^2)^2} - \frac{2}{1-2a^2}$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = 1 - \frac{2}{1-2a^2}$$

Since, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1$

$$\therefore 1 - \frac{2}{1-2a^2} < 1$$

$$\Rightarrow \frac{2}{2a^2 - 1} < 0 \Rightarrow 2a^2 < 1 \Rightarrow a^2 < \frac{1}{2}$$

Directions (Q.Nos. 31 and 32) *Read the following information carefully and answer the questions given below.*

$2x^2 + 3x - \alpha = 0$ has roots -2 and β while the equation $x^2 - 3mx + 2m^2 = 0$ has both roots positive, where $\alpha > 0$ and $\beta > 0$.

31. What is the value of α?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

(***c***) $\because -2$ and β are the roots of the equation

$$2x^2 + 3x - \alpha = 0$$

$$\therefore 2(-2)^2 + 3(-2) - \alpha = 0$$

$$\Rightarrow 8 - 6 - \alpha = 0 \Rightarrow \alpha = 2$$

32. If $\beta, 2, 2m$ are in GP, then what is the value of $\beta\sqrt{m}$?

(a) 1 (b) 2 (c) 4 (d) 6

(***a***) Since, $\alpha = 2$

$\therefore$ Equation becomes

$$2x^2 + 3x - 2 = 0$$

$$\Rightarrow 2x^2 + 4x - x - 2 = 0$$

$$\Rightarrow (2x-1)(x+2) = 0$$

$$\Rightarrow x = -2, x = \frac{1}{2}$$

$$\therefore \beta = \frac{1}{2}$$

The roots of second equation are given by

$$x^2 - 3mx + 2m^2 = 0$$

$$\Rightarrow x^2 - 2mx - mx + 2m^2 = 0$$

$$\Rightarrow (x-2m)(x-m) = 0$$

$$\Rightarrow x = 2m, x = m$$

Since, both roots are positive.

$$\therefore m > 0$$

$\because \beta, 2, 2m$ are in GP.

$\therefore 4 = 2m\beta \Rightarrow 4 = 2m \times \frac{1}{2} \Rightarrow m = 4$

$$\therefore \beta\sqrt{m} = \frac{1}{2}\sqrt{4} = \frac{1}{2} \times 2 = 1$$

33. If $c > 0$ and $4a + c < 2b$, then $ax^2 - bx + c = 0$ has a root in which one of the following intervals?

(a) (0, 2) (b) (2, 3)
(c) (3, 4) (d) (–2, 0)

(***a***) We have, $c > 0$ and $4a + c < 2b$

Let $f(x) = ax^2 - bx + c$

Then, $f(0) = c > 0$

$\Rightarrow f(2) = 4a - 2b + c \Rightarrow f(2) < 0$

$\therefore f(0) \cdot f(2) < 0$

Hence, one of the root is lie between (0, 2)

2016 (I)

Directions (Q. Nos. 34 and 35) *Read the following information carefully and answer the questions given below.*

Let α and β $(\alpha < \beta)$ be the roots of the equation $x^2 + bx + c = 0$, where $b > 0$ and $c < 0$.

34. Consider the following

1. $\beta < -\alpha$ 2. $\beta < |\alpha|$

Which of the above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

35. Consider the following

1. $\alpha + \beta + \alpha\beta > 0$
2. $\alpha^2\beta + \beta^2\alpha > 0$

Which of the above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

Solutions (Q. Nos. 34 and 35)

Given, α and β are the roots of equation $x^2 + bx + c = 0$.

$\therefore \quad \alpha + \beta = -b$ and $\alpha\beta = c$

As $\quad b > 0$

So, $\quad \alpha + \beta < 0 \Rightarrow \beta < -\alpha$

and given that $\alpha < \beta \Rightarrow \alpha < 0$ and $\beta > 0$

So, $\alpha\beta < 0$

34. ***(c)*** $\alpha + \beta < 0 \Rightarrow \beta < -\alpha$...(i)

$\therefore$ Statement 1 is correct.

As, $\quad \alpha < \beta \Rightarrow -\alpha > \beta$ ...(ii)

From Eqs. (i) and (ii), $\quad |\alpha| > \beta$

$\therefore$ Statement 2 is correct.

35. ***(b)*** As, $\alpha + \beta < 0$ and $\alpha\beta < 0$

$\Rightarrow \alpha + \beta + \alpha\beta < 0$

$\therefore$ Statement 1 is not correct.

Now, $\quad \alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$

As, $\alpha + \beta < 0$ and $\alpha\beta < 0$

$\Rightarrow \alpha\beta(\alpha + \beta) > 0$

$\therefore$ Statement 2 is correct.

36. If $x^2 - px + 4 > 0$ for all real values of x, then which one of the following is correct?

(a) $|p| < 4$ (b) $|p| \le 4$

(c) $|p| > 4$ (d) $|p| \ge 4$

(***a***) $\because x^2 - px + 4 > 0$

Here, $a > 0$ and $f(x) > 0$

$\therefore \quad D < 0$

$\therefore \quad p^2 - 16 < 0 \Rightarrow p^2 < 16 \Rightarrow |p| < 4$

37. If one root of the equation $(l - m)x^2 + lx + 1 = 0$ is double the other and l is real, then what is the greatest value of m?

(a) $-\frac{9}{8}$ (b) $\frac{9}{8}$

(c) $-\frac{8}{9}$ (d) $\frac{8}{9}$

(***b***) Given equation is

$(l - m)x^2 + lx + 1 = 0$

Let α and 2α be the roots of given equation. Then, we have

$$\alpha + 2\alpha = 3\alpha = \frac{-l}{l - m}$$

and $\quad \alpha \cdot 2\alpha = 2\alpha^2 = \frac{1}{l - m}$

$$\Rightarrow \quad \alpha = \frac{-l}{3(l - m)} \text{ and } 2\alpha^2 = \frac{1}{l - m}$$

$$\Rightarrow \quad 2 \cdot \left(\frac{-l}{3(l - m)}\right)^2 = \frac{1}{l - m}$$

$$\Rightarrow \quad \frac{2l^2}{9(l - m)^2} = \frac{1}{l - m}$$

$$\Rightarrow \quad 2l^2 = 9(l - m) \qquad [\because l \ne m]$$

$$\Rightarrow \quad 2l^2 - 9l + 9m = 0$$

$$\Rightarrow \quad l = \frac{9 \pm \sqrt{81 - 72m}}{4}$$

$\because l$ is real.

$\therefore \quad D \ge 0 \Rightarrow 81 - 72m \ge 0$

$$\Rightarrow \quad 81 \ge 72m$$

$$\Rightarrow \quad m \le \frac{81}{72} \Rightarrow m \le \frac{9}{8}$$

Hence, greatest value of m is $\frac{9}{8}$.

2015 (II)

38. The number of real roots of the equation $x^2 - 3|x| + 2 = 0$ is

(a) 4 (b) 3 (c) 2 (d) 1

(***a***) We have, $x^2 - 3|x| + 2 = 0$

$$\Rightarrow \quad |x|^2 - 3|x| + 2 = 0$$

$$\Rightarrow \quad |x|^2 - 2|x| - |x| + 2 = 0$$

$$\Rightarrow \quad (|x| - 2)(|x| - 1) = 0$$

$$\Rightarrow \quad |x| = 2 \text{ or } |x| = 1$$

$$\Rightarrow \quad x = \pm 2 \text{ or } x = \pm 1$$

$\therefore$ There are four real roots of the equation.

39. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of their squares, then

(a) $a^2 + b^2 = c^2$

(b) $a^2 + b^2 = a + b$

(c) $ab + b^2 = 2ac$

(d) $ab - b^2 = 2ac$

(***c***) We have, $ax^2 + bx + c = 0$

Sum of roots, *i.e.* $\alpha + \beta = -\frac{b}{a}$

and product of roots, *i.e.* $\alpha\beta = \frac{c}{a}$

It is given that $\alpha + \beta = \alpha^2 + \beta^2$

$$\Rightarrow \quad \alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\Rightarrow \quad -\frac{b}{a} = \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$$

$$\Rightarrow \quad -\frac{b}{a} = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$\Rightarrow \quad -\frac{b}{a} = \frac{b^2 - 2ac}{a^2}$$

$$\Rightarrow \quad -ab = b^2 - 2ac$$

$$\Rightarrow \quad b^2 = 2ac - ab$$

$$\Rightarrow \quad b^2 + ab = 2ac$$

40. If the roots of the equation $x^2 - nx + m = 0$ differ by 1, then

(a) $n^2 - 4m - 1 = 0$ (b) $n^2 + 4m - 1 = 0$

(c) $m^2 + 4n + 1 = 0$ (d) $m^2 - 4n - 1 = 0$

(***a***) We have, $x^2 - nx + m = 0$

$\alpha + \beta = n$, $\alpha\beta = m$ and given that $\alpha - \beta = 1$

We know that,

$(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$

$$\Rightarrow \quad n^2 - 1 = 4m$$

$$\Rightarrow \quad n^2 - 4m - 1 = 0$$

41. If a, b and c are the sides of a ΔABC, then $a^{\frac{1}{p}} + b^{\frac{1}{p}} - c^{\frac{1}{p}}$, where $p > 1$, is

(a) always negative (b) always positive

(c) always zero

(d) positive, if $1 < p < 2$ and negative if $p > 2$

(***b***) a, b, c are positive number.

$\Rightarrow a^{\frac{1}{p}}, b^{\frac{1}{p}}, c^{\frac{1}{p}}$ are positive number.

$$\Rightarrow \quad a^{\frac{1}{p}} + b^{\frac{1}{p}} > c^{\frac{1}{p}}$$

[sum of any two sides must be greater than third side]

$$\Rightarrow \quad a^{\frac{1}{p}} + b^{\frac{1}{p}} - c^{\frac{1}{p}} > 0$$

2015 (I)

42. In solving a problem that reduces to a quadratic equation, one student makes a mistake in the constant term and obtains 8 and 2 for roots. Another student makes a mistake only in the coefficient of first degree term and finds –9 and –1 for roots. The correct equation is

(a) $x^2 - 10x + 9 = 0$

(b) $x^2 + 10x + 9 = 0$

(c) $x^2 - 10x + 16 = 0$

(d) $x^2 - 8x - 9 = 0$

(***a***) Given, roots are 8 and 2, then the equation is

$x^2 -$ (sum of roots) x + product of roots = 0

$$\Rightarrow \quad x^2 - (8 + 2)x + 16 = 0$$

$$\Rightarrow \quad x^2 - 10x + 16 = 0 \qquad \text{...(i)}$$

[mistake in the constant term]

For roots –9 and –1, we have

$$x^2 - \{-9 + (-1)\}x + (-9) \times (-1) = 0$$

$$\Rightarrow \quad x^2 + 10x + 9 = 0 \qquad \text{...(ii)}$$

[mistake in the coefficient of first degree term]

But as per given information, we have the right equation as $x^2 - 10x + 9 = 0$.

43. If m and n are roots of the equation $(x + p)(x + q) - k = 0$, then roots of the equation $(x - m)(x - n) + k = 0$ are

(a) p and q
(b) $\frac{1}{p}$ and $\frac{1}{q}$
(c) $-p$ and $-q$
(d) $p+q$ and $p-q$

(c) Given, $(x+p)(x+q)-k=0$
$\Rightarrow \quad x^2+qx+px+pq-k=0$
$\Rightarrow \quad x^2+(p+q)x+pq-k=0 \quad \ldots(i)$
For m and n to be roots the equation should be
$x^2-(m+n)x+m\cdot n=0 \quad \ldots(ii)$
On comparing Eqs. (i) and (ii), we get
$p+q=-(m+n) \quad \ldots(iii)$
and $pq-k=m\cdot n \quad \ldots(iv)$
Also, $(x-m)(x-n)+k=0$
$\Rightarrow \quad x^2-nx-mx+mn+k=0$
$\Rightarrow \quad x^2-(m+n)x+mn+k=0$
$\Rightarrow \quad x^2+(p+q)x+pq-k+k=0$
[using Eqs. (iii) and (iv)]
$\Rightarrow \quad x^2+(p+q)x+pq=0$
$\Rightarrow x^2-[(-p)+(-q)]x+[(-p)(-q)]=0$
Hence, $-p$ and $-q$ are the required roots.

44. Every quadratic equation $ax^2+bx+c=0$, where $a,b,c\in R$, $a\neq 0$ has
(a) exactly one real root
(b) atleast one real root
(c) atleast two real roots
(d) atmost two real roots

(d) Every quadratic equation
$ax^2+bx+c=0,$
where $a,b,c\in R, a\neq 0$
has atmost two real roots.

45. If α,β are the roots of $ax^2+bx+c=0$ and $\alpha+h,\beta+h$ are the roots of $px^2+qx+r=0$, then what is h equal to?
(a) $\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$ (b) $\frac{1}{2}\left(-\frac{b}{a}+\frac{q}{p}\right)$
(c) $\frac{1}{2}\left(\frac{b}{p}+\frac{q}{a}\right)$ (d) $\frac{1}{2}\left(-\frac{b}{p}+\frac{q}{a}\right)$

(a) $\because \quad \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$
Also, $\alpha+h+\beta+h=-\frac{q}{p}$
$\Rightarrow \quad \alpha+\beta+2h=-\frac{q}{p}$
$\Rightarrow \quad 2h=-\frac{q}{p}+\frac{b}{a} \quad \left[\because \alpha+\beta=-\frac{b}{a}\right]$
$\Rightarrow \quad h=\frac{1}{2}\left[\frac{b}{a}-\frac{q}{p}\right]$

2014 (I)

46. If α and β are the roots of the equation $ax^2+bx+c=0$, where $a\neq 0$, then $(a\alpha+b)(a\beta+b)$ is equal to
(a) ab (b) bc (c) ca (d) abc

(c) Given that, α and β are the roots of the equation $ax^2+bx+c=0$, where $a\neq 0$.
Then, sum of roots $=\alpha+\beta=\frac{-b}{a}$
and product of roots $=\alpha\cdot\beta=\frac{c}{a}$
Now, we have
$(a\alpha+b)(a\beta+b)$
$=a^2(\alpha\beta)+ab(\alpha+\beta)+b^2$
$=a^2\left(\frac{c}{a}\right)+ab\left(-\frac{b}{a}\right)+b^2$
$=ac-b^2+b^2=ac$

47. The roots of the equation $2a^2x^2-2abx+b^2=0$, when $a<0$ and $b>0$ are
(a) sometimes complex
(b) always irrational
(c) always complex
(d) always real

(c) Given equation is
$2a^2x^2-2abx+b^2=0$
On comparing with $ax^2+bx+c=0$, we get $a=2a^2$, $b=-2ab$ and $c=b^2$
When, $a<0$ and $b>0$
$\therefore x=\frac{-(-2ab)\pm\sqrt{(-2ab)^2-4\cdot 2a^2\cdot b^2}}{2\cdot 2a^2}$
[by quadratic formula]
$=\frac{2ab\pm\sqrt{4a^2b^2-8a^2b^2}}{4a^2}$
$=\frac{2ab\pm\sqrt{-4a^2b^2}}{4a^2}=\frac{2ab\pm i\,2ab}{4a^2}$
$=\frac{2ab(1\pm i)}{4a^2}=\frac{b}{2a}(1\pm i)$
which shows that the roots of the given equation is always complex.

6

PERMUTATIONS AND COMBINATIONS

2019 (II)

1. What is
$C(47,4) + C(51,3) + C(50,3)$
$+ C(49,3) + C(48,3) + C(47,3)$
equal to?

(a) $C(47, 4)$ (b) $C(52, 5)$
(c) $C(52, 4)$ (d) $C(47, 5)$

(c) $C(47, 4) + C(51, 3) + C(50, 3)$
$+ C(49, 3) + C(48, 3) + C(47, 3)$
$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$
$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$
$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$
$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$
$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$
$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$
$= {}^{51}C_3 + {}^{51}C_4$
$= {}^{52}C_4 = C(52, 4)$

2. If $n!$ has 17 zeros, then what is the value of n?

(a) 95
(b) 85
(c) 80
(d) No such value of n exists

(b) We know that each interval of 5! is one zero.
i.e. 5! has one zero.
10! has two zeros.
$\therefore$ 85! has 17 zeros.
Hence, the value of n is 85.

3. What is the number of diagonals of an octagon?

(a) 48 (b) 40 (c) 28 (d) 20

(d) The number of vertices of an octagon = 8
$\therefore$ The number of points in a plane = 8
$\therefore$ Total number of straight line form by 8 points = ${}^8C_2$
[$\because$ 1 straight line form by 2 points]
$= \frac{8!}{2!6!} = \frac{8 \times 7}{2} = 28$
$\therefore$ The number of diagonals of an octagon = Total number of straight line form by 8 points – number of sides of octagon
$= 28 - 8 = 20$

4. If $P(n, r) = 2520$ and $C(n, r) = 21$, then what is the value of $C(n + 1, r + 1)$?

(a) 7 (b) 14
(c) 28 (d) 56

(c) If $P(n, r) = 2520$ and $C(n, r) = 21$,
$\therefore \quad {}^nP_r = 2520$
$\Rightarrow \quad \frac{n!}{(n-r)!} = 2520 \quad \ldots(i)$
and $\quad {}^nC_r = 21$
$\Rightarrow \quad \frac{n!}{r!\,(n-r)!} = 21 \quad \ldots(ii)$
From Eqs. (i) and (ii), we get
$\frac{2520}{r!} = 21$
$\Rightarrow \quad r! = \frac{2520}{21} = 120$
$\Rightarrow \quad r! = 5!$
$\therefore \quad r = 5$
Putting the value of r in Eq. (i),
$\frac{n!}{(n-5)!} = 2520$
$\Rightarrow \quad n(n-1)(n-2)(n-3)(n-4) = 7 \times 6 \times 5 \times 4 \times 3$
$\therefore \quad n = 7$
Now, $C(n + 1, r + 1) = {}^{n+1}C_{r+1}$
$= {}^{7+1}C_{5+1} = {}^8C_6$
$= \frac{8!}{6!2!} = \frac{8 \times 7}{2}$
$= 28$

2019 (I)

5. If $C(20, n + 2) = C(20, n - 2)$, then what is n equal to

(a) 8 (b) 10 (c) 12 (d) 16

(b) We have, $C(20, n + 2) = C(20, n - 2)$
$\Rightarrow \quad {}^{20}C_{n+2} = {}^{20}C_{n-2}$
$\Rightarrow \quad n + 2 + n - 2 = 20$
$[\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$
$\therefore \quad n = 10$

6. There are 10 points in a plane. No three of these points are in a straight line. What is the total number of straight lines which can be formed by joining the points?

(a) 90 (b) 45 (c) 40 (d) 30

(b) Given, 10 points in a plane where no three of these points are in straight line.
Total number of straight line formed from 10 points is
${}^{10}C_2 = \frac{10!}{2!8!} = \frac{10 \times 9}{2} = 45$

7. From 6 programmers and 4 typists, an office wants to recruit 5 people. What is the number of ways this can be done so as to recruit atleast one typist?

(a) 209 (b) 210 (c) 246 (d) 242

(c) We have,
6 programmers and 4 typists
Number of ways of 5 recruit people such that atleast one typist
$= {}^4C_1\,{}^6C_4 + {}^4C_2\,{}^6C_3 + {}^4C_3\,{}^6C_2 + {}^4C_4\,{}^6C_1$
$= 4 \times 15 + 6 \times 20 + 4 \times 15 + 1 \times 6$
$= 60 + 120 + 60 + 6 = 246$

8. How many three digit even numbers can be formed using the digits 1, 2, 3, 4 and 5 when repetition of digits is not allowed?
(a) 36 (b) 30 (c) 24 (d) 12

(**c**) Here, unit digit can be filled by 2 or 4. so number of ways is 2. Since repetition is not allowed therefore hundred place and ten place can be fill in ${}^4C_2 \times 2$ ways
$\therefore$ Total number of three digits even number $= 4 \times 3 \times 2 = 24$

2018 (II)

9. Let x be the number of integers lying between 2999 and 8001 which have atleast two digits equal.Then, x is equal to
(a) 2480 (b) 2481 (c) 2482 (d) 2483

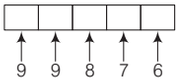
(b) We have, x be the number lying between 2999 and 8001
if repetition allowed,
Total numbers $= 5 \times 10 \times 10 \times 10 = 5000$
if repetition not allowed,
$\therefore$ Total numbers $= 5 \times 9 \times 8 \times 7 = 2520$
So, x = atleast two digit repeated
$= 5000 - 2520 + 1$
$= 2481$
[$\because$ add 1 because of number 8000]

10. There are 17 cricket players, out of which 5 players can bowl. In how many ways can a team of 11 players be selected, so as to include 3 bowlers?
(a) C (17, 11)
(b) C (12, 8)
(c) $C(17, 5) \times C(5, 3)$
(d) $C(5, 3) \times C$ (12, 8)

(d) There are 17 cricket players, out of which 5 players can bowl.
required number of ways $= {}^{12}C_8 \times {}^5C_3$
$= C$ (12, 8) $\times C$ (5, 3)

11. The total number of 5-digit numbers that can be composed of distinct digits from 0 to 9 is
(a) 45360 (b) 30240 (c) 27216 (d) 15120

(c) 5-digit number that can be composed by distinct digits from 0 to 9 is given as



$\therefore$ Required number $= 9 \times 9 \times 8 \times 7 \times 6$
$= 27216$

2018 (I)

12. How many numbers between 100 and 1000 can be formed with the digits 5, 6, 7, 8, 9, if the repetition of digits is not allowed?
(a) 3^5 (b) 5^3 (c) 120 (d) 60

(d) Number lying between 100 and 1000 are of three digit. Since the numbers are to be formed with 5, 6, 7, 8, 9 and repetition is not allowed, so total number of numbers $= 5 \times 4 \times 3 = 60$

13. How many four-digit numbers divisible by 10 can be formed using 1, 5, 0, 6, 7 without repetition of digits?
(a) 24 (b) 36 (c) 44 (d) 64

(a) We have to form four digit numbers which are divisible by 10 and using 1, 5, 0, 6, 7. Since numbers must be divisible by 10, so unit place must be zero.
$\therefore$ Total number of such numbers
= Permutations of three digits using 1, 5, 6, 7

$$= {}^4P_3 = \frac{4!}{(4-3)!} = 4! = 24$$

14. What is the number of triangles that can be formed by choosing the vertices from a set of 12 points in a plane, seven of which lie on the same straight line?
(a) 185 (b) 175
(c) 115 (d) 105

(a) Required number of triangle

$$= {}^{12}C_3 - {}^7C_3$$

$$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} - \frac{7 \times 6 \times 5}{3 \times 2 \times 1}$$

$= 220 - 35 = 185$

15. What is $C(n, r) + 2C(n, r-1) + C(n, r-2)$ equal to ?
(a) $C(n+1, r)$ (b) $C(n-1, r+1)$
(c) $C(n, r+1)$ (d) $C(n+2, r)$

(d) We have,
$C(n, r) + 2C(n, r-1) + C(n, r-2)$

$$= {}^nC_r + 2 \cdot {}^nC_{r-1} + {}^nC_{r-2}$$

$$= ({}^nC_r + {}^nC_{r-1}) + ({}^nC_{r-1} + {}^nC_{r-2})$$

$$= {}^{n+1}C_r + {}^{n+1}C_{r-1}$$

$$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$

$$= {}^{n+1+1}C_r$$

$$= {}^{n+2}C_r$$

$$= C(n+2, r)$$

2017 (II)

16. A tea party is arranged for 16 people along two sides of a long table with eight chairs on each side. Four particular men wish to sit on one particular side and two particular men on the other side. The number of ways they can be seated is
(a) $24 \times 8! \times 8!$ (b) $(8!)^3$
(c) $210 \times 8! \times 8!$ (d) 16!

(c) Since, four particular men want to sit on a particular side A (say) and two other particular men on the other side B. So, we are left with 10 guests out of which we choose 4 for side A and 6 for side B.
Hence, the number of selections for the two sides $= {}^{10}C_4 \times {}^6C_6$
Now, 8 persons on each side of the table can be arranged among themselves in 8! ways.
Hence, the total number of arrangements

$$= {}^{10}C_4 \times {}^6C_6 \times 8! \times 8!$$

$$= \frac{10!}{4!6!} \times 1 \times 8! \times 8!$$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times (8!)^2 = 210 \times (8!)^2$$

2017 (I)

17. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then, the number of subsets of A containing two or three elements is
(a) 45 (b) 120 (c) 165 (d) 330

(c) Given set
$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
The number of subsets of A containing two or three elements

$$= {}^{10}C_2 + {}^{10}C_3$$

$$= \frac{10 \times 9}{2 \times 1} + \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 45 + 120 = 165$$

18. Three-digit numbers are formed from the digits 1, 2 and 3 in such a way that the digits are not repeated. What is the sum of such three-digit numbers?
(a) 1233 (b) 1322 (c) 1323 (d) 1332

(d) Using digits 1, 2 and 3 total number of three-digit numbers formed
$= 3 \times 2 \times 1 = 6$ (if no digits are repeated)
Required numbers are

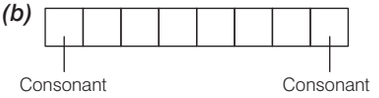
1	2	3	2	1	3	3	1	2
1	3	2	2	3	1	3	2	1

Hence, sum of above six numbers
$= 1332$

19. The number of different words (eight-letter words) ending and beginning with a consonant which can be made out of the letters of the word 'EQUATION' is

(a) 5200 (b) 4320 (c) 3000 (d) 2160

(b)



∵ There are three consonants in the word 'EQUATION'.

∴ First and last place of required word can be filled in ${}^3P_2$ ways $= \frac{3!}{(3-2)!} = 6$

and there are 5 vowels in the word 'EQUATION'.

∴ 6 places in between first and last placed can be filled by 6 letters in ${}^6P_6$ ways = 6! = 720

∴ Required number of different words

$= 6 \times 720 = 4320$

2016 (II)

20. What is the number of odd integers between 1000 and 9999 with no digit repeated?

(a) 2100 (b) 2120 (c) 2240 (d) 3331

(c) Odd integer between 1000 and 9999 with no digits repeated is

$8 \times 8 \times 7 \times 5 = 2240$

21. Out of 15 points in a plane, n points are in the same straight line. 445 triangles can be formed by joining these points. What is the value of n?

(a) 3 (b) 4 (c) 5 (d) 6

(c) We know that,

If n distinct points in a plane, in which m are collinear ($m \geq 3$), then the number of triangle formed $= {}^nC_3 - {}^mC_3$

From question,

There are 15 points in a plane in which n points are in same straight line, then

$${}^{15}C_3 - {}^nC_3 = 445$$

$$455 - {}^nC_3 = 445$$

$$\Rightarrow \quad {}^nC_3 = 455 - 445 = 10$$

$$\Rightarrow \quad \frac{n!}{3!\,(n-3)!} = 10$$

$$\Rightarrow \quad \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 10 \times 3!$$

$$\Rightarrow \quad n(n-1)(n-2) = 60$$

$$\Rightarrow \quad n(n-1)(n-2) = 5 \times 4 \times 3$$

$$\therefore \quad n = 5$$

22. A five- digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3 and 4 without repetition of digits. What is the number of ways this can be done?

(a) 96
(b) 48
(c) 32
(d) No number can be formed

(d) We are given to form five-digit numbers which are divisible by 3 with digits 0, 1, 2, 3 and 4.

But the sum of these digits is not divisible by 3.

∴ No number can be formed.

2016 (I)

23. What is the number of ways in which 3 holiday travel tickets are to be given to 10 employees of an organisation, if each employee is eligible for any one or more of the tickets?

(a) 60 (b) 120
(c) 500 (d) 1000

(d) Since, each ticket can be given to any one of 10 employees of an organisation.

∴ Required number of ways

$= 10 \times 10 \times 10 = 10^3$

$= 1000$

24. What is the number of four-digit decimal numbers (< l) in which no digit is repeated?

(a) 3024 (b) 4536
(c) 5040 (d) None of these

(b) Number of four-digit decimal numbers that can be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, when no digit is repeated.

$= 9 \times 9 \times 8 \times 7$

$= 4536$

25. What is the number of different messages that can be represented by three 0's and two 1's?

(a) 10 (b) 9
(c) 8 (d) 7

(a) Here,we are given three 0's and two 1's.

Hence, number of ways of different messages

$$= \frac{5!}{3! \times 2!}$$

$$= \frac{5 \times 4}{2} = 10$$

2015 (II)

26. If different words are formed with all the letters of the word 'AGAIN' and are arranged alphabetically among themselves as in a dictionary, the word at the 50th place will be

(a) NAAGI (b) NAAIG
(c) IAAGN (d) IAANG

(b) We have, AGAIN

The letter starts from AAGIN = 4! = 24

The letter starts from GAAIN $= \frac{4!}{2!} = 12$

The letter starts from IAAGN $= \frac{4!}{2!} = 12$

Total letter form is 48.

∴ 49th word is NAAGI and 50th word is NAAIG.

27. The number of ways in which a cricket team of 11 players be chosen out of a batch of 15 players so that the captain of the team is always included, is

(a) 165 (b) 364 (c) 1001 (d) 1365

(c) Total number of selection of 11 players out of 15 players in which captain is always included $= {}^{14}C_{10}$

$$= \frac{14!}{10!\,4!} = \frac{14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4} = 1001$$

28. A polygon has 44 diagonals. The number of its sides is

(a) 11 (b) 10 (c) 8 (d) 7

(a) Let the number of sides of the polygon be n.

Then, number of diagonals $= {}^nC_2 - n$

$$44 = {}^nC_2 - n$$

$$\Rightarrow \quad 44 = \frac{n!}{2!(n-2)!} - n$$

$$\Rightarrow \quad 44 = \frac{n(n-1)(n-2)!}{2!(n-2)!} - n$$

$$\Rightarrow \quad 44 = \frac{[n(n-1)]}{2} - n \Rightarrow 88 = n^2 - n - 2n$$

$$n^2 - 3n - 88 = 0$$

$\Rightarrow (n-11)(n+8) = 0 \Rightarrow n = 11$ and -8

Hence, polygon has 11 sides.

29. The number of ways in which 3 holiday tickets can be given to 20 employees of an organisation, if each employee is eligible for any one or more of the tickets, is

(a) 1140 (b) 3420 (c) 6840 (d) 8000

(d) Since, each employee is eligible for one or more ticket(s).

Hence, total number of ways $= 20^3$

$= 8000$

30. The number of 3-digit even numbers that can be formed from the digits 0, 1, 2, 3, 4 and 5, repetition of digits being not allowed, is

(a) 60 (b) 56 (c) 52 (d) 48

(c) *(**c**)* We have, digits 0, 1, 2, 3, 4 and 5

Even Number

(i) When 0 is at unit place

$\times \times \times$
$4 \quad 5 \quad 0$ = 20 Number

(ii) When 0 is not at unit place

$4 \quad 4 \quad 2$
$\times \times \times$ = 32 Number
2 or 4

Total even numbers = 20 + 32 = 52

2015 (I)

31. Let $A = \{1,2,3,4,5,6,7,8,9,10\}$. Then, the number of subsets of A containing exactly two elements is

(a) 20 (b) 40 (c) 45 (d) 90

(c) *(**c**)* Required number of subsets of A containing exactly two elements

$= {}^{10}C_2 = \dfrac{10 \times 9}{2} = \dfrac{90}{2} = 45$

32. How many words can be formed using all the letters of the word 'NATION', so that all the three vowels should never come together?

(a) 354 (b) 348
(c) 288 (d) None of these

(c) *(**c**)* Number of ways in which NATION can be arranged $= \dfrac{4!}{2!}$

Number of ways in which three vowels of word NATION come together

$= \dfrac{4! \times 3!}{2!}$.

Required number of words

$= \dfrac{6!}{2!} - \dfrac{4! \times 3!}{2!} = 360 - 72 = 288$

33. What is the number of ways in which one can post 5 letters in 7 letter boxes?

(a) 7^5 (b) 3^5
(c) 5^7 (d) 2520

(a) *(**a**)* We know that, the number of permutations of n different things taken r at a time allowing repetitions is n^r.

There are 5 letters and 7 letter boxes.

First letter can be put any 7 letters boxes = 7 ways

Similarly, 2nd, 3rd, 4th and 5th letters be put in 7 ways each, respectively.

∴ Required number of ways

$= 7 \times 7 \times 7 \times 7 \times 7$

$= 7^5$

34. What is the number of ways that a cricket team of 11 players can be made out of 15 players?

(a) 364 (b) 1001 (c) 1365 (d) 32760

(c) *(**c**)* 11 players can be made out of 15 players is

${}^{15}C_{11} = \dfrac{15!}{11!\,4!}$

$= \dfrac{15 \times 14 \times 13 \times 12 \times 11!}{11! \times 1 \times 2 \times 3 \times 4} = 1365$

2014 (I)

Directions (Q. Nos. 35-37) *Read the following information carefully and answer the questions given below.*
Given that, $C(n, r) : C(n, r+1) = 1:2$ and $C(n, r+1) : C(n, r+2) = 2:3$.

35. What is n equal to?

(a) 11 (b) 12 (c) 13 (d) 14

36. What is r equal to?

(a) 2 (b) 3 (c) 4 (d) 5

37. What is $P(n,r) : C(n,r)$ equal to?

(a) 6 (b) 24 (c) 120 (d) 720

(c) **Solutions (Q. Nos. 35-37)**

Given that, $C(n, r) : C(n, r+1) = 1:2$

$\Rightarrow \quad {}^nC_r : {}^nC_{r+1} = 1:2$

$\Rightarrow \quad \dfrac{\dfrac{n!}{r!\,(n-r)!}}{\dfrac{n!}{(r+1)!(n-r-1)!}} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{(r+1)!\,(n-r-1)!}{r!\,(n-r)!} = \dfrac{1}{2}$

$\Rightarrow \dfrac{(r+1)\cdot r!\,(n-r-1)!}{r!\,(n-r)(n-r-1)!} = \dfrac{1}{2} \Rightarrow \dfrac{r+1}{n-r} = \dfrac{1}{2}$

$\Rightarrow \quad 2r + 2 = n - r \quad \Rightarrow \quad n = 3r + 2 \ldots (i)$

and $C(n, r+1) : C(n, r+2) = 2 : 3$

$\Rightarrow \quad {}^nC_{r+1} : {}^nC_{r+2} = 2 : 3$

$\Rightarrow \quad \dfrac{\dfrac{n!}{(r+1)!\,(n-r-1)!}}{\dfrac{n!}{(r+2)!\,(n-r-2)!}} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{(r+2)!\,(n-r-2)!}{(r+1)!\,(n-r-1)!} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{(r+2)\cdot(r+1)!\,(n-r-2)!}{(r+1)!\,(n-r-1)\,(n-r-2)!} = \dfrac{2}{3}$

$\Rightarrow \dfrac{r+2}{n-r-1} = \dfrac{2}{3} \Rightarrow 3r + 6 = 2n - 2r - 2$

$\Rightarrow \quad 5r + 8 = 2n \quad \ldots (ii)$

On solving Eqs. (i) and (ii), we get $r = 4$.

35. (*d*) Now, put the value of r in Eq. (i), we get

$n = 3(4) + 2$
$n = 12 + 2$
$n = 14$

36. (*c*) Since, we have already get the value of r i.e. $r = 4$.

37. (*b*) We have, $P(n, r) : C(n, r) = {}^nP_r : {}^nC_r$

$= \dfrac{n!}{(n-r)!} : \dfrac{n!}{r!\,(n-r)!}$

$= 1 : \dfrac{1}{r!} = r! : 1 = 4! : 1 = 24 : 1 = 24$

38. Out of 7 consonants and 4 vowels, words are to be formed by involving 3 consonants and 2 vowels. The number of such words formed is

(a) 25200 (b) 22500 (c) 10080 (d) 5040

(a) *(**a**)* In out of 7 consonants, 3 consonants can be selected in ${}^7C_3$ ways.

In out of 4 vowels, 2 vowels can be selected in ${}^4C_2$ ways.

∴ Number of such words $= {}^7C_3 \times {}^4C_2 \times 5!$

[since, 5 letters can be selected in 5 ways]

$= 35 \times 6 \times 120 = 25200$

39. How many different words can be formed by taking four letters out of the letters of the word 'AGAIN', if each word has to start with A?

(a) 6 (b) 12
(c) 24 (d) None of these

(c) *(**c**)* A ... ... Fixed.

The word 'AGAIN' has five letters 2A, 1G, 1I, 1N. Since, A repeat two times and A is fixed at first position then, we have to arrange remaining 4 letters in three vacant position.

∴ Required number of ways

$= \underset{1}{(A}\ \underset{2}{A)}\ \underset{3}{G}\ \underset{4}{I}\ N = 4! = 24$

7

BINOMIAL THEOREM

2019 (II)

1. How many terms are there in the expansion of $(1+2x+x^2)^5+(1+4y+4y^2)^5$?

(a) 12 (b) 20 (c) 21 (d) 22

(*d*) Given expansion,

$(1+2x+x^2)^5+(1+4y+4y^2)^5$

$=[(1+x)^2]^5+[(1+2y)^2]^5$

$=(1+x)^{10}+(1+2y)^{10}$

∴ Total number of terms in given expansion.

$=(10+1)+(10+1)$

$=22$

[∵ total number of terms in expansion of $(1+x)^n = n+1$]

2. If the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$ is $184756x^{10}$, then what is the value of n?

(a) 10 (b) 8 (c) 5 (d) 4

(*a*) The middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$

$=\left(\frac{2n}{2}+1\right)$th term [∵ $2n$ is even]

$=(n+1)$th term.

According to the question,

Value of middle term $= 184756x^{10}$

$\Rightarrow {}^{2n}C_n\,(x^2)^{2n-n}\left(\frac{1}{x}\right)^n = 184756x^{10}$

[∵ $T_{r+1} = {}^nC_r\,x^{n-r}a^r$ in expansion of $(x+a)^n$]

$\Rightarrow {}^{2n}C_n(x)^{4n-2n-n} = 184756x^{10}$

$\Rightarrow {}^{2n}C_n(x)^n = 184756x^{10}$

Comparing the power of x both sides

$n = 10$

3. If the constant term in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then what can be the values of k?

(a) ±2 (b) ±3

(c) ±5 (d) ±9

(*b*) Let $(r+1)$th term in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is constant.

$\therefore\quad T_{r+1} = {}^{10}C_r\,(\sqrt{x})^{10-r}\left(\frac{-k}{x^2}\right)^r$

[∵ $T_{r+1} = {}^nC_r x^{n-r}a^r$ in expansion of $(x+a)^n$]

$\Rightarrow\ 405 = {}^{10}C_r\,(x)^{\frac{10-r}{2}-2r}\cdot(-k)^r$

$\Rightarrow 405 = {}^{10}C_r\,(x)^{\frac{10-5r}{2}}\cdot(-k)^r$ …(i)

For constant term

$\frac{10-5r}{2} = 0 \Rightarrow 10-5r = 0$

$\therefore\quad r = 2$

Putting the value of r, in Eq. (i),

$405 = {}^{10}C_2\cdot(-k)^2$

$\Rightarrow\quad 405 = \frac{10!}{2!8!}\times k^2$

$\Rightarrow\quad 405 = \frac{10\times 9}{2}.k^2$

$\Rightarrow\quad k^2 = \frac{405}{45}$

$\Rightarrow\quad k^2 = 9 \Rightarrow k = \pm 3$

2019 (I)

4. What is the number of terms in the expansion of $[(2x-3y)^2(2x+3y)^2]^2$?

(a) 4 (b) 5 (c) 8 (d) 16

(*b*) Given, $[(2x-3y)^2(2x+3y)^2]^2$

$=[4x^2-9y^2]^4$

∴ Total number of terms $= 4+1=5$

5. In the expansion of $(1+ax)^n$, the first three terms are respectively $1, 12x$ and $64x^2$. What is n equal to?

(a) 6 (b) 9

(c) 10 (d) 12

(*b*) Given, first three terms of expansion $(1+ax)^n$ is $1, 12x, 64x^2$,

Now,

$(1+ax)^n = 1+nax+\frac{n(n-1)}{2}a^2x^2+\ldots$

On equating first three terms, we get

$na = 12$ and $\frac{n(n-1)}{2}a^2 = 64$

On putting the value of a in $\frac{n(n-1)}{2}a^2 = 64$, we get

$\frac{n(n-1)}{2}\left(\frac{12}{n}\right)^2 = 64$

$\Rightarrow\quad \frac{144(n-1)}{2n} = 64$

$\therefore\quad n = 9$

2018 (II)

6. What is the coefficient of the middle term in the binomial expansion of $(2+3x)^4$?

(a) 6 (b) 12

(c) 108 (d) 216

(*d*) We have, $(2+3x)^4$

Here, $n = 4$, so middle term is

$\left(\frac{4}{2}+1\right)$th = 3rd term

$T_3 = {}^4C_2\times 2^2\times(3x)^2$

$[T_{r+1} = {}^nC_r a^r b^{n-r}]$

$=\frac{4\times 3}{2\times 1}\times 4\times 9x^2$

$T_3 = 216x^2$

Hence, coefficient of middle term is 216.

7. Let the coefficient of the middle term of the binomial expansion of $(1+x)^{2n}$ be α and those of two middle terms of the binomial expansion of $(1+x)^{2n-1}$ be β and γ. Which one of the following relations is correct?

(a) $\alpha > \beta + \gamma$ (b) $\alpha < \beta + \gamma$
(c) $\alpha = \beta + \gamma$ (d) $\alpha = \beta\gamma$

(c) We have, $(1+x)^{2n}$

Middle term $= \left(\frac{2n}{2}+1\right)$th term [$\because 2n$ is even]

$= (n+1)$th term

Coefficient of $(n+1)$th term $= {}^{2n}C_n$

$\alpha = {}^{2n}C_n$

Again, we have binomial expansion of $(1+x)^{2n-1}$ coefficient of middle terms are,

$\therefore \quad \beta = {}^{2n-1}C_n$ and $\gamma = {}^{2n-1}C_{n-1}$

Now, $\beta + \gamma = {}^{2n-1}C_n + {}^{2n-1}C_{n-1}$

[$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

$= {}^{2n}C_n$

Hence, $\alpha = \beta + \gamma$.

2018 (I)

8. In the expansion of $(1+x)^{43}$, if the coefficients of $(2r+1)$th and $(r+2)$ th terms are equal, then what is the values of $r(r \neq 1)$?

(a) 5 (b) 14 (c) 21 (d) 22

(b) We have, $(1+x)^{43}$

$\therefore$ General term, $T_{r+1} = {}^{43}C_r x^r$

Now, $T_{2r+1} = {}^{43}C_{2r}\, x^{2r}$

and $T_{r+2} = {}^{43}C_{r+1} x^{r+1}$

Now, according to the question

Coefficients of $(2r+1)$th and $(r+2)$th terms are equal

$\therefore \quad {}^{43}C_{2r} = {}^{43}C_{r+1} \Rightarrow 2r + r + 1 = 43$

[$\because$ if ${}^nC_x = {}^nC_y \Rightarrow x + y = n$]

$\Rightarrow \quad 3r = 42 \Rightarrow r = 14$

9. If the coefficients of a^m and a^n in the expansion of $(1+a)^{m+n}$ are α and β, then which one of the following is correct?

(a) $\alpha = 2\beta$ (b) $\alpha = \beta$
(c) $2\alpha = \beta$ (d) $\alpha = (m+n)\beta$

(b) We have $(1+a)^{m+n}$

$\therefore \quad T_{r+1} = {}^{m+n}C_r a^r$

$\therefore$ Coefficient of $a^m = {}^{m+n}C_m$ [$\because r = m$]

and coefficient of $a^n = {}^{m+n}C_n$ [$\because r = n$]

$\therefore \quad \alpha = {}^{m+n}C_m$

and $\beta = {}^{m+n}C_n = {}^{m+n}C_{m+n-n}$

[$\because {}^nC_r = {}^nC_{n-r}$]

$= {}^{m+n}C_m = \alpha \quad \therefore \ \alpha = \beta$

10. What is the number of non-zero terms in the expansion of $(1+2\sqrt{3}x)^{11} + (1-2\sqrt{3}x)^{11}$ (after simplification)?

(a) 4 (b) 5
(c) 6 (d) 11

(c) $(a+b)^n + (a-b)^n$, number of terms

$$= \begin{cases} \frac{n+2}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

$\therefore$ Number of terms in $(1+2\sqrt{3}x)^{11} + (1-2\sqrt{3}x)^{11}$

$= \frac{11+1}{2} = \frac{12}{2} = 6$ [$\because n = 11$, is odd]

2017 (II)

11. The number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$ after simplification is

(a) 202 (b) 101 (c) 51 (d) 50

(c) We know,

Number of terms in the expansion of $(x+a)^{100}$ will be 101 and number of terms in the expansion of $(x-a)^{100}$ also will be 101.

$\therefore$ In the expansion of $(x+a)^{100} + (x-a)^{100}$, 50 terms will be cancel out and 51 terms will be added up

$\therefore$ Number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$ will 51

12. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is

(a) 2^{26} (b) 2^{49} (c) 2^{50} (d) 2^{51}

(b) We know, the sum of the coefficients of $(1+x)^n$.

i.e. $C_0 + C_1 + C_2 + \ldots\ldots + C_n = 2^n$

$\therefore$ Sum of the coefficients of odd powers of $(1+x)^n$.

i.e. $C_1 + C_3 + C_5 + \ldots\ldots = \frac{2^n}{2}$

$\therefore$ Sum of the coefficients of odd powers of $(1+x)^{50} = \frac{2^{50}}{2} = 2^{49}$

2017 (I)

13. The value of $[C(7,0)+C(7,1)]+[C(7,1)+C(7,2)]+\ldots+[C(7,6)+C(7,7)]$ is

(a) 254 (b) 255 (c) 256 (d) 257

(a) $[C(7,0)+C(7,1)]$

$+ [C(7,1)+C(7,2)]$

$+ \ldots + [C(7,6)+C(7,7)]$

$= C(8,1) + C(8,2) + C(8,3) + C(8,4)$

$+ C(8,5) + C(8,6) + C(8,7)$

[$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

$= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4$

$+ {}^8C_3 + {}^8C_2 + {}^8C_1$

$= 2\,{}^8C_1 + 2\,{}^8C_2 + 2\,{}^8C_3 + {}^8C_4$

$= 2 \times 8 + 2 \times \frac{8\times 7}{2\times 1}$

$+ 2 \times \frac{8\times 7\times 6}{3\times 2\times 1} + \frac{8\times 7\times 6\times 5}{4\times 3\times 2\times 1}$

$= 16 + 56 + 2 \times 56 + 2 \times 7 \times 5$

$= 16 + 56 + 112 + 70 = 254$

14. The expansions of $(x-y)^n$, $n \geq 5$ is done in the descending powers of x. If the sum of the fifth and sixth terms is zero, then $\frac{x}{y}$ is equal to

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$
(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

(b) ${}^nC_4 x^{n-4}y^4 - {}^nC_5 x^{n-5}y^5 = 0$

$\Rightarrow {}^nC_4 x^{n-4}y^4 - {}^nC_5 x^{n-4} \cdot x^{-1}y^5 = 0$

$\Rightarrow \quad {}^nC_4 = {}^nC_5 \cdot \frac{y}{x} \Rightarrow \frac{x}{y} = \frac{{}^nC_5}{{}^nC_4}$

$\Rightarrow \quad \frac{x}{y} = \frac{n!}{5!(n-5)!} \times \frac{4!(n-4)!}{n!}$

$\therefore \quad \frac{x}{y} = \frac{n-4}{5}$

2016 (II)

15. What is ${}^{47}C_4 + {}^{51}C_3 + \sum_{j=2}^{5} {}^{52-j}C_3$ equal to?

(a) ${}^{52}C_4$ (b) ${}^{51}C_5$
(c) ${}^{53}C_4$ (d) ${}^{52}C_5$

(a) We have, ${}^{47}C_4 + {}^{51}C_3 + \sum_{j=2}^{5} {}^{52-j}C_3$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3$

$+ {}^{48}C_3 + {}^{47}C_3$

$= ({}^{48}C_4 - {}^{47}C_3) + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3$

$+ {}^{48}C_3 + {}^{47}C_3$

[$\because {}^nC_r = {}^{n+1}C_r - {}^nC_{r-1}$]

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ({}^{48}C_3 + {}^{48}C_4)$

$= {}^{51}C_3 + {}^{50}C_3 + ({}^{49}C_3 + {}^{49}C_4)$

$= {}^{51}C_3 + ({}^{50}C_3 + {}^{50}C_4)$

$= {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$

2015 (II)

16. In the expansion of $\left(\sqrt{x}+\frac{1}{3x^2}\right)^{10}$, the value of constant term (independent of x) is

(a) 5 (b) 8
(c) 45 (d) 90

(a) We have, $\left(\sqrt{x}+\frac{1}{3x^2}\right)^{10}$

$$T_{r+1}={}^{10}C_r(x)^{\frac{10-r}{2}}(3)^{-r}(x)^{-2r}$$

$$\Rightarrow \quad T_{r+1}={}^{10}C_r(3)^{-r}x^{\frac{10-5r}{2}}$$

Since, the term independent of x is

$$\frac{10-5r}{2}=0 \Rightarrow r=2$$

On putting $r=2$, we get

$$T_3={}^{10}C_2(3)^{-2}=\frac{10\times 9}{1\times 2}\times\frac{1}{9}=5$$

Directions (Q. Nos. 17 and 18) *Read the following information carefully and answer the questions given below.*

Consider the expansion of $(1+x)^{2n+1}$.

17. If the coefficients of x^r and x^{r+1} are equal in the expansion, then r is equal to

(a) n (b) $\frac{2n-1}{2}$ (c) $\frac{2n+1}{2}$ (d) $n+1$

(a) We have, $(1+x)^{2n+1}$

General term, $T_{r+1}={}^{(2n+1)}C_r x^r$

We have, ${}^{2n+1}C_r={}^{2n+1}C_{r+1}$

$\because \quad 2n+1=r+r+1$

$$2r=2n \Rightarrow r=n$$

18. The average of the coefficients of the two middle terms in the expansion is

(a) ${}^{2n+1}C_{n+2}$ (b) ${}^{2n+1}C_n$
(c) ${}^{2n+1}C_{n-1}$ (d) ${}^{2n}C_{n+1}$

(b) Since, $2n+1$ is odd.

Hence, $\frac{2n+1+1}{2}$ and $\frac{2n+1+3}{2}$ are two middle terms.

i.e. $(n+1)$ th and $(n+2)$ th term are two middle terms.

∴ Required average

$$=\frac{{}^{2n+1}C_n+{}^{2n+1}C_{n+1}}{2}=\frac{{}^{2n+1+1}C_{n+1}}{2}$$

$$=\frac{1}{2}{}^{2n+2}C_{n+1}$$

$$=\frac{1}{2}\cdot\frac{2n+2}{n+1}\cdot{}^{2n+1}C_n={}^{2n+1}C_n$$

19. The sum of the coefficients of all the terms in the expansion is

(a) 2^{2n-1} (b) 4^{n-1}
(c) 2×4^n (d) None of these

(c) To find the sum of coefficient of all terms

On putting $x=1$ in the given expression $(1+x)^{2n+1}$, we get

$$2^{2n+1}=2\cdot 2^{2n}=2\cdot 4^n$$

20. The coefficient of x^{99} in the expansion of $(x-1)(x-2)(x-3)\ldots(x-100)$ is

(a) 5050 (b) 5000
(c) – 5050 (d) – 5000

(c) We have,

$(x-1)(x-2)(x-3)\ldots(x-100)$

$=x^{100}-(1+2+3+\ldots+100)x^{99}+\ldots$

∴ Coefficient of x^{99} is

$$-\frac{100}{2}[1+100]=-50\times 101$$

$$=-5050$$

2015 (I)

21. What is $\sum_{r=0}^{1}{}^{n+r}C_n$ equal to?

(a) ${}^{n+2}C_1$ (b) ${}^{n+2}C_n$
(c) ${}^{n+3}C_n$ (d) ${}^{n+2}C_{n+1}$

(a) $\sum_{r=0}^{1}{}^{n+r}C_n=\sum_{r=0}^{1}{}^{n+r}C_{n+r-n}$

$$=\sum_{r=0}^{1}{}^{n+r}C_r={}^nC_0+{}^{n+1}C_1=1+\frac{n+1!}{n!1!}$$

$$=1+n+1=n+2={}^{n+2}C_1$$

2014 (II)

Directions (Q. Nos. 22-24) *Read the following information carefully and answer the questions given below.*

Consider the expansion $\left(x^2+\frac{1}{x}\right)^{15}$.

22. What is the independent term in the given expansion?

(a) 2103 (b) 3003 (c) 4503 (d) 3001

(b) We have, $\left(x^2+\frac{1}{x}\right)^{15}$

$$T_{r+1}={}^{15}C_r(x^2)^{15-r}\left(\frac{1}{x}\right)^r$$

$$={}^{15}C_r x^{30-2r-r}={}^{15}C_r x^{30-3r}$$

For independent term, $30-3r=0$

$$\Rightarrow \quad r=10$$

On putting $r=10$, we get independent term

$$T_{10+1}={}^{15}C_{10}=\frac{15!}{10!\,5!}$$

$$=\frac{15\times 14\times 13\times 12\times 11\times 10!}{10!\times 1\times 2\times 3\times 4\times 5}=3003$$

23. What is the ratio of coefficient of x^{15} to the term independent of x in the given expansion?

(a) 1 (b) 1/2 (c) 2/3 (d) 3/4

(a) For coefficient of x^{15},

$30-3r=15 \Rightarrow r=5$

So, the coefficient of x^{15} is ${}^{15}C_5$.

and coefficient of independent of x is

$$30-3r=0 \Rightarrow r=10$$

So, coefficient of independent of x is ${}^{15}C_{10}$.

∴ Required ratio $=\frac{{}^{15}C_5}{{}^{15}C_{10}}=\frac{{}^{15}C_5}{{}^{15}C_5}=1$

$[\because {}^nC_r={}^nC_{n-r}]$

24. Consider the following statements

1. There are 15 terms in the given expansion.
2. The coefficient of x^{12} is equal to that of x^3.

Which of the above statement(s) is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) **Statement 1** We know that, $(a+b)^n$ have total number of terms is $n+1$.

So, $\left(x^2+\frac{1}{x}\right)^{15}$ have 16 terms.

Hence, Statement 1 is false.

Statement 2 For coefficient of x^{12},

$$30-3r=12 \Rightarrow r=6$$

∴ The coefficient of x^{12} is ${}^{15}C_6$.

and for coefficient of x^3,

$30-3r=3 \Rightarrow r=9$

∴ The coefficient of x^3 is ${}^{15}C_9$.

∴ ${}^{15}C_6={}^{15}C_9$

Hence, Statement 2 is correct.

25. Consider the following statements

1. The term containing x^2 does not exist in the given expansion.
2. The sum of the coefficients of all the terms in the given expansion is 2^{15}.

Which of the above statement(s) is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) **Statement 1** For coefficient of x^2,

$$30 - 3r = 2$$

$$\Rightarrow \quad r = \frac{28}{3},\ r \notin N$$

So, the term which containing x^2 does not exist in the expansion.

Hence, Statement 1 is correct.

Statement 2 Now,

$$\left(x^2 + \frac{1}{x}\right)^{15} = {}^{15}C_0(x^2)^{15} + {}^{15}C_1(x^2)^{14}\left(\frac{1}{x}\right) + \ldots + {}^{15}C_{15}\left(\frac{1}{x}\right)^{15}$$

On putting $x = 1$ both sides, we get

$$(1+1)^{15} = {}^{15}C_0 + {}^{15}C_1 + \ldots + {}^{15}C_{15}$$

$$\Rightarrow \quad 2^{15} = {}^{15}C_0 + {}^{15}C_1 + \ldots + {}^{15}C_{15}$$

Hence, Statement 2 is correct.

26. What is the sum of the coefficients of the middle terms in the given expansion?

(a) $C(15, 9)$
(b) $C(16, 9)$
(c) $C(16, 8)$
(d) None of these

(c) We have, $\left(x^2 + \frac{1}{x}\right)^{15}$

Since, n is odd.

So, it has two middle terms T_8 and T_9.

$$\therefore T_8 + T_9 = {}^{15}C_7 + {}^{15}C_8$$

$$= {}^{16}C_8 \ [\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r]$$

$$= C(16, 8)$$

2014 (I)

Directions (Q. Nos. 27-29) *Read the following information carefully and answer the questions given below.*

In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, where n is a positive integer, the sum of the coefficients of x^5 and x^{10} is 0.

27. What is n equal to?

(a) 5
(b) 10
(c) 15
(d) None of the above

28. What is the value of the independent term?

(a) 5005 (b) 7200
(c) –5005 (d) –7200

29. What is the sum of the coefficients of the two middle terms?

(a) 0 (b) 1
(c) –1 (d) None of these

Solutions (Q.Nos. 27-29)

In the expansion of $(x + a)^n$,

General term, $T_{r+1} = {}^nC_r\, x^{n-r} \cdot a^r$

$\therefore$ In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$

General term,

$$T_{r+1} = {}^nC_r\,(x^3)^{n-r} \cdot \left(\frac{1}{x^2}\right)^r$$

$$= {}^nC_r \cdot x^{(3n-3r)} \cdot (-1)^r \cdot x^{-2r}$$

$$= {}^nC_r \cdot (-1)^r \cdot x^{(3n-5r)} \quad \ldots(i)$$

For the coefficient x^5,

On putting $3n - 5r = 5 \Rightarrow 5r = 3n - 5$

$$\therefore \quad r = \frac{3n}{5} - 1$$

$\therefore$ Coefficient of

$$x^5 = {}^nC_{\left(\frac{3n}{5}-1\right)} \cdot (-1)^{\left(\frac{3n}{5}-1\right)}$$

For the coefficient of x^{10},

On putting $3n - 5r = 10 \Rightarrow 5r = 3n - 10$

$$\therefore \quad r = \frac{3n}{5} - 2$$

$\therefore$ Coefficient of

$$x^{10} = {}^nC_{\left(\frac{3n}{5}-2\right)} \cdot (-1)^{\left(\frac{3n}{5}-2\right)}$$

Now, by condition the sum of the coefficients of x^5 and $x^{10} = 0$.

$$\Rightarrow \quad {}^nC_{\left(\frac{3n}{5}-1\right)} \cdot (-1)^{\left(\frac{3n}{5}-1\right)} + {}^nC_{\left(\frac{3n}{5}-2\right)} \cdot (-1)^{\left(\frac{3n}{5}-2\right)} = 0$$

$$\Rightarrow \quad (-1)^{\frac{3n}{5}}\left[{}^nC_{\left(\frac{3n}{5}-1\right)} \cdot (-1)^{-1} + {}^nC_{\left(\frac{3n}{5}-2\right)} \cdot (-1)^{-2}\right] = 0$$

$$\Rightarrow \quad -{}^nC_{\left(\frac{3n}{5}-1\right)} + {}^nC_{\left(\frac{3n}{5}-2\right)} = 0 \quad \ldots(ii)$$

27. (c) ${}^nC_{\left(\frac{3n}{5}-2\right)} = {}^nC_{\left(\frac{3n}{5}-1\right)}$ [from Eq. (ii)]

$$\Rightarrow \quad n = \left(\frac{3n}{5} - 2\right) + \left(\frac{3n}{5} - 1\right)$$

$$[\because {}^nC_x = {}^nC_y \Rightarrow n = x + y]$$

$$\Rightarrow \quad n = \frac{6n}{5} - 3 \quad \Rightarrow \frac{6n}{5} - n = 3$$

$$\Rightarrow \quad \frac{n}{5} = 3 \ \therefore \ n = 15$$

28. (c) For the independent term,

put $3n - 5r = 0$, [from Eq. (i)]

$$\Rightarrow \quad 5r = 3n = 3 \times 15 \quad [\because n = 15]$$

$$\Rightarrow \quad 5r = 3 \times 3 \times 5 \ \therefore \ r = 9$$

Now, put the value of r in Eq. (i), we get

$$T_{9+1} = {}^{15}C_9\,(-1)^9 \cdot x^{(3\times 15 - 5\times 9)}$$

$$\Rightarrow \quad T_{10} = -{}^{15}C_9 \cdot x^0 = -{}^{15}C_9$$

$$\Rightarrow \quad T_{10} = -{}^{15}C_6 \quad [\because {}^nC_r = {}^nC_{n-r}]$$

$$= -\frac{15!}{6!\,9!} \quad \left[\because {}^nC_r = \frac{n!}{r!\,(n-r)!}\right]$$

$$= -\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= -5005$$

So, the value of the independent term is – 5005.

29. (a) Since, $n = 15$

$\therefore$ Total term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$ is 16.

So, middle term $= \left(\frac{n}{2}\right)$th term and $\left(\frac{n}{2} + 1\right)$th term

$= \left(\frac{16}{2}\right)$th term and $\left(\frac{16}{2} + 1\right)$th term

= 8th term and 9th term

So, $\left(x^3 - \frac{1}{x^2}\right)^{15}$ has two middle terms.

$$T_8 = T_{(7+1)} = {}^{15}C_7\,(-1)^7\,x^{(3\times 15 - 5\times 7)}$$

$$= -{}^{15}C_7\,x^{10}; \quad \text{[from Eq. (i)]}$$

and $T_9 = T_{(8+1)} = {}^{15}C_8\,(-1)^8\,x^{(3\times 15 - 5\times 8)}$

$$= {}^{15}C_8\,x^5 \quad \text{[from Eq. (ii)]}$$

Now, the sum of the coefficients of the two middle terms

$$= -{}^{15}C_7 + {}^{15}C_8$$

$$= -{}^{15}C_7 + {}^{15}C_7 = 0 \quad [\because {}^nC_r = {}^nC_{n-r}]$$

8

LOGARITHMS

2019 (II)

1. If $x^{\log_7 x} > 7$ where $x > 0$, then which one of the following is correct?

(a) $x \in (0, \infty)$

(b) $x \in \left(\frac{1}{7}, 7\right)$

(c) $x \in \left(0, \frac{1}{7}\right) \cup (7, \infty)$

(d) $x \in \left(\frac{1}{7}, \infty\right)$

(**b**) $x^{\log_7 x} > 7$ where $x > 0$.

Taking log on base 7 both sides

$\log_7 x . \log_7 x > \log_7 7$

$[\because \log_a m^n = n \log_a m]$

$\Rightarrow \quad (\log_7 x)^2 > 1 \qquad [\because \log_a a = 1]$

$\Rightarrow \quad \log_7 x > (\pm 1)$

$\therefore \quad x > 7^1 \Rightarrow x > 7$

and $\quad x < 7^{-1} \Rightarrow x < \frac{1}{7}$

Hence, $x \in \left(\frac{1}{7}, 7\right)$

2018 (II)

2. What is the value of $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$ equal to ?

(a) $3 \log_2 7$ (b) $1 - 3 \log_2 7$

(c) $1 - 3 \log_7 2$ (d) $\frac{7}{8}$

(**c**) We have,

$$\sqrt{7\sqrt{7\sqrt{7}}} = 7^{\frac{1}{2}} \cdot 7^{\frac{1}{4}} \cdot 7^{\frac{1}{8}} = 7^{\frac{7}{8}}$$

Now, $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$

$= \log_7 \log_7 (7)^{\frac{7}{8}}$

$= \log_7 \frac{7}{8} \cdot \log_7 (7) \qquad [\because \log_a a = 1]$

$= \log_7 \left(\frac{7}{8}\right)$

$= \log_7 7 - \log_7 8$

$[\because \log \frac{m}{n} = \log m - \log n]$

$= \log_7 7 - \log_7 2^3 = 1 - 3 \log_7 2$

$[\because \log_b a^n = n \log_b a]$

3. What is the value of $\log_9 27 + \log_8 32$?

(a) $\frac{7}{2}$ (b) $\frac{19}{6}$

(c) 4 (d) 7

(**b**) We have,

$\log_9 27 + \log_8 32 = \log_{3^2} 3^3 + \log_{2^3} 2^5$

$= \frac{3}{2} \log_3 3 + \frac{5}{3} \log_2 2$

$\left[\because \log_{a^m} b^n = \frac{n}{m} \log_a b\right]$

$= \frac{3}{2} + \frac{5}{3} = \frac{19}{6}$

4. If $(0.2)^x = 2$ and $\log_{10} 2 = 0.3010$, then what is the value of x to the nearest tenth?

(a) –10.0

(b) –0.5

(c) –0.4

(d) –0.2

(**c**) We have, $(0.2)^x = 2$

Taking $\log_{10}$ on both side

$x \log_{10} 0.2 = \log_{10} 2$

$[\because \log a^{m^n} = n \log a^m]$

$\Rightarrow \quad x \log_{10}\left(\frac{2}{10}\right) = \log_{10} 2$

$\Rightarrow x [\log_{10} 2 - \log_{10} 10] = \log_{10} 2$

$\Rightarrow x [0.3010 - 1] = 0.3010 \quad [\because \log_a a = 1]$

$\therefore \quad x = -\frac{0.3010}{0.6990} = -0.43 \approx -0.4$

2018 (I)

5. If $n = (2017)!$, then what is $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{2017} n}$ equal to ?

(a) 0 (b) 1

(c) $\frac{n}{2}$ (d) n

(**b**) We have,

$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{2017} n}$

$= \log_n 2 + \log_n 3 + \log_n 4 + \ldots + \log_n 2017$

$\left[\because \log_b a = \frac{1}{\log_a b}\right]$

$= \log_n (2 \cdot 3 \cdot 4 \ldots\ldots 2017)$

$[\because \log a + \log b = \log ab]$

$= \log_n (1 \cdot 2 \cdot 3 \cdot 4 \ldots\ldots 2017)$

$= \log_n (2017)!$

$[\because n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1 = n!]$

$= \log_{(2017)!} (2017)! \qquad [\because n = 2017!]$

$= 1 \qquad [\because \log_a a = 1]$

6. If $x + \log_{15}(1 + 3^x) = x \log_{15} 5 + \log_{15} 12$, where x is an integer, then what is x equal to?

(a) – 3

(b) 2

(c) 1

(d) 3

(**c**) We have,

$x + \log_{15}(1 + 3^x) = x \log_{15} 5 + \log_{15} 12$

$\Rightarrow \log_{15} 15^x + \log_{15}(1 + 3^x)$

$= \log_{15} 5^x + \log_{15} 12$

$[\because \log_a a = 1 \text{ and } \log_b a^m = m \log_b a]$

$\Rightarrow \log_{15}[15^x(1+3^x)] = \log_{15}(5^x \times 12)$
$[\because \log a + \log b = \log ab]$
$\Rightarrow 15^x(1+3^x) = 12 \cdot 5^x$
$\Rightarrow 3^x(1+3^x) = 12$
$\Rightarrow y(1+y) = 12 \quad [\text{where, } y = 3^x]$
$\Rightarrow y^2 + y - 12 = 0$
$\Rightarrow (y+4)(y-3) = 0$
$\Rightarrow y = -4, 3$
$\Rightarrow 3^x = -4, 3$
$\Rightarrow 3^x = 3 \quad [\because 3^x \neq -4]$
$\therefore x = 1$

7. If $0 < a < 1$, the value of $\log_{10} a$ is negative. This is justified by
(a) Negative power of 10 is less than 1
(b) Negative power of 10 is between 0 and 1
(c) Negative power of 10 is positive
(d) Negative power of 10 is negative

(***b***) Let $\log_{10} a = x$
$\Rightarrow a = 10^x$
It is given that $0 < a < 1$
$\Rightarrow 0 < 10^x < 1$
$\Rightarrow x$ must be negative
$\therefore$ If $0 < a < 1$, the value of $\log_{10} a$ is negative implies that negative power of 10 is between 0 and 1.

2017 (II)

8. If $x + \log_{10}(1+2^x) = x\log_{10} 5 + \log_{10} 6$ then x is equal to
(a) 2, –3 (b) only 2 (c) 1 (d) 3

(c) $x + \log_{10}(1+2^x) = x\log_{10} 5 + \log_{10} 6$
$\Rightarrow x - x\log_{10} 5 = \log_{10} 6 - \log_{10}(1+2^x)$
$\Rightarrow x(1 - \log_{10} 5) = \log_{10} 6 - \log_{10}(1+2^x)$
$\Rightarrow x(\log_{10} 10 - \log_{10} 5) = \log_{10} 6 - \log_{10}(1+2^x) \quad [\because \log_{10} 10 = 1]$
$\Rightarrow x\left(\log_{10}\frac{10}{5}\right) = \log_{10}\frac{6}{1+2^x}$
$\left[\because \log_{10} a - \log_{10} b = \log_{10}\frac{a}{b}\right]$
$\Rightarrow x\log_{10} 2 = \log_{10}\frac{6}{1+2^x}$
For only $x = 1$, above equation is satisfied
$\therefore x = 1$ is only solution.

2015 (II)

9. If $\log_8 m + \log_8 \frac{1}{6} = \frac{2}{3}$, then m is equal to
(a) 24 (b) 18 (c) 12 (d) 4

(***a***) We have, $\log_8 m + \log_8 \frac{1}{6} = \frac{2}{3}$
$\Rightarrow \log_8 \frac{m}{6} = \frac{2}{3}$
$[\because \log_a m + \log_a n = \log_a(m \times n)]$
$\Rightarrow \frac{m}{6} = (8)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$
$\therefore m = 6 \times 4 = 24$

9
MATRICES

2019 (II)

1. If $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, then the expression A^3-2A^2 is

(a) a null matrix
(b) an identity matrix
(c) equal to A
(d) equal to $-A$

(**a**) $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

$\therefore A^2=A\cdot A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

$=\begin{bmatrix}1+1 & -1-1\\ -1-1 & 1+1\end{bmatrix}=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$

and $A^3=A^2\cdot A=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}\cdot\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

$=\begin{bmatrix}2+2 & -2-2\\ -2-2 & 2+2\end{bmatrix}=\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix}$

Now,

$A^3-2A^2=\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix}-2\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$

$=\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix}+\begin{bmatrix}-4 & 4\\ 4 & -4\end{bmatrix}$

$=\begin{bmatrix}4-4 & -4+4\\ -4+4 & 4-4\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

= a null matrix

2. If $A=\begin{pmatrix}1 & 2\\ 2 & 3\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$, then which one of the following is correct?

(a) Both *AB* and *BA* exist
(b) Neither *AB* nor *BA* exists
(c) *AB* exists but *BA* does not exist
(d) *AB* does not exist but *BA* exists

(**c**) We have, $A=\begin{pmatrix}1 & 2\\ 2 & 3\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$

order of $A=3\times2$ and order of $B=2\times2$

$\because$ Number of column of A = Number of row of B

$\therefore$ *AB* exists.

and number of column of $B\neq$ Number of raw of A

$\therefore$ *BA* does not exist.

Hence, *AB* exists but *BA* does not exist.

2019 (I)

3. If $B=\begin{bmatrix}3 & 2 & 0\\ 2 & 4 & 0\\ 1 & 1 & 0\end{bmatrix}$, then what is adjoint of B equal to?

(a) $\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ -2 & -1 & 8\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0 & -2\\ 0 & 0 & -1\\ 0 & 0 & 8\end{bmatrix}$

(c) $\begin{bmatrix}0 & 0 & 2\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$ (d) It does not exist

(**a**) We have, $B=\begin{bmatrix}3 & 2 & 0\\ 2 & 4 & 0\\ 1 & 1 & 0\end{bmatrix}$

Co-factor of B,

$B_{11}=0, B_{12}=0, B_{13}=-2$
$B_{21}=0, B_{22}=0, B_{23}=-1$
$B_{31}=0, B_{32}=0, B_{33}=8$

adj $B=\begin{bmatrix}B_{11} & B_{12} & B_{13}\\ B_{21} & B_{22} & B_{23}\\ B_{31} & B_{32} & B_{33}\end{bmatrix}'$

$=\begin{bmatrix}0 & 0 & -2\\ 0 & 0 & -1\\ 0 & 0 & 8\end{bmatrix}'=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ -2 & -1 & 8\end{bmatrix}$

4. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then the matrix A is a/an

(a) singular matrix
(b) involutory matrix
(c) nilpotent matrix
(d) idempotent matrix

(**b**) We have, $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

$|A|=-1$

Since, $|A|\neq0$

Hence, *A* is not singular.

$A^2=A\cdot A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$A^2=I$

Hence, A is involutory matrix.

5. If $\begin{bmatrix}x & -3i & 1\\ y & 1 & i\\ 0 & 2i & -i\end{bmatrix}=6+11i$, then what are the values of x and y respectively?

(a) –3, 4 (b) 3, 4
(c) 3, – 4 (d) –3, –4

(**a**) We have, $\begin{vmatrix}x & -3i & 1\\ y & 1 & i\\ 0 & 2i & -i\end{vmatrix}=6+11i$

$\Rightarrow x(-i+2)-y(-3-2i)=6+11i$

$\Rightarrow 2x+3y+(-x+2y)i=6+11i$

On equating real and imaginary parts, on both sides,

we get $2x+3y=6$ ...(i)

and $-x+2y=11$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x=-3$

and $y=4$

6. If A is an identity matrix of order 3, then its inverse (A^{-1})

(a) is equal to null matrix
(b) is equal to A
(c) is equal to $3A$
(d) does not exist

(**b**) Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$

7. A is a square matrix of order 3 such that its determinant is 4. What is the determinant of its transpose?

(a) 64 (b) 36
(c) 32 (d) 4

(**d**) Given, $|A| = 4$

$\therefore \quad |A'| = 4 \quad [\because |A| = |A'|]$

2018 (II)

8. A square matrix A is called orthogonal, if

(a) $A = A^2$ (b) $A' = A^{-1}$
(c) $A = A^{-1}$ (d) $A = A'$

where A' is the transpose of A

(**b**) A square matrix is called an orthogonal matrix, if $AA' = I$

multiply. by A^{-1}

$\therefore \quad A^{-1}(AA') = A^{-1}I$

$\Rightarrow \quad IA' = A^{-1} \Rightarrow A' = A^{-1}$

9. If A and B are two invertible square matrices of same order, then what is $(AB)^{-1}$ equal to?

(a) $B^{-1}A^{-1}$ (b) $A^{-1}B^{-1}$ (c) $B^{-1}A$ (d) $A^{-1}B$

(a) If A and B are two invertible square matrices of same order, then

$$(AB)^{-1} = B^{-1}A^{-1}$$

10. Consider the following in respect of matrices A, B and C of same order.

1. $(A + B + C)' = A' + B' + C'$
2. $(AB)' = A'B'$
3. $(ABC)' = C'B'A'$

Where, A' is the transpose of the matrix A. Which of the above are correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(**c**) Given that A, B and C are matrices of same order

Statement 1

$(A + B + C)' = A' + B' + C' \quad [\because (A + B)' = A' + B']$

So, Statement 1 is correct.

Statement 2

We know that, $(AB)' = B'A'$

Hence, Statement 2 is incorrect.

Statement 3

$(ABC)' = C'B'A' \quad [\because (AB)' = B'A']$

Hence, Statement 3 is correct.

11. Consider the following in respect of matrices A and B of same order.

1. $A^2 - B^2 = (A + B)(A - B)$
2. $(A - I)(I + A) = O \Leftrightarrow A^2 = I$

where, I is the identity matrix and O is the null matrix.

Which of the above is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**b**) **Statement 1**

RHS $= (A + B)(A - B)$

$= A^2 - AB + BA - B^2 \quad [\because AB \neq BA]$

$\neq A^2 - B^2$

Statement 1 is not correct.

Statement 2

$(A - I)(I + A) = 0$

$\Rightarrow \quad AI + A^2 - I^2 - IA = 0 \quad [\because AI = IA = A]$

$\Rightarrow \quad A + A^2 - I^2 - A = 0$

$\Rightarrow \quad A^2 = I \quad [\because I^2 = I]$

Statement 2 is correct.

12. What is the adjoint of the matrix $\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix}$?

(a) $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(**a**) We have,

$A = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$

$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Now, $C_{11} = \cos\theta$

$C_{12} = -\sin\theta$

$C_{21} = -\sin\theta$

$C_{22} = \cos\theta$

$\text{adj } A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^T$

$= \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

13. What should be the value of x, so that the matrix $\begin{pmatrix} 2 & 4 \\ -8 & x \end{pmatrix}$ does not have an inverse?

(a) 16 (b) –16
(c) 8 (d) –8

(**b**) Let, $A = \begin{bmatrix} 2 & 4 \\ -8 & x \end{bmatrix}$

Matrix does not have an inverse, if

$|A| = 0$

$\Rightarrow \quad 2x + 32 = 0$

$\Rightarrow \quad 2x = -32$

$\Rightarrow \quad x = -\frac{32}{2}$

$\Rightarrow \quad x = -16$

2018 (I)

14. If $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ and $A^2 - kA - I_2 = O$, where I_2 is the 2×2 identity matrix, then what is the value of k ?

(a) 4 (b) – 4
(c) 8 (d) – 8

(a) We have,

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1\cdot1 + 2\cdot2 & 1\cdot2 + 2\cdot3 \\ 2\cdot1 + 3\cdot2 & 2\cdot2 + 3\cdot3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$

Now, it is given that,

$A^2 - kA - I_2 = 0$

$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 4 & 8 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix}$

$\Rightarrow \quad k = 4$

15. What is the inverse of the matrix $A=\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$?

(a) $\begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$

(b) $\begin{pmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{pmatrix}$

(c) $\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{pmatrix}$

(d) $\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$

(**a**) We have,

$$A=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$|A| = 1[\cos^2\theta - (-\sin^2\theta)] = 1 \neq 0$

$$C_{11}=\begin{vmatrix}\cos\theta & 0\\ 0 & 1\end{vmatrix}=\cos\theta$$

$$C_{12}=-\begin{vmatrix}-\sin\theta & 0\\ 0 & 1\end{vmatrix}=\sin\theta$$

$$C_{13}=\begin{vmatrix}-\sin\theta & \cos\theta\\ 0 & 0\end{vmatrix}=0$$

$$C_{21}=-\begin{vmatrix}\sin\theta & 0\\ 0 & 1\end{vmatrix}=-\sin\theta$$

$$C_{22}=\begin{vmatrix}\cos\theta & 0\\ 0 & 1\end{vmatrix}=\cos\theta$$

$$C_{23}=-\begin{vmatrix}\cos\theta & \sin\theta\\ 0 & 0\end{vmatrix}=0$$

$$C_{31}=\begin{vmatrix}\sin\theta & 0\\ \cos\theta & 0\end{vmatrix}=0$$

$$C_{32}=-\begin{vmatrix}\cos\theta & 0\\ -\sin\theta & 0\end{vmatrix}=0$$

$$C_{33}=\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}=\cos^2\theta+\sin^2\theta=1$$

$$\therefore \text{adj}A=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}^T$$

$$=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{1}{|A|}\text{adj}(A)$$

$$A^{-1}=\frac{1}{1}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

2017 (II)

16. If $A=\begin{pmatrix}-2 & 2\\ 2 & -2\end{pmatrix}$, then which one of the following is correct?

(a) $A^2=-2A$ (b) $A^2=-4A$

(c) $A^2=-3A$ (d) $A^2=4A$

(**b**) We have, $A=\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}$

$$\therefore \quad A^2=A\cdot A=\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}4+4 & -4-4\\ -4-4 & 4+4\end{bmatrix}$$

$$=\begin{bmatrix}8 & -8\\ -8 & 8\end{bmatrix}=-4\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}=-4A$$

17. The matrix A has x rows and $x+5$ columns. The matrix B has y rows and $11-y$ columns. Both AB and BA exist. What are the values of x and y respectively?

(a) 8 and 3 (b) 3 and 4

(c) 3 and 8 (d) 8 and 8

(**c**) We know, two matrices P and Q can be multiplied (i.e., PQ exist) only when number of columns of P(pre-multiplier) is equal to the number of rows of q (post multiplier).

$\therefore$ For AB and BA both to be exist

$$x+5=y$$

and $$11-y=x$$

By solving above two equations,

$$x=3$$

and $$y=8.$$

18. If A is a square matrix, then the value of adj A^T – (adj $A)^T$ is equal to

(a) A

(b) $2|A|I$, where I is the identity matrix

(c) null matrix whose order is same as that of A

(d) unit matrix whose order is same as that of A

(**c**) We know, in square matrix A

$$\text{adj } A^T=(\text{adj } A)^T$$

$\Rightarrow$ adj A^T – (adj $A)^T=0$

$\therefore$ adj A^T – (adj $A)^T$ is null matrix, whose order is same as that of A.

19. The adjoint of the matrix $A=\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 0\\ 0 & 3 & 1\end{bmatrix}$ is

(a) $\begin{bmatrix}-1 & 6 & 2\\ -2 & 1 & -4\\ 6 & 3 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 6 & -2\\ -2 & 1 & 4\\ 6 & -3 & 1\end{bmatrix}$

(c) $\begin{bmatrix}6 & 1 & 2\\ 4 & -1 & 2\\ 6 & 3 & -1\end{bmatrix}$ (d) $\begin{bmatrix}-6 & 2 & 1\\ 4 & -2 & 1\\ 3 & 1 & -6\end{bmatrix}$

(**b**) We have,

$$A=\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 0\\ 0 & 3 & 1\end{bmatrix}$$

$\therefore \quad C_{11}=1-3\times0=1$

$C_{12}=-(2-0)=-2$

$C_{13}=2\times3-0\times1=6$

$C_{21}=-(0\times1-3\times2)=6$

$C_{22}=1\times1-0\times2=1$

$C_{23}=-(1\times3-0\times1)=-3$

$C_{31}=0-1\times2=-2$

$C_{32}=-(1\times0-2\times2)=4$

$C_{33}=1\times1-2\times0=1$

$$\therefore \quad \text{adj}\,A=\begin{bmatrix}1 & -2 & 6\\ 6 & 1 & -3\\ -2 & 4 & 1\end{bmatrix}^T$$

$$=\begin{bmatrix}1 & 6 & -2\\ -2 & 1 & 4\\ 6 & -3 & 1\end{bmatrix}$$

20. If a, b, c are non-zero real numbers, then the inverse of the matrix $A=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$ is equal to

(a) $\begin{bmatrix}a^{-1} & 0 & 0\\ 0 & b^{-1} & 0\\ 0 & 0 & c^{-1}\end{bmatrix}$

(b) $\frac{1}{abc}\begin{bmatrix}a^{-1} & 0 & 0\\ 0 & b^{-1} & 0\\ 0 & 0 & c^{-1}\end{bmatrix}$

(c) $\frac{1}{abc}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

(d) $\frac{1}{abc}\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$

(a) We have, $A=\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}$

$\therefore$ $A=\text{diag}\,(a\ \ b\ \ c)$

We know that, the inverse of a diagonal matrix diag $(a_1, a_2, a_3, \ldots\ldots a_n)$

$=\text{diag}\,(a_1^{-1}, a_2^{-1}, a_3^{-1}, \ldots\ldots a_n^{-1})$

$\therefore$ Inverse of matrix A

i.e. $A^{-1}=\text{diag}\,(a^{-1}\ b^{-1}\ c^{-1})$

$$=\begin{bmatrix}a^{-1}&0&0\\0&b^{-1}&0\\0&0&c^{-1}\end{bmatrix}$$

2017 (I)

21. If $A=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}$, then what is AA^T equal to (where, A^T is transpose of A)?

(a) Null matrix (b) Identity matrix
(b) A (c) $-A$

(b) Given, $A=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}$

$$\Rightarrow\quad A^T=\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$$

Now $AA^T=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$

$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha\\-\sin\alpha\cos\alpha+\cos\alpha\sin\alpha\\-\cos\alpha\sin\alpha+\sin\alpha\cos\alpha\\\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

$=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I$, which is an identity matrix.

22. $A=\begin{bmatrix}x+y&y\\x&x-y\end{bmatrix}$, $B=\begin{bmatrix}3\\-2\end{bmatrix}$ and $C=\begin{bmatrix}4\\-2\end{bmatrix}$.

If $AB=C$, then what is A^2 equal to?

(a) $\begin{bmatrix}4&8\\-4&-16\end{bmatrix}$ (b) $\begin{bmatrix}4&-4\\8&-16\end{bmatrix}$
(c) $\begin{bmatrix}-4&-8\\4&12\end{bmatrix}$ (d) $\begin{bmatrix}-4&-8\\8&12\end{bmatrix}$

(d) Given, $AB=C$

$$\Rightarrow\quad\begin{bmatrix}x+y&y\\x&x-y\end{bmatrix}\begin{bmatrix}3\\-2\end{bmatrix}=\begin{bmatrix}4\\-2\end{bmatrix}$$

$$\Rightarrow\quad\begin{bmatrix}3x+3y-2y\\3x-2x+2y\end{bmatrix}=\begin{bmatrix}4\\-2\end{bmatrix}$$

$$\Rightarrow\quad\begin{bmatrix}3x+y\\x+2y\end{bmatrix}=\begin{bmatrix}4\\-2\end{bmatrix}$$

$\Rightarrow 3x+y=4$ and $x+2y=-2$

On solving above two equations, we get

$x=2$ and $y=-2$

$$\therefore\quad A=\begin{bmatrix}2-2&-2\\2&2+2\end{bmatrix}=\begin{bmatrix}0&-2\\2&4\end{bmatrix}$$

$$\Rightarrow\quad A^2=A\cdot A=\begin{bmatrix}0&-2\\2&4\end{bmatrix}\cdot\begin{bmatrix}0&-2\\2&4\end{bmatrix}$$

$$=\begin{bmatrix}0-4&0-8\\0+8&-4+16\end{bmatrix}=\begin{bmatrix}-4&-8\\8&12\end{bmatrix}$$

23. Consider the set A of all matrices of order 3×3 with entries 0 or 1 only. Let B be the subset of A consisting of all matrices whose determinant is 1. Let C be the subset of A consisting of all matrices whose determinant is -1. Then which one of the following is correct?

(a) C is empty
(b) B has as many elements as C
(c) $A=B\cup C$
(d) B has thrice as many elements as C

(b) $\because$ All matrices contained in set B have determinant 1 and all matrices contained in set C have determinant -1.

$\therefore$ By symmetry, number of elements in B = Number of elements in C

24. If $A=\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}$, then what is A^3 equal to?

(a) $\begin{bmatrix}\cos3\theta&\sin3\theta\\-\sin3\theta&\cos3\theta\end{bmatrix}$
(b) $\begin{bmatrix}\cos^3\theta&\sin^3\theta\\-\sin^3\theta&\cos^3\theta\end{bmatrix}$
(c) $\begin{bmatrix}\cos3\theta&-\sin3\theta\\\sin3\theta&\cos3\theta\end{bmatrix}$
(d) $\begin{bmatrix}\cos^3\theta&-\sin^3\theta\\\sin^3\theta&\cos^3\theta\end{bmatrix}$

(a) $A^2=\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}$

$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta\\-\sin\theta\cos\theta-\cos\theta\sin\theta\\\cos\theta\sin\theta+\sin\theta\cos\theta\\-\sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos2\theta&\sin2\theta\\-\sin2\theta&\cos2\theta\end{bmatrix}$$

$$A^3=A^2\cdot A=\begin{bmatrix}\cos2\theta&\sin2\theta\\-\sin2\theta&\cos2\theta\end{bmatrix}\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos2\theta\cdot\cos\theta-\sin2\theta\sin\theta\\-\sin2\theta\cos\theta-\cos2\theta\sin\theta\\\cos2\theta\cdot\sin\theta+\sin2\theta\cdot\cos\theta\\-\sin2\theta\sin\theta+\cos2\theta\cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos3\theta&\sin3\theta\\-\sin3\theta&\cos3\theta\end{bmatrix}$$

25. What is the order of

$$[x\ y\ z]\begin{bmatrix}a&h&g\\h&b&f\\g&f&c\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}?$$

(a) 3×1 (b) 1×1
(c) 1×3 (d) 3×3

(b) $[x\ y\ z]\begin{bmatrix}a&h&g\\h&b&f\\g&f&c\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$

Order of $[x\ y\ z]\begin{bmatrix}a&h&g\\h&b&f\\g&f&c\end{bmatrix}$

$=1\times3$

Since, order of $\begin{bmatrix}x\\y\\z\end{bmatrix}=3\times1$

$\therefore$ Order of resultant martrix

$=1\times1$

26. If $A=\begin{bmatrix}0&1\\1&0\end{bmatrix}$, then the value of A^4 is

(a) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ (b) $\begin{bmatrix}1&1\\0&0\end{bmatrix}$
(c) $\begin{bmatrix}0&0\\1&1\end{bmatrix}$ (d) $\begin{bmatrix}0&1\\1&0\end{bmatrix}$

(a) $A^2=A\cdot A=\begin{bmatrix}0&1\\1&0\end{bmatrix}\cdot\begin{bmatrix}0&1\\1&0\end{bmatrix}$

$$=\begin{bmatrix}0+1&0+0\\0+0&1+0\end{bmatrix}$$

$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$A^4=A^2\cdot A^2$

$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}1+0&0+0\\0+0&0+1\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

2016 (II)

27. If $A=\begin{bmatrix}1&-1\\2&3\end{bmatrix}$ and $B=\begin{bmatrix}2&3\\-1&-2\end{bmatrix}$, then which of the following is/are correct?

I. $AB\,(A^{-1}B^{-1})$ is a unit matrix.
II. $(AB)^{-1} = A^{-1}\,B^{-1}$.

Select the correct answer using the code given below.

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(**d**) We have,

$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$\therefore\ AB = \begin{bmatrix} 1\times 2+(-1)\times(-1) & 1\times 3+(-1)\times(-2) \\ 2\times 2+3\times(-1) & 2\times 3+3\times(-2) \end{bmatrix}$

$= \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ -5 & -5 \end{bmatrix}$

Clearly, $AB \neq BA$

Now, $AB\,(A^{-1}B^{-1}) = AB\,(BA)^{-1}$ $[\because (AB)^{-1} = B^{-1}A^{-1}]$

$AB\,(BA)^{-1}$ is a unit matrix if and only if $AB = BA$ but $AB \neq BA$. So, $AB\,(A^{-1}B^{-1})$ is not a unit matrix. Since, $AB \neq BA$, therefore $(AB)^{-1} \neq A^{-1}B^{-1}$

2016 (I)

28. Consider the following in respect of the matrix

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

1. $A^2 = -A$ 2. $A^3 = 4A$

Which of the above is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**b**) Given, $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

1. Now, $A^2 = A \cdot A$

$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \neq -A$

2. $A^3 = A^2 \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} -2-2 & 2+2 \\ 2+2 & -2-2 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix}$

$= 4\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow A^3 = 4A$

Hence, only 2 is correct.

2015 (II)

29. The matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & x-1 & 1 \\ 2 & 7 & x-3 \end{bmatrix}$ will have inverse for every real number x except for

(a) $x = \dfrac{11 \pm \sqrt{5}}{2}$ (b) $x = \dfrac{9 \pm \sqrt{5}}{2}$

(c) $x = \dfrac{11 \pm \sqrt{3}}{2}$ (d) $x = \dfrac{9 \pm \sqrt{3}}{2}$

(**a**) We have, $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & x-1 & 1 \\ 2 & 7 & x-3 \end{bmatrix}$

A has inverse, if $|A| \neq 0$

$\therefore\quad |A| = 1\{(x-1)(x-3)-7\} - 3(x-3-2) + 2\{7-2(x-1)\} \neq 0$

$\Rightarrow x^2 - 11x + 29 \neq 0$

$\therefore\quad x \neq \dfrac{11 \pm \sqrt{121-116}}{2}$

$\Rightarrow x \neq \dfrac{11 \pm \sqrt{5}}{2}$

30. If $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -3 & 4 \end{bmatrix}$, then the matrix X for which $2X + 3A = 0$ holds true is

(a) $\begin{bmatrix} -\frac{3}{2} & 0 & -3 \\ -3 & -\frac{9}{2} & -6 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{3}{2} & 0 & -3 \\ 3 & -\frac{9}{2} & -6 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{2} & 0 & 3 \\ 3 & \frac{9}{2} & 6 \end{bmatrix}$

(d) $\begin{bmatrix} -\frac{3}{2} & 0 & 3 \\ -3 & \frac{9}{2} & -6 \end{bmatrix}$

(**d**) Given, $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -3 & 4 \end{bmatrix}$

We have, $2X + 3A = 0$

$\Rightarrow\quad X = -\dfrac{3}{2}A$

$\Rightarrow\quad X = -\dfrac{3}{2}\begin{bmatrix} 1 & 0 & -2 \\ 2 & -3 & 4 \end{bmatrix}$,

$X = \begin{bmatrix} -\frac{3}{2} & 0 & 3 \\ -3 & \frac{9}{2} & -6 \end{bmatrix}$

31. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$, then which of the following is/are correct?

1. A and B commute.
2. AB is a null matrix.

Select the correct answer using the codes given below.

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**b**) We have, $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

and $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$

$\therefore\ AB = \begin{bmatrix} -1+6-5 & -2+12-10 & -1+6-5 \\ -2-18+20 & -4-36+40 & -2-18+20 \\ -3-12+15 & -6-24+30 & -3-12+15 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Hence, AB is null matrix.

32. Which one of the following matrices is an elementary matrix?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 2 \end{bmatrix}$

(**b**) Let $A = \begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$|A| = 1(1-0) - 5(0-0) + 0(0-0)$

$= 1$

$\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is anelementary matrix. Since, the value of determinant of the given matrix is 1.

33. If $A=\begin{bmatrix}2 & 7\\ 1 & 5\end{bmatrix}$, then what is $A+3A^{-1}$ equal to?

(a) $3I$ (b) $5I$
(c) $7I$ (d) None of these

where, I is the identity matrix of order 2.

(**c**) We have, $A=\begin{bmatrix}2 & 7\\ 1 & 5\end{bmatrix}$

$\therefore |A| = 10-7=3$

Now, $A^{-1}=\frac{1}{|A|}\text{adj}(A)$

$\therefore \quad A^{-1}=\frac{1}{3}\begin{bmatrix}5 & -1\\ -7 & 2\end{bmatrix}^T=\frac{1}{3}\begin{bmatrix}5 & -7\\ -1 & 2\end{bmatrix}$

$\therefore A+3A^{-1}=\begin{bmatrix}2 & 7\\ 1 & 5\end{bmatrix}+3\times\frac{1}{3}\begin{bmatrix}5 & -7\\ -1 & 2\end{bmatrix}$

$=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=7I$

34. The matrix $\begin{bmatrix}0 & -4+i\\ 4+i & 0\end{bmatrix}$ is

(a) symmetric (b) skew-symmetric
(c) hermitian (d) skew-hermitian

(**d**) A square matrix A is said to be skew-hermitian, if $A^*=-A$ or $a_{ij}=-\bar{a}_{ji}, \forall\, i$ and j.

Here, $a_{12}=-4+i$ and $a_{21}=4+i$

Now, $a_{21}=-(\overline{-4+i})=-(-i-4)$
$=4+i$

Hence, the given matrix is skew-hermitian matrix.

35. If $X=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, $B=\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$ and $A=\begin{bmatrix}p & q\\ r & s\end{bmatrix}$ satisfy the equation $AX=B$, then the matrix A is equal to

(a) $\begin{bmatrix}-7 & 26\\ 1 & -5\end{bmatrix}$ (b) $\begin{bmatrix}7 & 26\\ 4 & 17\end{bmatrix}$
(c) $\begin{bmatrix}-7 & -4\\ 26 & 13\end{bmatrix}$ (d) $\begin{bmatrix}-7 & 26\\ -6 & 23\end{bmatrix}$

(**a**) $\because \quad AX=B$

$\therefore \quad \begin{bmatrix}p & q\\ r & s\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}=\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}3p+q & -4p-q\\ 3r+s & -4r-s\end{bmatrix}=\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$

$\Rightarrow \quad 3p+q=5 \quad \ldots$ (i)

and $\quad -4p-q=2 \quad \ldots$ (ii)

On solving Eqs. (i) and (ii), we get

$-p=7 \Rightarrow p=-7$

Now, $q=5+21=26$,

$3r+s=-2 \quad \ldots$(iii)

Also, $\quad -4r-s=1 \quad \ldots$ (iv)

On solving Eqs. (iii) and (iv), we get

$-r=-1 \Rightarrow r=1$ and $s=-2-3=-5$

$\therefore \quad A=\begin{bmatrix}-7 & 26\\ 1 & -5\end{bmatrix}$

36. Let $A=\begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix}$, $B=\begin{bmatrix}2\\ -1\end{bmatrix}$ and $C=\begin{bmatrix}3\\ 2\end{bmatrix}$. If $AB=C$, then what is A^2 equal to?

(a) $\begin{bmatrix}6 & -10\\ 4 & 26\end{bmatrix}$ (b) $\begin{bmatrix}-10 & 5\\ 4 & 24\end{bmatrix}$
(c) $\begin{bmatrix}-5 & -6\\ -4 & -20\end{bmatrix}$ (d) $\begin{bmatrix}-5 & -7\\ -5 & 20\end{bmatrix}$

(**a**) We have, $AB=C$

$\therefore \quad \begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix}\begin{bmatrix}2\\ -1\end{bmatrix}=\begin{bmatrix}3\\ 2\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}2x+2y-y\\ 4x \quad -x+y\end{bmatrix}=\begin{bmatrix}3\\ 2\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}2x+y\\ 3x+y\end{bmatrix}=\begin{bmatrix}3\\ 2\end{bmatrix}$

$\Rightarrow \quad 2x+y=3 \quad \ldots$(i)

and $\quad 3x+y=2 \quad \ldots$(ii)

On solving Eqs. (i) and (ii), we get

$x=2-3=-1$

$\therefore \quad y=5$

$\therefore \quad A^2=\begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix}^2$

$=\begin{bmatrix}-1+5 & 5\\ 2\times(-1) & -1-5\end{bmatrix}^2$

$=\begin{bmatrix}4 & 5\\ -2 & -6\end{bmatrix}\begin{bmatrix}4 & 5\\ -2 & -6\end{bmatrix}$

$=\begin{bmatrix}16-10 & 20-30\\ -8+12 & -10+36\end{bmatrix}=\begin{bmatrix}6 & -10\\ 4 & 26\end{bmatrix}$

37. If $E(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then $E(\alpha)E(\beta)$ is equal to

(a) $E(\alpha\beta)$ (b) $E(\alpha-\beta)$
(c) $E(\alpha+\beta)$ (d) $-E(\alpha+\beta)$

(**c**) Given, $E(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\therefore \quad E(\alpha)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

and $\quad E(\beta)=\begin{bmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{bmatrix}$

$\therefore \quad E(\alpha)E(\beta)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{bmatrix}$

$\begin{bmatrix}\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta & \cos\alpha\cdot\sin\beta+\sin\alpha\cdot\cos\beta\\ -\sin\alpha\cdot\cos\beta-\sin\beta\cdot\cos\alpha & -\sin\alpha\cdot\sin\beta+\cos\alpha\cdot\cos\beta\end{bmatrix}$

$=\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta)\\ -\sin(\alpha+\beta) & \cos(\alpha+\beta)\end{bmatrix}=E(\alpha+\beta)$

38. If A is an orthogonal matrix of order 3 and $B=\begin{bmatrix}1 & 2 & 3\\ -3 & 0 & 2\\ 2 & 5 & 0\end{bmatrix}$, then which of the following is/are correct?

1. $|AB|=\pm 47$ 2. $AB=BA$

Select the correct answer using the code given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**d**) $\because A$ is orthogonal matrix.

Then, $AA'=I$

So, neither 1 nor 2 statements are not correct.

2014 (II)

39. If, A and B are two matrices such that $AB=A$ and $BA=B$. Then, which of the following statements are correct?

1. $A^2=A$
2. $B^2=B$
3. $(AB)^2=AB$

Select the correct answer using the code given below.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(**d**) 1. We have, $AB=A$

$\therefore \quad A^2=(AB)\cdot(AB)=A\cdot(BA)\,B$

$=ABB \quad [\because BA=B]$

$=AB=A \quad [\because AB=A]$

2. $B^2=(BA)\cdot(BA)=B\cdot(AB)\cdot A$

$=BA\cdot A \quad [\because AB=A]$

$=BA=B \quad [\because BA=B]$

3. $(AB)^2=(AB)\cdot(AB)$

$=A\cdot(BA)\,B=A\cdot B\cdot B \quad [\because BA=B]$

$=AB=A \quad [\because AB=A]$

Hence, 1, 2 and 3 statements are correct.

40. If the matrix A is such that $\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}A=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}$, then what is A equal to?

(a) $\begin{bmatrix}1 & 4\\ 0 & -1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 4\\ 0 & 1\end{bmatrix}$
(c) $\begin{bmatrix}-1 & 4\\ 0 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -4\\ 0 & -1\end{bmatrix}$

(**a**) $\because \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

Let $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$, then $A = B^{-1} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$|B| = 1$ …(i)

$\therefore \quad B^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

$\left[\because A^{-1} = \frac{1}{|A|} \text{adj}\,(A)\right]$

From Eq. (i),

$$A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$$

41. From the matrix equation $AB = AC$, where A, B and C are the square matrices of same order, we can conclude $B = C$ provided

(a) A is non-singular
(b) A is singular
(c) A is symmetric
(d) A is skew-symmetric

(**a**) A is non-singular, if $|A| \neq 0$

A^{-1} exists $\therefore \quad AB = AC$

Pre multiply both sides by A^{-1}

$A^{-1}(AB) = A^{-1}(AC)$

$(A^{-1}A)B = (A^{-1}A)\,C$

[by association law]

$IB = IC \quad [\because A^{-1}A = I]$

$B = C \quad [IB = B,\ IC = C]$

42. If $A = \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is symmetric, then what is x equal to?

(a) 2 (b) 3 (c) –1 (d) 5

(**d**) Since, a square matrix $A = [a_{ij}]_{n \times m}$, is said to be symmetric, if $A' = A$

$\therefore \quad A = A'$

$$\Rightarrow \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix} = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$$

On comparing, we get

$2x - 3 = x + 2$

$\therefore \quad x = 5$

2014 (I)

43. Consider the following statements in respect of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

I. The matrix A is skew-symmetric.
II. The matrix A is symmetric.
III. The matrix A is invertible.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only III
(c) I and II (d) II and III

(**a**) Given matrix, $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

Now, $A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}^T$

$$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = -A$$

$\Rightarrow A = -A^T$

So, A is skew-symmetric matrix.

and $|A| = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

$= 0 - 1(0 - 6) + 2(-3)$

$= 6 - 6 = 0$

Since, $|A| = 0$ *i.e.,* A is singular matrix.

So, A cannot be an invertible matrix.

44. Consider two matrices

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{bmatrix}$$

Which one of the following is correct?

(a) B is the right inverse of A
(b) B is the left inverse of A
(c) B is the both sided inverse of A
(d) None of the above

(**b**) Given matrices, $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ and

$B = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{bmatrix}$

Taking option (a),

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{bmatrix}_{2 \times 3}$$

$$= \begin{bmatrix} 1+4 & 2+2 & -4-8 \\ 2+2 & 4+1 & -8-4 \\ 1+2 & 2+1 & -4-4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 & -12 \\ 4 & 5 & -12 \\ 3 & 3 & -8 \end{bmatrix}_{3 \times 3}$$

Taking option (b),

$$BA = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}_{3 \times 2}$$

$$= \begin{bmatrix} 1+4-4 & 2+2-4 \\ 2+2-4 & 4+1-4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$$

Now, we observe that B is not the right inverse of A but B is the left inverse of A.

45. If A is any matrix, then the product AA is defined only when A is a matrix of order $m \times n$, where

(a) $m > n$ (b) $m < n$
(c) $m = n$ (d) $m \leq n$

(**c**) Given that, A is any matrix.

Then, the product AA is defined only when A is a matrix of order $m \times n$ where, $m = n$. *i.e.,* A must be a square matrix.

$A \times A = (m \times n)\,(m \times n)$

$= (m \times n)\,(n \times n)$,

if $m = n$

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DETERMINANTS

2019 (II)

1. If $x + a + b + c = 0$, then what is the value of $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$?

(a) 0 (b) $(a + b + c)^2$
(c) $a^2 + b^2 + c^2$ (d) $a + b + c - 2$

(*a*) Given, $x + a + b + c = 0$

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$$

$$= \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

[by $C_1 \to C_1 + C_2 + C_3$]

$$= (x + a + b + c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

$[x + a + b + c$ common from $C_1] = 0$

$[\because x + a + b + c = 0]$

2. What are the values of x that satisfy the equation

$$\begin{vmatrix} x & 0 & 2 \\ 2x & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3x & 0 & 2 \\ x^2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0 ?$$

(a) $-2 \pm \sqrt{3}$ (b) $-1 \pm \sqrt{3}$
(c) $-1 \pm \sqrt{6}$ (d) $-2 \pm \sqrt{6}$

(*d*) Given equation,

$$\begin{vmatrix} x & 0 & 2 \\ 2x & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3x & 0 & 2 \\ x^2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow x(2 - 1) - 0 + 2(2x - 2) + 3x(2 - 1)$
$- 0 + 2(x^2 - 0) = 0$

[expression w.r.t. first row]

$\Rightarrow x + 4x - 4 + 3x + 2x^2 = 0$
$\Rightarrow 2x^2 + 8x - 4 = 0$
$\Rightarrow x^2 + 4x - 2 = 0$

$$\Rightarrow x = \frac{-4 \pm \sqrt{16 - 4(1)(-2)}}{2}$$

$$= \frac{-4 \pm \sqrt{24}}{2} = \frac{-4 \pm 2\sqrt{6}}{2}$$

$$= -2 \pm \sqrt{6}$$

3. What is the value of the determinant $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$?

(a) 0 (b) 12 (c) 24 (d) 36

(*c*) Given determinant

$$= \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 6 & 12 & 48 \end{vmatrix}$$

[by $C_2 \to C_2 - 2C_1$, $C_3 \to C_3 - 3C_2$]

$= 1(96 - 72) - 0 + 0$

[expression w.r.t. first row]

$= 24$

2018 (II)

4. For a square matrix A, which of the following properties hold?

1. $(A^{-1})^{-1} = A$
2. $\det(A^{-1}) = \dfrac{1}{\det A}$
3. $(\lambda A)^{-1} = \lambda A^{-1}$, where λ is a scalar

Select the correct answer using the code given below.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(*d*) For a square matrix A

Statement 1 $(A^{-1})^{-1} = A$ Statement 1 is true.

Statement 2 $\det(A^{-1}) = \dfrac{1}{\det A}$

Statement 2 is correct.

Statement 3 $(\lambda A)^{-1} = \lambda A^{-1}$, where λ is a scalar. So, Statement 3 is correct.

5. Which one of the following factors does the expansion of the determinant $\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix}$ contain ?

(a) $x - 3$ (b) $x - y$ (c) $y - 3$ (d) $x - 3y$

(*a*) We have,

$$\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix} \quad [\text{by } C_1 \to C_1 - C_3]$$

$$= \begin{vmatrix} x-3 & y & 3 \\ x^2-9 & 5y^3 & 9 \\ x^3-27 & 10y^5 & 27 \end{vmatrix}$$

$(x - 3)$ common by C_1

$$= (x-3)\begin{vmatrix} 1 & y & 3 \\ x+3 & 5y^3 & 9 \\ x^2+9+3x & 10y^5 & 27 \end{vmatrix}$$

Hence, $(x - 3)$ is a factor of given determinant.

6. If $a + b + c = 0$, then one of the solutions of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

(a) $x = a$ (b) $x = \sqrt{\dfrac{3(a^2 + b^2 + c^2)}{2}}$

(c) $x = \sqrt{\dfrac{2(a^2 + b^2 + c^2)}{3}}$

(d) $x = 0$

(***d***) We have,

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$R_1 \to R_1 + R_2 + R_3$$

$$\Rightarrow \begin{vmatrix} a+b+c-x & a+b+c-x & a+b+c-x \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -x & -x & -x \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow (-x)\begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$\Rightarrow x = 0$

Hence, $x = 0$ is a solution.

7. The system of equations

$$2x + y - 3z = 5$$
$$3x - 2y + 2z = 5$$
and $$5x - 3y - z = 16$$

(a) is inconsistent
(b) is consistent, with a unique solution
(c) is consistent, with infinitely many solutions
(d) has its solution lying along *X*-axis in three-dimensional space

(***b***) The system of equations

$2x + y - 3z = 5$
$3x - 2y + 2z = 5$
and $5x - 3y - z = 16$

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & -2 & 2 \\ 5 & -3 & -1 \end{bmatrix}$$

$$|A| = 2[-2(-1) - 2(-3)] - 1[3(-1) - 2(5)] + (-3)[3(-3) - (-2)(5)]$$
$$= 2(8) - 1(-13) - 3(1)$$
$$= 16 + 13 - 3 = 26 \neq 0$$

So, system is consistent with unique solution.

8. If u, v and w (all positive) are the pth, qth and rth terms of a GP, then the determinant of the matrix $\begin{pmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{pmatrix}$ is

(a) 0
(b) 1
(c) $(p-q)(q-r)(r-p)$
(d) $\ln u \times \ln v \times \ln w$

(***a***) Given that u, v and w are the pth, qth and rth term of GP

$\therefore u = aR^{p-1}$, $v = aR^{q-1}$ $[\because a_n = aR^{n-1}]$ and $w = aR^{r-1}$.

We have,

$$\begin{vmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{vmatrix} = \begin{vmatrix} \ln aR^{p-1} & p & 1 \\ \ln aR^{q-1} & q & 1 \\ \ln aR^{r-1} & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \ln a + (p-1)\ln R & p & 1 \\ \ln a + (q-1)\ln R & q & 1 \\ \ln a + (r-1)\ln R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \ln a & p & 1 \\ \ln a & q & 1 \\ \ln a & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\ln R & p & 1 \\ (q-1)\ln R & q & 1 \\ (r-1)\ln R & r & 1 \end{vmatrix}$$

$$= \ln a \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \ln R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

$$= 0 + \ln R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix} [\because C_1 = C_3]$$

$[\because C_1 = C_2]$

$= 0$

9. Let matrix B be the adjoint of a square matrix A, I be the identity matrix of same order as A. If $k(\neq 0)$ is the determinant of the matrix A, then what is AB equal to ?

(a) I (b) kI (c) k^2I (d) $(1/k)I$

(***b***) Given,

$B = \text{adj } A$, $I =$ Identity matrix

$|A| = K$

$\therefore AB = A(\text{adj } A) = |A| I = kI$

10. What is the determinant of the matrix $\begin{pmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{pmatrix}$?

(a) $(x-y)(y-z)(z-x)$
(b) $(x-y)(y-z)$
(c) $(y-z)(z-x)$
(d) $(z-x)^2(x+y+z)$

(***d***) We have,

$$\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$R_1 \to R_1 + R_2 + R_3$$

$$\begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & x & z+x \\ y & z & x+y \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - 2C_1$

$$= (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ z & x-z & x-z \\ y & z-y & x-y \end{vmatrix}$$

$$= (x+y+z)[1(x-z)(x-y) - (x-z)(z-y)]$$
$$= (x+y+z)[(x-z)(x-y-z+y)]$$
$$= (x+y+z)(x-z)(x-z)$$
$$= (x+y+z)(z-x)^2$$

11. If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1 \\ \sin A + \sin^2 A & \sin B + \sin^2 B & 1+\sin C \\ & & \sin C + \sin^2 C \end{vmatrix} = 0,$$

then which one of the following is correct?

(a) The triangle ABC is isosceles
(b) The triangle ABC is equilateral
(c) The triangle ABC is scalene
(d) No conclusion can be drawn with regard to the nature of the triangle

(***b***) We have,

$$\begin{bmatrix} 1 & 1 \\ 1+\sin A & 1+\sin B \\ \sin A + \sin^2 A & \sin B + \sin^2 B \\ 1 & \\ 1+\sin C & \\ \sin C + \sin^2 C & \end{bmatrix} = 0$$

$$R_1 \to R_1 - R_2, R_3 \to R_3 - R_2$$

$$\begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin^2 A - 1 & \sin^2 B - 1 & \sin^2 C - 1 \end{vmatrix} = 0$$

$$R_2 \to R_2 + R_1$$

$$\begin{vmatrix} -\sin A & -\sin B & -\sin C \\ 1 & 1 & 1 \\ -\cos^2 A & -\cos^2 B & -\cos^2 C \end{vmatrix} = 0$$

$$R_3 \to R_3 + R_2$$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ 1 & 1 & 1 \\ 1-\cos^2 A & 1-\cos^2 B & 1-\cos^2 C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ 1 & 1 & 1 \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0$$

$$[C_1 \to C_1 - C_2, C_2 \to C_2 - C_3]$$

$$\Rightarrow \begin{vmatrix} \sin A - \sin B & \sin B - \sin C & \sin C \\ 0 & 0 & 1 \\ \sin^2 A - \sin^2 B & \sin^2 B - \sin^2 C & \sin^2 C \end{vmatrix} = 0$$

$$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)$$

$$\begin{bmatrix} 1 & 1 & \sin C \\ 0 & 0 & 0 \\ \sin A + \sin C & \sin B + \sin C & \sin^2 C \end{bmatrix}$$

$\therefore \sin A - \sin B = 0$ or $\sin B - \sin C = 0$
$\Rightarrow \sin A = \sin B$ and $\sin B = \sin C$
$\Rightarrow A = B$ and $B = C$
$\therefore A = B = C$
Hence, triangle ABC is an equilateral.

2017 (II)

12. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + kz = k^2$ has no solution if k equals.
(a) 0 (b) 1 (c) –1 (d) –2

(**d**) We have,
$$kx + y + z = 1$$
$$x + ky + z = k$$
$$x + y + kz = k^2$$
For no solution, $\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$
$\Rightarrow k(k^2 - 1) - 1(k - 1) + 1(1 - k) = 0$
$\Rightarrow k^3 - k - k + 1 + 1 - k = 0$
$\Rightarrow k^3 - 3k + 2 = 0$
$\Rightarrow (k + 2)(k - 1)(k - 1) = 0 \Rightarrow k = 1, -2$
But for $k = 1$, first two equations will be identical
$\therefore k = -2$

13. The value of the determinant $\begin{vmatrix} 1-\alpha & \alpha-\alpha^2 & \alpha^2 \\ 1-\beta & \beta-\beta^2 & \beta^2 \\ 1-\gamma & \gamma-\gamma^2 & \gamma^2 \end{vmatrix}$ is equal to
(a) $(\alpha - \beta)(\beta - \gamma)(\alpha - \gamma)$
(b) $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$
(c) $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha + \beta + \gamma)$
(d) 0

(**b**) We have
$$\begin{vmatrix} 1-\alpha & \alpha-\alpha^2 & \alpha^2 \\ 1-\beta & \beta-\beta^2 & \beta^2 \\ 1-\gamma & \gamma-\gamma^2 & \gamma^2 \end{vmatrix}$$
On applying $C_1 \to C_1 + C_2 + C_3$, $C_2 \to C_2 + C_3$, we get
$$= \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$
On applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get
$$= \begin{vmatrix} 0 & (\alpha-\beta) & (\alpha-\beta)(\alpha+\beta) \\ 0 & (\beta-\gamma) & (\beta-\gamma)(\beta+\gamma) \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$
On taking common $(\alpha - \beta)$ and $(\beta - \alpha)$ from R_1 and R_2 respectively, we get
$$= (\alpha-\beta)(\beta-\gamma)\begin{vmatrix} 0 & 1 & \alpha+\beta \\ 0 & 1 & \beta+\gamma \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$
On expanding along C_1, we get
$$= (\alpha - \beta)(\beta - \gamma) \cdot 1[\beta + \gamma - \alpha - \beta]$$
$$= (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$$

14. If $p + q + r = a + b + c = 0$, then the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ equals
(a) 0 (b) 1
(c) $pa + qb + rc$
(d) $pa + qb + rc + a + b + c$

(**a**) We have,
$p + q + r = 0$ and $a + b + c = 0$
Now, $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$
On expanding along R_1, we get
$= pa[a^2qr - bcp^2] - qb[caq^2 - b^2pr] + rc[c^2pq - abr^2]$
$= a^3pqr - abcp^3 - abcq^3 + pqrb^3 + pqrc^3 - abcr^3$
$= pqr(a^3 + b^3 + c^3) - abc(p^3 + q^3 + r^3)$
$= pqr(3abc) - abc(3pqr)$
$[\because$ if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz]$
$= 3abcpqr - 3abcpqr = 0$

2017 (I)

15. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $\det(A^3) = 125$, then α is equal to
(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 5

(**c**) $\det(A^3) = 125 = 5^3$
$\Rightarrow \det(A) = 5 \quad [\because \det(A^n) = (\det A)^n]$
$\Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 5$
$\Rightarrow \alpha^2 - 4 = 5$
$\Rightarrow \alpha^2 = 5 + 4 = 9$
$\Rightarrow \alpha = \pm 3$

16. If B is a non-singular matrix and A is a square matrix, then the value of $\det(B^{-1}AB)$ is equal to
(a) $\det(B)$ (b) $\det(A)$
(c) $\det(B^{-1})$ (d) $\det(A^{-1})$

(**b**) $|B^{-1}\ AB| = |B^{-1}||AB|$
$= \frac{1}{|B|}|A||B| = |A| \qquad [\because |B| \neq 0]$

17. If $a \neq b \neq c$, then one value of x which satisfies the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$ is given by
(a) a (b) b (c) c (d) 0

(**d**) $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$
$\Rightarrow -(x - a)\begin{vmatrix} x+a & x-c \\ x+b & 0 \end{vmatrix} + (x - b)\begin{vmatrix} x+a & 0 \\ x+b & x+c \end{vmatrix} = 0$
$\Rightarrow -(x - a)\{-(x + b)(x - c)\} + (x - b)(x + a)(x + c) = 0$
$\Rightarrow (x - a)(x + b)(x - c) + (x - b)(x + a)(x + c) = 0$
If we put $x = 0$ in LHS
$\Rightarrow (0 - a)(0 + b)(0 - c) + (0 - b)(0 + a)(0 + c)$
$= abc - abc = 0 =$ RHS
which satisfies the above equation
Hence, $x = 0$.

18. The equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$
(a) have the unique solution
(b) have infinitely many solutions
(c) are inconsistent
(d) None of the above

(**a**) Here $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{bmatrix}$
$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$
Now,
$|A| = 1\begin{vmatrix} 1 & 3 \\ 5 & 9 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 5 & 9 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 5 & 5 \end{vmatrix}$
$= 1(9 - 15) - 2(18 - 15) + 3(10 - 5)$
$= 1(-6) - 2(3) + 3(5)$
$= -6 - 6 + 15 = -12 + 15 = 3$
$\Rightarrow |A| \neq 0$
Hence, the given system of equations, has a unique solution.

19. What is the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix}$?
(a) $1 + x + y + z$ (b) $2xyz$
(c) $x^2y^2z^2$ (d) $2x^2y^2z^2$

(c) We have, $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix}$

$C_1 \to C_1 - C_2$

$= \begin{vmatrix} 0 & 1 & 1 \\ -xyz & 1+xyz & 1 \\ 0 & 1 & 1+xyz \end{vmatrix}$

$= xyz[(1+xyz) - 1]$ [expanding along C_1]

$= x^2y^2z^2$

20. If $\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = 0$, then which one of the following is correct?

(a) $\frac{x}{y}$ is one of the cube roots of unity
(b) x is one of the cube roots of unity
(c) y is one of the cube roots of unity
(d) $\frac{x}{y}$ is one of the cube roots of –1

(d) Given, $\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = 0$

$\Rightarrow \quad x(x^2) - y(-y^2) = 0$

$\Rightarrow \quad x^3 + y^3 = 0 \Rightarrow x^3 = -y^3$

$\Rightarrow \quad (x/y)^3 = -1 \Rightarrow x/y = \sqrt[3]{-1}$

2016 (II)

Directions (Q.Nos. 21 and 22) For the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$ and $2x + 3y + \lambda z = \mu$

21. Under what condition does the above system of equations have infinitely many solutions?

(a) $\lambda = 5$ and $\mu \neq 9$ (b) $\lambda = 5$ and $\mu = 9$
(c) $\lambda = 9$ and $\mu = 5$ (d) $\lambda = 9$ and $\mu \neq 5$

(b) We have,

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

and $2x + 3y + \lambda z = \mu$

Now, $D = \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix}$

For infinite solution, $D = 0$

$\therefore 2(3\lambda + 6) - 3(7\lambda + 4) + 5(21 - 6) = 0$

$\Rightarrow \quad 6\lambda + 12 - 21\lambda - 12 + 75 = 0$

$\Rightarrow \quad \lambda = 5$

and $\begin{vmatrix} 2 & 3 & 9 \\ 7 & 3 & 8 \\ 2 & 3 & \mu \end{vmatrix} = 0$

$\Rightarrow 2(3\mu - 24) - 3(7\mu - 16) + 9(21 - 6) = 0$

$\Rightarrow \quad 6\mu - 48 - 21\mu + 48 + 135 = 0$

$\therefore \quad \mu = 9$

22. Under what condition does the above system of equations have unique solutions?

(a) $\lambda = 5$ and $\mu = 9$
(b) $\lambda \neq 5$ and $\mu = 7$ only
(c) $\lambda \neq 5$ and μ has any real value
(d) λ has any real value and $\mu \neq 9$

(c) For unique solution,

$D \neq 0$

$\therefore \lambda \neq 5$ and μ has any real value.

23. If A is a square matrix of order 3 and $\det A = 5$, then what is $\det\{2(A)^{-1}\}$ equal to?

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{8}{5}$ (d) $\frac{1}{40}$

(c) Given, A is a square matrix of order 3 and

$|A| = 5$

$\therefore \quad |2A^{-1}| = 2^3 |A^{-1}| = \frac{8}{|A|} = \frac{8}{5}$

Directions (Q. Nos. 24 and 25)

Let $ax^3 + bx^2 + cx + d = \begin{vmatrix} x+1 & 2x & 3x \\ 2x+3 & x+1 & x \\ 2-x & 3x+4 & 5x-1 \end{vmatrix}$

24. What is the value of c?

(a) –1 (b) 34
(c) 35 (d) 50

(c) Let $\Delta(x) = ax^3 + bx^2 + cx + d$

$= \begin{vmatrix} x+1 & 2x & 3x \\ 2x+3 & x+1 & x \\ 2-x & 3x+4 & 5x-1 \end{vmatrix}$

$\Rightarrow \quad \Delta'(x) = 3ax^2 + 2bx + c$

$= \begin{vmatrix} 1 & 2x & 3x \\ 2 & x+1 & x \\ -1 & 3x+4 & 5x-1 \end{vmatrix} + \begin{vmatrix} x+1 & 2 & 3x \\ 2x+3 & 1 & x \\ 2-x & 3 & 5x-1 \end{vmatrix} + \begin{vmatrix} x+1 & 2x & 3 \\ 2x+3 & x+1 & 1 \\ 2-x & 3x+4 & 5 \end{vmatrix}$

$\Rightarrow \Delta'(0) = c = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 2 & 3 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 3 \\ 3 & 1 & 1 \\ 2 & 4 & 5 \end{vmatrix}$

$\therefore c = (-1) + (-1)(1 - 6) + 31$

$= -1 + 5 + 31$

$= 35$

25. What is the value of $a + b + c + d$?

(a) 62 (b) 63 (c) 65 (d) 68

(b) $\Delta(1) = a + b + c + d = \begin{vmatrix} 2 & 2 & 3 \\ 5 & 2 & 1 \\ 1 & 7 & 4 \end{vmatrix}$

$= 2(8 - 7) - 2(20 - 1) + 3(35 - 2)$

$= 2 - 2(19) + 3(33) = 2 - 38 + 99$

$= 63$

26. If $m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $n = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then what is the value of the determinant of $m\cos\theta - n\sin\theta$?

(a) –1 (b) 0 (c) 1 (d) 2

(c) Given, $m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $n = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\therefore \quad m\cos\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$

and $\quad n\sin\theta = \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

$\therefore \quad m\cos\theta - n\sin\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\therefore$ Determinant of $(m\cos\theta - n\sin\theta)$

$= \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$= \cos^2\theta + \sin^2\theta = 1$

27. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then which of the following are correct?

I. $f(\theta) \times f(\phi) = f(\theta + \phi)$.
II. The value of the determinant of the matrix $f(\theta) \times f(\phi)$ is 1.
III. The determinant of $f(x)$ is an even function.

Select the correct answer using the code given below.

(a) I and II (b) II and III
(c) I and III (d) I, II and III

(d) Given,

$f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad f(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $\quad f(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore f(\theta)\cdot f(\phi)$

$$= \begin{bmatrix} \cos\theta\cdot\cos\phi - \sin\theta\cdot\sin\phi + 0 & -\cos\theta\cdot\sin\phi - \sin\theta\cdot\cos\phi + 0 & 0+0+0 \\ \sin\theta\cdot\cos\phi + \cos\theta\sin\phi + 0 & -\sin\theta\cdot\sin\phi + \cos\theta\cdot\cos\phi + 0 & 0+0+0 \\ 0 & 0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= f(\theta + \phi)$

Now, determinant of $f(\theta) \times f(\phi)$

$$= \begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= \cos^2(\theta + \phi) + \sin^2(\theta + \phi) = 1$

Since, determinant of

$$f(x) = \begin{vmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= \cos^2 x + \sin^2 x = 1$

$\therefore$ Determinant of $f(x)$ is an even function.

28. Which of the following are correct in respect of the system of equations $x + y + z = 8$, $x - y + 2z = 6$ and $3x - y + 5z = k$?

I. They have no solution, if $k = 15$.
II. They have infinitely many solutions, if $k = 20$.
III. They have unique solution, if $k = 25$.

Select the correct answer using the code given below.

(a) I and II (b) II and III
(c) I and III (d) I, II and III

(a) Given equations are

$$x + y + z = 8 \quad \ldots (i)$$
$$x - y + 2z = 6 \quad \ldots (ii)$$
$$3x - y + 5z = k \quad \ldots (iii)$$

Here, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -1 & 5 \end{vmatrix}$

$= 1(-5 + 2) - 1(5 - 6) + 1(-1 + 3)$
$= -3 + 1 + 2 = 0$

$$D_1 = \begin{vmatrix} 8 & 1 & 1 \\ 6 & -1 & 2 \\ k & -1 & 5 \end{vmatrix}$$

$= 8(-5 + 2) - 1(30 - 2k) + 1(-6 + k)$
$= -24 - 30 + 2k - 6 + k$
$= 3k - 60$

$$D_2 = \begin{vmatrix} 1 & 8 & 1 \\ 1 & 6 & 2 \\ 3 & k & 5 \end{vmatrix}$$

$= 1(30 - 2k) - 8(5 - 6) + 1(k - 18)$
$= 30 - 2k - 40 + 48 + k - 18 = 20 - k$

$$D_3 = \begin{vmatrix} 1 & 1 & 8 \\ 1 & -1 & 6 \\ 3 & -1 & k \end{vmatrix}$$

$= 1(-k + 6) - 1(k - 18) + 8(-1 + 3)$
$= -k + 6 - k + 18 + 16 = 40 - 2k$

For, $k = 15$

$D = 0$ and $D_1 \neq 0$, $D_2 \neq 0$ and $D_3 \neq 0$.

$\therefore$ No solution for $k = 15$.

For $k = 20 \Rightarrow D_1 = D_2 = D_3 = 0$

and $D = 0$

$\therefore$ Infinitely many solution for $k = 20$.

Since, $D = 0$

$\therefore$ System have no unique solution for any value of k.

2016 (I)

29. Which of the following determinants have value 'zero'?

1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ 2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

3. $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

Select the correct answer using the code given below.

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(d) 1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 5 \\ 7 & 7 & 9 \\ 5 & 5 & 3 \end{vmatrix} = 0$

$[C_1 \to C_1 - 8C_3]$

[$\because$ two columns of determinant are same, then value of determinant is zero.]

2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$

$[C_3 \to C_2 + C_3]$

$$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = 0$$

[$\because$ two columns of determinant are same, then value of determinant is zero]

3. $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

$= 0(0 + a^2) - c(0 + ab) + b(ac - 0)$

[expanding with respect to R_1]

$= 0 - c(ab) + b(ac) = -abc + abc = 0$

Hence, 1, 2 and 3 determinants have value 'zero'.

30. The system of linear equations $kx + y + z = 1$, $x + ky + z = 1$ and $x + y + kz = 1$ has a unique solution under which one of the following conditions?

(a) $k \neq 1$ and $k \neq -2$
(b) $k \neq 1$ and $k \neq 2$
(c) $k \neq -1$ and $k \neq -2$
(d) $k \neq -1$ and $k \neq 2$

(a) Given linear equations

$kx + y + z = 1$,
$x + ky + z = 1$ and $x + y + kz = 1$

For unique solution, $\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} \neq 0$

$\Rightarrow k(k^2 - 1) - 1(k - 1) + 1(1 - k) \neq 0$
$\Rightarrow k(k+1)(k-1) - (k-1) - (k-1) \neq 0$
$\Rightarrow k(k-1)(k+1) - 2(k-1) \neq 0$
$\Rightarrow (k-1)[k^2 + k - 2] \neq 0$
$\Rightarrow (k-1)(k-1)(k+2) \neq 0$
$\Rightarrow (k-1)^2 (k+2) \neq 0$
$\Rightarrow k \neq 1$ and $k \neq -2$

2015 (II)

31. If A is an invertible matrix of order n and k is any positive real number, then the value of $[\det(kA)]^{-1} \det(A)$ is

(a) k^{-n} (b) k^{-1} (c) k^n (d) nk

(a) $[\det(kA)]^{-1} \det(A)$

$$= \frac{1}{\det(kA)} \times \det(A) \quad [\because a^{-m} = \frac{1}{a^m}]$$

$$= \frac{1}{k^n \det(A)} \times \det(A) = \frac{1}{k^n} = k^{-n}$$

32. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, where $a \neq b \neq c$, then the value of abc

(a) cannot be less than 1
(b) is greater than -8
(c) is less than -8
(d) must be greater than 8

(b) Let $\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$

$= a(bc - 1) - 1(c - 1) + 1(1 - b)$
$= abc - a - b - c + 2$

$\because \quad \Delta > 0$

$\therefore \quad abc - a - b - c + 2 > 0$

$\Rightarrow abc + 2 > a + b + c$

$\Rightarrow \quad abc + 2 > 3\,(abc)^{1/3}$

$[\because a \neq b \neq c \Rightarrow$ AM of $a, b, c >$ GM of $a, b, c \Rightarrow \dfrac{a+b+c}{3} > (abc)^{1/3}$

$\Rightarrow a + b + c > 3\,(abc)^{1/3}]$

Now, let $x = (abc)^{1/3}$, then we have

$x^3 + 2 > 3x$

$\Rightarrow \quad x^3 - 3x + 2 > 0$

$\Rightarrow (x-1)^2 (x+2) > 0$

$\Rightarrow \quad x + 2 > 0 \quad [\because (x-1)^2 > 0]$

$\Rightarrow \quad x > -2$

$\Rightarrow \quad (abc)^{1/3} > -2$

$\Rightarrow \quad abc > -8$

33. Consider the following statements in respect of the determinant $\begin{vmatrix} \cos^2\frac{\alpha}{2} & \sin^2\frac{\alpha}{2} \\ \sin^2\frac{\beta}{2} & \cos^2\frac{\beta}{2} \end{vmatrix}$

where α, β are complementary angles.

1. The value of the determinant is $\frac{1}{\sqrt{2}}\cos\left(\frac{\alpha-\beta}{2}\right)$.
2. The maximum value of the determinant is $\frac{1}{\sqrt{2}}$.

Which of the above statement(s) is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) 1. We have, $\Delta = \begin{vmatrix} \cos^2\frac{\alpha}{2} & \sin^2\frac{\alpha}{2} \\ \sin^2\frac{\beta}{2} & \cos^2\frac{\beta}{2} \end{vmatrix}$

$= \cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2} - \sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}$

$= \left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$

$= \cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$= \cos\left(\frac{\alpha-\beta}{2}\right)\cos 45°$

$[\because \alpha + \beta = 90°]$

$= \frac{1}{\sqrt{2}}\cos\left(\frac{\alpha-\beta}{2}\right)$

2. The maximum value of $\cos\frac{\alpha-\beta}{2}$ is 1.

$\therefore$ The maximum value of determinant is $\frac{1}{\sqrt{2}}$.

Hence, both statements are correct.

34. If a, b and c are real numbers, then the value of the determinant $\begin{vmatrix} 1-a & a-b-c & b+c \\ 1-b & b-c-a & c+a \\ 1-c & c-a-b & a+b \end{vmatrix}$ is

(a) 0
(b) $(a-b)(b-c)(c-a)$
(c) $(a+b+c)^2$
(d) $(a+b+c)^3$

(a) We have,

$\Delta = \begin{vmatrix} 1-a & a-b-c & b+c \\ 1-b & b-c-a & c+a \\ 1-c & c-a-b & a+b \end{vmatrix}$

On applying $C_2 \to C_2 + C_3$, we get

$\Delta = \begin{vmatrix} 1-a & a & b+c \\ 1-b & b & c+a \\ 1-c & c & a+b \end{vmatrix}$

Now, applying operations $C_1 \to C_1 + C_2$ and $C_3 \to C_3 + C_2$ and taking common $a + b + c$ from C_3, we get

$\Delta = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$

[$\because C_1$ and C_3 are identical]

Hence, determinant is 0.

2015 (I)

35. Consider the following in respect of two non-singular matrices A and B of same order

I. $\det(A+B) = \det(A) + \det(B)$
II. $(A+B)^{-1} = A^{-1} + B^{-1}$

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(d) If $A = B + C$, then it is not necessary that $\det(A) = \det(B) + \det(C)$

Also, $(A+B)^{-1} = A^{-1} + B^{-1}$ is false.

36. The value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ is

(a) $x + y$ (b) $x - y$
(c) xy (d) $1 + x + y$

(c) Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix}$ $\begin{bmatrix} \because R_2 \to R_2 - R_1 \\ \text{and } R_3 \to R_3 - R_1 \end{bmatrix}$

$= 1\,(xy - 0)$

$= xy$ [expanding along C_1]

2014 (II)

37. If A and B are square matrices of second order such that $|A| = -1$ and $|B| = 3$, then what is $|3AB|$ equal to?

(a) 3 (b) –9
(c) –27 (d) None of these

(c) A and B are square matrices of order 2.

We know that, $|kA| = k^n |A|$, where n is order of matrix A.

$\therefore \quad |3AB| = 3^2 |A||B|$

$[\because |AB| = |A||B|]$

$= 9(-1)(3) [\because |A| = -1, |B| = 3]$

$= -27$

38. If $a \neq b \neq c$ are all positive, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) non-negative (b) non-positive
(c) negative (d) positive

(c) Let $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$

$[\because C_1 \to C_1 + C_2 + C_3]$

$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$

[on taking $(a+b+c)$ common from C_1]

$= (a+b+c)\,[1(bc - a^2) - b(b-a) + c(a-c)]$

$= (a+b+c)\,[bc - a^2 - b^2 + ab + ac - c^2]$

$= (a+b+c)\,[-(a^2 + b^2 + c^2 - ab - bc - ca)]$

$= -\frac{1}{2}(a+b+c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2]$

= Negative value

39. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, where $i = \sqrt{-1}$, then what is x equal to?

(a) 3 (b) 2
(c) 1 (d) 0

(d) Let $\Delta = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$= 6i\,[3i^2 + 3] + 3i\,[4i + 20] + 1\,[12 - 60i]$

$= 6i\,[-3 + 3] + 12i^2 + 60i + 12 - 60i$

$= -12 + 12 = 0 = x + iy$

$\therefore \quad x = 0$

40. Consider the following statements

1. Determinant is a square matrix.
2. Determinant is a number associated with a square matrix.

Which of the above statement(s) is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) 1. We know that, determinant is not a square matrix, so it is not a true statement.

2. It is true that, determinant is a number associated with a square matrix.

Hence, Statement 2 is correct.

41. If A is an invertible matrix, then what is $\det(A^{-1})$ equal to?

(a) $\det(A)$ (b) $\frac{1}{\det(A)}$
(c) 1 (d) None of these

(b) $\det(A^{-1}) = \frac{1}{\det(A)}$

42. If $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then which one of the following is correct?

(a) $\frac{a}{b}$ is one of the cube roots of unity
(b) $\frac{a}{b}$ is one of the cube roots of -1
(c) a is one of the cube roots of unity
(d) b is one of the cube roots of unity

(b) $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = a(a^2 - 0) - b(-b^2) + 0$

[expanding along R_1]

$= a^3 + b^3 = 0$

$\Rightarrow \quad a^3 = -b^3 \Rightarrow \left(\frac{a}{b}\right)^3 = -1$

Hence, $\frac{a}{b}$ is one of the cube roots of -1.

2014 (I)

43. One of the roots of $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$ is

(a) abc (b) $a + b + c$
(c) $-(a + b + c)$ (d) $-abc$

(c) Given that,

$$\Delta = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$$

Using operation $C_1 \to C_1 + C_2 + C_3$,

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

Again using operations,

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$,

$$(x + a + b + c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

Now, expanding along C_1,

$(x + a + b + c) \cdot 1 \cdot (x^2) = 0$

$\Rightarrow \quad x^2 \cdot (x + a + b + c) = 0$

$\Rightarrow \quad x + a + b + c = 0 \quad [\because x \neq 0]$

$\Rightarrow \quad x = -(a + b + c)$

44. The determinant of an odd order skew-symmetric matrix is always

(a) zero
(b) one
(c) negative
(d) depends on the matrix

(a) We know that, elements of principal diagonals of a skew-symmetric matrix are all zero.

i.e., $a_{ii} = -a_{ii}$

$\Rightarrow \quad 2a_{ii} = 0$

or $a_{ii} = 0$ for all values of i.

e.g., (i) $A = \begin{bmatrix} 0 & -a \\ a & 0 \end{bmatrix}_{2\times 2}$

$\Rightarrow |A| = \begin{vmatrix} 0 & -a \\ a & 0 \end{vmatrix} = 0 + a^2 = a^2 \neq 0$

(ii) $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & -c \\ -b & c & 0 \end{bmatrix}_{3\times 3}$

$\Rightarrow |A| = \begin{vmatrix} 0 & a & b \\ -a & 0 & -c \\ -b & c & 0 \end{vmatrix}$

$= 0 - a(-bc) + b(-ac)$

$= abc - abc = 0$

Here, we see that, determinant of an even order skew-symmetric matrix is not zero but an odd order skew-symmetric matrix is always zero.

45. If any two adjacent rows or columns of a determinant are interchanged in position, the value of the determinant

(a) becomes zero
(b) remains the same
(c) changes its sign
(d) is doubled

(c) If any two adjacent rows or columns of a determinant are interchanged in position, then the value of the determinant changes its sign. e.g.,

(i) Let $\Delta = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$

If $R_1 \leftrightarrow R_2$, then

$\Delta' = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1 = -\Delta$

$\Rightarrow \quad \Delta' = -\Delta$

(ii) $\Delta = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$

If $C_1 \leftrightarrow C_2$, then

$\Delta' = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1 = -\Delta$

$\Rightarrow \Delta' = -\Delta$

11

TRIGONOMETRIC RATIOS OF ANGLES AND EQUATIONS

2019 (II)

1. What is $\tan 25° \tan 15° + \tan 15° \tan 50° + \tan 25° \tan 50°$ equal to?

(a) 0 (b) 1
(c) 2 (d) 4

(b) $\because \tan 50° = \tan(90° - 40°)$

$\Rightarrow \tan 50° = \cot 40°$

$\Rightarrow \tan 50° = \dfrac{1}{\tan 40°}$

$\Rightarrow \tan 50° = \dfrac{1}{\tan (25° + 15°)}$

$\Rightarrow \tan 50° = \dfrac{1 - \tan 25° \tan 15°}{\tan 25° + \tan 15°}$

$\Rightarrow \tan 25° \tan 50° + \tan 15° \tan 50° = 1 - \tan 25° \tan 15°$

$\Rightarrow \tan 25° \tan 15° + \tan 15° \tan 50° + \tan 25° \tan 50° = 1$

2. What is $\cot A + \operatorname{cosec} A$ equal to?

(a) $\tan\left(\dfrac{A}{2}\right)$ (b) $\cot\left(\dfrac{A}{2}\right)$
(c) $2\tan\left(\dfrac{A}{2}\right)$ (d) $2\cot\left(\dfrac{A}{2}\right)$

(b) $\cot A + \operatorname{cosec} A$

$= \dfrac{\cos A}{\sin A} + \dfrac{1}{\sin A} = \dfrac{\cos A + 1}{\sin A}$

$= \dfrac{2\cos^2 \dfrac{A}{2} - 1 + 1}{2 \sin \dfrac{A}{2} \cos \dfrac{A}{2}}$

$= \dfrac{2 \cos^2 \dfrac{A}{2}}{2 \sin \dfrac{A}{2} \cos \dfrac{A}{2}} = \dfrac{\cos \dfrac{A}{2}}{\sin \dfrac{A}{2}}$

$= \cot\left(\dfrac{A}{2}\right)$

3. What is $\cot\left(\dfrac{A}{2}\right) - \tan\left(\dfrac{A}{2}\right)$ equal to?

(a) $\tan A$ (b) $\cot A$
(c) $2 \tan A$ (d) $2 \cot A$

(d) $\cot \dfrac{A}{2} - \tan \dfrac{A}{2}$

$= \dfrac{\cos \dfrac{A}{2}}{\sin \dfrac{A}{2}} - \dfrac{\sin \dfrac{A}{2}}{\cos \dfrac{A}{2}}$

$= \dfrac{\cos^2 \dfrac{A}{2} - \sin^2 \dfrac{A}{2}}{\sin \dfrac{A}{2} \cos \dfrac{A}{2}}$

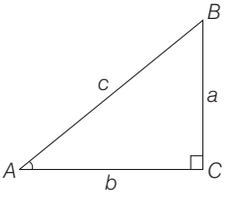
$= \dfrac{2 \cos A}{2 \sin \dfrac{A}{2} \cos \dfrac{A}{2}}$

$= \dfrac{2 \cos A}{\sin A} = 2 \cot A$

4. If angle C of a triangle ABC is a right angle, then what is $\tan A + \tan B$ equal to ?

(a) $\dfrac{a^2 - b^2}{ab}$ (b) $\dfrac{a^2}{bc}$
(c) $\dfrac{b^2}{ca}$ (d) $\dfrac{c^2}{ab}$

(d) In ΔABC, $\angle C = 90°$



$\therefore\ c^2 = a^2 + b^2$

[by pythagoras theorum] … (i)

$\tan A = \dfrac{a}{b}$, $\tan B = \dfrac{b}{a}$

Now, $\tan A + \tan B = \dfrac{a}{b} + \dfrac{b}{a}$

$= \dfrac{a^2 + b^2}{ab} = \dfrac{c^2}{ab}$ [from Eq. (i)]

5. What is $\cos 80° + \cos 40° - \cos 20°$ equal to?

(a) 2 (b) 1
(c) 0 (d) –19

(c) $\cos 80° + \cos 40° - \cos 20°$

$= 2 \cos \dfrac{80° + 40°}{2} . \cos \dfrac{80° - 40°}{2} - \cos 20°$

$= 2 \cos 60° \cos 20° - \cos 20°$

$= 2 \times \dfrac{1}{2} \cos 20° - \cos 20°$

$= \cos 20° - \cos 20° = 0$

6. If $2 \tan A = 3 \tan B = 1$, then what is $\tan (A - B)$ equal to?

(a) $\dfrac{1}{5}$ (b) $\dfrac{1}{6}$
(c) $\dfrac{1}{7}$ (d) $\dfrac{1}{9}$

(c) Given, $2 \tan A = 3 \tan B = 1$

$\therefore \tan A = \dfrac{1}{2}$, $\tan B = \dfrac{1}{3}$

Now, $\tan (A - B)$

$= \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \dfrac{\dfrac{1}{2} - \dfrac{1}{3}}{1 + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{3}\right)} = \dfrac{\dfrac{3-2}{6}}{\dfrac{6+1}{6}}$

$= \dfrac{1}{7}$

7. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then what is the value of $\cot (A - B)$?

(a) $\dfrac{1}{x} + \dfrac{1}{y}$ (b) $\dfrac{1}{y} - \dfrac{1}{x}$
(c) $\dfrac{xy}{x + y}$ (d) $1 + \dfrac{1}{xy}$

(a) Given, $\tan A - \tan B = x$ … (i)

and $\cot B - \cot A = y$ … (ii)

From Eq. (i), $\tan A - \tan B = x$

$\Rightarrow \quad \frac{1}{\cot A} - \frac{1}{\cot B} = x$

$\Rightarrow \quad \frac{\cot B - \cot A}{\cot A \cot B} = x$

$\Rightarrow \quad \cot A \cot B = \frac{y}{x}$ [from Eq. (ii)]

Now, $\cot (A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

$= \frac{\frac{y}{x} + 1}{y} = \frac{y + x}{xy}$

$= \frac{1}{x} + \frac{1}{y}$

8. What is the minimum value of $\frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$ where $a > 0$ and $b > 0$?

(a) $(a + b)^2$ (b) $(a - b)^2$
(c) $a^2 + b^2$ (d) $|a^2 + b^2|$

(*) Let $p = \frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$

$= a^2 \sec^2 x + b^2 \operatorname{cosec}^2 x - 2ab \sec x \operatorname{cosec} x + 2ab \sec x \operatorname{cosec} x$

$= (a \sec x - b \operatorname{cosec} x)^2 + 2ab \sec x \operatorname{cosec} x$

For minimum value of p,

$a \sec x - b \operatorname{cosec} x = 0$

$\Rightarrow \quad a \sec x = b \operatorname{cosec} x$

$\Rightarrow \frac{\sec x}{\operatorname{cosec} x} = \frac{b}{a} \Rightarrow \tan x = \frac{b}{a}$

$\therefore \quad \sin x = \frac{b}{\sqrt{a^2 + b^2}}, \cos x = \frac{a}{\sqrt{a^2 + b^2}}$

$\therefore$ Minimum value of p

$= \frac{a^2 (a^2 + b^2)}{a^2} + \frac{b^2 (a^2 + b^2)}{b^2}$

$= 2(a^2 + b^2)$

9. What is $\sin (\alpha + \beta) - 2 \sin \alpha \cos \beta + \sin (\alpha - \beta)$ equal to?

(a) 0 (b) $2 \sin \alpha$
(c) $2 \sin \beta$ (d) $\sin \alpha + \sin \beta$

(a) $\sin (\alpha + \beta) - 2 \sin \alpha \cos \beta + \sin (\alpha - \beta)$

$= \sin \alpha \cos \beta + \cos \alpha \sin \beta - 2 \sin \alpha \cos \beta + \sin \alpha \cos \beta - \cos \alpha \sin \beta = 0$

10. If $\operatorname{cosec} \theta = \frac{29}{21}$, where $0 < \theta < 90°$, then what is the value of $4 \sec \theta + 4 \tan \theta$?

(a) 5 (b) 10
(c) 15 (d) 20

(b) Given, $\operatorname{cosec} \theta = \frac{29}{21}$

where, $0 < \theta < 90°$

$\because \operatorname{cosec} \theta = \frac{H}{P} = \frac{29}{21} = k$ (let)

$\therefore \quad H = 29k, P = 21k$

$\therefore \quad B = \sqrt{(H)^2 - (P)^2}$

$= \sqrt{(29k)^2 - (21k)^2}$

$= \sqrt{841k^2 - 441k^2}$

$= \sqrt{400k^2} = 20k$

$\therefore \sec \theta = \frac{H}{B} = \frac{29k}{20k} = \frac{29}{20}$

and $\tan \theta = \frac{P}{B} = \frac{21k}{20k} = \frac{21}{20}$

Now, $4 \sec \theta + 4 \tan \theta$

$= 4 \times \frac{29}{20} + 4 \times \frac{21}{20}$

$= 4 \times \frac{50}{20} = 10$

11. Consider the following statements

1. $\cos \theta + \sec \theta$ can never be equal to 1.5.
2. $\tan \theta + \cot \theta$ can never be less than 2.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) We know that, $-1 \le \cos \theta \le 1$ and $-1 \le \sec \theta \le \infty$ but $\cos \theta = \sec \theta$ if $\theta = 0$ and $\theta = 180°$

$\therefore \quad -2 \le \cos \theta + \sec \theta \le \infty$

So, $\cos \theta + \sec \theta = 1.5$ is possible.

and again $0 \le \tan \theta \le \infty$ and $0 \le \cot \theta \le \infty$, but $\tan \theta = \cot \theta$ if $\theta = 45°$

$\therefore \quad 2 \le \tan \theta + \cot \theta \le \infty$

So, $\tan \theta + \cot \theta$ can never be less than 2.

Hence, only the Statement 2 is correct.

12. What is the length of the chord of a unit circle which subtends an angle θ at the centre?

(a) $\sin \left(\frac{\theta}{2}\right)$ (b) $\cos \left(\frac{\theta}{2}\right)$
(c) $2 \sin \left(\frac{\theta}{2}\right)$ (d) $2 \cos \left(\frac{\theta}{2}\right)$

(c) Given, radius of circle = 1 unit

Angle subtends at the centre of circle by chord = θ

We know that, length of chord

$= 2r \sin \frac{\theta}{2} = 2 \times 1 \sin \frac{\theta}{2}$

$= 2 \sin \frac{\theta}{2}$

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13. What is the least value of $25 \operatorname{cosec}^2 x + 36 \sec^2 x$?

(a) 1 (b) 11 (c) 120 (d) 121

(d) Given, $25 \operatorname{cosec}^2 x + 36 \sec^2 x$

$= 25 (1 + \cot^2 x) + 36 (1 + \tan^2 x)$

$= 25 + 25 \cot^2 x + 36 + 36 \tan^2 x$

$= 25 + 36 + 25 \cot^2 x + 36 \tan^2 x$

$= 61 + (5 \cot x - 6 \tan x)^2 + 2 \times 5 \times 6$

$\ge 61 + 60 = 121$ [$\because$ minimum value of $(5 \cot x - 6 \tan x)^2 = 0$]

$\therefore$ Minimum value of $25 \operatorname{cosec}^2 x + 36 \sec^2 x = 121$

14. What is the value of $\frac{\sin 34° \cos 236° - \sin 56° \sin 124°}{\cos 28° \cos 88° + \cos 178° \sin 208°}$?

(a) –2 (b) –1
(c) 2 (d) 1

(a) We have,

$\frac{\sin 34° \cos 236° - \sin 56° \sin 124°}{\cos 28° \cos 88° + \cos 178° \sin 208°}$

$= \frac{\sin 34° \cos (180° + 56°) - \sin 56° \sin (90° + 34°)}{\cos 28° \cos 88° + \cos (90° + 88°) \sin (180° + 28°)}$

$= \frac{-\sin 34° \cos 56° - \sin 56° \cos 34°}{\cos 28° \cos 88° + \sin 88° \sin 28°}$

$= \frac{-\sin (56° + 34°)}{\cos (88° - 28°)} = \frac{-\sin 90°}{\cos 60°}$

$= \frac{-1}{\frac{1}{2}} = -2$

15. $\tan 54°$ can be expressed as

(a) $\frac{\sin 9° + \cos 9°}{\sin 9° - \cos 9°}$ (b) $\frac{\sin 9° - \cos 9°}{\sin 9° + \cos 9°}$
(c) $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$
(d) $\frac{\sin 36°}{\cos 36°}$

(c) We have, $\tan 54° = \tan (45° + 9°)$

$= \frac{\tan 45° + \tan 9°}{1 - \tan 45° \tan 9°} = \frac{1 + \tan 9°}{1 - \tan 9°}$

$= \frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$

Directions (Q. Nos. 16-18) *Read the given information carefully and answer the given questions.*

If $p = X \cos \theta - Y \sin \theta$, $q = X \sin \theta + Y \cos \theta$ and $p^2 + 4pq + q^2 = AX^2 + BY^2, 0 \le \theta \le \frac{\pi}{2}$.

16. What is the value of θ?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(c) We have,

$p = X\cos\theta - Y\sin\theta$ ...(i)

$q = X\sin\theta + Y\cos\theta$ ...(ii)

and $p^2 + 4pq + q^2 = AX^2 + BY^2$ ...(iii)

From Eqs. (i) and (ii), we get

$p^2 + q^2 = (X\cos\theta - Y\sin\theta)^2 + (X\sin\theta + Y\cos\theta)^2$

$\Rightarrow p^2 + q^2 = X^2 + Y^2$

and $pq = (X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta)$

$\Rightarrow pq = (X^2 - Y^2)\sin\theta\cos\theta + XY\cos 2\theta$

$\therefore p^2 + 4pq + q^2 = X^2 + Y^2 + 2(X^2 - Y^2)\sin 2\theta + 4XY\cos 2\theta$

Given, $p^2 + 4pq + q^2 = AX^2 + BY^2$

$\therefore X^2 + Y^2 + 2(X^2 - Y^2)\sin 2\theta + 4XY\cos 2\theta = AX^2 + 2BY^2$

Coefficient of $XY = 0$

$\therefore \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{4}$

17. What is the value of A?

(a) 4 (b) 3 (c) 2 (d) 1

(b) $X^2 + Y^2 + 2(X^2 - Y^2)\sin\frac{\pi}{2} = AX^2 + BY^2$

$\Rightarrow X^2 + Y^2 + 2X^2 - 2Y^2 = AX^2 + BY^2$

$\Rightarrow 3X^2 - Y^2 = AX^2 + BY^2$

$\therefore A = 3, B = -1$

18. What is the value of B ?

(a) –1 (b) 0 (c) 1 (d) 2

(a) $B = -1$

Directions (Q. Nos. 19 and 20) *Read the given information carefully and answer the given questions.*

It is given that $\cos(\theta - \alpha) = a$, $\cos(\theta - \beta) = b$.

19. What is $\cos(\alpha - \beta)$ equal to ?

(a) $ab + \sqrt{1-a^2}\sqrt{1-b^2}$

(b) $ab - \sqrt{1-a^2}\sqrt{1-b^2}$

(c) $a\sqrt{1-a^2} - b\sqrt{1-a^2}$

(d) $a\sqrt{1-b^2} + b\sqrt{1-a^2}$

(a) Given

$\cos(\theta - \alpha) = a$

$\cos(\theta - \beta) = b$

$\cos(\alpha - \beta) = \cos\{(\theta - \beta) - (\theta - \alpha)\}$

$= \cos(\theta - \beta)\cos(\theta - \alpha) + \sin(\theta - \beta)\sin(\theta - \alpha)$

$= ab + \sqrt{1-a^2}\sqrt{1-b^2}$

20. What is $\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$ equal to?

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $b^2 - a^2$ (d) $-(a^2 + b^2)$

(a) $\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$

$= 1 - \cos^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$

$= 1 - (ab + \sqrt{1-a^2}\sqrt{1-b^2})^2 + 2ab(ab + \sqrt{1-a^2}\sqrt{1-b^2})$

$= 1 - [a^2b^2 + (1-a^2)(1-b^2)] + 2ab\sqrt{1-a^2}\sqrt{1-b^2} + 2a^2b^2 + 2ab(\sqrt{1-a^2})(\sqrt{(1-b^2)}$

$= 1 - a^2b^2 - 1 + a^2 + b^2 - a^2b^2 - 2ab\sqrt{1-a^2}\sqrt{1-b^2} + 2a^2b^2 + 2ab\sqrt{1-a^2}\sqrt{1-b^2}$

$= a^2 + b^2$

21. If $\sin\alpha + \cos\alpha = p$, then what is $\cos^2(2\alpha)$ equal to?

(a) p^2 (b) $p^2 - 1$

(c) $p^2(2 - p^2)$ (d) $p^2 + 1$

(c) We have, $\sin\alpha + \cos\alpha = p$

$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = p^2$

$\Rightarrow 1 + \sin 2\alpha = p^2$

$\Rightarrow \sin 2\alpha = p^2 - 1$

$\Rightarrow \sin^2 2\alpha = (p^2 - 1)^2$

$\Rightarrow 1 - \cos^2 2\alpha = p^4 - 2p^2 + 1$

$\Rightarrow \cos^2 2\alpha = 2p^2 - p^4$

$\therefore \cos^2 2\alpha = p^2(2 - p^2)$

22. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then what is the value of $(\theta + \phi)$?

(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(c) Given, $\tan\theta = \frac{1}{2}$, $\tan\phi = \frac{1}{3}$

$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

$\Rightarrow \tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}}$

$\Rightarrow \tan(\theta + \phi) = \frac{3+2}{6-1} = \frac{5}{5} = 1$

$\Rightarrow \tan(\theta + \phi) = 1$

$\Rightarrow \theta + \phi = \tan^{-1} 1 = \frac{\pi}{4}$

23. If $\cos A = \frac{3}{4}$, then what is the value of $\sin\left(\frac{A}{2}\right)\sin\left(\frac{3A}{2}\right)$?

(a) $\frac{5}{8}$ (b) $\frac{5}{16}$ (c) $\frac{5}{24}$ (d) $\frac{7}{32}$

(b) Given, $\cos A = \frac{3}{4}$

Now, $\sin\frac{A}{2}\sin\frac{3A}{2} = \frac{1}{2}\left(2\sin\frac{A}{2}\sin\frac{3A}{2}\right)$

$= \frac{1}{2}\left[\cos\left(\frac{A}{2} - \frac{3A}{2}\right) - \cos\left(\frac{A}{2} + \frac{3A}{2}\right)\right]$

$= \frac{1}{2}[\cos A - \cos 2A]$

$= \frac{1}{2}(\cos A - 2\cos^2 A + 1)$

$= \frac{1}{2}\left[\frac{3}{4} - 2\times\frac{9}{16} + 1\right]$

$= \frac{1}{2}\left[\frac{3}{4} - \frac{9}{8} + 1\right] = \frac{1}{2}\left[\frac{6 - 9 + 8}{8}\right] = \frac{5}{16}$

24. What is the value of $\tan 75° + \cot 75°$?

(a) 2 (b) 4 (c) $2\sqrt{3}$ (d) $4\sqrt{3}$

(b) We have, $\tan 75° + \cot 75°$

$= \tan(90° - 15°) + \cot(90° - 15°)$

$= \cot 15° + \tan 15° = \frac{\cos 15°}{\sin 15°} + \frac{\sin 15°}{\cos 15°}$

$= \frac{\cos^2 15° + \sin^2 15°}{\sin 15°\cos 15°}$

$= \frac{1}{\sin 15°\cos 15°} = \frac{2}{2\sin 15°\cos 15°}$

$= \frac{2}{\sin 30°} = \frac{2}{\frac{1}{2}} = 4$

25. What is the value of $\cos 46°\cos 47°\cos 48°\cos 49°\cos 50°.....\cos 135°$?

(a) –1 (b) 0

(c) 1 (d) Greater than 1

(b) We have,

$\cos 46°\cos 47°\cos 48°\cos 49°\cos 50°\ldots\cos 90°\ldots\cos 135° = 0$ [$\because \cos 90° = 0$]

26. If $\sin 2\theta = \cos 3\theta$, where $0 < \theta < \frac{\pi}{2}$, then what is $\sin\theta$ equal to ?

(a) $\frac{\sqrt{5}+1}{4}$ (b) $\frac{\sqrt{5}-1}{4}$

(c) $\frac{\sqrt{5}+1}{16}$ (d) $\frac{\sqrt{5}-1}{16}$

(b) Given, $\sin 2\theta = \cos 3\theta$

$\Rightarrow 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow 2\sin\theta = 4\cos^2\theta - 3$ [$\because \cos\theta \neq 0$]

$\Rightarrow 2\sin\theta = 4(1 - \sin^2\theta) - 3$

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-2 \pm \sqrt{4+16}}{2\times 4}$

$\Rightarrow \sin\theta = \frac{-2 \pm 2\sqrt{5}}{2\times 4} = \frac{-1 \pm \sqrt{5}}{4}$

$\therefore \sin\theta = \frac{\sqrt{5}-1}{4}$ $\left[\because \theta \in \left[0, \frac{\pi}{2}\right]\right]$

27. If the roots of the equation $x^2 + px + q = 0$ are $\tan 19°$ and $\tan 26°$, then which one of the following is correct ?

(a) $q - p = 1$ (b) $p - q = 1$
(c) $p + q = 2$ (d) $p + q = 3$

(a) Given, $\tan 19°$ and $\tan 26°$ are roots of $x^2 + px + q = 0$.

$\therefore \tan 19° + \tan 26° = -p$

$\tan 19° \cdot \tan 26° = q$

$$\tan(19° + 26°) = \frac{\tan 19° + \tan 26°}{1 - \tan 19° \tan 26°}$$

$$\Rightarrow \quad \tan 45° = \frac{-p}{1-q} \Rightarrow 1 = \frac{-p}{1-q}$$

$$\Rightarrow \quad 1 - q = -p$$

$$\therefore \quad q - p = 1$$

28. What is $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 - \sec^2\alpha \sec^2\beta$ equal to?

(a) 0 (b) 1 (c) 2 (d) 4

(a) We have,

$(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 - \sec^2\alpha \sec^2\beta$

$= 1 + \tan^2\alpha \tan^2\beta + 2\tan\alpha \tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha \tan\beta - \sec^2\alpha \sec^2\beta$

$= 1 + \tan^2\alpha \tan^2\beta + \tan^2\alpha + \tan^2\beta - \sec^2\alpha \sec^2\beta$

$= (1 + \tan^2\alpha)(1 + \tan^2\beta) - \sec^2\alpha \sec^2\beta$

$= \sec^2\alpha \sec^2\beta - \sec^2\alpha \sec^2\beta = 0$

29. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then what is $(\cos\theta - \sin\theta)$ equal to ?

(a) $-\sqrt{2}\cos\theta$ (b) $-\sqrt{2}\sin\theta$
(c) $\sqrt{2}\sin\theta$ (d) $2\sin\theta$

(c) Given, $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$ ...(i)

Now, $\cos\theta - \sin\theta$

$= \cos\theta - (\sqrt{2} - 1)\cos\theta$ [from Eq. (i)]

$= \cos\theta[1 - (\sqrt{2} - 1)] = \cos\theta\,[2 - \sqrt{2}]$

$= \cos\theta \cdot \sqrt{2}\,(\sqrt{2} - 1)$

$= \sqrt{2}\sin\theta$ [using Eq. (i)]

30. If $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$, then in which quadrant does θ lie?

(a) First (b) Second
(c) Third (d) Fourth

(c) We know that, if θ lies in third quadrant then, $\sin\theta < 0$ and $\tan\theta > 0$.

31. If $p = \operatorname{cosec}\theta - \cot\theta$ and $q = (\operatorname{cosec}\theta + \cot\theta)^{-1}$, then which one of the following is correct?

(a) $pq = 1$ (b) $p = q$
(c) $p + q = 1$ (d) $p + q = 0$

(b) Given, $p = \operatorname{cosec}\theta - \cot\theta$

$q = (\operatorname{cosec}\theta + \cot\theta)^{-1}$

$$\Rightarrow \quad q = \left(\frac{1}{\operatorname{cosec}\theta + \cot\theta}\right)\left(\frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}\theta - \cot\theta}\right)$$

$$\Rightarrow \quad q = \operatorname{cosec}\theta - \cot\theta$$

$$\therefore \quad q = p$$

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32. What is $\frac{2\tan\theta}{1 + \tan^2\theta}$ equal to ?

(a) $\cos 2\theta$ (b) $\tan 2\theta$
(c) $\sin 2\theta$ (d) cosec 2θ

(c) We have,

$$\frac{2\tan\theta}{1 + \tan^2\theta} = \frac{\frac{2\sin\theta}{\cos\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}} = \frac{2\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 2\sin\theta\cos\theta = \sin 2\theta$

33. If $\sec(\theta - \alpha)$, $\sec\theta$ and $\sec(\theta + \alpha)$ are in AP, where $\cos\alpha \neq 1$, then what is the value of $\sin^2\theta + \cos\alpha$?

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

(a) If $\sec(\theta - \alpha)$, $\sec\theta$ and $\sec(\theta + \alpha)$ are in AP, then

$$2\sec\theta = \sec(\theta - \alpha) + \sec(\theta + \alpha)$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{1}{\cos(\theta - \alpha)} + \frac{1}{\cos(\theta + \alpha)}$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{\cos(\theta + \alpha) + \cos(\theta - \alpha)}{\cos(\theta - \alpha)\cdot\cos(\theta + \alpha)}$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{2\cos\left(\frac{\theta + \alpha + \theta - \alpha}{2}\right)\cos\left(\frac{\theta + \alpha - \theta + \alpha}{2}\right)}{\cos^2\theta - \sin^2\alpha}$$

$\left[\because \cos C + \cos D = 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)\right]$

[and $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$]

$$\Rightarrow \quad \frac{2}{\cos\theta} = \frac{2\cos\theta\cos\alpha}{\cos^2\theta - \sin^2\alpha}$$

$$\Rightarrow \cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha$$

$$\Rightarrow \quad \sin^2\alpha = \cos^2\theta - \cos^2\theta\cos\alpha$$

$$\Rightarrow \quad \sin^2\alpha = \cos^2\theta(1 - \cos\alpha)$$

$$\Rightarrow \quad \cos^2\theta = \frac{\sin^2\alpha}{1 - \cos\alpha}$$

$$= \frac{1 - \cos^2\alpha}{1 - \cos\alpha} = \frac{(1 + \cos\alpha)(1 - \cos\alpha)}{1 - \cos\alpha}$$

$$\Rightarrow \quad \cos^2\theta = 1 + \cos\alpha$$

$$\Rightarrow \quad 1 - \sin^2\theta = 1 + \cos\alpha$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$\Rightarrow \quad \sin^2\theta + \cos\alpha = 0$$

34. If $A + B + C = 180°$, then what is $\sin 2A - \sin 2B - \sin 2C$ equal to?

(a) $-4\sin A \sin B \sin C$
(b) $-4\cos A \sin B \cos C$
(c) $-4\cos A \cos B \sin C$
(d) $-4\sin A \cos B \cos C$

(d) We have,

$\sin 2A - \sin 2B - \sin 2C$

$= \sin 2A - [\sin 2B + \sin 2C]$

$= \sin 2A - \left[2\sin\left(\frac{2B + 2C}{2}\right)\cos\left(\frac{2B - 2C}{2}\right)\right]$

$\left[\because \sin C + \sin D = 2\sin\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)\right]$

$= 2\sin A\cos A - [2\sin(B + C)\cos(B - C)]$

$[\because \sin 2A = 2\sin A\cos A]$

$= 2\sin A\cos A - [2\sin(180° - A)\cos(B - C)]$

$[\because A + B + C = 180°]$

$= 2\sin A\cos A - 2\sin A\cos(B - C)$

$[\because \sin(180° - \theta) = \sin\theta]$

$= 2\sin A[\cos A - \cos(B - C)]$

$= 2\sin A[\cos(180 - (B + C)) - \cos(B - C)]$

$= 2\sin A[-\cos(B + C) - \cos(B - C)]$

$= -2\sin A[\cos(B + C) + \cos(B - C)]$

$= -2\sin A[2\cos B\cos C]$

$[\because \cos C + \cos D = 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)]$

$= -4\sin A\cos B\cos C$

35. A is an angle in the fourth quadrant it satisfies the trigonometric equation $3(3 - \tan^2 A - \cot A)^2 = 1$. Which one of the following is a value of A ?

(a) 300° (b) 315° (c) 330° (d) 345°

(a) We have, $3(3 - \tan^2 A - \cot A)^2 = 1$

Checking through options

$A = 300°$

So, $3[3 - \tan^2 300° - \cot(300)°]^2$

$= 3[3 - \tan^2(360° - 60°) - \cot(360° - 60)°]^2$

$= 3[3 - \tan^2 60° + \cot 60°]^2$

$$= 3\left[3 - 3 + \frac{1}{\sqrt{3}}\right]^2 = 3 \times \frac{1}{3} = 1$$

So, option (a) is correct

36. What is/are the solution(s) of the trigonometric equation $\text{cosec}\, x + \cot x = \sqrt{3}$ where $0 < x < 2\pi$?

(a) $\frac{5\pi}{3}$ only (b) $\frac{\pi}{3}$ only

(c) π only (d) $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

(b) We have,

$$\text{cosec}\, x + \cot x = \sqrt{3}$$

$$\Rightarrow \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \sqrt{3}$$

$$\Rightarrow \frac{1 + \cos x}{\sin x} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow 1 + \cos x = \sqrt{3} \sin x$$

$$\Rightarrow \sqrt{3}\sin x - \cos x = 1$$

$$\Rightarrow \frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x = \frac{1}{2}$$

$$\Rightarrow \sin x \cos 30° - \cos x . \sin 30° = \sin 30°$$

$$\Rightarrow \sin (x - 30°) = \sin 30°$$

$$\Rightarrow x - 30° = 30°$$

$$\Rightarrow x = 60° = \frac{\pi}{3}$$

37. If $\theta = \frac{\pi}{8}$, then what is the value of $(2\cos\theta + 1)^{10} (2\cos 2\theta - 1)^{10}$ $(2\cos\theta - 1)^{10}$ $(2\cos 4\theta - 1)^{10}$?

(a) 0 (b) 1 (c) 2 (d) 4

(b) If $\theta = \frac{\pi}{8}$, then

$$(2\cos\theta + 1)^{10} (2\cos 2\theta - 1)^{10}$$

$$(2\cos\theta - 1)^{10} (2\cos 4\theta - 1)^{10}$$

$$= (2\cos\theta + 1)^{10} (2\cos\theta - 1)^{10}$$

$$(2\cos 2\theta - 1)^{10} (2\cos 4\theta - 1)^{10}$$

$$= (4\cos^2\theta - 1)^{10} (2\cos 2\theta - 1)^{10}$$

$$(2\cos 4\theta - 1)^{10}$$

$$= [2(2\cos^2\theta) - 1]^{10} (2\cos 2\theta - 1)^{10}$$

$$(2\cos 4\theta - 1)^{10}$$

$$= [2(1 + \cos 2\theta) - 1]^{10} (2\cos 2\theta - 1)^{10}$$

$$(2\cos 4\theta - 1)^{10}$$

$$= (2\cos 2\theta + 1)^{10} (2\cos 2\theta - 1)^{10}$$

$$(2\cos 4\theta - 1)^{10}$$

$$= \left[2\cos\frac{\pi}{4} + 1\right]^{10} \left(2\cos\frac{\pi}{4} - 1\right)^{10}$$

$$\left(2\cos\frac{\pi}{2} - 1\right)^{10} \quad \left[\because \theta = \frac{\pi}{8}\right]$$

$$= \left[2 \times \frac{1}{\sqrt{2}} + 1\right]^{10} \left(2 \cdot \frac{1}{\sqrt{2}} - 1\right)^{10} (0 - 1)^{10}$$

$$= (\sqrt{2} + 1)^{10} (\sqrt{2} - 1)^{10} \times 1$$

$$= [(\sqrt{2})^2 - (1)^2]^{10} \times 1 = (2 - 1)^{10} = 1$$

38. If $\cos\alpha$ and $\cos\beta$ $(0 < \alpha < \beta < \pi)$ are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of $\sec\alpha \times \sec\beta$?

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

(a) Given,

$$4x^2 - 3 = 0$$

$$\Rightarrow \quad 4x^2 = 3 \Rightarrow x^2 = \frac{3}{4}$$

$$\Rightarrow \quad x = \pm \frac{\sqrt{3}}{2}$$

So, $\cos\alpha = \frac{\sqrt{3}}{2}$

and $\cos\beta = \frac{-\sqrt{3}}{2}$

Now, $\sec\alpha \cdot \sec\beta = \frac{2}{\sqrt{3}} \times \left(\frac{-2}{\sqrt{3}}\right) = \frac{-4}{3}$

39. If $A = \sin^2\theta + \cos^4\theta$, then for all real θ, which one of the following is correct?

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$

(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

(b) We have

$$A = \sin^2\theta + \cos^4\theta$$

$$= \sin^2\theta + (1 - \sin^2\theta)^2$$

Let $\sin^2\theta = x$

$\Rightarrow A(x) = x^2 - x + 1, 0 \le x \le 1$

$$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

Now, $A(0) = A(1) = 1$

So, $\frac{3}{4} \le A \le 1$

2018 (I)

40. What is $\frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x}$ equal to?

(a) $\sin x$ (b) $\cos x$

(c) $\tan x$ (d) $\cot x$

(c) Given, $\frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\cos\frac{5x + 3x}{2} . \sin\frac{5x - 3x}{2}}{2\cos\frac{5x + 3x}{2} . \cos\frac{5x - 3x}{2}}$$

$$\left[\because \sin C - \sin D = 2\cos\left(\frac{C + D}{2}\right) \cdot \sin\left(\frac{C - D}{2}\right) \text{ and } \cos C + \cos D = 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)\right]$$

$$= \frac{2\cos 4x \sin x}{2\cos 4x \cos x}$$

$$= \frac{\sin x}{\cos x} = \tan x$$

41. What is $\sin 105° + \cos 105°$ equal to ?

(a) $\sin 50°$ (b) $\cos 50°$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

(c) We have, $\sin 105° + \cos 105°$

$$= \sin(90° + 15°) + \cos 105°$$

$$= \cos 15° + \cos 105°$$

$$[\because \sin(90° + \theta) = \cos\theta]$$

$$= 2\cos\left(\frac{105° + 15°}{2}\right)\cos\left(\frac{105° - 15°}{2}\right)$$

$$\left[\because \cos C + \cos D = 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)\right]$$

$$= 2\cos 60° \cos 45° = 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$\left[\because \cos 60° = \frac{1}{2}, \cos 45° = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}}$$

42. If $x, x - y$ and $x + y$ are the angles of a triangle (not an equilateral triangle) such that $\tan(x - y)$, $\tan x$ and $\tan(x + y)$ are in GP, then what is x equal to?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

(a) Given

$\tan(x - y), \tan x$ and $\tan(x + y)$ are in GP.

$$\therefore \quad \tan^2 x = \tan(x - y)\tan(x + y)$$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} = \frac{\sin(x - y)\sin(x + y)}{\cos(x - y)\cos(x + y)}$$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} = \frac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$$

$$\Rightarrow \sin^2 x \cos^2 x - \sin^2 x \sin^2 y = \sin^2 x \cos^2 x - \sin^2 y \cos^2 x$$

$$\Rightarrow \sin^2 y (\cos^2 x - \sin^2 x) = 0$$

$$\Rightarrow \quad \cos^2 x - \sin^2 x = 0$$

$$\because \quad \sin^2 y \neq 0, \text{ if } y = 0$$

Then, triangle is equilateral.

$$\Rightarrow \tan^2 x = 1 \Rightarrow \tan x = \pm 1 \Rightarrow x = \frac{\pi}{4}$$

43. ABC is a triangle inscribed in a circle with centre O. Let $\alpha = \angle BAC$, where $45° < \alpha < 90°$. Let $\beta = \angle BOC$. Which one of the following is correct?

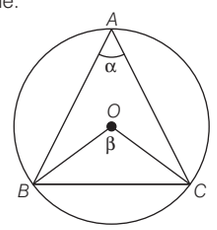
(a) $\cos\beta = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$

(b) $\cos\beta = \frac{1 + \tan^2\alpha}{1 - \tan^2\alpha}$

(c) $\cos\beta = \frac{2\tan\alpha}{1 + \tan^2\alpha}$

(d) $\sin\beta = 2\sin^2\alpha$

(a) We know that angle subtended by a chord at centre is always double the angle subtended by it at any other part of the circle.



$\therefore \quad \beta = 2\alpha$

$\Rightarrow \quad \cos\beta = \cos 2\alpha$

$\Rightarrow \quad \cos\beta = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$

$\left[\because \cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right]$

44. If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$, then what is $\dfrac{\tan x}{\tan y}$ equal to ?

(a) $\dfrac{a}{b}$ (b) $\dfrac{b}{a}$

(c) $\dfrac{a+b}{a-b}$ (d) $\dfrac{a-b}{a+b}$

(a) We have,

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

On using componendo and dividendo rule, we get

$$\frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$\Rightarrow$

$$\frac{2\sin\left(\dfrac{x+y+x-y}{2}\right)\cos\left(\dfrac{x+y-x+y}{2}\right)}{2\cos\left(\dfrac{x+y+x-y}{2}\right)\sin\left(\dfrac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$$

$\Rightarrow \quad \dfrac{\sin x\cos y}{\cos x\sin y} = \dfrac{a}{b}$

$\Rightarrow \quad \dfrac{\tan x}{\tan y} = \dfrac{a}{b}$

45. If $\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$, where $0 < \beta < \alpha < 2\pi$, then which one of the following is correct?

(a) $\alpha = \pi - \beta$
(b) $\alpha = \pi + \beta$
(c) $\alpha = 2\pi - \beta$
(d) $2\alpha = \pi + 2\beta$

(b) We have,

$\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$

$\therefore (\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 = 0$

$\Rightarrow \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = 0$

$\Rightarrow (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 0$

$\Rightarrow \quad 1 + 1 + 2\cos(\alpha - \beta) = 0$

$\Rightarrow \quad 2\cos(\alpha - \beta) = -2$

$\Rightarrow \quad \cos(\alpha - \beta) = -1$

$\Rightarrow \quad \alpha - \beta = \pi$

$\Rightarrow \quad \alpha = \beta + \pi$

46. Suppose $\cos A$ is given. If only one value of $\cos\left(\dfrac{A}{2}\right)$ is possible, then A must be

(a) an odd multiple of 90°
(b) a multiple of 90°
(c) an odd multiple of 180°
(d) a multiple of 180°

(c) We know that,

$\cos A = 2\cos^2\dfrac{A}{2} - 1$

Since, $\cos A$ is given and $\cos\dfrac{A}{2}$ has only one solution. So, A must be odd multiple of 180°.

47. If $\cos\alpha + \cos\beta + \cos\gamma = 0$, where $0 < \alpha \le \dfrac{\pi}{2}, 0 < \beta \le \dfrac{\pi}{2}$, $0 < \gamma \le \dfrac{\pi}{2}$, then what is the value of $\sin\alpha + \sin\beta + \sin\gamma$?

(a) 0 (b) 3

(c) $\dfrac{5\sqrt{2}}{2}$ (d) $\dfrac{3\sqrt{2}}{2}$

(b) We have,

$\cos\alpha + \cos\beta + \cos\gamma = 0,$

$0 < \alpha \le \dfrac{\pi}{2}, 0 < \beta \le \dfrac{\pi}{2}, 0 < \gamma \le \dfrac{\pi}{2}$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow \quad \alpha = \beta = \gamma = \dfrac{\pi}{2}$

$\therefore \quad \sin\alpha + \sin\beta + \sin\gamma$

$= \sin\dfrac{\pi}{2} + \sin\dfrac{\pi}{2} + \sin\dfrac{\pi}{2}$

$\left[\because \text{if } \theta \in \left[0, \dfrac{\pi}{2}\right] \Rightarrow \cos\dfrac{\pi}{2} = 0\right]$

$= 1 + 1 + 1 = 3$

48. The maximum value of $\sin\left(x + \dfrac{\pi}{5}\right) + \cos\left(x + \dfrac{\pi}{5}\right)$, where $x \in \left(0, \dfrac{\pi}{2}\right)$, is attained at

(a) $\dfrac{\pi}{20}$ (b) $\dfrac{\pi}{15}$ (c) $\dfrac{\pi}{10}$ (d) $\dfrac{\pi}{2}$

(a) Let

$f(x) = \sin\left(x + \dfrac{\pi}{5}\right) + \cos\left(x + \dfrac{\pi}{5}\right)$

$= \sqrt{2}\left[\dfrac{1}{\sqrt{2}}\sin\left(x + \dfrac{\pi}{5}\right) + \dfrac{1}{\sqrt{2}}\cos\left(x + \dfrac{\pi}{5}\right)\right]$

$= \sqrt{2}\left[\sin\left(x + \dfrac{\pi}{5}\right)\cos\dfrac{\pi}{4} + \cos\left(x + \dfrac{\pi}{5}\right)\sin\dfrac{\pi}{4}\right]$

$= \sqrt{2}\sin\left(x + \dfrac{\pi}{5} + \dfrac{\pi}{4}\right)$

$f(x)$ attains maximum value, when

$x + \dfrac{\pi}{5} + \dfrac{\pi}{4} = \dfrac{\pi}{2}$

$\Rightarrow \quad x = \dfrac{\pi}{20}$

49. What is the maximum value of $16\sin\theta - 12\sin^2\theta$?

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{3}$ (c) $\dfrac{16}{3}$ (d) 4

(c) Let $f(x) = 16\sin\theta - 12\sin^2\theta$

$= -12\left[\sin^2\theta - \dfrac{16}{12}\sin\theta\right]$

$= -12\left[\sin^2\theta - \dfrac{4}{3}\sin\theta\right]$

$= -12\left[\left(\sin\theta - \dfrac{2}{3}\right)^2 - \dfrac{4}{9}\right]$

$= -12\left(\sin\theta - \dfrac{2}{3}\right)^2 + \dfrac{16}{3}$

$\therefore \quad f(x) \le \dfrac{16}{3}$

$\therefore$ Maximum value of $f(x) = \dfrac{16}{3}$

2017 (II)

50. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is equal to

(a) –1 (b) 0 (c) 1 (d) 4

(d) We have,

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= \tan 9° - \tan 27° - \tan(90° - 27°) + \tan(90° - 9°)$

$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$

$= (\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$= \left[\dfrac{\sin 9°}{\cos 9°} + \dfrac{\cos 9°}{\sin 9°}\right] - \left[\dfrac{\sin 27°}{\cos 27°} + \dfrac{\cos 27°}{\sin 27°}\right]$

$= \left[\dfrac{\sin^2 9° + \cos^2 9°}{\sin 9°\cos 9°}\right] - \left[\dfrac{\sin^2 27° + \cos^2 27°}{\sin 27°\cos 27°}\right]$

$= \dfrac{1}{\sin 9°\cos 9°} - \dfrac{1}{\sin 27°\cos 27°}$

$= \dfrac{2}{\sin 18°} - \dfrac{2}{\sin 54°} = 2\left[\dfrac{\sin 54° - \sin 18°}{\sin 18°\sin 54°}\right]$

$= 2\left[\dfrac{2\cos 36°\sin 18°}{\sin 18°\sin(90° - 36°)}\right]$

$= \dfrac{4\cos 36°\sin 18°}{\sin 18°\cos 36°} = 4$

51. The value of $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$ is equal to

(a) 4 (b) 2 (c) 1 (d) –4

(a) We have, $\sqrt{3}\,\text{cosec}\ 20° - \sec 20°$

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$$

$$= \frac{4\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{2\sin 20° \cos 20°}$$

$$= 4\left(\frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\sin 40°}\right)$$

$$= \frac{4\sin(60° - 20°)}{\sin 40°}$$

$$= \frac{4\sin 40°}{\sin 40°} = 4$$

52. Angle α is divided into two parts A and B such that $A - B = x$ and $\tan A : \tan B = p : q$. The value of $\sin x$ is equal to

(a) $\frac{(p+q)\sin\alpha}{p-q}$ (b) $\frac{p\sin\alpha}{p+q}$

(c) $\frac{p\sin\alpha}{p-q}$ (d) $\frac{(p-q)\sin\alpha}{p+q}$

(d) We have,

$$A + B = \alpha \text{ and } A - B = x$$

$$\therefore \quad A = \frac{\alpha + x}{2}$$

and $$B = \frac{\alpha - x}{2}$$

Now, according to question

$$\frac{\tan A}{\tan B} = \frac{p}{q} \Rightarrow \frac{\tan\left(\frac{\alpha + x}{2}\right)}{\tan\left(\frac{\alpha - x}{2}\right)} = \frac{p}{q}$$

$$\Rightarrow \frac{\sin\left(\frac{\alpha + x}{2}\right)\cos\left(\frac{\alpha - x}{2}\right)}{\sin\left(\frac{\alpha - x}{2}\right)\cos\left(\frac{\alpha + x}{2}\right)} = \frac{p}{q}$$

$$\Rightarrow \frac{2\sin\left(\frac{\alpha + x}{2}\right)\cos\left(\frac{\alpha - x}{2}\right)}{2\sin\left(\frac{\alpha - x}{2}\right)\cos\left(\frac{\alpha + x}{2}\right)} = \frac{p}{q}$$

$$\Rightarrow \frac{\sin\alpha + \sin x}{\sin\alpha - \sin x} = \frac{p}{q}$$

On applying componendo and dividendo , we get

$$\frac{\sin\alpha + \sin x + \sin\alpha - \sin x}{\sin\alpha + \sin x - \sin\alpha + \sin x} = \frac{p+q}{p-q}$$

$$\frac{\sin\alpha}{\sin x} = \frac{p+q}{p-q}$$

$$\Rightarrow \quad \sin x = \frac{p-q}{p+q}\sin\alpha$$

53. $\sqrt{1+\sin A} = -\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$ is true, if

(a) $\frac{3\pi}{2} < A < \frac{5\pi}{2}$ only (b) $\frac{\pi}{2} < A < \frac{3\pi}{2}$ only

(c) $\frac{3\pi}{2} < A < \frac{7\pi}{2}$ (d) $0 < A < \frac{3\pi}{2}$

(a) We have,

$$\sqrt{1+\sin A}$$

$$= \sqrt{\sin^2\frac{A}{2} + \cos^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$= \sqrt{\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)^2} = \left|\sin\frac{A}{2} + \cos\frac{A}{2}\right|$$

$$= \begin{cases} \sin\frac{A}{2} + \cos\frac{A}{2}, \\ -\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right), \end{cases}$$

$$2n\pi - \frac{\pi}{4} \le A \le 2n\pi + \frac{3\pi}{4}$$

otherwise

So, $\sqrt{1+\sin A} = -\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$ is true when $\frac{3\pi}{4} < \frac{A}{2} < \frac{5\pi}{4} \Rightarrow \frac{3\pi}{2} < A < \frac{5\pi}{2}$

54. In triangle ABC, if

$$\frac{\sin^2 A + \sin^2 B + \sin^2 C}{\cos^2 A + \cos^2 B + \cos^2 C} = 2,$$

then the triangle is

(a) right-angled (b) equilateral

(c) isosceles (d) obtuse-angled

(a) We have,

$$\frac{\sin^2 A + \sin^2 B + \sin^2 C}{\cos^2 A + \cos^2 B + \cos^2 C} = 2$$

$$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2\cos^2 A + 2\cos^2 B + 2\cos^2 C$$

$$\Rightarrow 1 - \cos^2 A + 1 - \cos^2 B + 1 - \cos^2 C = 2\cos^2 A + 2\cos^2 B + 2\cos^2 C$$

$$\Rightarrow \quad 3 = 3\cos^2 A + 3\cos^2 B + 3\cos^2 C$$

$$\Rightarrow \quad \cos^2 A + \cos^2 B + \cos^2 C = 1$$

$$\Rightarrow \frac{1+\cos 2A}{2} + \frac{1+\cos 2B}{2} + \frac{1+\cos 2C}{2} = 1$$

$$\Rightarrow \quad 3 + \cos 2A + \cos 2B + \cos 2C = 2$$

$$\Rightarrow \quad \cos 2A + \cos 2B + \cos 2C = -1$$

$$\Rightarrow \quad \cos 2A + \cos 2B = -(1 + \cos 2C)$$

$$\Rightarrow 2\cos(A+B)\cos(A-B) = -2\cos^2 C$$

$$\Rightarrow 2\cos(180° - C)\cos(A-B) = -2\cos^2 C$$

$$\Rightarrow \quad -\cos C\cos(A-B) = -\cos^2 C$$

$$\Rightarrow \quad \cos(A-B) = \cos C$$

$$\Rightarrow \quad A - B = C$$

$$\Rightarrow \quad A = B + C$$

Again, $A + B + C = \pi$

$$\Rightarrow \quad A + A = \pi$$

$$\Rightarrow \quad 2A = \pi$$

$$\Rightarrow \quad A = \frac{\pi}{2}$$

$\therefore \Delta ABC$ is a right-angled triangle.

55. Match List-I with List-II and select the correct answer using the code given below the lists

List-I (Function)		List-II (Maximum value)
A. $\sin x + \cos x$	1.	$\sqrt{10}$
B. $3\sin x + 4\cos x$	2.	$\sqrt{2}$
C. $2\sin x + \cos x$	3.	5
D. $\sin x + 3\cos x$	4.	$\sqrt{5}$

Code

	A	B	C	D		A	B	C	D
(a)	2	3	1	4	(b)	2	3	4	1
(c)	3	2	1	4	(d)	3	2	4	1

(b) If $f(x) = a\sin x + b\cos x$

$$\therefore \quad -\sqrt{a^2+b^2} \le f(x) \le \sqrt{a^2+b^2}$$

$$\therefore \quad -\sqrt{1^2+1^2} \le \sin x + \cos x \le \sqrt{1^2+1^2}$$

$$\Rightarrow \quad -\sqrt{2} \le \sin x + \cos x \le \sqrt{2}$$

$$-\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2+4^2}$$

$$\Rightarrow \quad -5 \le 3\sin x + 4\cos x \le 5$$

$$-\sqrt{2^2+1^2} \le 2\sin x + \cos x \le \sqrt{2^2+1^2}$$

$$\Rightarrow -\sqrt{5} \le 2\sin x + \cos x \le \sqrt{5}$$

$$-\sqrt{1^2+3^2} \le \sin x + 3\cos x \le \sqrt{1^2+3^2}$$

$$\Rightarrow -\sqrt{10} \le \sin x + 3\cos x \le \sqrt{10}$$

2017 (I)

56. If $\sin A = \frac{3}{5}$, where $450° < A < 540°$, then $\cos\frac{A}{2}$ is equal to

(a) $\frac{1}{\sqrt{10}}$ (b) $-\sqrt{\frac{3}{10}}$

(c) $\frac{\sqrt{3}}{\sqrt{10}}$ (d) None of these

(a) $\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - (3/5)^2}$

$$= \sqrt{1 - 9/25}$$

$$= \sqrt{16/25} = \pm\, 4/5$$

Since $450° < A < 540°$

$$\therefore \quad \cos A = -4/5$$

Now, $\cos A = 2\cos^2 A/2 - 1$

$$\Rightarrow \quad \frac{-4}{5} = 2\cos^2 A/2 - 1$$

$$\Rightarrow \quad \frac{-4}{5} + 1 = 2\cos^2\frac{A}{2}$$

$$\Rightarrow \quad \frac{1}{5} = 2\cos^2 A/2$$

$$\Rightarrow \quad \cos A/2 = \frac{1}{\sqrt{10}}$$

57. What is $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°}$ equal to?

(a) 0 (b) 1 (c) 2 (d) 4

(d) $\frac{1}{\sin10^\circ}-\frac{\sqrt{3}}{\cos10^\circ}$

$=\frac{\cos10^\circ-\sqrt{3}\sin10^\circ}{\sin10^\circ\cos10^\circ}$

$=\frac{2\left(\frac{1}{2}\cos10^\circ-\frac{\sqrt{3}}{2}\sin10^\circ\right)}{\frac{1}{2}\times2\sin10^\circ\cos10^\circ}$

$=\frac{2(\sin30^\circ\cos10^\circ-\cos30^\circ\sin10^\circ)}{\frac{1}{2}\times\sin20^\circ}$

$=\frac{4\cdot\sin(30^\circ-10^\circ)}{\sin20^\circ}=\frac{4\sin20^\circ}{\sin20^\circ}=4$

58. The maximum value of $\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$ in the interval $\left(0,\frac{\pi}{2}\right)$ is attained at

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(a) $\sin(x+\pi/6)+\cos(x+\pi/6)$

$=\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin(x+\pi/6)+\frac{1}{\sqrt{2}}\cos(x+\pi/6)\right]$

$=\sqrt{2}\left[\sin\frac{\pi}{4}\sin(x+\pi/6)+\cos\pi/4\cos(x+\pi/6)\right]$

$=\sqrt{2}[\cos(x+\pi/6-\pi/4)]$

$=\sqrt{2}\cos(x-\pi/12)$

for maximum value, $x=\pi/12$

59. If $K=\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, then what is the value of K?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(c) $K=\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$

$=\sin\left(\frac{180^\circ}{18}\right)\sin\left(\frac{5\times180^\circ}{18}\right)\sin\left(\frac{7\times180^\circ}{18}\right)$ $[\because \pi^c=180^\circ]$

$=\sin10^\circ\cdot\sin50^\circ\cdot\sin70^\circ$

$=\sin10^\circ\cdot\sin50^\circ\cdot\cos20^\circ$

$=\frac{1}{2}[(2\sin10^\circ\sin50^\circ)]\cdot\cos20^\circ$

$=\frac{1}{2}[\cos(10^\circ-50^\circ)-\cos(10^\circ+50^\circ)]\cdot\cos20^\circ$

$=\frac{1}{2}\left[\cos40^\circ-\frac{1}{2}\right]\cdot\cos20^\circ$

$=\frac{1}{2}\left[\frac{1}{2}(2\cos40^\circ\cos20^\circ)-\frac{1}{2}\cos20^\circ\right]$

$=\frac{1}{2}\left[\frac{1}{2}\{\cos(40^\circ+20)+\cos(40^\circ-20^\circ)\}-\frac{1}{2}\cos20^\circ\right]$

$=\frac{1}{2}\left[\frac{1}{2}\{\cos60^\circ+\cos20^\circ\}-\frac{1}{2}\cos20^\circ\right]$

$=\frac{1}{2}\left[\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\cos20^\circ-\frac{1}{2}\cos20^\circ\right]$

$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

60. The expression $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}$ is equal to

(a) $\tan\left(\frac{\alpha+\beta}{2}\right)$ (b) $\cot\left(\frac{\alpha+\beta}{2}\right)$

(c) $\sin\left(\frac{\alpha+\beta}{2}\right)$ (d) $\cos\left(\frac{\alpha+\beta}{2}\right)$

(a) $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}$

$=\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$

$=\tan\left(\frac{\alpha+\beta}{2}\right)$

61. If $\sin\theta=3\sin(\theta+2\alpha)$, then the value of $\tan(\theta+\alpha)+2\tan\alpha$ is equal to

(a) –1 (b) 0
(c) 1 (d) 2

(b) $\sin\theta=3\sin(\theta+2\alpha)$

$\Rightarrow \frac{\sin\theta}{\sin(\theta+2\alpha)}=\frac{3}{1}$

On applying componendo and dividendo rule, we get

$\frac{\sin\theta+\sin(\theta+2\alpha)}{\sin\theta-\sin(\theta+2\alpha)}=\frac{3+1}{3-1}$

$\Rightarrow \frac{2\sin\left(\frac{(\theta+\theta+2\alpha)}{2}\right)\cos\left(\frac{\theta-\theta-2\alpha}{2}\right)}{2\cos\left(\frac{\theta+\theta+2\alpha}{2}\right)\sin\left(\frac{\theta-\theta-2\alpha}{2}\right)}=\frac{4}{2}$

$\Rightarrow -\frac{\sin(\theta+\alpha)\cos\alpha}{\cos(\theta+\alpha)\sin\alpha}=2$

$\Rightarrow \tan(\theta+\alpha)\cdot\cot\alpha=-2$

$\Rightarrow \tan(\theta+\alpha)=-2\tan\alpha$

$\Rightarrow \tan(\theta+\alpha)+2\tan\alpha=0$

62. What is the value of $\tan18^\circ$?

(a) $\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$ (b) $\frac{\sqrt{5}-1}{\sqrt{10+\sqrt{5}}}$

(c) $\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$ (d) $\frac{\sqrt{10+\sqrt{5}}}{\sqrt{5}-1}$

(a) $\tan18^\circ=\frac{\sin18^\circ}{\cos18^\circ}$

$=\frac{\sqrt{5}-1}{4}\div\frac{\sqrt{10+2\sqrt{5}}}{4}=\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$

63. If $\tan(\alpha+\beta)=2$ and $\tan(\alpha-\beta)=1$, then $\tan(2\alpha)$ is equal to

(a) –3 (b) –2 (c) $-\frac{1}{3}$ (d) 1

(a) $\tan(2\alpha)=\tan(\alpha+\beta+\alpha-\beta)$

$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}$

$=\frac{2+1}{1-2\times1}=\frac{3}{-1}=-3$

64. Consider the following for triangle ABC

1. $\sin\left(\frac{B+C}{2}\right)=\cos\left(\frac{A}{2}\right)$
2. $\tan\left(\frac{B+C}{2}\right)=\cot\left(\frac{A}{2}\right)$
3. $\sin(B+C)=\cos A$
4. $\tan(B+C)=-\cot A$

Which of the above are correct?

(a) 1 and 3 (b) 1 and 2
(c) 1 and 4 (d) 2 and 3

(b) In ΔABC, we have,

$\angle A+\angle B+\angle C=180^\circ$

$\Rightarrow \frac{B+C}{2}=\frac{180^\circ}{2}-\frac{A}{2}=90^\circ-\frac{A}{2}$

1. $\sin\left(\frac{B+C}{2}\right)=\sin\left(90^\circ-\frac{A}{2}\right)=\cos\frac{A}{2}$

So, 1 is correct.

2. $\tan\left(\frac{B+C}{2}\right)=\tan\left(90^\circ-\frac{A}{2}\right)=\cot(A/2)$

So, 2 is correct.

3. $B+C=180^\circ-A^\circ$

$\Rightarrow \sin(B+C)=\sin(180^\circ-A)$

$\Rightarrow \sin(B+C)=\sin A$

So, 3 is not correct.

4. $B+C=180^\circ-A$

$\Rightarrow \tan(B+C)=\tan(180^\circ-A)$

$\Rightarrow \tan(B+C)=-\tan A$

So, 4 is not correct.

65. If $\sec\theta-\operatorname{cosec}\theta=\frac{4}{3}$, then what is $(\sin\theta-\cos\theta)$ equal to?

(a) Only –2
(b) Only $\frac{1}{2}$
(c) Both –2 and $\frac{1}{2}$
(d) Neither $\frac{1}{2}$ nor –2

(b) $\sec\theta-\operatorname{cosec}\theta=\frac{4}{3}$

$\Rightarrow \frac{1}{\cos\theta}-\frac{1}{\sin\theta}=\frac{4}{3}$

$\Rightarrow \frac{\sin\theta-\cos\theta}{\sin\theta\cos\theta}=\frac{4}{3}$ … (i)

On squaring both sides, we get

$$\Rightarrow \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\sin^2\theta \cdot \cos^2\theta} = \frac{16}{9}$$

$$\Rightarrow 9(1 - 2\sin\theta\cos\theta) = 16\sin^2\theta\cos^2\theta$$

$$\Rightarrow 9 - 18\sin\theta\cos\theta = 16\sin^2\theta\cos^2\theta$$

$$\Rightarrow 16\sin^2\theta\cos^2\theta + 18\sin\theta\cos\theta - 9 = 0$$

Let $\sin\theta\cos\theta = x$, then we get

$$\Rightarrow 16x^2 + 18x - 9 = 0$$

$$\Rightarrow x = \frac{-18 \pm \sqrt{(18)^2 - 4\times16\times(-9)}}{2\times16}$$

$$\Rightarrow x = \frac{-18 \pm \sqrt{324 + 576}}{32}$$

$$\Rightarrow x = \frac{-18 \pm \sqrt{900}}{32}$$

$$\Rightarrow x = \frac{-18 \pm 30}{32}$$

On using + ve sign, we get

$$x = \frac{-18 + 30}{32} = \frac{12}{32} = \frac{3}{8}$$

If $x = 3/8$

$\Rightarrow \sin\theta\cos\theta = 3/8$

Then, $\sin\theta - \cos\theta = \frac{4}{3} \times \frac{3}{8} = \frac{1}{2}$ [from Eq. (i)]

$$x \neq \frac{-3}{2}$$

[$\because \sin\theta\cos\theta$ lie between $\frac{-1}{2}$ to $\frac{1}{2}$]

66. What is the maximum value of the function $f(x) = 4\sin^2 x + 1$?

(a) 5 (b) 3 (c) 2 (d) 1

(a) Given,

$$f(x) = 4\sin^2 x + 1$$

We know,

$$0 \le \sin^2 x \le 1$$

$$\therefore \quad 0 \le 4\sin^2 x \le 4$$

$$1 \le 4\sin^2 x + 1 \le 4 + 1$$

$\therefore$ Maximum value of $f(x) = 5$

2016 (II)

67. $\sin A + 2\sin 2A + \sin 3A$ is equal to which of the following?

I. $4\sin 2A \cos^2\left(\frac{A}{2}\right)$

II. $2\sin 2A\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)^2$

III. $8\sin A\cos A\cos^2\left(\frac{A}{2}\right)$

Select the correct answer using the code given below.

(a) I and II (b) II and III
(c) I and III (d) I, II and III

(c) We have,

$$\sin A + 2\sin 2A + \sin 3A$$

$$= \sin A + \sin 3A + 2\sin 2A$$

$$= 2\sin 2A\cos A + 2\sin 2A$$

$$\left[\because \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right]$$

$$= 2\sin 2A\,(1 + \cos A)$$

$$= 2\sin 2A \cdot 2\cos^2\frac{A}{2}$$

$$= 4\sin 2A\cos^2\frac{A}{2} \quad \text{[part I]}$$

$$= 4 \cdot 2\sin A\cos A\cos^2\frac{A}{2}$$

$$= 8\sin A\cos A\cos^2\frac{A}{2} \quad \text{[part III]}$$

68. If $x = \sin 70° \cdot \sin 50°$ and $y = \cos 60° \cdot \cos 80°$, then what is xy equal to?

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

(a) We have,

$$x = \sin 70°\sin 50°$$

$$y = \cos 60°\cos 80°$$

$$\therefore \quad xy = \sin 70°\sin 50°\cos 60°\cos 80°$$

$$= \cos 20°\cos 40°\cos 60°\cos 80° \quad [\because \sin(90° - \theta) = \cos\theta]$$

$$= \cos 60°\cos 20°\cos(60° - 20°)\cos(60° + 20°)$$

$$= \frac{1}{2} \times \frac{\cos 3\times 20°}{4} \quad [\because \cos x\cos(60° - x)\cos(60° + x) = \frac{\cos 3x}{4}]$$

$$= \frac{1}{8}\cos 60° = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$$

69. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 = 4$, then what is the value of $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4$?

(a) 0 (b) 1 (c) 2 (d) 4

(a) We have,

$$\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 = 4$$

Since, maximum value of $\sin\theta = 1$

$$\therefore \quad \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = \sin\theta_4 = 1$$

$$\Rightarrow \quad \theta_1 = \theta_2 = \theta_3 = \theta_4 = 90°$$

$$\therefore \quad \cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 = 0 \quad [\because \cos 90° = 0]$$

70. What is the value of $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$?

(a) $\frac{1}{2}$ (b) $\frac{1}{2} + \frac{1}{2\sqrt{2}}$
(c) $\frac{1}{2} - \frac{1}{2\sqrt{2}}$ (d) $\frac{1}{8}$

(d) $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$

$$= \left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left\{1 + \cos\left(\pi - \frac{3\pi}{8}\right)\right\}\left\{1 + \cos\left(\pi - \frac{\pi}{8}\right)\right\}$$

$$= \left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{\pi}{8}\right)$$

$$= \left(1 - \cos^2\frac{\pi}{8}\right)\left(1 - \cos^2\frac{3\pi}{8}\right)$$

$$= \sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8} = \left(\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right)^2$$

$$= \left\{\sin\frac{\pi}{8}\sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right\}^2 = \left(\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)^2$$

$$= \frac{1}{4}\left(\sin\frac{\pi}{4}\right)^2 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

71. If $x\cos\theta + y\sin\theta = z$, then what is the value of $(x\sin\theta - y\cos\theta)^2$?

(a) $x^2 + y^2 - z^2$ (b) $x^2 - y^2 - z^2$
(c) $x^2 - y^2 + z^2$ (d) $x^2 + y^2 + z^2$

(a) We have,

$$x\cos\theta + y\sin\theta = z$$

On squaring both sides, we get

$$x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta = z^2$$

$$\Rightarrow x^2(1 - \sin^2\theta) + y^2(1 - \cos^2\theta) + 2xy\sin\theta\cos\theta = z^2$$

$$\Rightarrow \quad x^2 + y^2 - z^2 = x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta$$

$$\Rightarrow \quad x^2 + y^2 - z^2 = (x\sin\theta - y\cos\theta)^2$$

$$\therefore (x\sin\theta - y\cos\theta)^2 = x^2 + y^2 - z^2$$

72. If $\sin 18° = \frac{\sqrt{5} - 1}{4}$, then what is the value of $\sin 81°$?

(a) $\frac{\sqrt{3 + \sqrt{5}} + \sqrt{5 - \sqrt{5}}}{4}$

(b) $\frac{\sqrt{3 + \sqrt{5}} + \sqrt{5 + \sqrt{5}}}{4}$

(c) $\frac{\sqrt{3 - \sqrt{5}} + \sqrt{5 - \sqrt{5}}}{4}$

(d) $\frac{\sqrt{3 + \sqrt{5}} - \sqrt{5 - \sqrt{5}}}{4}$

(a) Given, $\sin 18° = \frac{\sqrt{5} - 1}{4}$

Now, $(\sin 18°)^2 = 1 - \cos^2 18°$

$$\Rightarrow \quad \left(\frac{\sqrt{5} - 1}{4}\right)^2 = 1 - \cos^2 18°$$

$$\Rightarrow \quad \frac{5+1-2\sqrt{5}}{16} = 1-\cos^2 18°$$

$$\Rightarrow \quad \frac{6-2\sqrt{5}}{16} = 1-\cos^2 18°$$

$$\Rightarrow \quad \cos^2 18° = 1-\left(\frac{6-2\sqrt{5}}{16}\right)$$

$$\Rightarrow \quad \cos^2 18° = \frac{10+2\sqrt{5}}{16}$$

$$\Rightarrow \quad \cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

Since, $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$ $[\because \theta = 18°]$

$$\therefore \quad \cos 9° = \sqrt{\frac{1+\frac{\sqrt{10+2\sqrt{5}}}{4}}{2}}$$

$$\Rightarrow \cos(90° - 81°) = \sqrt{\frac{(4+\sqrt{10+2\sqrt{5}})}{8}}$$

$$\Rightarrow \quad \sin 81° = \sqrt{\frac{8+2\sqrt{10+2\sqrt{5}}}{16}}$$

$$= \sqrt{\frac{3+\sqrt{5}+5-\sqrt{5}+2}{16} + \frac{\sqrt{3+\sqrt{5}}\cdot\sqrt{5-\sqrt{5}}}{16}}$$

$$= \frac{\sqrt{(3+\sqrt{5})}+\sqrt{5-\sqrt{5}}}{4}$$

73. What is $\dfrac{1-\tan 2°\cot 62°}{\tan 152° - \cot 88°}$ equal to?

(a) $\sqrt{3}$ (b) $-\sqrt{3}$
(c) $\sqrt{2}-1$ (d) $1-\sqrt{2}$

(*b*) We have, $\dfrac{1-\tan 2°\cot 62°}{\tan 152° - \cot 88°}$

$$= \frac{1-\tan(90°-88°)\cot 62°}{\tan(180°-28°)-\cot 88°}$$

$$= \frac{1-\cot 88°\cot 62°}{-\tan 28° - \cot 88°}$$

$$= \frac{1-\cot 88°\cot 62°}{-\tan(90°-62°)-\cot 88°}$$

$$= \frac{1-\cot 88°\cot 62°}{-\cot 62°-\cot 88°} = \frac{\cot 62°\cot 88°-1}{\cot 62°+\cot 88°}$$

$$= \cot(62°+88°)$$

$$= \cot 150° = -\sqrt{3}$$

2016 (I)

Directions (Q. Nos. 74-76) *Consider the function* $f(\theta) = 4(\sin^2\theta + \cos^4\theta)$

74. What is the maximum value of the function $f(\theta)$?

(a) 1 (b) 2
(c) 3 (d) 4

75. What is the minimum value of the function $f(\theta)$?

(a) 0 (b) 1 (c) 2 (d) 3

76. Consider the following statements

1. $f(\theta) = 2$ has no solution.
2. $f(\theta) = \frac{7}{2}$ has a solution.

Which of the above statement(s) is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

Solutions (Q. Nos. 74-76)

$$\because \quad f(\theta) = 4(\sin^2\theta + \cos^4\theta)$$

Also, $1 - 2\sin^2\theta = \cos 2\theta$

$$\Rightarrow \quad \sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad ...(i)$$

and $2\cos^2\theta - 1 = \cos 2\theta$

$$\Rightarrow \quad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\Rightarrow \quad \cos^4\theta = \left(\frac{1+\cos 2\theta}{2}\right)^2 \qquad ...(ii)$$

$$\therefore \quad f(\theta) = 4(\sin^2\theta + \cos^4\theta)$$

$$= 4\left\{\frac{1-\cos 2\theta}{2} + \left(\frac{1+\cos 2\theta}{2}\right)^2\right\}$$

$$= 4\left\{\frac{1-\cos 2\theta}{2} + \frac{1+\cos^2 2\theta + 2\cos 2\theta}{4}\right\}$$

$$= 4\left\{\frac{2-2\cos 2\theta+1+\cos^2 2\theta+2\cos 2\theta}{4}\right\}$$

$$= 4\left\{\frac{3+\cos^2 2\theta}{4}\right\} = 3+\cos^2 2\theta$$

74. (*d*) $\therefore f(\theta)_{max} = 3 + 1 = 4$

75. (*d*) $f(\theta)_{min} = 3 + 0 = 3$

76. (*c*) Given, $f(\theta) = 4(\sin^2\theta + \cos^4\theta)$

$$= 4[\sin^2\theta + (\cos^2\theta)^2]$$
$$= 4[\sin^2\theta + (1-\sin^2\theta)^2]$$
$$= 4[\sin^2\theta + 1 + \sin^4\theta - 2\sin^2\theta]$$
$$= 4[\sin^4\theta - \sin^2\theta + 1]$$

1. When $f(\theta) = 2$

$$\Rightarrow \quad 4(\sin^4\theta - \sin^2\theta + 1) = 2$$

$$\Rightarrow \quad \sin^4\theta - \sin^2\theta + 1 = \frac{1}{2}$$

$$\Rightarrow \quad \sin^4\theta - \sin^2\theta + \frac{1}{2} = 0$$

$$\Rightarrow \quad 2\sin^4\theta - 2\sin^2\theta + 1 = 0$$

Let $\sin^2\theta = x$, then above equation becomes $2x^2 - 2x + 1 = 0$

Now, $b^2 - 4ac = (-2)^2 - 4\times 2\times 1$

$$= 4 - 8 = -4 < 0$$

$\Rightarrow f(\theta) = 2$ has no solution.

2. When $f(\theta) = \frac{7}{2}$

$$\Rightarrow \quad 4(\sin^4\theta - \sin^2\theta + 1) = \frac{7}{2}$$

$$\Rightarrow \quad \sin^4\theta - \sin^2\theta + 1 = \frac{7}{8}$$

$$\Rightarrow \quad \sin^4\theta - \sin^2\theta + \frac{1}{8} = 0$$

$$\Rightarrow \quad 8\sin^4\theta - 8\sin^2\theta + 1 = 0$$

Let $\sin^2\theta = x$, then $8x^2 - 8x + 1 = 0$

Now, $b^2 - 4ac = (-8)^2 - 4\times 8\times 1$

$$= 64 - 32 = 32 > 0$$

$$\Rightarrow x = \frac{-(-8)\pm\sqrt{32}}{2\times 8}$$

$$= \frac{8\pm 4\sqrt{2}}{16} = \frac{2\pm\sqrt{2}}{4}$$

$$\Rightarrow \sin^2\theta = \frac{2\pm\sqrt{2}}{4} \Rightarrow \sin\theta = \frac{\sqrt{2\pm\sqrt{2}}}{2}$$

Using – ve sign,

$$\sin\theta = \frac{\sqrt{2-\sqrt{2}}}{2} = \sin 22\frac{1}{2}°$$

$$\Rightarrow \quad \theta = 22\frac{1}{2}°$$

Hence, $f(\theta) = \frac{7}{2}$ has a solution.

Using +ve sign, $\sin\theta = \frac{\sqrt{2+\sqrt{2}}}{2} < 1$, so it gives a solution.

Directions (Q. Nos. 77 and 78)

Consider a ΔABC *in which* $\cos A + \cos B + \cos C = \sqrt{3}\sin\frac{\pi}{3}$

77. What is the value of $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(*c*) Given, in ΔABC,

$$\cos A + \cos B + \cos C = \sqrt{3}\sin\frac{\pi}{3}$$

$$\Rightarrow \quad \cos A + \cos B + \cos C = \sqrt{3}\times\frac{\sqrt{3}}{2} = \frac{3}{2}$$

We know that,

$$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\Rightarrow \quad 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{3}{2}$$

$$\Rightarrow \quad 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\Rightarrow \quad \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{1}{8}$$

78. What is the value of $\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{16}$ (d) None of these

(d) $\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)$

$= \cos\left(90° - \frac{C}{2}\right)\cos\left(90° - \frac{A}{2}\right)\cos\left(90° - \frac{B}{2}\right)$

[$\because A + B + C = 180°$]

$= \sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$

$= \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{1}{8}$ [from Q. No. 65]

Directions (Q. Nos. 79 and 80) *Given that* $\tan\alpha$ *and* $\tan\beta$ *are the roots of the equation* $x^2 + bx + c = 0$ *with* $b \neq 0$.

79. What is $\tan(\alpha + \beta)$ equal to?

(a) $b(c-1)$ (b) $c(b-1)$

(c) $c(b-1)^{-1}$ (d) $b(c-1)^{-1}$

(d) Given, $x^2 + bx + c = 0, b \neq 0$

$\tan\alpha + \tan\beta = -b$

and $\tan\alpha\tan\beta = c$

Now, $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{-b}{1-c} = b(c-1)^{-1}$

80. What is $\sin(\alpha+\beta)\sec\alpha\sec\beta$ equal to?

(a) b (b) $-b$ (c) c (d) $-c$

(b) $\because$ We know that,

$\tan\alpha + \tan\beta = -b$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = -b$

$\Rightarrow \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta} = -b$

$\Rightarrow \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = -b$

$\Rightarrow \sin(\alpha+\beta)\sec\alpha\sec\beta = -b$

Directions (Q. Nos. 81-82) *Consider the equation* $k\sin x + \cos 2x = 2k - 7$

81. If the equation possesses solution, then what is the minimum value of k?

(a) 1 (b) 2

(c) 4 (d) 6

82. If the equation possesses solution, then what is the maximum value of k?

(a) 1 (b) 2 (c) 4 (d) 6

(Solutions Q. Nos. 81 and 82) Given, equation is $k\sin x + \cos 2x = 2k - 7$

$\Rightarrow k\sin x + 1 - 2\sin^2 x = 2k - 7$

$\Rightarrow -2\sin^2 x + k\sin x + 8 - 2k = 0$

$\Rightarrow 2\sin^2 x - k\sin x + 2k - 8 = 0$

$\therefore \sin x = \frac{k \pm \sqrt{k^2 - 16k + 64}}{4}$

$\Rightarrow \sin x = \frac{k \pm \sqrt{(k-8)^2}}{4}$

$\Rightarrow \sin x = \frac{k \pm (k-8)}{4}$

$\Rightarrow \sin x = \frac{2k-8}{4}$ and $\sin x = 2$

As, $-1 \le \sin x \le 1$

$\Rightarrow -1 \le \frac{2k-8}{4} \le 1 \Rightarrow -4 \le 2k - 8 \le 4$

$\Rightarrow 4 \le 2k \le 12 \Rightarrow 2 \le k \le 6$

$\therefore$ Minimum value of $k = 2$

and maximum value of $k = 6$

81. *(b)* The minimum value of k is 2.

82. *(d)* The maximum value of k is 6.

83. Consider the following statements

1. If ABC is an equilateral triangle, then $3\tan(A+B)\tan C = 1$.
2. If ABC is a triangle in which $A = 78°, B = 66°$, then $\tan\left(\frac{A}{2} + C\right) < \tan A$
3. If ABC is any triangle, then $\tan\left(\frac{A+B}{2}\right)\sin\left(\frac{C}{2}\right) < \cos\left(\frac{C}{2}\right)$

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2

(c) 1 and 2 (d) 2 and 3

(b) 1. Since, ABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60°$

Now, consider

$3\tan(A+B)\tan C = 3\tan 120°\tan 60°$

$= 3\tan(180° - 60°)\tan 60°$

$= (-3)\tan 60°\tan 60°$

$= (-3)\cdot\sqrt{3}\cdot\sqrt{3} = -9$

2. Consider,

$\tan\left(\frac{A}{2} + C\right) = \tan\left(\frac{78°}{2} + 36°\right)$

[$\because \angle C = 180° - \angle A - \angle B$

$= 180° - 78° - 66° = 36°$]

$= \tan 75° < \tan 78°$

[$\because$ value of $\tan\theta$ increases, as θ varying from 0° to 90°]

$< \tan A$

Hence, $\tan\left(\frac{A}{2} + C\right) < \tan A$

3. Consider, $\tan\left(\frac{A+B}{2}\right)\sin\frac{C}{2}$

$= \tan\left(\frac{180° - C}{2}\right)\sin\frac{C}{2}$

$= \tan\left(90° - \frac{C}{2}\right)\sin\frac{C}{2}$

$= \cot\frac{C}{2}\sin\frac{C}{2}$

$= \frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}\cdot\sin\frac{C}{2}$

$= \cos\frac{C}{2} \not< \cos\frac{C}{2}$

84. If $A = (\cos 12° - \cos 36°)(\sin 96° + \sin 24°)$ and $B = (\sin 60° - \sin 12°)(\cos 48° - \cos 72°)$, then what is $\frac{A}{B}$ equal to?

(a) –1 (b) 0

(c) 1 (d) 2

(c) We have, $A = (\cos 12° - \cos 36°)(\sin 96° + \sin 24°)$

and $B = (\sin 60° - \sin 12°)(\cos 48° - \cos 72°)$

Now, consider

$A = (\cos 12° - \cos 36°)(\sin 96° + \sin 24°)$

$= -2\sin 24°\sin(-12°)(2\sin 60°\cos 36°)$

$= 4\sin 24°\sin 12°\sin 60°\cos 36°$

and $B = 2\cos 36°\sin 24°(-2\sin 60°\sin(-12°))$

$= 4\sin 24°\sin 12°\sin 60°\cos 36°$

Clearly, $\frac{A}{B} = 1$

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85. The value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \sin^2 20° + \ldots + \sin^2 90°$ is

(a) 7 (b) 8 (c) 9 (d) $\frac{19}{2}$

(d) $\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots + \sin^2 90°$

$= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + \ldots + \sin^2 45° + \ldots + \sin^2 90°$

$= \sin^2 5° + \sin^2(90°-5°) + \sin^2 10°$
$+ \sin^2(90°-10°) + \ldots + \sin^2 90°$
$= (\sin^2 5° + \cos^2 5°) + (\sin^2 10° + \cos^2 10°)$
$+ \ldots + \sin^2 90°$
$= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$
$+ \left(\frac{1}{\sqrt{2}}\right)^2 + 1$
$= 9 + \frac{1}{2} = \frac{19}{2}$

86. On simplifying $\frac{\sin^3 A + \sin 3A}{\sin A} + \frac{\cos^3 A - \cos 3A}{\cos A}$ we get

(a) $\sin 3A$ (b) $\cos 3A$
(c) $\sin A + \cos A$ (d) 3

(d) $\frac{\sin^3 A + \sin 3A}{\sin A} + \frac{\cos^3 A - \cos 3A}{\cos A}$

$= \frac{\sin^3 A + (3\sin A - 4\sin^3 A)}{\sin A}$
$+ \frac{\cos^3 A - (4\cos^3 A - 3\cos A)}{\cos A}$
$[\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta, \cos 3\theta$
$= 4\cos^3\theta - 3\cos\theta]$
$= (-3\sin^2 A + 3) + (-3\cos^2 A + 3)$
$= 6 - 3(\sin^2 A + \cos^2 A) = 6 - 3(1) = 3$

87. If $\sin x + \sin y = a$ and $\cos x + \cos y = b$, then $\tan^2\left(\frac{x+y}{2}\right) + \tan^2\left(\frac{x-y}{2}\right)$ is equal to

(a) $\frac{a^4 + b^4 + 4b^2}{a^2b^2 + b^4}$ (b) $\frac{a^4 - b^4 + 4b^2}{a^2b^2 + b^4}$
(c) $\frac{a^4 - b^4 + 4a^2}{a^2b^2 + a^4}$ (d) None of these

(b) We have,

$\sin x + \sin y = a$ ...(i)
$\cos x + \cos y = b$ ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

$1 + 1 + 2(\sin x \cdot \sin y + \cos x \cdot \cos y) = a^2 + b^2$
$\Rightarrow 2 + 2\cos(x - y) = a^2 + b^2$
$\Rightarrow \cos(x - y) = \frac{a^2 + b^2 - 2}{2}$ ...(iii)

Now, on dividing Eq. (i) by Eq. (ii), we get

$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{a}{b}$
$\Rightarrow \tan\frac{x+y}{2} = \frac{a}{b}$ ...(iv)

Now, $\tan^2\left(\frac{x+y}{2}\right) + \tan^2\left(\frac{x-y}{2}\right)$
$= \frac{a^2}{b^2} + \frac{1 - \cos(x-y)}{1 + \cos(x-y)}$
$= \frac{a^2}{b^2} + \frac{1 - \frac{a^2 + b^2 - 2}{2}}{1 + \frac{a^2 + b^2 - 2}{2}}$
$= \frac{a^2}{b^2} + \frac{4 - a^2 - b^2}{a^2 + b^2}$
$= \frac{a^4 + a^2b^2 + 4b^2 - a^2b^2 - b^4}{a^2b^2 + b^4}$
$= \frac{a^4 - b^4 + 4b^2}{a^2b^2 + b^4}$

88. If $p = \tan\left(-\frac{11\pi}{6}\right), q = \tan\left(\frac{21\pi}{4}\right)$ and $r = \cot\left(\frac{283\pi}{6}\right)$, then which of the following is/are correct?

1. The value of $p \times r$ is 2.
2. p, q and r are in GP.

Select the correct answer using the code given below.

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(b) $p = \tan\left(-\frac{11\pi}{6}\right) = -\tan\frac{11\pi}{6}$
$= -\tan\left(2\pi - \frac{\pi}{6}\right)$
$= \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$
$q = \tan\frac{21\pi}{4} = \tan\left(5\pi + \frac{\pi}{4}\right)$
$= \tan\frac{\pi}{4} = 1$
$r = \cot\left(\frac{283\pi}{6}\right) = \cot\left(47\pi + \frac{\pi}{6}\right)$
$= \cot\frac{\pi}{6} = \sqrt{3}$

1. $p \times r = \frac{1}{\sqrt{3}} \times \sqrt{3} = 1$ which is incorrect.
2. p, q and r are in GP.

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Directions (Q. Nos. 89 and 90) *Let* α *be the root of the equation* $25\cos^2\theta + 5\cos\theta - 12 = 0$, *where* $\frac{\pi}{2} < \alpha < \pi$.

89. What is $\tan\alpha$ equal to?

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$
(c) $-\frac{4}{3}$ (d) $-\frac{4}{5}$

90. What is $\sin 2\alpha$ equal to?

(a) $\frac{24}{25}$ (b) $-\frac{24}{25}$ (c) $-\frac{5}{12}$ (d) $-\frac{21}{25}$

Solutions (Q. Nos. 89 and 90)

89. **(a)** $25\cos^2\theta + 5\cos\theta - 12 = 0$
$\Rightarrow 25\cos^2\theta + 20\cos\theta - 15\cos\theta - 12 = 0$
$\Rightarrow 5\cos\theta(5\cos\theta + 4) - 3(5\cos\theta + 4) = 0$
$\Rightarrow (5\cos\theta - 3)(5\cos\theta + 4) = 0$
$\Rightarrow \cos\theta = \frac{3}{5}$ or $\cos\theta = -\frac{4}{5}$
$\because \alpha \in \left(\frac{\pi}{2}, \pi\right)$

Since, α is a root of the given equation.

$\therefore \cos\alpha = \frac{3}{5}$ and $\cos\alpha = -\frac{4}{5}$
$\therefore$ We take $\cos\alpha = \frac{-4}{5}$
$\Rightarrow \tan\alpha = -\frac{3}{4}$

90. **(b)** From the above solution, we have
$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$
$= 2 \times \frac{3}{5} \times \left(-\frac{4}{5}\right) = -\frac{24}{25}$

91. $(1 - \sin A + \cos A)^2$ is equal to

(a) $2(1 - \cos A)(1 + \sin A)$
(b) $2(1 - \sin A)(1 + \cos A)$
(c) $2(1 - \cos A)(1 - \sin A)$
(d) None of the above

(b) $(1 - \sin A + \cos A)^2$
$= 1 + \sin^2 A + \cos^2 A - 2\sin A$
$- 2\sin A\cos A + 2\cos A$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2$
$+ 2ab + 2bc + 2ca]$
$= 2 - 2\sin A - \sin 2A + 2\cos A$
$= 2(1 + \cos A) - 2\sin A(1 + \cos A)$
$= 2(1 - \sin A)(1 + \cos A)$

92. What is $\frac{\cos\theta}{1 - \tan\theta} + \frac{\sin\theta}{1 - \cot\theta}$ equal to?

(a) $\sin\theta - \cos\theta$ (b) $\sin\theta + \cos\theta$
(c) $2\sin\theta$ (d) $2\cos\theta$

(b) $\frac{\cos\theta}{1 - \tan\theta} + \frac{\sin\theta}{1 - \cot\theta}$
$= \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}}$
$= \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}} + \frac{\sin\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}}$
$= \frac{\cos\theta \times \cos\theta}{\cos\theta - \sin\theta} + \frac{\sin\theta \times \sin\theta}{\sin\theta - \cos\theta}$
$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$

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93. What is $\sqrt{1+\sin 2\theta}$ equal to?

(a) $\cos\theta - \sin\theta$ (b) $\cos\theta + \sin\theta$
(c) $2\cos\theta + \sin\theta$ (d) $\cos\theta + 2\sin\theta$

(b) Consider, $\sqrt{1+\sin 2\theta}$

$= \sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}$

$= \sqrt{(\sin\theta + \cos\theta)^2} = \sin\theta + \cos\theta$

94. If $\cot A = 2$ and $\cot B = 3$, then what is the value of $\cot(A+B)$?

(a) $\frac{\pi}{6}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(d) We have, $\cot A = 2$ and $\cot B = 3$

Consider,

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} = \frac{6-1}{2+3} = \frac{5}{5} = 1$$

$$\Rightarrow \cot(A+B) = \cot\left(\frac{\pi}{4}\right) \Rightarrow A + B = \frac{\pi}{4}$$

95. What is $\sin^2 66\frac{1^\circ}{2} - \sin^2 23\frac{1^\circ}{2}$ equal to

(a) $\sin 47^\circ$ (b) $\cos 47^\circ$
(c) $2\sin 47^\circ$ (d) $2\cos 47^\circ$

(b) Consider, $\sin^2 66\frac{1^\circ}{2} - \sin^2 23\frac{1^\circ}{2}$

$$= \left[\sin\left(90^\circ - 23\frac{1^\circ}{2}\right)\right]^2 - \sin^2 23\frac{1^\circ}{2}$$

$$= \cos^2 23\frac{1^\circ}{2} - \sin^2 23\frac{1^\circ}{2} = \cos 2\left(23\frac{1^\circ}{2}\right)$$

$[\because \cos 2A = \cos^2 A - \sin^2 A]$

$$= \cos\left[2 \times \left(\frac{47}{2}\right)^\circ\right] = \cos 47^\circ$$

96. What is $\frac{\cos 7x - \cos 3x}{\sin 7x - 2\sin 5x + \sin 3x}$ equal to?

(a) $\tan x$ (b) $\cot x$
(c) $\tan 2x$ (d) $\cot 2x$

(b) $\frac{\cos 7x - \cos 3x}{\sin 7x - 2\sin 5x + \sin 3x}$

$$= \frac{-2\sin\frac{7x+3x}{2}\cdot\sin\frac{7x-3x}{2}}{2\sin\frac{7x+3x}{2}\cdot\cos\frac{7x-3x}{2} - 2\sin 5x}$$

$\left[\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\right.$

$\left.\text{and } \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right)\right]$

$$= \frac{-2\sin 5x\cdot\sin 2x}{2\sin 5x\cos 2x - 2\sin 5x}$$

$$= \frac{-2\sin 5x\cdot\sin 2x}{-2\sin 5x\,[1-\cos 2x]}$$

$$= \frac{\sin 2x}{1 - 1 + 2\sin^2 x}$$

$[\because \cos 2A = 1 - 2\sin^2 A]$

$$= \frac{2\sin x\cos x}{2\sin^2 x} = \cot x$$

97. In a ΔABC, if $\sin A - \cos B = \cos C$, then what is B equal to?

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(c) In a ΔABC, we have

$\sin A - \cos B = \cos C$

$\Rightarrow \sin A = \cos B + \cos C$

$$\Rightarrow 2\sin\frac{A}{2}\cdot\cos\frac{A}{2} = 2\cos\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)$$

$\left[\because \sin 2A = 2\sin A\cdot\cos A\right.$

$\left.\text{and } \cos B + \cos C = 2\cos\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)\right]$

$$\Rightarrow 2\sin\frac{A}{2}\cdot\cos\frac{A}{2} = 2\cos\left(90^\circ - \frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)$$

$\left[\because A + B + C = 180^\circ \Rightarrow \left(\frac{B+C}{2}\right) = 90^\circ - \frac{A}{2}\right]$

$$\Rightarrow 2\sin\frac{A}{2}\cdot\cos\frac{A}{2} = 2\sin\frac{A}{2}\cdot\cos\left(\frac{B-C}{2}\right)$$

$[\because \cos(90^\circ - \theta) = \sin\theta]$

$$\Rightarrow \cos\frac{A}{2} = \cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \frac{A}{2} = \frac{B-C}{2}$$

$\Rightarrow A + C = B$ ...(i)

Also, $A + C = 180^\circ - B$ ...(ii)

So, $180^\circ - B = B \Rightarrow 2B = 180^\circ$

$\therefore B = 90^\circ = \frac{\pi}{2}$

98. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then what is $\frac{\tan x}{\tan y}$ equal to?

(a) $\frac{b}{a}$ (b) $\frac{a}{b}$ (c) ab (d) 1

(b) $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

Applying componendo and dividendo, we get

$$\frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$$

$$\Rightarrow \frac{2\sin x\cdot\cos y}{2\cos x\cdot\sin y} = \frac{2a}{2b}$$

$$\Rightarrow \tan x\cdot\cot y = \frac{a}{b}$$

$$\therefore \frac{\tan x}{\tan y} = \frac{a}{b}$$

99. If $\sin A\sin(60^\circ - A)\sin(60^\circ + A) = k\sin 3A$, then what is k equal to?

(a) 1/4 (b) 1/2
(c) 1 (d) 4

(a) We have,

$\sin A\cdot\sin(60^\circ - A)\sin(60^\circ + A) = k\cdot\sin 3A$

$$\Rightarrow \sin A\cdot\frac{\sin 3A}{4\sin A} = k\cdot\sin 3A$$

$\left[\because \sin(60^\circ + A)\cdot\sin(60^\circ - A) = \frac{\sin 3A}{4\sin A}\right]$

$$\Rightarrow \frac{\sin 3A}{4} = k\cdot\sin 3A$$

$$\therefore k = \frac{1}{4}$$

100. The line $y = \sqrt{3}$ meets the graph $y = \tan x$, where $x \in \left(0, \frac{\pi}{2}\right)$, in k points. What is k equal to?

(a) One
(b) Two
(c) Three
(d) Infinity

(a) We have, $y = \sqrt{3}$ ...(i)

Also, to meet the graph of $y = \tan x$ and $y = \sqrt{3}$, we should have

$\sqrt{3} = \tan x$

$\Rightarrow \tan x = \tan 60^\circ$

$\Rightarrow x = 60^\circ$ $\left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

Hence, one intersecting point possible in given domain i.e. $k = 1$.

101. Which one of the following is one of the solutions of the equation $\tan 2\theta\cdot\tan\theta = 1$?

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(b) We have, $\tan 2\theta \cdot \tan \theta = 1$

$\Rightarrow \quad \frac{2\tan\theta}{1-\tan^2\theta} \cdot \tan\theta = 1$

$\left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$

$\Rightarrow 2\tan^2\theta = 1 - \tan^2\theta \Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \quad \tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$

$\Rightarrow \quad \tan^2\theta = \tan^2(30°) = \tan^2\left(\frac{\pi}{6}\right)$

$\therefore \quad \theta = \frac{\pi}{6}$

Directions (Q. Nos. 102-104) *Read the following information carefully and answer the questions given below.*

Given, $16\sin^5 x = p\sin 5x + q\sin 3x + r\sin x$

102. What is the value of p?

(a) 1 (b) 2 (c) –1 (d) –2

103. What is the value of q?

(a) 3 (b) 5 (c) 10 (d) –5

104. What is the value of r?

(a) 5 (b) 8 (c) 10 (d) –10

(Solutions Q. Nos. 102-104)

Consider,

$16\sin^5 x = 16(\sin^2 x)^2 \cdot \sin x$

$= 16\left(\frac{1-\cos 2x}{2}\right)^2 \cdot \sin x$

$= 4(1 + \cos^2 2x - 2\cos 2x)\cdot \sin x$

$= 4\left(1 + \frac{1+\cos 4x}{2} - 2\cos 2x\right)\cdot \sin x$

$= \frac{4}{2}(3 + \cos 4x - 4\cos 2x)\cdot \sin x$

$= (6 + 2\cos 4x - 8\cos 2x)\sin x$

$= 6\sin x + 2\sin x\cos 4x - 8\cos 2x\cdot \sin x$

$= 6\sin x + \sin 5x - \sin 3x - 4(\sin 3x - \sin x)$

$[\because 2\cos A \sin B = \sin(A+B) - \sin(A-B)]$

$= 6\sin x + \sin 5x - \sin 3x - 4\sin 3x + 4\sin x$

$= \sin 5x - 5\sin 3x + 10\sin x$

102. (a) Clearly, $p = 1$

103. (d) Clearly, $q = -5$

104. (c) Clearly, $r = 10$

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105. Consider the follwoing statements

I. $\sin|x| + \cos|x|$ is always positive.

II. $\sin(x^2) + \cos(x^2)$ is always positive.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(d) I. Let $f_1(x) = \sin|x| + \cos|x|$

We know that, the value of $\sin|x|$ and $\cos|x|$ depend on its angles.

$\because [0, \pi/2] \rightarrow \sin x > 0$ and $\cos x > 0$

$[\pi/2, \pi] \rightarrow \sin x > 0$ and $\cos x < 0$

$\left[\pi, \frac{3\pi}{2}\right] \rightarrow \sin x < 0$ and $\cos x < 0$

$[3\pi/2, 2\pi] \rightarrow \sin x < 0$ and $\cos x > 0$

We see that, in interval $x \in [\pi, 3\pi/2]$, the value of $\sin|x| + \cos|x|$ is always negative.

So, it is not necessary that $\sin|x| + \cos|x|$ is not always positive.

II. Given that, let

$f_2(x) = \sin(x^2) + \cos(x^2)$

If we take the values of x^2 between any value which lies in the interval $\left[\pi, \frac{3\pi}{2}\right]$, then value of $f_2(x) = \sin(x^2) + \cos(x^2)$ is always negative.

If $x^2 = 225° \Rightarrow x = 15°$, then $f_2(x) = (\sin x^2 + \cos x^2) < 0$

So, it's also not necessary that $\sin x^2 + \cos x^2$ is not always positive.

Note If $x \in [-\pi/2, 0] \rightarrow \sin x < 0$ and $\cos x > 0$ but $\sin|x| + \cos|x|$ is always positive.

If we take $x = -15° \Rightarrow x^2 = 225°$, then $\sin x^2 + \cos x^2$ is negative.

106. What is value of

$$\frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A}?$$

(a) $\sec A - \tan A$ (b) $2\sec A \cdot \tan A$

(c) $4\sec A \cdot \tan A$ (d) $4\operatorname{cosec} A \cdot \cot A$

(c) $\frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A}$

$= \frac{(1+\sin A)^2 - (1-\sin A)^2}{(1-\sin A)(1+\sin A)}$

$= \frac{(1 + \sin^2 A + 2\sin A) - (1 + \sin^2 A - 2\sin A)}{(1-\sin^2 A)}$

$[\because (a+b)^2 = a^2 + 2ab + b^2, (a-b)^2 = a^2 - 2ab + b^2, (a-b)(a+b) = (a^2 - b^2)]$

$= \frac{(1 + \sin^2 A + 2\sin A - 1 - \sin^2 A + 2\sin A)}{\cos^2 A}$

$= \frac{4\sin A}{\cos^2 A} = 4\frac{\sin A}{\cos A}\cdot\frac{1}{\cos A}$ $[\because \sin^2 A + \cos^2 A = 1]$

$= 4\tan A \cdot \sec A$

$\left[\because \tan A = \frac{\sin A}{\cos A} \text{ and } \sec A = \frac{1}{\cos A}\right]$

107. What is value of

$$\frac{\cot 224° - \cot 134°}{\cot 226° + \cot 316°}?$$

(a) $-\operatorname{cosec} 88°$ (b) $-\operatorname{cosec} 2°$

(c) $-\operatorname{cosec} 44°$ (d) $-\operatorname{cosec} 46°$

(b) $\frac{\cot 224° - \cot 134°}{\cot 226° + \cot 316°}$

$= \frac{\cot(360° - 136°) - \cot(90° + 44°)}{\cot(360° - 134) + \cot(360° - 44°)}$

$[\because \cot(360° - \theta) = -\cot\theta \text{ and } \cot(90° + \theta) = -\tan\theta]$

$= \frac{-\cot 136° + \tan 44°}{-\cot 134° - \cot 44°}$

$= \frac{-\cot(90° + 46°) + \tan 44°}{-\cot(90° + 44°) - \cot 44°}$

$= \frac{\tan 46° + \tan 44°}{\tan 44° - \cot(90° - 46°)}$

$[\because \cot(90° + \theta) = -\tan\theta]$

$= \frac{\tan 46° + \tan 44°}{\tan 44° - \tan 46°}$

$[\because \cot(90° - \theta) = \tan\theta]$

$= \frac{\left(\frac{\sin 46°}{\cos 46°} + \frac{\sin 44°}{\cos 44°}\right)}{\left(\frac{\sin 44°}{\cos 44°} - \frac{\sin 46°}{\cos 46°}\right)}$

$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{\sin 46°\cdot\cos 44° + \sin 44°\cdot\cos 46°}{\sin 44°\cdot\cos 46° - \sin 46°\cdot\cos 44°}$

$$= \frac{\sin(46° + 44°)}{\sin(44° - 46°)}$$

$$\left[\because \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B, \quad \sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B\right]$$

$$= \frac{\sin 90°}{\sin(-2)°} = \frac{1}{-\sin 2°}$$

$$\left[\because \sin 90° = 1, \sin(-\theta) = -\sin\theta \text{ and } \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$$

$$= -\operatorname{cosec} 2°$$

108. If $A + B + C = \pi$, then what is $\cos(A + B) + \cos C$ equal to?

(a) 0 (b) $2\cos C$
(c) $\cos C - \sin C$ (d) $2\sin C$

(a) Given that,

$$A + B + C = \pi \quad \ldots(i)$$

Now, $\cos(A + B) + \cos C$
$= \cos(\pi - C) + \cos C$ [from Eq. (i)]
$= -\cos C + \cos C = 0$
[$\because$ in second quadrant, $\cos\theta < 0$]

109. What is $\cos 20° + \cos 100° + \cos 140°$ equal to?

(a) 2 (b) 1
(c) 1/2 (d) 0

(d) We have,

$\cos 20° + \cos 100° + \cos 140°$
$= (\cos 140° + \cos 20°) + \cos 100°$

$$= 2\cos\left(\frac{140° + 20°}{2}\right) \cdot \cos\left(\frac{140° - 20°}{2}\right) + \cos 100°$$

$$\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}\right]$$

$$= 2\cos\left(\frac{160°}{2}\right) \cdot \cos\left(\frac{120°}{2}\right) + \cos 100°$$

$$= 2\cos(80°) \cdot \cos 60° + \cos 100°$$

$$= 2.\cos 80° \cdot \frac{1}{2} + \cos 100°$$

$$= \cos 80° + \cos 100°$$

$$= 2\cos\left(\frac{80° + 100°}{2}\right) \cdot \cos\left(\frac{100 - 80°}{2}\right)$$

$$\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}\right]$$

$$= 2\cos\left(\frac{180°}{2}\right) \cdot \cos\left(\frac{20°}{2}\right)$$

$$= 2\cos 90° \cdot \cos 10°$$
$$= 2 \times 0 \times \cos 10° = 0$$

110. What is $\sin^2(3\pi) + \cos^2(4\pi) + \tan^2(5\pi)$ equal to?

(a) 0 (b) 1
(c) 2 (d) 3

(b) We have,

$$\sin^2(3\pi) + \cos^2(4\pi) + \tan^2(5\pi)$$
$$= \sin^2(2\pi + \pi) + \cos^2(2\pi + 2\pi) + \tan^2(4\pi + \pi)$$
$$= \sin^2\pi + \cos^2 2\pi + \tan^2\pi$$

[$\because \sin(2\pi + \theta) = \sin\theta, \cos(2\pi + \theta) = \cos\theta \tan(4\pi + \theta) = \tan\theta$]

$$= \sin^2\left(\frac{\pi}{2} + \frac{\pi}{2}\right) + \cos^2\left(\frac{3\pi}{2} + \frac{\pi}{2}\right) + \tan^2\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$\left[\because \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta, \quad \cos\left(\frac{3\pi}{2} + \theta\right) = \sin\theta, \quad \tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta\right]$$

$$= \cos^2\frac{\pi}{2} + \sin^2\frac{\pi}{2} + \cot^2\left(\frac{\pi}{2}\right)$$

$$= (0)^2 + (1)^2 + (0)^2 = 0 + 1 + 0 = 1$$

111. The complete solution of $3\tan^2 x = 1$ is given by

(a) $x = n\pi \pm \frac{\pi}{3}$ (b) $x = n\pi + \frac{\pi}{3}$
(c) $x = n\pi \pm \frac{\pi}{6}$ (d) $x = n\pi + \frac{\pi}{6}$

where, $n \in Z$

(c) Given that, $3\tan^2 x = 1$

$$\Rightarrow \tan^2 x = \frac{1}{3}$$

$$\Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan x = \tan(\pm\pi/6)$$

$$\therefore \quad x = n\pi \pm \frac{\pi}{6}, \text{ where, } n \in Z$$

112. What is the value of $\cos 36°$?

(a) $\frac{\sqrt{5} - 1}{4}$ (b) $\frac{\sqrt{5} + 1}{4}$
(c) $\frac{\sqrt{10 + 2\sqrt{5}}}{4}$ (d) $\frac{\sqrt{10 - 2\sqrt{5}}}{4}$

(b) We take, $5\theta = 180°$ [$\because \theta = 36°$]

$$\Rightarrow \quad 3\theta + 2\theta = 180°$$
$$\Rightarrow \quad 2\theta = 180° - 3\theta$$

Taking cos both sides, we get

$$\cos 2\theta = \cos(180° - 3\theta)$$
$$\Rightarrow \quad \cos 2\theta = -\cos 3\theta$$
$$\Rightarrow 2\cos^2\theta - 1 = -(4\cos^3\theta - 3\cos\theta)$$
$$\Rightarrow 2\cos^2\theta - 1 = -4\cos^3\theta + 3\cos\theta$$
$$\Rightarrow 4\cos^3\theta + 2\cos^2\theta - 3\cos\theta - 1 = 0$$
$$\Rightarrow 4\cos^2\theta(\cos\theta + 1) - 2\cos\theta(\cos\theta + 1) - 1(\cos\theta + 1) = 0$$
$$\Rightarrow (\cos\theta + 1)(4\cos^2\theta - 2\cos\theta - 1) = 0$$
$$\Rightarrow \quad \cos\theta + 1 = 0$$
$$\text{or } 4\cos^2\theta - 2\cos\theta - 1 = 0$$
$$\Rightarrow \quad \cos\theta = -1$$

$$\text{or } \cos\theta = \frac{2 \pm \sqrt{4 - 4 \times (4)(-1)}}{2 \times 4}$$

$$\Rightarrow \cos\theta = \frac{2 \pm \sqrt{4 + 16}}{8} = \frac{2 \pm \sqrt{20}}{8}$$

$$\Rightarrow \cos\theta = \frac{2 \pm 2\sqrt{5}}{8}$$

$$\Rightarrow \cos\theta = \frac{1 \pm \sqrt{5}}{4} \quad [\because \cos\theta \in [-1, 1]]$$

Put $\theta = 36°$, we get

$$\cos 36° = \frac{\sqrt{5} + 1}{4} \quad [\because \cos 36° > 0]$$

113. Consider the following statements

I. Value of $\sin\theta$ oscillates between -1 and 1.
II. Value of $\cos\theta$ oscillates between 0 and 1.

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(a) I. We know that, $\sin\theta \in [-1, 1]$; $\theta \in R$
i.e. the value of $\sin\theta$ lies between -1 to 1.

II. We know that, $\cos\theta \in [-1, 1]$, $\theta \in R$
i.e. the value of $\cos\theta$ also lies between -1 to 1.

114. Consider the following statements

I. $n\left(\sin^2 67\frac{1°}{2} - \sin^2 22\frac{1°}{2}\right) > 1$ for all positive integers $n \geq 2$.
II. If x is any positive real number, then $nx > 1$ for all positive integers $n \geq 2$.

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(a) I. We have,

$$n\left(\sin^2\frac{135°}{2} - \sin^2\frac{45°}{2}\right)$$
$$= n\left(\sin\frac{135°}{2} + \sin\frac{45°}{2}\right)\left(\sin\frac{135°}{2} - \sin\frac{45°}{2}\right)$$

$$= n\left[\left[2\sin\frac{\left(\frac{135°}{2}+\frac{45°}{2}\right)}{2}\cdot\cos\frac{\left(\frac{135°}{2}-\frac{45°}{2}\right)}{2}\right]\right.$$

$$\left.\left[2\cos\frac{\left(\frac{135°+45°}{2}\right)}{2}\cdot\sin\frac{\left(\frac{135°}{2}-\frac{45°}{2}\right)}{2}\right]\right]$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right.$$

$$\left.\text{and } \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= n\left[2\sin\left(\frac{90°}{2}\right)\cdot\cos\left(\frac{45°}{2}\right)\right],$$

$$\left[2\cos\left(\frac{90°}{2}\right)\cdot\sin\left(\frac{45°}{2}\right)\right]$$

$$= 2n\left(2\sin\frac{45°}{2}\cdot\cos\frac{45°}{2}\right)$$

$$(\sin 45°\cdot\cos 45°)$$

$$= 2n\cdot\sin\left(2\cdot\frac{45°}{2}\right)\cdot\left(\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)$$

$$= 2n\cdot\sin 45\cdot\frac{1}{2} = n\cdot\frac{1}{\sqrt{2}} = \frac{n}{\sqrt{2}}$$

$\Rightarrow \frac{n}{\sqrt{2}} > 1$ for all positive integers $n \geq 2$.

II. Given that,

x is any positive real number i.e. $x \in R^+ \Rightarrow x \in (0, \infty)$

Then, $n\,x > 1,\ \forall\, n \geq 2 \Rightarrow\ n > \frac{1}{x},\ \forall\ n \geq 2$

Since, $x \in (0, \infty)$, then we take $x = 1$,

$\Rightarrow n > 1$ but n is always greater or equal to 2 for all x positive numbers.

Hence, Statement II is false.

115. Consider the following statements

I. If 3θ is an acute angle such that $\sin 3\theta = \cos 2\theta$, then the measurement of θ in radian equals to $\frac{\pi}{10}$.

II. One radian is the angle subtended at the centre of a circle by an arc of the same circle whose length is equal to the diameter of that circle.

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(a) I. Given that, $\sin 3\theta = \cos 2\theta$

$\Rightarrow \sin 3\theta = \sin\left(\frac{\pi}{2} - 2\theta\right)$

[$\because 3\theta$ is an acute angle]

$\Rightarrow\ 3\theta = \frac{\pi}{2} - 2\theta\ \Rightarrow\ 5\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{10}$

II. By definition of radian, "A radian is the angle subtended at the centre of a circle by an arc, whose length is equal to the radius of the circle".

The number of radians in an angle subtended by an arc of circle at the centre is equal to $\frac{\text{arc}}{\text{radius}}$.

Hence, Statement II is false.

12

INVERSE CIRCULAR FUNCTIONS

2019 (II)

1. What is $\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\}$ equal to?

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{1}{9}$

(**b**) $\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\}$

$= \tan\left\{\tan^{-1}\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right\}$

$\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$= \tan\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right) = \frac{2\times 9}{3\times 8} = \frac{3}{4}$

2019 (I)

2. What is the value of $\sin^{-1}\frac{4}{5} + \sec^{-1}\frac{5}{4} - \frac{\pi}{2}$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 0

(**d**) We have,

$\sin^{-1}\frac{4}{5} + \sec^{-1}\frac{5}{4} - \frac{\pi}{2}$

$= \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} - \frac{\pi}{2}$

$\left[\because \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$

$= \frac{\pi}{2} - \frac{\pi}{2} = 0 \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

3. If $\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-q^2}{1+q^2} = \tan^{-1}\frac{2x}{1-x^2}$, then what is x equal to?

(a) $\frac{p+q}{1+pq}$ (b) $\frac{p-q}{1+pq}$

(c) $\frac{pq}{1+pq}$ (d) $\frac{p+q}{1-pq}$

(**b**) Given,

$\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}p - 2\tan^{-1}q = 2\tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1}x$

$\therefore x = \frac{p-q}{1+pq}$

2018 (II)

4. Consider the following values of x

1. 8 2. –4 3. $\frac{1}{6}$ 4. $-\frac{1}{4}$

Which of the above values of x is/are the solution(s) of the equation

$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$?

(a) Only 3 (b) 2 and 3

(c) 1 and 4 (d) Only 4

(**a**) We have,

$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{2x+3x}{1-2x\cdot 3x}\right) = \frac{\pi}{4}$

$\left[\because \tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right),\ ab \le 1\right]$

$\Rightarrow \frac{5x}{1-6x^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 1 - 6x^2 = 5x$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow (6x-1)(x+1) = 0$

$\therefore x = -1, \frac{1}{6}$

So, $x = \frac{1}{6}$

2018 (I)

5. If $\sin x = \frac{1}{\sqrt{5}}$, $\sin y = \frac{1}{\sqrt{10}}$, where $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, then what is $(x + y)$ equal to ?

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) 0

(**c**) We have,

$\sin x = \frac{1}{\sqrt{5}}$ and $\sin y = \frac{1}{\sqrt{10}}$

$\Rightarrow x = \sin^{-1}\frac{1}{\sqrt{5}}$ and $y = \sin^{-1}\frac{1}{\sqrt{10}}$

Now, $x + y = \sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{10}}$

$= \sin^{-1}\left[\frac{1}{\sqrt{5}}\sqrt{1-\left(\frac{1}{\sqrt{10}}\right)^2} + \frac{1}{\sqrt{10}}\sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2}\right]$

$\left[\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]\right.$

$$= \sin^{-1}\left[\frac{1}{\sqrt{5}}\sqrt{1-\frac{1}{10}} + \frac{1}{\sqrt{10}}\sqrt{1-\frac{1}{5}}\right]$$
$$= \sin^{-1}\left[\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} \times \frac{2}{\sqrt{5}}\right]$$
$$= \sin^{-1}\left[\frac{5}{\sqrt{5} \times \sqrt{10}}\right] = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{\pi}{4} \qquad \left[\because \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$$

6. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

(***c***) We have, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$$
$$= \sin^{-1}\sin\frac{\pi}{3} \quad [\because \sin(\pi - \theta) = \sin\theta]$$
$$= \frac{\pi}{3} \left[\because \sin^{-1}\sin\theta = \theta, \text{ if } \theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right]$$

7. What is $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$ equal to ?

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(***b***) We have

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$$
$$= \tan^{-1}\left[\frac{\frac{1}{4}+\frac{3}{5}}{1-\frac{1}{4}\times\frac{3}{5}}\right]$$
$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}, \; xy < 1\right]$$
$$= \tan^{-1}\left[\frac{\frac{5+12}{20}}{\frac{20-3}{20}}\right] = \tan^{-1}\left(\frac{17}{17}\right)$$
$$= \tan^{-1}1 = \frac{\pi}{4} \qquad \left[\because \tan^{-1}1 = \frac{\pi}{4}\right]$$

2017 (II)

8. The value of $\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is equal to

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(***b***) We have,

$$\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$
$$= \tan^{-1}\frac{3/5}{\sqrt{1-\left(\frac{3}{5}\right)^2}} + \tan^{-1}\frac{1}{7}$$
$$\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$$
$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$$
$$= \tan^{-1}\left[\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right] = \tan^{-1}\left(\frac{21+4}{28-3}\right)$$
$$= \tan^{-1}\frac{25}{25} = \tan^{-1}1 = \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$
$$= \frac{\pi}{4}$$

9. The principal value of $\sin^{-1}x$ lies in the interval

(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(c) $\left[0, \frac{\pi}{2}\right]$ (d) $[0, \pi]$

(***b***) Range of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2017 (I)

10. Let x, y, z be positive real numbers such that x, y, z are in GP and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in AP. Then which one of the following is correct?

(a) $x = y = z$ (b) $xz = 1$
(c) $x \neq y$ and $y = z$ (d) $x = y$ and $y \neq z$

(***a***) Since, $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in AP.

$$\Rightarrow \quad 2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$$
$$\Rightarrow \quad \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$
$$\Rightarrow \quad \frac{2y}{1-xz} = \frac{x+z}{1-xz} \quad (\because y^2 = xz)$$
$$\Rightarrow \quad 2y = x + z$$

$\Rightarrow x, y, z$ are in AP, but x, y, z are in GP

$\therefore \quad x = y = z$

2016 (II)

11. What is the value of $\cos(2\cos^{-1}0.8)$?

(a) 0.81 (b) 0.56
(c) 0.48 (d) 0.28

(***d***) Let $2\cos^{-1}0.8 = \theta$

$$\Rightarrow \quad \cos^{-1}0.8 = \theta/2$$
$$\Rightarrow \quad 0.8 = \cos\theta/2$$
$$\Rightarrow \quad (0.8)^2 = \cos^2\theta/2$$
$$\Rightarrow \quad (0.8)^2 = \frac{1+\cos\theta}{2}$$
$$[\because 1 + \cos\theta = 2\cos^2\theta/2]$$
$$\Rightarrow \quad 0.64 \times 2 = 1 + \cos\theta$$
$$\Rightarrow \quad 1.28 - 1 = \cos\theta$$
$$\Rightarrow \quad 0.28 = \cos\theta$$
$$\therefore \quad \cos(2\cos^{-1}0.8) = \cos\theta = 0.28$$

2016 (I)

12. Consider the following statements

1. There exists $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which $\tan^{-1}(\tan\theta) \neq \theta$.
2. $\sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{2\sqrt{2}\,(\sqrt{3}-1)}{15}\right)$

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***b***) $\tan^{-1}(\tan\theta) = \theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\therefore$ Statement 1 is incorrect.

Now, $\sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right)$

$$= \sin^{-1}\left[\frac{1}{3}\sqrt{1-\left(\frac{1}{5}\right)^2} - \frac{1}{5}\sqrt{1-\left(\frac{1}{3}\right)^2}\right]$$
$$= \sin^{-1}\left[\frac{1}{3} \times \sqrt{\frac{24}{25}} - \frac{1}{5}\sqrt{\frac{8}{9}}\right]$$
$$= \sin^{-1}\left[\frac{1}{3} \times \frac{\sqrt{24}}{5} - \frac{1}{5} \times \frac{\sqrt{8}}{3}\right]$$
$$= \sin^{-1}\left[\frac{\sqrt{24}-\sqrt{8}}{15}\right] = \sin^{-1}\left[\frac{\sqrt{8}\,(\sqrt{3}-1)}{15}\right]$$
$$= \sin^{-1}\left[\frac{2\sqrt{2}\,(\sqrt{3}-1)}{15}\right]$$

$\therefore$ Statement 2 is correct.

13. Consider the following statements

1. $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \pi$
2. Their exist, $x, y \in [-1, 1]$, where $x \neq y$ such that $\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(*d*) We know that, $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$\therefore \quad \tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

$\therefore$ Statement 1 is incorrect.

Let $x = y$

Given that, $\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$

This is true for $x \in [-1, 1]$.

$\therefore$ Statement 2 is also incorrect.

2015 (II)

14. The value of $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$ is

(a) $-\frac{7}{17}$ (b) $\frac{5}{16}$

(c) $\frac{5}{4}$ (d) $\frac{7}{17}$

(*a*) $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$

$\because \quad 2\tan^{-1}\frac{1}{5} = \tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right)$

$\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}, \text{ for } |x| < 1\right]$

$= \tan^{-1}\left(\frac{5}{12}\right)$

Now, $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$

$= \tan\left(\tan^{-1}\frac{5}{12} - \frac{\pi}{4}\right)$

$= \frac{\tan\left(\tan^{-1}\frac{5}{12}\right) - \tan\frac{\pi}{4}}{1 + \tan\left(\tan^{-1}\frac{5}{12}\right)\cdot\tan\frac{\pi}{4}}$

$\left[\because \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\cdot\tan B}\right]$

$= \frac{\frac{5}{12} - 1}{1 + \frac{5}{12}(1)} = \frac{-7}{17}$

15. Consider the following statements

1. $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{3}{5} = \frac{\pi}{2}$
2. $\tan^{-1}\sqrt{3} + \tan^{-1}1 = -\tan^{-1}(2+\sqrt{3})$

Which of the above statements(s) is/are correct?

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

(*c*) From Statement 1,

$\text{LHS} = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{3}{5}$

$= \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5}$

$\left[\because \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{4}{5}\right]$

$= \frac{\pi}{2} = \text{RHS}$

From Statement 2,

$\text{LHS} = \tan^{-1}\sqrt{3} + \tan^{-1}1$

$= \tan^{-1}\left(\frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}\right)$

$= \tan^{-1}\left(\frac{3+1+2\sqrt{3}}{1-3}\right)$

$= \tan^{-1}\left(\frac{2+\sqrt{3}}{-1}\right)$

$= -\tan^{-1}(2+\sqrt{3})$

2015 (I)

16. The equation $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ is satisfied by

(a) $x = 1$ (b) $x = -1$

(c) $x = 0$ (d) $x = \frac{1}{2}$

(*c*) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}\left(\frac{1+x+1-x}{1-(1+x)(1-x)}\right) = \frac{\pi}{2}$

$\left[\because \tan^{-1}(A+B) = \tan^{-1}\left(\frac{A+B}{1-AB}\right)\right]$

$\Rightarrow \quad \tan^{-1}\left(\frac{2}{1-1+x^2}\right) = \frac{\pi}{2}$

$\Rightarrow \quad \frac{2}{x^2} = \tan\frac{\pi}{2} \Rightarrow \frac{2}{x^2} = \frac{1}{0}$

$\therefore \quad x^2 = 0 \Rightarrow x = 0$

Directions (Q. Nos. 17-19) *Consider* $x = 4\tan^{-1}\left(\frac{1}{5}\right)$, $y = \tan^{-1}\left(\frac{1}{70}\right)$ and $z = \tan^{-1}\left(\frac{1}{99}\right)$.

17. What is x equal to?

(a) $\tan^{-1}\left(\frac{60}{119}\right)$ (b) $\tan^{-1}\left(\frac{120}{119}\right)$

(c) $\tan^{-1}\left(\frac{90}{169}\right)$ (d) $\tan^{-1}\left(\frac{170}{169}\right)$

(*b*) Given, $x = 4\tan^{-1}\left(\frac{1}{5}\right) = 2\cdot 2\tan^{-1}\left(\frac{1}{5}\right)$

$= 2\cdot\tan^{-1}\left(\frac{2/5}{24/25}\right)$

$\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$= 2\tan^{-1}(5/12)$

$= \tan^{-1}\left(\frac{10/12}{1-\frac{25}{144}}\right)$

$\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$= \tan^{-1}\left(\frac{10/12}{119/144}\right)$

$= \tan^{-1}\left(\frac{144\times 10}{12\times 119}\right)$

$\Rightarrow \quad x = \tan^{-1}\left(\frac{120}{119}\right)$

18. What is $x - y$ equal to?

(a) $\tan^{-1}\left(\frac{828}{845}\right)$ (b) $\tan^{-1}\left(\frac{8287}{8450}\right)$

(c) $\tan^{-1}\left(\frac{8281}{8450}\right)$ (d) $\tan^{-1}\left(\frac{8287}{8471}\right)$

(*c*) $x - y = \tan^{-1}\left(\frac{120}{119}\right) - \tan^{-1}\left(\frac{1}{70}\right)$

$\left[\because \tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+A\cdot B}\right)\right]$

$= \tan^{-1}\left(\frac{\frac{120}{119} - \frac{1}{70}}{1+\frac{120}{119}\times\frac{1}{70}}\right)$

$= \tan^{-1}\left(\frac{120\times 70 - 119}{119\times 70 + 120}\right)$

$= \tan^{-1}\left(\frac{8400-119}{8330+120}\right)$

$= \tan^{-1}\left(\frac{8281}{8450}\right)$

19. What is $x - y + z$ equal to?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

(*d*) $x - y + z = \tan^{-1}\left(\frac{8281}{8450}\right) + \tan^{-1}\left(\frac{1}{99}\right)$

$= \tan^{-1}\left(\frac{\frac{8281}{8450} + \frac{1}{99}}{1-\frac{8281\times 1}{8450\times 99}}\right)$

$\left[\because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-A\cdot B}\right)\right]$

$= \tan^{-1}\left(\frac{99\times 8281 + 8450}{8450\times 99 - 8281}\right)$

$= \tan^{-1}\left(\frac{828269}{828269}\right)$

$= \tan^{-1}1$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$

2014 (II)

20. What is $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}$ equal to?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(a) $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}$

Let $\sin^{-1}\frac{4}{5}=\theta$

$\Rightarrow \sin\theta=\frac{4}{5}$

$\Rightarrow \cos\theta=\frac{3}{5}$

$\Rightarrow \theta=\cos^{-1}\frac{3}{5}$

$\therefore \sin^{-1}\frac{3}{5}+\cos^{-1}\frac{3}{5}=\frac{\pi}{2}$

2014 (I)

21. Consider the following statements

I. $\tan^{-1}1+\tan^{-1}(0.5)=\pi/2$

II. $\sin^{-1}(1/3)+\cos^{-1}(1/3)=\pi/2$

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(b) I. $\tan^{-1}1+\tan^{-1}(0.5)$

$\Rightarrow \tan^{-1}1+\tan^{-1}\left(\frac{1}{2}\right)$

$=\tan^{-1}1+\cot^{-1}\left(\frac{1}{1/2}\right)$

$\left[\because \tan^{-1}A=\cot^{-1}\frac{1}{A}\right]$

$=\tan^{-1}1+\cot^{-1}2\neq\pi/2$

$[\because \tan^{-1}A+\cot^{-1}A=\pi/2, \forall A\in R]$

II. $\sin^{-1}\left(\frac{1}{3}\right)+\cos^{-1}\left(\frac{1}{3}\right)=\pi/2$

$[\because \sin^{-1}A+\cos^{-1}A=\pi/2, \forall A\in R]$

22. What is $\sin^{-1}\sin\left(\frac{3\pi}{5}\right)$ equal to?

(a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$

(c) $\frac{\pi}{5}$ (d) None of these

(b) We have,

$\sin^{-1}\sin\left(\frac{3\pi}{5}\right)=\sin^{-1}\sin\left(\frac{\pi}{2}+\frac{\pi}{10}\right)$

$\left[\because \sin^{-1}\sin x=x, \forall x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$

$=\sin^{-1}\cos\frac{\pi}{10}$

$=\sin^{-1}\sin\left(\frac{\pi}{2}-\frac{\pi}{10}\right)$

$\left[\because \cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)\right]$

$=\frac{\pi}{2}-\frac{\pi}{10}$

$=\frac{5\pi-\pi}{10}=\frac{4\pi}{10}=\frac{2\pi}{5}$

23. If x and y are positive and $xy>1$, then what is $\tan^{-1}x+\tan^{-1}y$ equal to?

(a) $\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(b) $\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(c) $\pi-\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(d) $\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(b) We know that,

If x and y are positive *i.e.*, $x>0$, $y>0$ and $xy>1$, then

$\tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

13

HEIGHT AND DISTANCE

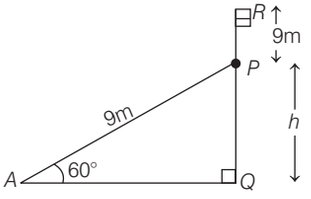
2019 (II)

1. A ladder 9m long reaches a point 9m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the flagstaff is 60°. What is the height of the flagstaff?

(a) 9 m (b) 10.5 m (c) 13.5 m (d) 15 m

(*) Let AP be a ladder and QR be a vertical flagstaff. P is a point 9 m below the top on flagstaff. A is the foot of ladder and h is the height of point P from the ground.

$\therefore AP = 9$ m, $PR = 9$ m, $PQ = h$m



In $\Delta\ AQP$, $\sin\theta = \frac{PQ}{AP}$

$\Rightarrow \sin 60° = \frac{h}{9} \Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{9}$

$\Rightarrow 9\sqrt{3} = 2h \Rightarrow h = \frac{9\sqrt{3}}{2}$

$= \frac{9 \times 1.73}{2} = \frac{15.57}{2} = 7.7$ m

$\therefore$ Height of flagstaff

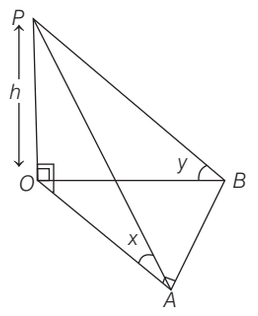
$= h + 9 = 7.7 + 9 = 16.7$ m

2019 (I)

2. The angle of elevation of a tower of height h from a point A due South of it is x and from a point B due East of A is y. If $AB = z$, then which one of the following is correct ?

(a) $h^2(\cot^2 y - \cot^2 x) = z^2$
(b) $z^2(\cot^2 y - \cot^2 x) = h^2$
(c) $h^2(\tan^2 y - \tan^2 x) = z^2$
(d) $z^2(\tan^2 y - \tan^2 x) = h^2$

(*a*)



Here, OP be the tower,

$OA = h \cot x$

$OB = h \cot y$

In right-angled ΔOAB,

$h^2 \cot^2 y = z^2 + h^2 \cot^2 x$

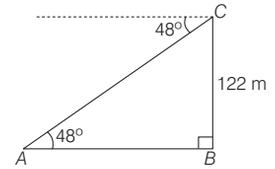
$\therefore \quad z^2 = h^2 (\cot^2 y - \cot^2 x)$

2018 (II)

3. A balloon is directly above one end of a bridge. The angle of depression of the other end of the bridge from the ballon is 48°. If the height of the balloon above the bridge is 122 m, then what is the length of the bridge?

(a) 122 sin 48° m (b) 122 tan 42° m
(c) 122 cos 48° m (d) 122 tan 48° m

(*b*) AB is a bridge and C is a balloon.



$\tan 48° = \frac{BC}{AB} \Rightarrow \tan 48° = \frac{122}{AB}$

$AB = \frac{122}{\tan 48°} = 122 \cot 48°$

$= 122 \cot (90° - 42°)$

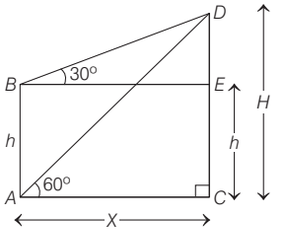
$[\because \cot (90° - \theta) = \tan\theta]$

$= 122 \tan 42°$

4. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation $\frac{\pi}{6}$ and $\frac{\pi}{3}$ respectively. What is the height of the hill?

(a) $2h$ (b) $\frac{3h}{2}$ (c) h (d) $\frac{h}{2}$

(*b*) Given, height of building AB is h let height of hill CD be H



In ΔACD

$\tan 60° = \frac{H}{x} \Rightarrow \sqrt{3} = \frac{H}{x}$

$\Rightarrow \quad x = \frac{H}{\sqrt{3}}$ ...(i)

In ΔBDE

$\tan 30° = \frac{H - h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{H - h}{x}$

$\Rightarrow \quad x = \sqrt{3}(H - h)$

$\Rightarrow \quad \frac{H}{\sqrt{3}} = \sqrt{3}(H - h)$

$\Rightarrow \quad H = 3(H - h) \Rightarrow H = 3H - 3h$

$\Rightarrow \quad 2H = 3h$

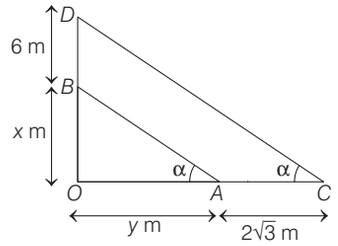
$\therefore \quad H = \frac{3}{2}h$

2018 (I)

5. If a flage-staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then what is the angle that the sun makes with the ground ?

(a) 60° (b) 45° (c) 30° (d) 15°

(**a**) Let OB and BD be the tower and flag-staff respectively OA and AC be the shadow of tower and flag-staff respectively.



Again let α be the angle that sun makes with the ground.

$\therefore \quad \angle OAB = \angle OCD = \alpha$

Now, in ΔOAB

$$\tan\alpha = \frac{x}{y} \qquad \ldots (i)$$

and in ΔOCD

$$\tan\alpha = \frac{x+6}{y+2\sqrt{3}} \qquad \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$\frac{x}{y} = \frac{x+6}{y+2\sqrt{3}}$$

$$\Rightarrow xy + 2\sqrt{3}x = xy + 6y$$

$$\Rightarrow \frac{x}{y} = \sqrt{3}$$

$$\Rightarrow \tan\alpha = \sqrt{3} \qquad \text{[from Eq. (i)]}$$

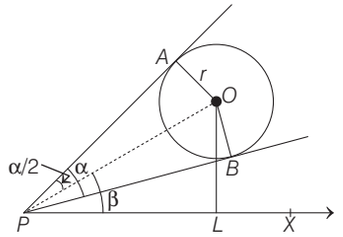
$$\therefore \alpha = 60°$$

6. A spherical balloon of radius r subtends an angle α at the eye of an observer, while the angle of elevation of its centre is β. What is the height of the centre of the balloon (neglecting the height of the observer)?

(a) $\dfrac{r\sin\beta}{\sin\left(\frac{\alpha}{2}\right)}$ (b) $\dfrac{r\sin\beta}{\sin\left(\frac{\alpha}{4}\right)}$

(c) $\dfrac{r\sin\left(\frac{\beta}{2}\right)}{\sin\alpha}$ (d) $\dfrac{r\sin\alpha}{\sin\left(\frac{\beta}{2}\right)}$

(**a**) Let O be the centre of the balloon, P be the eye of the observer and $\angle APB$ be the angle subtendd by the balloon at the eye of the observer. $\angle APB = \alpha$



$$\therefore \quad \angle APO = \angle BPO = \frac{\alpha}{2}$$

In $\Delta OAP \quad \sin\frac{\alpha}{2} = \frac{OA}{OP} \Rightarrow \sin\frac{\alpha}{2} = \frac{r}{OP}$

$$\Rightarrow \quad OP = r \operatorname{cosec}\frac{\alpha}{2} \qquad \ldots (i)$$

In $\Delta OPL, \quad \sin\beta = \frac{OL}{OP}$

$$\Rightarrow \quad OL = OP\sin\beta$$

$$\Rightarrow \quad OL = r \operatorname{cosec}\frac{\alpha}{2}.\sin\beta$$

[from Eq. (i)]

$$\therefore \quad OL = \frac{r\sin\beta}{\sin\left(\frac{\alpha}{2}\right)}$$

2017 (II)

7. The angle of elevation of a stationary cloud from a point 25 m above a lake is 15° and the angle of depression of its image in the lake is 45°. The height of the cloud above the lake level is

(a) 25 m (b) $25\sqrt{3}$ m
(c) 50 m (d) $50\sqrt{3}$ m

(**b**) Let AB be the surface of the lake and P be the point of observation such that $AP = 25$ m. Let C be the position of the cloud and C' be its reflection in the lake. The $CB = C'B$. Let PM be perpendicular from P on CB. Then, $\angle CPM = 15°$ and $\angle C'PM = 45°$. Let $CM = h$, then $CB = h + 25$ consequently, $C'B = h + 25$. In ΔCMP, we have

$$\tan 15° = \frac{CM}{PM}$$

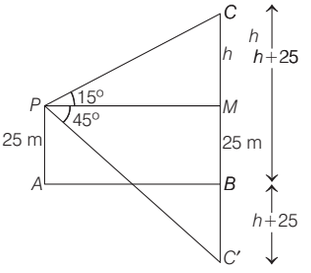
$$\Rightarrow \quad 2 - \sqrt{3} = \frac{h}{PM}$$

$$\Rightarrow \quad PM = \frac{h}{2-\sqrt{3}} \qquad \ldots (i)$$

In $\Delta PMC'$, we have

$$\tan 45° = \frac{C'M}{PM} \Rightarrow 1 = \frac{h+50}{PM}$$

$$\Rightarrow \quad PM = h + 50 \qquad \ldots (ii)$$



From Eqs. (i) and (ii), we get

$$\frac{h}{2-\sqrt{3}} = h + 50$$

$$\Rightarrow \quad h = h(2-\sqrt{3}) + 50(2-\sqrt{3})$$

$$\Rightarrow h - 2h + \sqrt{3}h = 50(2-\sqrt{3})$$

$$\Rightarrow \quad h(\sqrt{3}-1) = 50(2-\sqrt{3})$$

$$\Rightarrow \quad h = \frac{50(2-\sqrt{3})}{(\sqrt{3}-1)}$$

$\therefore$ Height of the cloud above the lake level

$$= h + 25 = \frac{100 - 50\sqrt{3}}{\sqrt{3}-1} + 25$$

$$= \frac{100 - 50\sqrt{3} + 25\sqrt{3} - 25}{\sqrt{3}-1}$$

$$= \frac{75 - 25\sqrt{3}}{(\sqrt{3}-1)} = \frac{25(3-\sqrt{3})}{\sqrt{3}-1}$$

$$= \frac{25(3-\sqrt{3})}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{25(3\sqrt{3} + 3 - 3 - \sqrt{3})}{3-1} = \frac{25(2\sqrt{3})}{2}$$

$$= 25\sqrt{3} \text{ m}$$

8. The angles of elevation of the top of a tower from the top and foot of a pole are respectively 30° and 45°. If h_T is the height of the tower and h_P is the height of the pole, then which of the following are correct?

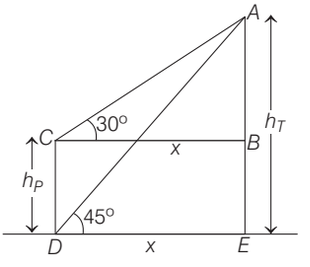
1. $\dfrac{2h_Ph_T}{3+\sqrt{3}} = h_P^2$ 2. $\dfrac{h_T - h_P}{\sqrt{3}+1} = \dfrac{h_P}{2}$

3. $\dfrac{2(h_P + h_T)}{h_P} = 4 + \sqrt{3}$

Select the correct answer using the code given below.

(a) 1 and 3 only (b) 2 and 3 only
(c) 1 and 2 only (d) 1, 2 and 3

(**c**)



Let CD be the pole and AE be the tower.

Now, In ΔABC,

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h_T - h_P}{x}$$

$$\Rightarrow \quad x = \sqrt{3}(h_T - h_P) \qquad \ldots (i)$$

Now, in ΔAED,

$$\tan 45° = \frac{AE}{DE} \Rightarrow 1 = \frac{h_T}{x}$$

$$\Rightarrow \quad x = h_T \qquad \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$h_T = \sqrt{3}(h_T - h_P)$$

$$\Rightarrow \quad h_T = \sqrt{3}h_T - \sqrt{3}h_P$$

$$\Rightarrow (\sqrt{3}-1)h_T = \sqrt{3}\,h_P$$

$$\Rightarrow \quad h_T = \frac{\sqrt{3}}{\sqrt{3}-1}h_P$$

1. $\frac{2h_P\, h_T}{3+\sqrt{3}} = \frac{2}{3+\sqrt{3}} h_P \cdot \frac{\sqrt{3}}{\sqrt{3}-1} h_P$

$= \frac{2\sqrt{3}}{3\sqrt{3}-3+3-\sqrt{3}} h_p^2$

$= \frac{2\sqrt{3}}{2\sqrt{3}} h_p^2 = h_p^2$

2. $\frac{h_T - h_P}{\sqrt{3}+1} = \frac{\frac{\sqrt{3}}{\sqrt{3}-1}h_P - h_P}{\sqrt{3}+1}$

$= \frac{\sqrt{3}h_P - (\sqrt{3}-1)h_P}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{\sqrt{3}h_P - \sqrt{3}h_P + h_P}{3-1} = \frac{h_P}{2}$

3. $\frac{2(h_P + h_T)}{h_P} = \frac{2\left(h_P + \frac{\sqrt{3}}{\sqrt{3}-1}h_P\right)}{h_P}$

$= 2\left(\frac{\sqrt{3}-1+\sqrt{3}}{\sqrt{3}-1}\right)$

$= 2\left(\frac{2\sqrt{3}-1}{\sqrt{3}-1}\right) = \frac{4\sqrt{3}-2}{\sqrt{3}-1}$

$= \frac{(4\sqrt{3}-2)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{12+4\sqrt{3}-2\sqrt{3}-2}{3-1}$

$= \frac{10+2\sqrt{3}}{2} = 5+\sqrt{3}$

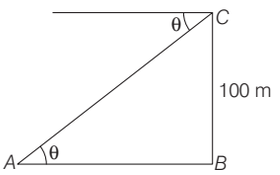
2017 (I)

9. From the top of a lighthouse, 100 m high, the angle of depression of a boat is $\tan^{-1}\left(\frac{5}{12}\right)$. What is the distance between the boat and the lighthouse?

(a) 120 m (b) 180 m
(c) 240 m (d) 360 m

(**c**) Let C be the top of a lighthouse and the angle of depression of a boat be θ from C then

$\theta = \tan^{-1}\frac{5}{12}$ or $\tan\theta = \frac{5}{12}$



In ΔBAC, $\tan\theta = \frac{100}{AB}$

$\Rightarrow \quad \frac{5}{12} = \frac{100}{AB}$ [given, $\tan\theta = \frac{5}{12}$]

$\Rightarrow \quad AB = \frac{12\times 100}{5}$

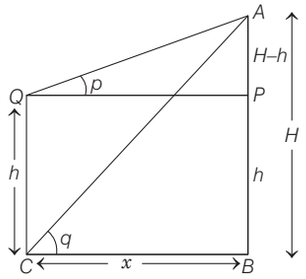
$\therefore \quad AB = 240$ m

2016 (II)

10. The top of a hill when observed from the top and bottom of a building of height h is at angles of elevation p and q respectively. What is the height of the hill?

(a) $\frac{h\cot q}{\cot q - \cot p}$ (b) $\frac{h\cot p}{\cot p - \cot q}$
(c) $\frac{2h\tan p}{\tan p - \tan q}$ (d) $\frac{2h\tan q}{\tan q - \tan p}$

(**b**) Let the height of hill AB be H and distance between building and hill be x.



Given, $CQ = BP = h$ = height of building,

$\angle AQP = p$

and $\angle ACB = q$

In ΔABC, $\tan q = \frac{H}{x}$

$\Rightarrow \quad x = \frac{H}{\tan q}$ … (i)

In ΔAPQ,

$\tan p = \frac{H-h}{x} \Rightarrow x = \frac{H-h}{\tan p}$ … (ii)

From Eqs. (i) and (ii), we get

$\frac{H}{\tan q} = \frac{H-h}{\tan p}$

$\Rightarrow \quad H\tan p = H\tan q - h\tan q$

$\Rightarrow \quad H(\tan q - \tan p) = h\tan q$

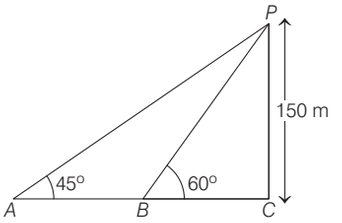
$\Rightarrow \quad H = \frac{h\tan q}{\tan q - \tan p}$

$\therefore \quad H = \frac{h\cot p}{\cot p - \cot q}$

11. A moving boat is observed from the top of a cliff of 150 m height. The angle of depression of the boat changes from 60° to 45° in 2 minutes. What is the speed to the boat in metres per hour?

(a) $\frac{4500}{\sqrt{3}}$ (b) $\frac{4500(\sqrt{3}-1)}{\sqrt{3}}$
(c) $4500\sqrt{3}$ (d) $\frac{4500(\sqrt{3}+1)}{\sqrt{3}}$

(**b**) Given, height of cliff is 150 m.



Let P is top of the cliff PC then

$PC = 150$ m.

Let A and B are the position of boat

In ΔPBC, $\tan 60° = \frac{PC}{BC} \Rightarrow \sqrt{3} = \frac{150}{BC}$

$\Rightarrow \quad BC = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$ m

In ΔPAC, $\tan 45° = \frac{PC}{AC}$

$\Rightarrow \quad 1 = \frac{150}{AC}$

$\Rightarrow \quad AC = 150$ m

Now, distance traced in 2 min,

$AB = AC - BC = 150 - 50\sqrt{3}$

$\Rightarrow \quad AB = 50(3-\sqrt{3})$

$\therefore$ Speed of boat in metres per hour

$= \frac{\text{Distance}}{\text{Time taken in hour}} = \frac{50(3-\sqrt{3})\times 60}{2}$

$= \frac{4500(\sqrt{3}-1)}{\sqrt{3}}$

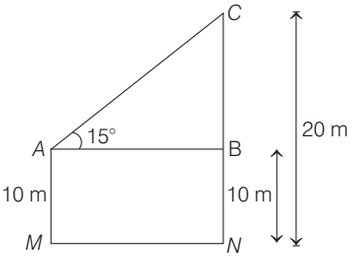
2015 (II)

12. Two poles are 10 m and 20 m high. The line joining their tops makes an angle of 15° with the horizontal. The distance between the poles is approximately equal to

(a) 36.3 m (b) 37.3 m
(c) 38.3 m (d) 39.3 m

(**b**) Here, $CB = CN - BN = 20 - 10 = 10$ m

Now, in ΔABC, $\tan 15° = \frac{BC}{AB}$



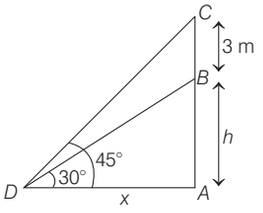
$\Rightarrow \quad AB = \frac{BC}{\tan 15°} = \frac{10}{2-\sqrt{3}} = 37.3$ m

[$\because \tan 15° = 2-\sqrt{3}$]

13. A vertical tower standing on a levelled field is mounted with a vertical flag staff of length 3 m. From a point on the field, the angles of elevation of the bottom and tip of the flag staff are 30° and 45°, respectively. Which one of the following gives the best approximation to the height of the tower?

(a) 3.90 m (b) 4.00 m
(c) 4.10 m (d) 4.25 m

(**c**) Let AB be tower of height h m and BC is flag.



In ΔDAB, $\tan 30° = \frac{h}{x}$

$\Rightarrow \quad x = \sqrt{3}\,h$ ...(i)

Again, in ΔDAC,

$$\tan 45° = \frac{h+3}{x} \Rightarrow x = h + 3$$

$\Rightarrow \quad h = x - 3 = \sqrt{3}\,h - 3$ [from Eq. (i)]

$\Rightarrow \quad h(1-\sqrt{3}) = -3$

$\therefore \quad h = \frac{3}{\sqrt{3}-1} = \frac{3(\sqrt{3}+1)}{2}$

$= 1.5 \times 2.732 = 4.098$ m

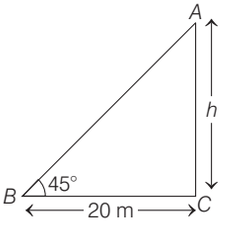
$= 4.10$ m (approx)

2015 (I)

14. The angle of elevation of the top of a tower from a point 20 m away from its base is 45°. What is the height of the tower?

(a) 10 m (b) 20 m (c) 30 m (d) 40 m

(**b**) Let AC be tower of height h m.



In ΔABC, $\tan 45° = \frac{h}{20}$

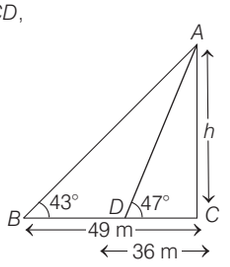
$\Rightarrow \quad h = 20$ m

15. The angles of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at distances 49 m and 36 m are 43° and 47° respectively. What is the height of the tower?

(a) 40 m (b) 42 m (c) 45 m (d) 47 m

(**b**) Let the height of tower AC be h m.

In ΔACD,



$$\tan 47° = \frac{h}{36} \quad \ldots(i)$$

and in ΔABD, $\tan 43° = \frac{h}{49}$ ...(ii)

$\Rightarrow \quad \frac{1}{\cot 43°} = \frac{h}{49} \Rightarrow \frac{49}{\cot 43°} = h$

$\Rightarrow \quad h = \frac{49}{\cot(90° - 47°)}$

$\Rightarrow \quad h = \frac{49}{\tan 47°}$

$\Rightarrow \quad h = \frac{49}{\left(\frac{h}{36}\right)}$ [from Eq. (i)]

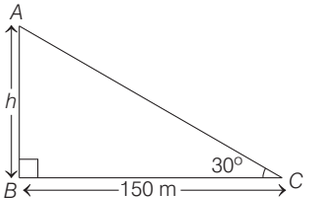
$\Rightarrow \quad h^2 = 49 \times 36$

$\therefore \quad h = 7 \times 6 = 42$ m

16. A lamp post stands on a horizontal plane. From a point situated at a distance 150 m from its foot, the angle of elevation of the top is 30°. What is the height of the lamp post?

(a) 50 m (b) $50\sqrt{3}$ m
(c) $\frac{50}{\sqrt{3}}$ m (d) 100 m

(**b**) Let the height of lamp post AB be h m and C is a point situated at a distance of 150 m from its foot B.



In ΔABC, we have

$$\tan 30° = \frac{h}{150} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{150}$$

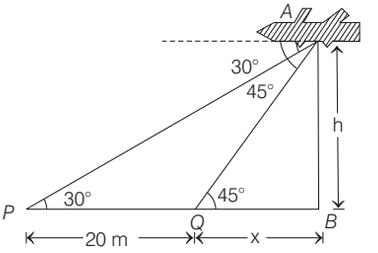
$\therefore \quad h = \frac{150}{\sqrt{3}} = \frac{150 \times \sqrt{3}}{3} = 50\sqrt{3}$ m

2014 (I)

17. From an aeroplane above a straight road the anlges of depression of two positions at a distance 20 m apart on the road are observed to be 30° and 45°. The height of the aeroplane above the ground is

(a) $10\sqrt{3}$ m
(b) $10(\sqrt{3}-1)$ m
(c) $10(\sqrt{3}+1)$ m
(d) 20 m

(**c**) Let the height of the aeroplane above the ground is h m and $QB = x$ m.



Given that, $PQ = 20$ m, $\angle APB = 30°$ and $\angle AQB = 45°$

Now, in ΔAQB,

$$\tan 45° = \frac{AB}{QB} = \frac{h}{x}$$

$\Rightarrow \quad 1 = \frac{h}{x}$

$\Rightarrow \quad x = h$ ... (i)

and in ΔAPB,

$$\tan 30° = \frac{AB}{PB} = \frac{h}{PQ + QB}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20 + x}$

$\Rightarrow \sqrt{3}\,h = 20 + x = 20 + h$ [from Eq. (i)]

$\Rightarrow \quad (\sqrt{3}\,h - h) = 20$

$\Rightarrow \quad h(\sqrt{3}-1) = 20$

$\therefore \quad h = \frac{20}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$

[by rationalisation]

$$\frac{20(\sqrt{3}+1)}{(3-1)} = \frac{20(\sqrt{3}+1)}{2} = 10(\sqrt{3}+1)$$

Hence, the required height is

$10(\sqrt{3}+1)$ m.

14

PROPERTIES OF TRIANGLES

2019 (II)

1. If the angles of a triangle ABC are in AP and $b : c = \sqrt{3} : \sqrt{2}$, then what is the measure of angle A?

(a) 30° (b) 45° (c) 60° (d) 75°

(**d**) Angles of a triangle ABC are in AP, then $2B = A + C$

We know that, $A + B + C = 180°$

$\Rightarrow \quad 3B = 180° \Rightarrow B = 60°$

By sine rule, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin A}{a} = \frac{\sin 60°}{b} = \frac{\sin C}{c}$

Take II and III, $\frac{\sin 60°}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin 60°}{\sin C} = \frac{b}{c} \Rightarrow \frac{\sqrt{3}/2}{\sin C} = \frac{\sqrt{3}}{\sqrt{2}}$

$[\because$ Given, $b : c = \sqrt{3} : \sqrt{2}]$

$\Rightarrow \quad \sin C = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin C = \sin 45° \Rightarrow C = 45°$

$\therefore \quad A = 180° - (B + C)$

$= 180° - (60° + 45°) = 75°$

2019 (I)

2. If the angles of a triangle ABC are in the ratio $1 : 2 : 3$, then the corresponding sides are in the ratio

(a) 1 : 2 : 3 (b) 3 : 2 : 1
(c) $1 : \sqrt{3} : 2$ (d) $1 : \sqrt{3} : \sqrt{2}$

(**c**) We have, angle of triangle ABC are in the ratio 1 : 2 : 3

$\therefore \quad x + 2x + 3x = 180°$

$\Rightarrow \quad x = 30°$

$\therefore$ Angles of triangle are 30°, 60°, 90°.

We know that, sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 30°} = \frac{b}{\sin 60°} = \frac{c}{\sin 90°}$$

$$\Rightarrow \quad \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{1}$$

$\therefore \quad a : b : c = 1 : \sqrt{3} : 2$

2018 (II)

3. Consider the following statements:

Statement I If the line segment joining the points $P(m, n)$ and $Q(r, s)$ subtends an angle α at the origin, then

$$\cos\alpha = \frac{ms - nr}{\sqrt{(m^2 + n^2)(r^2 + s^2)}}.$$

Statement II In any triangle ABC, it is true that

$a^2 = b^2 + c^2 - 2bc \cos A$.

Which one of the following is correct in respect of the above two statements?

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of statement I.
(b) Both Statement I and Statement II are true, but Statement II is not the correct explanation II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

(**d**) **Statement I** If the line segment joining the point $P(m, n)$ and $Q(r, s)$ subtends angle α at origin, then

$$\cos\alpha = \frac{mr + ns}{\sqrt{m^2 + n^2}\sqrt{r^2 + s^2}}$$

So, Statement I is not correct

Statement II In any triangle ABC

$a^2 = b^2 + c^2 - 2bc \cos A$

Statement II is correct.

4. What is the area of the triangle with vertices

$$\left(x_1, \frac{1}{x_1}\right), \left(x_2, \frac{1}{x_2}\right), \left(x_3, \frac{1}{x_3}\right)?$$

(a) $|(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)|$

(b) 0

(c) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{x_1 x_2 x_3}\right|$

(d) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1 x_2 x_3}\right|$

(**d**) Area of triangle whose vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Area of ΔABC

$$A\left(x_1, \frac{1}{x_1}\right), B\left(x_2, \frac{1}{x_2}\right), C\left(x_3, \frac{1}{x_3}\right)$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & \frac{1}{x_1} & 1 \\ x_2 & \frac{1}{x_2} & 1 \\ x_3 & \frac{1}{x_3} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x_1 & \frac{1}{x_1} & 1 \\ x_2 & \frac{1}{x_2} & 1 \\ x_3 & \frac{1}{x_3} & 1 \end{vmatrix}$$

$$\Rightarrow \quad \frac{1}{2x_1 x_2 x_3}\begin{vmatrix} x_1^2 & 1 & x_1 \\ x_2^2 & 1 & x_2 \\ x_3^2 & 1 & x_3 \end{vmatrix}$$

$$= \left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1 x_2 x_3}\right|$$

2018 (I)

5. In a ΔABC, if $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then what is $\angle B$ equal to ?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

(**b**) We have, $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$

Now, from sine formula

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\frac{2}{3}}{2} = \frac{\sin B}{3}$$

$$\Rightarrow \quad \sin B = 1$$

$$\therefore \quad B = \frac{\pi}{2} \qquad \left[\because \sin\frac{\pi}{2} = 1\right]$$

2017 (II)

6. In a triangle ABC, $a - 2b + c = 0$. The value of $\cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right)$ is

(a) $\frac{9}{2}$ (b) 3 (c) $\frac{3}{2}$ (d) 1

(**b**) We have,

$$a - 2b + c = 0$$

$$\Rightarrow \quad a + c = 2b \qquad \ldots(i)$$

Now, we know that

$$\cot\frac{A}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$$

and $\cot\frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$$\therefore \cot\frac{A}{2}\cdot\cot\frac{C}{2}$$

$$= \sqrt{\frac{s(s-a)}{(s-b)(s-c)} \times \frac{s(s-c)}{(s-a)(s-b)}}$$

$$= \frac{s}{s-b} = \frac{(a+b+c)/2}{\frac{(a+b+c)}{2} - b}$$

$$= \frac{(a+b+c)}{a+b+c-2b} = \frac{2b+b}{2b-b}$$

[from Eq. (i)]

$$= \frac{3b}{b} = 3$$

2015 (II)

Directions (Q. Nos. 7 and 8) *Consider a ΔABC satisfying*

$$2a\sin^2\left(\frac{C}{2}\right) + 2c\sin^2\left(\frac{A}{2}\right) = 2a + 2c - 3b$$

7. The sides of the triangle are in

(a) GP (b) AP
(c) HP
(d) Neither in GP nor AP nor in HP

(**b**) We have, $2a\sin^2\frac{C}{2} + 2c\sin^2\frac{A}{2}$

$$= 2a + 2c - 3b$$

$$\Rightarrow a(1 - \cos C) + c(1 - \cos A) = 2a + 2c - 3b$$

$$\Rightarrow (a + c) - (a\cos C + c\cos A) = 2a + 2c - 3b$$

$$\Rightarrow a + c - b = 2a + 2c - 3b$$

$$[\because b = a\cos C + c\cos A]$$

$$\Rightarrow -a - c = -2b \Rightarrow a + c = 2b$$

Hence, a, b, c are in AP.

8. $\sin A, \sin B, \sin C$ are in

(a) GP (b) AP
(c) HP
(d) Neither in GP nor in AP nor in HP

(**b**) Since, a, b, c are in AP.

So, $K\sin A$, $K\sin B$, $K\sin C$ are in AP. (by sine rule)

We know that if each term of an AP is multiplied or divide by a non-zero constant K, then resulting sequence is also an AP.

$\therefore$ $\sin A, \sin B, \sin C$ are in AP.

2015 (I)

9. In a ΔABC, $a = (1+\sqrt{3})$ cm, $b = 2$ cm and $\angle C = 60°$, then the other two angles are

(a) 45° and 75° (b) 30° and 90°
(c) 105° and 15° (d) 100° and 20°

(**a**) Here, $a = 1 + \sqrt{3}$,

$$b = 2, \angle C = 60°$$

$$\therefore \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \quad \cos 60° = \frac{(1+\sqrt{3})^2 + (2)^2 - c^2}{2(1+\sqrt{3})\cdot 2}$$

$$\Rightarrow \quad \frac{1}{2} \times 2 \times 2 \times (1+\sqrt{3}) = 1 + 3 + 2\sqrt{3} + 4 - c^2$$

$$\Rightarrow \quad 2 + 2\sqrt{3} = 4 + 2\sqrt{3} + 4 - c^2$$

$$\Rightarrow \quad c^2 = 8 + 2\sqrt{3} - 2 - 2\sqrt{3} = 6$$

$$\Rightarrow \quad c = \sqrt{6}$$

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{4 + 6 - (1+\sqrt{3})^2}{2 \times 2 \times \sqrt{6}} = \frac{10 - 1 - 3 - 2\sqrt{3}}{4\sqrt{6}}$$

$$= \frac{6 - 2\sqrt{3}}{4\sqrt{6}} = \frac{3 - \sqrt{3}}{2\sqrt{6}} = \frac{3\sqrt{6} - \sqrt{18}}{12}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4} = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$= \cos 75°$$

$\therefore$ $A = 75°$, $B = 45°$

and $C = 60°$

2014 (II)

10. In a ΔABC, if $c = 2$, $A = 45°$ and $a = 2\sqrt{2}$, then what is C equal to?

(a) 30° (b) 15°
(c) 45° (d) None of these

(**a**) By sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\therefore \quad \frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\Rightarrow \quad \sin C = \frac{c \cdot \sin A}{a} = \frac{2 \cdot \sin 45°}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} = \sin 30°$$

$$\therefore \quad C = 30°$$

11. Consider the following statements

I. There exists no ΔABC for which $\sin A + \sin B = \sin C$.

II. If the angles of a triangle are in the ratio 1 : 2 : 3, then its sides will be in the ratio $1 : \sqrt{3} : 2$.

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(**c**) I. Given that, $\sin A + \sin B = \sin C$

$$(ak) + (bk) = (ck)$$

$$\Rightarrow \quad a + b = c$$

$$\left[\text{by sine rule, } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k\right]$$

i.e., the sum of two sides of ΔABC is equal to the third side.

But it is not possible, because by triangle inequality, the sum of the length of two sides of a triangle is always greater than the length of the third side.

Hence, there exists no ΔABC for which $\sin A + \sin B = \sin C$.

II. Given that, the ratio of the angles of a triangle are $A : B : C = 1 : 2 : 3$.

Let $A = \alpha$, $B = 2\alpha$ and $C = 3\alpha$

We know that, $A + B + C = 180°$

[since, sum of all interior angles of a triangle is 180°]

$$\Rightarrow \quad \alpha + 2\alpha + 3\alpha = 180°$$

$$\Rightarrow \quad 6\alpha = 180° \Rightarrow \alpha = 30°$$

$$\therefore \quad A = 30°, B = 60° \text{ and } C = 90°$$

So, the required ratio of its sides,

$$a : b : c = \sin A : \sin B : \sin C$$

[by sine rule]

$$= \sin 30° : \sin 60° : \sin 90°$$

$$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1$$

$$= 1 : \sqrt{3} : 2$$

15

COORDINATE SYSTEM AND STRAIGHT LINE

2019 (II)

1. What is the distance between the points $P(m\cos 2\alpha, m\sin 2\alpha)$ and $Q(m\cos 2\beta, m\sin 2\beta)$?

(a) $|2m\sin(\alpha-\beta)|$
(b) $|2m\cos(\alpha-\beta)|$
(c) $|m\sin(2\alpha-2\beta)|$
(d) $|m\sin(2\alpha-2\beta)|$

(**a**) Given points, $p(m\cos 2\alpha, m\sin 2\alpha)$ and $Q(m\cos 2\beta, m\sin 2\beta)$

$$\therefore PQ = \left|\sqrt{(m\cos 2\beta - m\cos 2\alpha)^2 + (m\sin 2\beta - m\sin 2\alpha)^2}\right|$$

[$\because$ Distance between two points (x_1, y_1) and (x_2, y_2)

$$= \left|\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\right|$$

$$= \left|\sqrt{\begin{matrix} m^2\cos^2 2\beta + m^2\cos^2 2\alpha \\ -2m^2\cos 2\beta\cos 2\alpha + m^2\sin^2 2\beta \\ +m^2\sin^2 2\alpha - 2m^2\sin 2\beta\sin 2\alpha \end{matrix}}\right|$$

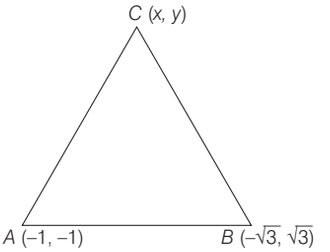
$$= \left|m\sqrt{\begin{matrix} (\cos^2 2\beta + \sin^2 2\beta) \\ +(\cos^2 2\alpha + \sin^2 2\alpha) \\ -2(\cos 2\beta\cos 2\alpha \\ +\sin 2\beta\sin 2\alpha) \end{matrix}}\right|$$

$$= |m\sqrt{1+1-2\cos(2\alpha-2\beta)}|$$
$$= |m\sqrt{2[1-\cos 2(\alpha-\beta)]}|$$
$$= \left|m\sqrt{2\left[1-1+2\sin^2\frac{2(\alpha-\beta)}{2}\right]}\right|$$
$$= |m\sqrt{2\times 2\sin^2(\alpha-\beta)}|$$
$$= |2m\sin(\alpha-\beta)|$$

2. An equilateral triangle has one vertex at $(-1,-1)$ and another vertex at $(-\sqrt{3}, \sqrt{3})$. The third vertex may lie on

(a) $(-\sqrt{2}, \sqrt{2})$ (b) $(\sqrt{2}, -\sqrt{2})$
(c) (1, 1) (d) (1, – 1)

(**c**) Consider two vertices of an equilateral triangle are $A(-1,-1)$ and $B(-\sqrt{3}, \sqrt{3})$. Let third vertex x be $C(x, y)$.



$\because \Delta ABC$ is equilateral

$$\therefore \quad AC = AB$$
$$\sqrt{(x+1)^2 + (y+1)^2} = \sqrt{(-\sqrt{3}+1)^2 + (\sqrt{3}+1)^2}$$
$$\Rightarrow \quad x^2 + 1 + 2x + y^2 + 1 + 2y = 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}$$
$$\Rightarrow \quad x^2 + y^2 + 2x + 2y + 2 = 8$$
$$\Rightarrow \quad x^2 + y^2 + 2x + 2y = 6$$

From option only point (1, 1) is satisfying of it equation. Hence, the third vertex may lie on (1, 1).

3. The equation $ax + by + c = 0$ represents a straight line

(a) for all real numbers, a, b and c
(b) only when $a \neq 0$
(c) only when $b \neq 0$
(d) only when at least one of a and b is non-zero.

(**d**) The equation $ax + by + c = 0$ represents a straight line only when at least one of a and b is non zero.

4. What is the angle between the lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\beta - y\cos\beta = a$?

(a) $\beta - \alpha$ (b) $\pi + \beta - \alpha$
(c) $\frac{(\pi + 2\beta + 2\alpha)}{2}$ (d) $\frac{(\pi - 2\beta + 2\alpha)}{2}$

(**d**) The equations of given lines

$x\cos\alpha + y\sin\alpha = a$ … (i)
and $x\sin\beta - y\cos\beta = a$ … (ii)

Slope of Eq. (i), $m_1 = \frac{-\cos\alpha}{\sin\alpha} = -\cot\alpha$

$$= \tan\left(\frac{\pi}{2} + \alpha\right)$$

Slope of Eq. (ii), $m_2 = \frac{-\sin\beta}{-\cos\beta} = \tan\beta$

Let θ be the angle between the lines, then

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$
$$= \frac{\tan\left(\frac{\pi}{2} + \alpha\right) - \tan\beta}{1 + \tan(\pi - \alpha)\tan\beta}$$
$$\tan\theta = \tan\left(\frac{\pi}{2} + \alpha - \beta\right)$$
$$\theta = \frac{\pi}{2} + \alpha - \beta = \frac{\pi + 2\alpha - 2\beta}{2}$$

2019 (I)

5. Consider the following statements

1. For an equation of a line, $x\cos\theta + y\sin\theta = p$, in normal form, the length of the perpendicular from the point (α,β) to the line is $|\alpha\cos\theta + \beta\sin\theta + p|$.

The length of the perpendicular from the point (α,β) to the line $\frac{x}{a} + \frac{y}{b} = 1$ is $\left|\frac{a\alpha + b\beta - ab}{\sqrt{a^2 + b^2}}\right|$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*d*) **1.** Equation of line
$x\cos\theta + y\sin\theta = p$
Perpendicular distance from (α,β) to the given line is

$$\left|\frac{\alpha\cos\theta + \beta\sin\theta - p}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$$

$= (\alpha\cos\theta + \beta\sin\theta - p)$
Hence, Statement 1 is incorrect.
2. Length of the perpendicular from the point (α,β) to the line $\frac{x}{a} + \frac{y}{b} = 1$ is

$$\left|\frac{\frac{\alpha}{a} + \frac{\beta}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| = \left|\frac{\alpha b + a\beta - ab}{\sqrt{a^2 + b^2}}\right|$$

Hence, Statement 2 is incorrect.
∴ Neither 1 nor 2.

6. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. What is the value of c?
(a) 2 (b) –2 (c) 4 (d) –4

(*d*) The points (1, 3) and (5, 1) are two opposite vertex of rectangle. The other two vertices lie on the line $y = 2x + c$.
∴ The mid point of vertices lie on the line
i.e. $\left(\frac{1+5}{2}, \frac{3+1}{2}\right) \equiv (3, 2)$ lie on the line

$$y = 2x + c$$
$$\therefore \quad 2 = 2(3) + c \Rightarrow c = -4$$

7. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b?
(a) 1 (b) 3 (c) 6 (d) $\frac{1}{2}$

(*c*) The lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent.

$$\therefore \quad \begin{vmatrix} 3 & 4 & -1 \\ 1 & -1 & -5 \\ 5 & b & -3 \end{vmatrix} = 0$$

$\Rightarrow 3(3 + 5b) - 4(-3 + 25) - 1(b + 5) = 0$
$\Rightarrow \quad 9 + 15b + 12 - 100 - b - 5 = 0$
$\Rightarrow \quad 14b = 84$
$\therefore \quad b = 6$

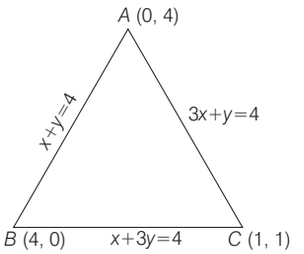
8. What is the equation of the straight line which is perpendicular to $y = x$ and passes through (3, 2)?
(a) $x - y = 5$ (b) $x + y = 5$
(c) $x + y = 1$ (d) $x - y = 1$

(*b*) Equation of line perpendicular to $y = x$ is $x + y = \lambda$.
Since, this line is passes through (3, 2)
$\therefore \quad 3 + 2 = \lambda \Rightarrow \lambda = 5$
Hence, equation of required line is $x + y = 5$.

9. The straight lines $x + y - 4 = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form a triangle, which is
(a) isosceles (b) right angled
(c) equilateral (d) scalene

(*a*) Given, equation of line

$$x + y = 4 \quad \ldots(i)$$
$$3x + y = 4 \quad \ldots(ii)$$
$$x + 3y = 4 \quad \ldots(iii)$$



On solving Eqs. (i) and (ii), we get
$x = 0, y = 4, A = (0, 4)$
On solving Eqs. (i) and (iii), we get

$x = 4, y = 0, B = (4, 0)$
On solving Eqs. (ii) and (iii), we get
$x = 1, y = 1, C = (1, 1)$
Clearly, $AC = BC$
∴ Triangle is an isosceles.

2018 (II)

10. The angle between the two lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ is given by $\tan^{-1}\theta$. What is θ equal to?
(a) $\left|\frac{lm' - l'm}{ll' - mm'}\right|$ (b) $\left|\frac{lm' + l'm}{ll' + mm'}\right|$
(c) $\left|\frac{lm' - l'm,}{ll' + mm'}\right|$ (d) $\left|\frac{lm' + l'm}{ll' - mm'}\right|$

(*c*) Given straight lines,

$$lx + my + n = 0 \quad \ldots(i)$$
$$m_1 = -\frac{l}{m}$$

and $l'x + m'y + n' = 0$

$$m_2 = -\frac{l'}{m'}$$

angle between both lines is θ

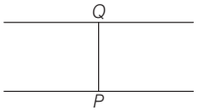
$$\therefore \tan\theta = \left|\frac{-\frac{l}{m} - \frac{l'}{m'}}{1 + \frac{ll'}{mm'}}\right| \quad \left[\because \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|\right]$$

$$= \frac{\left|\frac{-lm' - l'm}{mm'}\right|}{\frac{mm' + ll'}{mm'}} = \left|\frac{lm' - l'm}{ll' + mm'}\right|$$

11. Consider the following statements
1. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is $\frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$
2. The distance between the lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.
3. The distance between the lines $x = c_1$ and $x = c_2$ is $|c_1 - c_2|$.

Which of the above statements are correct?
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(*d*) **Statement 1** Given lines,
$y = mx + c_1$ and $y = mx + c_2$
both lines are parallel

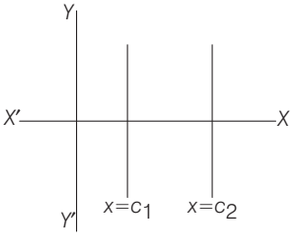


$$PQ = \left|\frac{c_1 - c_2}{\sqrt{1 + m^2}}\right|$$

So, Statement 1 is correct
Statement 2 for $ax + by + c_1 = 0$
and $ax + by + c_2 = 0$
both lines are parallel so,

$$\therefore \quad D = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

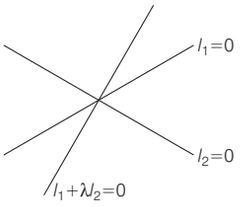
Statement 2 is correct
Statement 3 $x = c_1$
$x = c_2$
$D = |c_1 - c_2|$
Statement 3 is correct



12. What is the equation of straight line passing through the point of intersection of the lines $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$ and parallel to the line $4x + 5y - 6 = 0$?
(a) $20x + 25y - 54 = 0$
(b) $25x + 20y - 54 = 0$
(c) $4x + 5y - 54 = 0$
(d) $4x + 5y - 45 = 0$

› ***(a)*** Given lines,

$$\frac{x}{2}+\frac{y}{3}=1 \quad \text{and} \quad \frac{x}{3}+\frac{y}{2}=1$$



Equation of line passing through the point of intersection are $l_1 + \lambda l_2 = 0$

$(3x + 2y - 6) + \lambda (2x + 3y - 6) = 0$ ...(i)

Slope of line

$$\therefore \quad \frac{-(3+2\lambda)}{2+3\lambda}=-\frac{4}{5}$$

$$\Rightarrow \quad 15+10\lambda=8+12\lambda$$

$$\Rightarrow \quad 7=2\lambda \Rightarrow \lambda=\frac{7}{2}$$

$$(3x+2y-6)+\frac{7}{2}(2x+3y-6)=0$$

$$\Rightarrow 2(3x+2y-6)+14x+21y-42=0$$

$$\Rightarrow 6x+4y-12+14x+21y-42=0$$

$$\therefore \quad 20x+25y-54=0$$

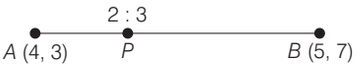
2018 (I)

13. What is the distance between the points which divide the line segment joining $(4, 3)$ and $(5, 7)$ internally and externally in the ratio $2 : 3$?

(a) $\frac{12\sqrt{17}}{5}$ (b) $\frac{13\sqrt{17}}{5}$ (c) $\frac{\sqrt{17}}{5}$ (d) $\frac{6\sqrt{17}}{5}$

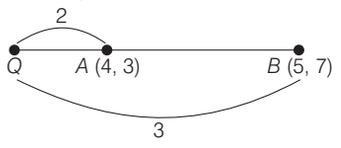
› ***(a)*** Let P and Q be the points which divides $A(4, 3)$ and $B(5, 7)$ internally and externally in the ratio $2 : 3$ respectively.

$$\therefore P=\left(\frac{2\times5+3\times4}{2+3},\frac{2\times7+3\times3}{2+3}\right)$$



$$=\left(\frac{22}{5},\frac{23}{5}\right)$$

$$\text{and } Q=\left(\frac{2\times5-3\times4}{2-3},\frac{2\times7-3\times3}{2-3}\right)$$



$= (2, -5)$

∴ Required distance = PQ

$$=\sqrt{\left(2-\frac{22}{5}\right)^2+\left(-5-\frac{23}{5}\right)^2}=\frac{12}{5}\sqrt{17}$$

14. What is the angle between the straight lines $(m^2 - mn)\, y = (mn + n^2)\, x + n^3$ and $(mn + m^2)\, y = (mn - n^2)x + m^3$, where $m > n$?

(a) $\tan^{-1}\left(\frac{2mn}{m^2+n^2}\right)$

(b) $\tan^{-1}\left(\frac{4m^2n^2}{m^4-n^4}\right)$

(c) $\tan^{-1}\left(\frac{4m^2n^2}{m^4+n^4}\right)$

(d) 45°

› (***b***) Given equations of lines are

$(m^2 - mn)\, y = (mn + n^2)\, x + n^3$

and $(mn + m^2)y = (mn - n^2)\, x + m^3$

Given equation of lines can be written as

$$y=\frac{mn+n^2}{m^2-mn}x+\frac{n^3}{m^2-mn}$$

and $$y=\frac{mn-n^2}{mn+m^2}x+\frac{m^3}{mn+m^2}$$

Let m_1 and m_2 be the slopes of given lines.

$$\therefore \quad m_1=\frac{mn+n^2}{m^2-mn} \text{ and } m_2=\frac{mn-n^2}{mn+m^2}$$

If θ is the angle between these lines, then

$$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$$

$$=\frac{\frac{mn+n^2}{m^2-mn}-\frac{mn-n^2}{mn+m^2}}{1+\frac{mn+n^2}{m^2-mn}\cdot\frac{mn-n^2}{mn+m^2}}$$

$$=\frac{(mn+n^2)(mn+m^2)-(mn-n^2)(m^2-mn)}{(m^2-mn)(mn+m^2)+(mn+n^2)(mn-n^2)}$$

$$=\frac{m^2n^2+m^3n+mn^3+m^2n^2-m^3n+m^2n^2}{m^4-m^2n^2+m^2n^2+m^2n^2-n^4}$$

$$+m^2n^2-mn^3=\frac{4m^2n^2}{m^4-n^4}$$

$$\therefore \quad \theta=\tan^{-1}\left(\frac{4m^2n^2}{m^4-n^4}\right)$$

15. What is the equation of the straight line cutting-off an intercept 2 from the negative direction of Y-axis and inclined at 30° with the positive direction of X-axis ?

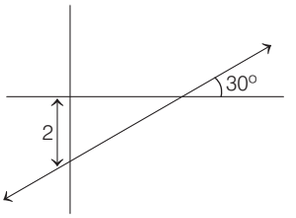
(a) $x - 2\sqrt{3}y - 3\sqrt{2} = 0$

(b) $x + 2\sqrt{3}y - 3\sqrt{2} = 0$

(c) $x + \sqrt{3}y - 2\sqrt{3} = 0$

(d) $x - \sqrt{3}y - 2\sqrt{3} = 0$

› (***d***) From the given figure, it is clear that



Slope of line = $\tan 30° = \frac{1}{\sqrt{3}}$ and line passes through the point $(0, -2)$.

∴ Equation of line is

$$y-(-2)=\frac{1}{\sqrt{3}}(x-0)$$

$$\Rightarrow \quad y+2=\frac{1}{\sqrt{3}}x$$

$$\Rightarrow \quad \sqrt{3}\,y+2\sqrt{3}=x$$

$$\therefore \quad x-\sqrt{3}y-2\sqrt{3}=0$$

16. What is the equation of the line passing through the point of intersection of the lines $x + 2y - 3 = 0$ and $2x - y + 5 = 0$ and parallel to the line $y - x + 10 = 0$?

(a) $7x - 7y + 18 = 0$

(b) $5x - 7y + 18 = 0$

(c) $5x - 5y + 18 = 0$

(d) $x - y + 5 = 0$

› (***c***) Equation of line passing through intersection point of lines $x + 2y - 3 = 0$ and $2x - y + 5 = 0$ is

$x + 2y - 3 + \lambda (2x - y + 5) = 0$ ...(i)

$\Rightarrow (1 + 2\lambda)x + (2 - \lambda)y + 5\lambda - 3 = 0$

$$\therefore \text{ Slope of above line} = -\frac{(1+2\lambda)}{(2-\lambda)}$$

Since line is parallel to $y - x + 10 = 0$

$$\Rightarrow \quad \frac{-(1+2\lambda)}{(2-\lambda)}=\frac{-(-1)}{1}$$

$$\Rightarrow \quad -(1+2\lambda)=2-\lambda$$

$$\Rightarrow \quad -1-2\lambda=2-\lambda$$

$$\Rightarrow \quad -\lambda=3$$

$$\Rightarrow \quad \lambda=-3$$

On putting $\lambda = -3$ in Eq. (i), we get

$x + 2y - 3 - 3(2x - y + 5) = 0$

$$\therefore \quad 5x-5y+18=0$$

Which is equation of required line.

17. Consider the following statements

I. The length p of the perpendicular from the origin to the line $ax + by = c$ satisfies the relation $p^2=\frac{c^2}{a^2+b^2}$.

II. The length p of the perpendicular from the origin

to the line $\frac{x}{a}+\frac{y}{b}=1$ satisfied the relation $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

III. The length p of the perpendicular from the origin to the line $y=mx+c$ satisfies the relation $\frac{1}{p^2}=\frac{1+m^2+c^2}{c^2}$.

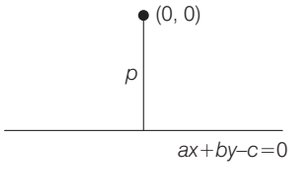
Which of the above is/are correct?

(a) I, II and III (b) I only
(c) I and II (d) II only

(**c**) We know that perpendicular distance of the line $Ax+By+C=0$ from the point $(x_1, y_1,)$ is given as

$$\text{Distance}=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$$

Statement I

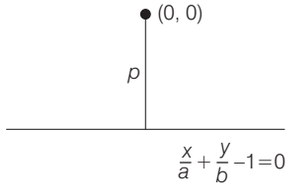


$$\therefore \quad p=\left|\frac{a\cdot 0+b\cdot 0-c}{\sqrt{a^2+b^2}}\right|$$

$$\Rightarrow \quad p=\frac{c}{\sqrt{a^2+b^2}} \Rightarrow p^2=\frac{c^2}{a^2+b^2}$$

It is true.

Statement II

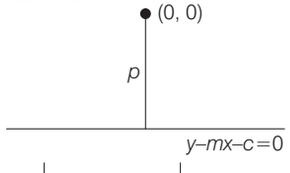


$$\therefore \quad p=\left|\frac{\frac{0}{a}+\frac{0}{b}-1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}\right|$$

$$\Rightarrow \quad p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} \Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

It is true.

Statement III



$$\therefore \quad p=\left|\frac{0-m\times 0-c}{\sqrt{(-m)^2+(1)^2}}\right|$$

$$\Rightarrow \quad p=\frac{c}{\sqrt{m^2+1}} \Rightarrow \frac{1}{p^2}=\frac{1+m^2}{c^2}$$

It is false.

18. What is the equation of the straight line passing through the point (2, 3) and making an intercept on the positive Y-axis equal to twice its intercept on the positive X-axis?

(a) $2x+y=5$ (b) $2x+y=7$
(c) $x+2y=7$ (d) $2x-y=1$

(**b**) Let the equation of line be

$$\frac{x}{a}+\frac{y}{b}=1$$

It is given that, $b=2a$ and line passes through the point (2, 3).

$$\therefore \quad \frac{2}{a}+\frac{3}{2a}=1$$

$$\Rightarrow \quad \frac{4+3}{2a}=1$$

$$\Rightarrow \quad 7=2a$$

$$\Rightarrow \quad a=\frac{7}{2}$$

$$\Rightarrow \quad b=2a=2\times\frac{7}{2}=7$$

$\therefore$ Equation of line is $\frac{x}{7/2}+\frac{y}{7}=1$

$$\Rightarrow \quad 2x+y=7$$

19. The perpendiculars that fall from any point of the straight line $2x+11y=5$ upon the two straight lines $24x+7y=20$ and $4x-3y=2$ are

(a) 12 and 4 respectively
(b) 11 and 5 respectively
(c) Equal to each other
(d) Not equal to each other

(**c**) Let $(-3, 1)$ be a point on $2x+11y=5$

Now, perpendicular from $(-3, 1)$ on $24x+7y=20$

$$=\left|\frac{24(-3)+7(1)-20}{\sqrt{(24)^2+(7)^2}}\right|$$

$$=\left|\frac{-72+7-20}{\sqrt{576+49}}\right|=\left|\frac{-85}{25}\right|=\frac{17}{5}$$

Again, perpendicular from $(-3, 1)$ on $4x-3y=2$

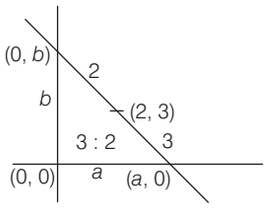
$$=\left|\frac{4(-3)-3(1)-2}{\sqrt{4^2+(-3)^2}}\right|=\left|\frac{-12-3-2}{\sqrt{16+9}}\right|=\frac{17}{5}$$

$\therefore$ Both perpendicular are equal to each other.

20. The equation of the line, when the portion of it intercepted between the axes is divided by the point (2, 3) in the ratio of 3 : 2, is

(a) Either $x+y=4$ or $9x+y=12$
(b) Either $x+y=5$ or $4x+9y=30$
(c) Either $x+y=4$ or $x+9y=12$
(d) Either $x+y=5$ or $9x+4y=30$

(**d**) **Case I**



From above figure,

$$\frac{2a+3\times 0}{2+3}=2 \text{ and } \frac{2\times 0+3b}{2+3}=3$$

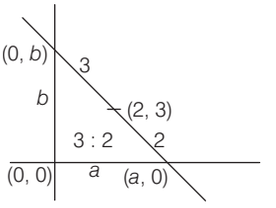
$$\Rightarrow \quad 2a=10 \text{ and } 3b=15$$

$$\Rightarrow \quad a=5 \text{ and } b=5$$

$\therefore$ Equation of line is $\frac{x}{5}+\frac{y}{5}=1$

$$\Rightarrow \quad x+y=5$$

Case II



From above figure,

$$\frac{3a+2\times 0}{3+2}=2 \text{ and } \frac{3\times 0+2b}{3+2}=3$$

$$\Rightarrow \quad 3a=10 \text{ and } 2b=15$$

$$\Rightarrow \quad a=\frac{10}{3} \text{ and } b=\frac{15}{2}$$

$\therefore$ Equation of line is

$$\frac{x}{10/3}+\frac{y}{15/2}=1$$

$$\Rightarrow \quad 9x+4y=30$$

21. What is the distance between the straight lines $3x+4y=9$ and $6x+8y=15$?

(a) $\frac{3}{2}$ (b) $\frac{3}{10}$
(c) 6 (d) 5

(**b**) Given equation of straight lines are

$$3x+4y=9 \quad \ldots \text{(i)}$$

and $\quad 6x+8y=15$

$$\Rightarrow \quad 3x+4y=\frac{15}{2} \quad \ldots \text{(ii)}$$

$$\therefore \text{ Required distance}=\frac{\left|9-\frac{15}{2}\right|}{\sqrt{3^2+4^2}}$$

[$\because$ distance between two lines $ax+by=c_1$ and $ax+by=c_2$ is given by $\left|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right|$]

$$=\frac{\frac{3}{2}}{5}=\frac{3}{10}$$

2017 (II)

22. The points $(a, b), (0, 0), (-a, -b)$ and (ab, b^2) are

(a) the vertices of a parallelogram
(b) the vertices of a rectangle
(c) the vertices of a square
(d) collinear

(***d***) All given point lie on the line having equation $ay = bx$.
$\therefore$ All point are collinear.

23. The angle between the lines $x + y - 3 = 0$ and $x - y + 3 = 0$ is α and the acute angle between the lines $x - \sqrt{3}y + 2\sqrt{3} = 0$ and $\sqrt{3}x - y + 1 = 0$ is β. Which one of the following is correct?

(a) $\alpha = \beta$ (b) $\alpha > \beta$
(c) $\alpha < \beta$ (d) $\alpha = 2\beta$

(***b***) We have, equation of lines

$$x + y - 3 = 0$$

and $$x - y + 3 = 0$$

$\therefore$ Slopes of the above lines are -1 and 1 respectively.

Since product of their slope is -1

$\therefore$ Angle between these lines is 90°.

$\therefore$ $\alpha = 90°$

Also, angle between the lines
$x - \sqrt{3}y + 2\sqrt{3} = 0$ and $\sqrt{3}x - y + 1 = 0$

$$m_1 = \frac{1}{\sqrt{3}}, m_2 = \sqrt{3}$$

$$\beta = \tan^{-1}\left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}\right|$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

$\therefore$ $\alpha > \beta$

24. The distance of the point $(1, 3)$ from the line $2x + 3y = 6$, measured parallel to the line $4x + y = 4$, is

(a) $\frac{5}{\sqrt{13}}$ units (b) $\frac{3}{\sqrt{17}}$ units
(c) $\sqrt{17}$ units (d) $\frac{\sqrt{17}}{2}$ units

(***d***) The slope of the line $4x + y = 4$ is -4.

$\therefore$ Equation of a line passing through (1, 3) with slope -4 is

$$y - 3 = -4(x - 1)$$

$$\Rightarrow \quad y - 3 = -4x + 4$$

$$\Rightarrow \quad 4x + y = 7$$

The coordinate of the point of intersection of the lines

$4x + y = 7$ and $2x + 3y = 6$ is $\left(\frac{3}{2}, 1\right)$.

$\therefore$ Required distance

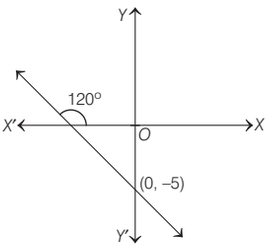
$$= \sqrt{\left(\frac{3}{2} - 1\right)^2 + (1 - 3)^2}$$

$$= \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2} \text{ units}$$

25. The equation of straight line which cuts off an intercept of 5 units on negative direction of Y-axis and makes and angle 120° with positive direction of X-axis is

(a) $y + \sqrt{3}x + 5 = 0$ (b) $y - \sqrt{3}x + 5 = 0$
(c) $y + \sqrt{3}x - 5 = 0$ (d) $y - \sqrt{3}x - 5 = 0$

(***a***)



We have, Slope $= m = \tan 120° = -\sqrt{3}$

and y-intercept $= c = -5$

$\therefore$ Equation of line is

$$y = mx + c$$

$$\Rightarrow \quad y = -\sqrt{3}x - 5$$

or $\sqrt{3}x + y + 5 = 0$

26. The equation of the line passing through the point $(2, 3)$ and the point of intersection of lines $2x - 3y + 7 = 0$ and $7x + 4y + 2 = 0$ is

(a) $21x + 46y - 180 = 0$
(b) $21x - 46y + 96 = 0$
(c) $46x + 21y - 155 = 0$
(d) $46x - 21y - 29 = 0$

(***b***) Since the line passes through the intersection point of the lines

$2x - 3y + 7 = 0$ and $7x + 4y + 2 = 0$

$\therefore$ Equation of required line is

$(2x - 3y + 7) + \lambda(7x + 4y + 2) = 0$ ...(i)

Now, required line passes through (2, 3).

So, put (2,3) in equation of line given in Eq. (i)

$(2 \times 2 - 3 \times 3 + 7) + \lambda(7 \times 2 + 4 \times 3 + 2) = 0$

$$\Rightarrow \quad (4 - 9 + 7) + \lambda(14 + 12 + 2) = 0$$

$$\Rightarrow \quad 2 + \lambda(28) = 0$$

$$\Rightarrow \quad \lambda = \frac{-1}{14}$$

$\therefore$ Equation of the required line is

$$2x - 3y + 7 + \left(-\frac{1}{14}\right)(7x + 4y + 2) = 0$$

$$\Rightarrow \quad 28x - 42y + 98 - 7x - 4y - 2 = 0$$

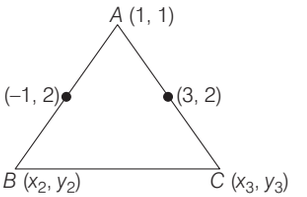
$$\therefore \quad 21x - 46y + 96 = 0$$

2017 (I)

27. If a vertex of a triangle is $(1, 1)$ and the midpoints of two sides of the triangle through this vertex are $(-1, 2)$ and $(3, 2)$, then the centroid of the triangle is

(a) $\left(-\frac{1}{3}, \frac{7}{3}\right)$ (b) $\left(-1, \frac{7}{3}\right)$
(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{3}\right)$

(***d***) Let $A(1, 1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertice of ΔABC



then, $\frac{x_2 + 1}{2} = -1 \Rightarrow x_2 = -3$

$$\frac{y_2 + 1}{2} = 2 \Rightarrow y_2 = 4 - 1 = 3$$

and also, $\frac{x_3 + 1}{2} = 3 \Rightarrow x_3 = 5$

$$\frac{y_3 + 1}{2} = 2 \Rightarrow y_3 = 3$$

Now, centroid of the triangle

$$= \left(\frac{1 - 3 + 5}{3}, \frac{1 + 3 + 3}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{7}{3}\right) = \left(1, \frac{7}{3}\right)$$

28. The incentre of the triangle with vertices $A(1, \sqrt{3})$, $B(0, 0)$ and $C(2, 0)$ is

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

(***d***) $AB = \sqrt{(1 - 0)^2 + (\sqrt{3} - 0)^2} = 2$

$$BC = \sqrt{(0 - 2)^2 + (0 - 0)^2} = 2$$

$$AC = \sqrt{(1 - 2)^2 + (\sqrt{3} - 0)^2} = 2$$

$\Rightarrow \Delta ABC$ is an equilateral triangle.

Since, in equilateral triangle incentre coincide with centroid of the triangle.

$\therefore$ Incentre of the triangle

$$= \left(\frac{1 + 0 + 2}{3}, \frac{\sqrt{3} + 0 + 0}{3}\right)$$

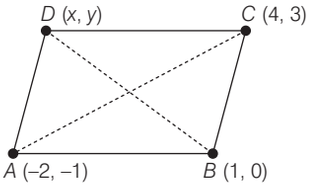
$$= \left(\frac{3}{3}, \frac{\sqrt{3}}{3}\right)$$

$$= \left(1, \frac{1}{\sqrt{3}}\right)$$

29. If the three consecutive vertices of a parallelogram are $(-2,-1)$, $(1,0)$ and $(4,3)$, then what are the coordinates of the fourth vertex?

(a) (1, 2) (b) (1, 0) (c) (0, 0) (d) (1, – 1)

(**a**) Let fourth vertex of the parallelogram is $D(x, y)$



Since diagonals of parallelogram bisects each other

$$\left(\frac{x+1}{2}, \frac{y+0}{2}\right)=\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right)$$

$$\Rightarrow \quad \left(\frac{x+1}{2}, \frac{y}{2}\right)=(1,1)$$

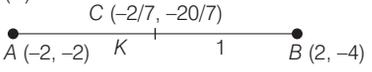
$$\therefore \quad \frac{x+1}{2}=1 \Rightarrow x=1$$

and $\quad \frac{y}{2}=1 \Rightarrow y=2$

30. What is the ratio in which the point $C\left(-\frac{2}{7}, -\frac{20}{7}\right)$ divides the line joining the points $A(-2,-2)$ and $B(2,-4)$?

(a) 1 : 3 (b) 3 : 4 (c) 1 : 2 (d) 2 : 3

(**b**)



Let point C divides line joining AB in the ratio $K:1$

$\therefore$ Coordinates of

$$C \equiv \left(\frac{2K+1(-2)}{K+1}, \frac{(-4)K+1(-2)}{K+1}\right)$$

$$\equiv \left(\frac{2K-2}{K+1}, \frac{-4K-2}{K+1}\right)$$

$\because$ We have, $C \equiv \left(\frac{-2}{7}, \frac{-20}{7}\right)$

$$\therefore \quad \frac{2K-2}{K+1}=\frac{-2}{7} \Rightarrow K=\frac{3}{4}$$

$\therefore$ Point C divides line joining AB in the ratio 3 : 4.

31. What is the equation of the straight line parallel to $2x+3y+1=0$ and passes through the point $(-1,2)$?

(a) $2x + 3y - 4 = 0$ (b) $2x + 3y - 5 = 0$
(c) $x + y - 1 = 0$ (d) $3x - 2y + 7 = 0$

(**a**) Equation of a line parallel to

$$2x + 3y + 1 = 0 \quad \ldots(i)$$

will be $\quad 2x + 3y + k = 0 \quad \ldots(ii)$

$\because$ Line (ii) passes through $(-1, 2)$

$\therefore 2(-1) + 3(2) + K = 0 \Rightarrow K = -4$

$\therefore$ Equation of a line parallel to line (i) will be $2x + 3y - 4 = 0$

32. What is the acute angle between the pair of straight lines $\sqrt{2}x+\sqrt{3}y=1$ and $\sqrt{3}x+\sqrt{2}y=2$?

(a) $\tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$ (b) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(c) $\tan^{-1}(3)$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(**a**) Given, $\sqrt{2}x + \sqrt{3}y = 1 \quad \ldots$ (i)

$$\sqrt{3}x + \sqrt{2}y = 2 \quad \ldots \text{(ii)}$$

Slope of line (i),

$$m_1 = -\frac{\sqrt{2}}{\sqrt{3}}$$

and slope of line (ii),

$$m_2 = \frac{-\sqrt{3}}{\sqrt{2}}$$

$\therefore$ Acute angle between line (i) and (ii)

$$= \tan^{-1}\left[\frac{\left(\frac{-\sqrt{2}}{\sqrt{3}}\right)-\left(\frac{-\sqrt{3}}{\sqrt{2}}\right)}{1+\left(\frac{-\sqrt{2}}{\sqrt{3}}\right)\left(\frac{-\sqrt{3}}{\sqrt{2}}\right)}\right]$$

$$= \tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$$

33. If the centroid of a triangle formed by $(7, x)$, $(y, -6)$ and $(9,10)$ is $(6, 3)$, then the values of x and y are respectively

(a) 5, 2 (b) 2, 5 (c) 1, 0 (d) 0, 0

(**a**) Given,

Centroid of a triangle formed by $(7, x)$, $(y, -6)$ and $(9, 10)$ is $(6, 3)$

$$\therefore \quad \left(\frac{7+y+9}{3}, \frac{x+(-6)+10}{3}\right) \equiv (6, 3)$$

$$\therefore \quad \frac{7+y+9}{3}=6$$

and $\quad \frac{x-6+10}{3}=3$

$$\Rightarrow \quad y = 2 \text{ and } x = 5$$

2016 (II)

34. An equilateral triangle has one vertex at $(0, 0)$ and another at $(3, \sqrt{3})$. What are the coordinates of the third vertex?

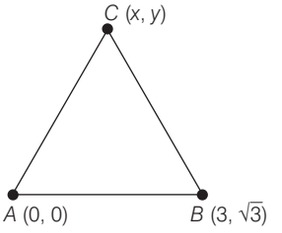
(a) $(0, 2\sqrt{3})$ only

(b) $(3, -\sqrt{3})$ only

(c) $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$

(d) Neither $(0, 2\sqrt{3})$ nor $(3, -\sqrt{3})$

(**c**) Let coordinates of third vertex C be (x, y).



Then,

$$AC = \sqrt{(x-0)^2+(y-0)^2}$$

$$= \sqrt{x^2+y^2}$$

$$AB = \sqrt{(3-0)^2+(\sqrt{3}-0)^2} = \sqrt{12}$$

and $BC = \sqrt{(x-3)^2+(y-\sqrt{3})^2}$

Since, triangle is equilateral.

$$\therefore \quad AC = AB$$

$$\Rightarrow \quad \sqrt{x^2+y^2} = \sqrt{12}$$

$$\Rightarrow \quad x^2 + y^2 = 12 \quad \ldots \text{(i)}$$

Also, $\quad AC = BC$

$$\Rightarrow \quad \sqrt{x^2+y^2} = \sqrt{(x-3)^2+(y-\sqrt{3})^2}$$

$$\Rightarrow \quad x^2 + y^2 = x^2 + 9 - 6x + y^2 + 3 - 2\sqrt{3}y$$

$$\Rightarrow \quad -6x - 2\sqrt{3}y + 12 = 0$$

$$\Rightarrow \quad y = \frac{6-3x}{\sqrt{3}} \quad \ldots \text{(ii)}$$

On putting the value of y in Eq. (i), we get

$$x^2 + \left(\frac{6-3x}{\sqrt{3}}\right)^2 = 12$$

$$\Rightarrow \quad x^2 + 12 + 3x^2 - 12x = 12$$

$$\Rightarrow \quad 4x^2 - 12x = 0$$

$$\Rightarrow \quad 4x(x-3) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = 3$$

$$\therefore \quad y = \frac{6-0}{\sqrt{3}} = 2\sqrt{3}$$

or $\quad y = \frac{6-3\times 3}{\sqrt{3}} = -\sqrt{3}$

Hence, the coordinates of C are $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$.

35. What is the equation of the right bisector of the line segment joining $(1, 1)$ and $(2, 3)$?

(a) $2x + 4y - 11 = 0$
(b) $2x - 4y - 5 = 0$
(c) $2x - 4y - 11 = 0$
(d) $x - y + 1 = 0$

(**a**) Let the points be A (1, 1) and B (2, 3)

Slope of line segment AB

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{2-1} = 2$$

$\therefore$ Slope of line perpendicular to

$$AB = -\frac{1}{2}$$

Coordinates of mid-point of line segment AB

$$= \left(\frac{2+1}{2}, \frac{3+1}{2}\right) = \left(\frac{3}{2}, 2\right)$$

$\therefore$ Equation of right bisector of the line segment AB is

$$(y-2) = -\frac{1}{2}\left(x - \frac{3}{2}\right)$$

$\Rightarrow \quad 2y - 4 = -x + 3/2$

$\therefore \quad 2x + 4y - 11 = 0$

36. If the point (a, a) lies between the lines $|x + y| = 2$, then which one of the following is correct?

(a) $|a| < 2$ (b) $|a| < \sqrt{2}$

(c) $|a| < 1$ (d) $|a| < \frac{1}{\sqrt{2}}$

(c) (***c***) Given,

$$|x + y| = 2 \Rightarrow x + y = \pm 2$$

When point (a, a) lie on the line,

$$x + y = \pm 2$$

$\therefore \quad a + a = \pm 2$

$\Rightarrow \quad 2a = \pm 2$

$\Rightarrow \quad a = \pm 1$

Hence, a lies in between -1 and 1,

i.e. $|a| < 1$

37. What is the equation of the straight line which passes through the point of intersection of the straight lines $x + 2y = 5$ and $3x + 7y = 17$ and is perpendicular to the straight line $3x + 4y = 10$?

(a) $4x + 3y + 2 = 0$

(b) $4x - y + 2 = 0$

(c) $4x - 3y - 2 = 0$

(d) $4x - 3y + 2 = 0$

(d) (***d***) Given,

$$x + 2y = 5 \quad \ldots (i)$$

$$3x + 7y = 17 \quad \ldots (ii)$$

$$3x + 4y = 10 \quad \ldots (iii)$$

Equation of straight line which passes through point of intersection of lines (i) and (ii) will be

$$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$$

$\Rightarrow \quad (3\lambda + 1)x + (7\lambda + 2)y - (17\lambda + 5) = 0 \quad \ldots (iv)$

$\because$ Line (iv) is perpendicular to line (iii).

$\therefore$ Slope of line (iii) $\times$ Slope of line (iv)

$$= -1$$

$$\Rightarrow \quad \left(\frac{-3}{4}\right) \times \left[\frac{-(3\lambda + 1)}{(7\lambda + 2)}\right] = -1$$

$$\Rightarrow \quad 9\lambda + 3 = -28\lambda - 8$$

$$\Rightarrow \quad 37\lambda = -11$$

$$\Rightarrow \quad \lambda = -\frac{11}{37}$$

$\therefore$ Equation of required line will be

$$(x + 2y - 5) + \frac{-11}{37}(3x + 7y - 17) = 0$$

$$\Rightarrow \quad 4x - 3y + 2 = 0$$

38. If (a, b) is at unit distance from the line $8x + 6y + 1 = 0$, then which of the following conditions are correct?

I. $3a - 4b - 4 = 0$

II. $8a + 6b + 11 = 0$

III. $8a + 6b - 9 = 0$

Select the correct answer using the code given below.

(a) I and II (b) II and III

(c) I and III (d) I, II and III

(b) (***b***) Given, distance between point (a, b) to the line $8x + 6y + 1 = 0$ is 1 unit.

$$\therefore \quad \frac{|8a + 6b + 1|}{\sqrt{(8)^2 + (6)^2}} = 1$$

$$\Rightarrow \quad \frac{|8a + 6b + 1|}{10} = 1$$

$$\Rightarrow \quad |8a + 6b + 1| = 10$$

$$\Rightarrow \quad 8a + 6b + 1 = \pm 10$$

$\therefore \quad 8a + 6b - 9 = 0$ or

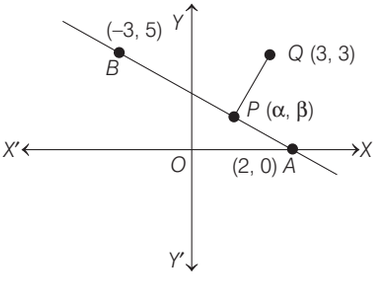
$8a + 6b + 11 = 0$

39. A straight line cuts off an intercept of 2 units on the positive direction of X-axis and passes through the point $(-3, 5)$. What is the foot of the perpendicular drawn from the point (3, 3) on this line?

(a) (1, 3) (b) (2, 0)

(c) (0, 2) (d) (1, 1)

(d) (***d***) Let $P(\alpha, \beta)$ be the foot of perpendicular drawn from the point (3, 3) to line AB.



Slope of line $AB = \frac{5 - 0}{-3 - 2} = -1$

Equation of line AB is

$$y - 0 = -1(x - 2)$$

$\Rightarrow \quad x + y - 2 = 0 \quad \ldots (i)$

Slope of line PQ perpendicular to

$$AB = \frac{-1}{-1} = 1$$

Equation of line PQ is

$$y - 3 = 1(x - 3)$$

$\Rightarrow \quad x - y = 0 \quad \ldots (ii)$

On solving Eqs. (i) and (ii), we get

$$x = 1, y = 1$$

Hence, $(\alpha, \beta) = (1, 1)$

2016 (I)

Directions (Q. Nos. 40-42) *Consider a parallelogram whose vertices are* $A(1, 2)$, $B(4, y)$, $C(x, 6)$ *and* $D(3, 5)$ *taken in order.*

40. What is the value of $AC^2 - BD^2$?

(a) 25 (b) 30

(c) 36 (d) 40

41. What is the point of intersection of the diagonals?

(a) $\left(\frac{7}{2}, 4\right)$ (b) (3, 4)

(c) $\left(\frac{7}{2}, 5\right)$ (d) (3, 5)

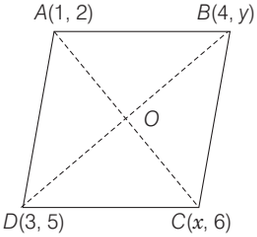
42. What is the area of the parallelogram?

(a) $\frac{7}{2}$ sq units (b) 4 sq units

(c) $\frac{11}{2}$ sq units (d) 7 sq units

Solutions (Q. Nos. 40-42) Since, in a parallelogram diagonals bisect each other.

$\therefore$ Mid-point of AC = Mid-point of DB



$$\Rightarrow \quad \left(\frac{x+1}{2}, \frac{6+2}{2}\right) \equiv \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \quad \frac{x+1}{2} = \frac{7}{2} \Rightarrow x = 6$$

and $\quad y + 5 = 6 + 2 \Rightarrow y = 3$

$\therefore$ Coordinates of O are

$$\left(\frac{7}{2}, \frac{8}{2}\right) \text{ i.e. } \left(\frac{7}{2}, 4\right).$$

Now, $\quad AC^2 = (6-1)^2 + (6-2)^2$

$$= 25 + 16 = 41$$

and $BD^2 = (4-3)^2 + (3-5)^2$
$= 1 + 4 = 5$

$\therefore \quad AC^2 - BD^2 = 41 - 5 = 36$

40. ***(c)*** 41. ***(a)***

42. (***d***) Area of parallelogram
= Area of ΔABD + Area of ΔBCD

$$= \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 4 & 3 & 1 \\ 3 & 5 & 1 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} 4 & 3 & 1 \\ 6 & 6 & 1 \\ 3 & 5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\begin{vmatrix} 1 & 2 & 1 \\ 4 & 3 & 1 \\ 3 & 5 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 3 & 1 \\ 6 & 6 & 1 \\ 3 & 5 & 1 \end{vmatrix}\right]$$

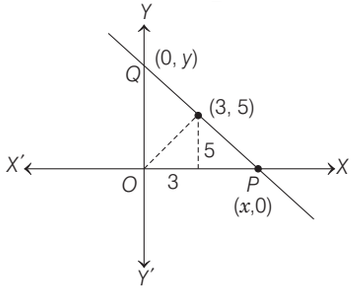
$$= \frac{1}{2}[\{1(3-5) - 2(4-3) + 1(20-9)\}$$
$$+ \{4(6-5) - 3(6-3) + 1(30-18)\}]$$
$$= \frac{1}{2}[7 + 7] = \frac{14}{2} = 7 \text{ sq units}$$

43. A straight line intersects x and y axes at P and Q, respectively. If $(3, 5)$ is the middle point of PQ, then what is the area of the ΔPOQ?

(a) 12 sq units (b) 15 sq units
(c) 20 sq units (d) 30 sq units

(***d***) Let coordinates of P and Q are $(x, 0)$ and $(0, y)$, respectively.

$\therefore$ Mid-point of PQ is $\frac{x+0}{2} = 3 \Rightarrow x = 6$



and $\frac{0+y}{2} = 5 \Rightarrow y = 10$

$\therefore$ Area of $\Delta POQ = \frac{1}{2} \times OP \times OQ$

$= \frac{1}{2} \times 6 \times 10 = 30$ sq units

Directions (Q. Nos. 44 and 45)

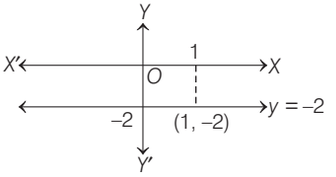
Consider the two lines $x + y + 1 = 0$ *and* $3x + 2y + 1 = 0$

44. What is the equation of the line passing through the point of intersection of the given lines and parallel to X-axis?

(a) $y + 1 = 0$ (b) $y - 1 = 0$
(c) $y - 2 = 0$ (d) $y + 2 = 0$

(***d***) Given, lines are $x + y + 1 = 0$ ...(i)
and $3x + 2y + 1 = 0$ ...(ii)

Point of intersection of these two lines are $(1, -2)$.

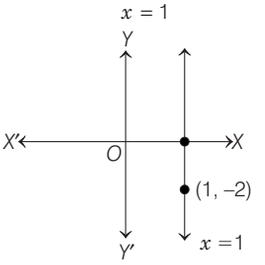


$\therefore$ Required equation of line $y = -2$ or $y + 2 = 0$

45. What is the equation of the line passing through the point of intersection of the given lines and parallel to Y-axis?

(a) $x + 1 = 0$ (b) $x - 1 = 0$
(c) $x - 2 = 0$ (d) $x + 2 = 0$

(***b***) Equation of line parallel to Y-axis



or $x - 1 = 0$

46. $(a, 2b)$ is the mid-point of the line segment joining the points $(10, -6)$ and $(k, 4)$. If $a - 2b = 7$, then what is the value of k?

(a) 2 (b) 3
(c) 4 (d) 5

(***a***) Since, $(a, 2b)$ is the mid-point of the line segment joining the points $(10, -6)$ and $(k, 4)$, therefore we have

$$(a, 2b) = \left(\frac{10+k}{2}, \frac{-6+4}{2}\right)$$

$$\Rightarrow \quad (a, 2b) = \left(\frac{10+k}{2}, -1\right)$$

$$\Rightarrow \quad a = \frac{10+k}{2} \text{ and } 2b = -1$$

$$\Rightarrow \quad a = \frac{10+k}{2} \text{ and } b = -\frac{1}{2}$$

Also, it is given that $a - 2b = 7$

$$\Rightarrow \quad \frac{10+k}{2} - 2\left(-\frac{1}{2}\right) = 7$$

$$\Rightarrow \quad 10 + k + 2 = 14 \Rightarrow k = 2$$

47. What is the acute angle between the lines represented by the equations $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$?

(a) 30° (b) 45°
(c) 60° (d) 75°

(***a***) Given lines are
$y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$

$\therefore \quad y = \sqrt{3}x + 5$ ...(i)

and $\sqrt{3}y = x - 6$

$$\Rightarrow \quad y = \frac{x}{\sqrt{3}} - \left(\frac{6}{\sqrt{3}}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$m_1 = \sqrt{3} \text{ and } m_2 = \frac{1}{\sqrt{3}}$$

Hence, acute angle $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}\right| = \left|\frac{\frac{3-1}{\sqrt{3}}}{2}\right| = \left|\frac{\frac{2}{\sqrt{3}}}{2}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \tan 30°$$

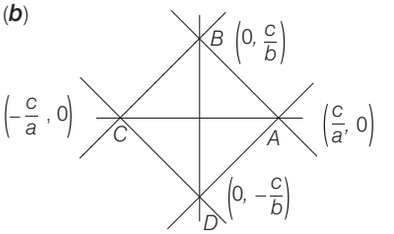
$\therefore \quad \theta = 30°$

2015 (II)

48. The area of the figure formed by the lines $ax + by + c = 0, ax - by + c = 0$, $ax + by - c = 0$ and $ax - by - c = 0$ is

(a) $\frac{c^2}{ab}$ (b) $\frac{2c^2}{ab}$
(c) $\frac{c^2}{2ab}$ (d) $\frac{c^2}{4ab}$

(***b***)



Clearly, $ABCD$ is a parallelogram and its area

$= \frac{1}{2} \times$ product of diagonals

$$= \frac{1}{2} \times \frac{2c}{b} \times \frac{2c}{a} = \frac{2c^2}{ab}$$

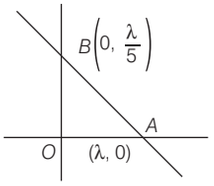
49. If a line is perpendicular to the line $5x - y = 0$ and forms a triangle of area 5 sq units with coordinate axes, then its equation is

(a) $x + 5y \mp 5\sqrt{2} = 0$
(b) $x - 5y \pm 5\sqrt{2} = 0$
(c) $5x + y \pm 5\sqrt{2} = 0$
(d) $5x - y \pm 5\sqrt{2} = 0$

(***a***) Equation of line perpendicular to $5x - y = 0$ is $x + 5y = \lambda$

We have, area of $\Delta OAB = 5$

$$\frac{1}{2}\lambda \times \frac{\lambda}{5} = 5$$



$$\lambda^2 = 50 \Rightarrow \lambda = \pm 5\sqrt{2}$$

$\therefore$ Equation of lines are $x + 5y = \pm 5\sqrt{2}$

or $x + 5y \mp 5\sqrt{2} = 0$

50. The three lines $4x + 4y = 1$, $8x - 3y = 2$, $y = 0$ are

(a) the sides of an isosceles triangle
(b) concurrent
(c) mutually perpendicular
(d) the sides of an equilateral triangle

(**b**) Point of intersection of $y = 0$ and $4x + 4y = 1$ is $\left(\frac{1}{4}, 0\right)$ and $\left(\frac{1}{4}, 0\right)$ lies on the line $8x - 3y = 2$.

Hence, the given lines are concurrent.

51. The line $3x + 4y - 24 = 0$ intersects the X-axis at A and Y-axis at B. Then, the circumcentre of the ΔOAB, where O is the origin, is

(a) (2, 3) (b) (3, 3)
(c) (4, 3) (d) None of these

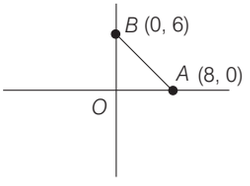
(**c**) Given, $3x + 4y - 24 = 0$

Its X-intercept $A = (8, 0)$

and Y-intercept $B = (0, 6)$

Since, ΔOAB is right angled triangle.

So, mid-point of hypotenuse is the circumcentre.



Hence, circumcentre = (4, 3)

$$\left[\because \frac{8+0}{2}, \frac{0+6}{2} = (4, 3)\right]$$

52. The product of the perpendiculars from the two points $(\pm 4, 0)$ to the line $3x\cos\phi + 5y\sin\phi = 15$ is

(a) 25 (b) 16 (c) 9 (d) 8

(**c**) $3x\cos\phi + 5y\sin\phi = 15$

Distance from (x_1, y_1) to $ax + by + c = 0$

$$= \frac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$$

Length of perpendicular from point $(\pm 4, 0)$,

$$p_1 = \left|\frac{12\cos\phi - 15}{\sqrt{9\cos^2\phi + 25\sin^2\phi}}\right| \quad \ldots(i)$$

$$p_2 = \left|\frac{-12\cos\phi - 15}{\sqrt{9\cos^2\phi + 25\sin^2\phi}}\right| \quad \ldots(ii)$$

On multiplying Eqs. (i) and (ii), we get

$$p_1 p_2 = \left|\frac{-(12\cos\phi - 15)(12\cos\phi + 15)}{9\cos^2\phi + 25\sin^2\phi}\right|$$

$$= \left|\frac{(12\cos\phi)^2 - (15)^2}{9\cos^2\phi + 25\sin^2\phi}\right|$$

$$= \left|\frac{144\cos^2\phi - 225}{9 + 16\sin^2\phi}\right|$$

$$= \left|\frac{144(1 - \sin^2\phi) - 225}{9 + 16\sin^2\phi}\right|$$

$$= \left|\frac{-9(9 + 16\sin^2\phi)}{9 + 16\sin^2\phi}\right| = 9$$

53. Two straight lines passing through the point $A(3, 2)$ cut the line $2y = x + 3$ and X-axis perpendicularly at P and Q, respectively. The equation of the line PQ is

(a) $7x + y - 21 = 0$ (b) $x + 7y + 21 = 0$
(c) $2x + y - 8 = 0$ (d) $x + 2y + 8 = 0$

(**a**) Given, $2y = x + 3$ $\ldots(i)$

A line passes through $A(3, 2)$ and having slope -2

[since, line is perpendicular to line having slope $\frac{1}{2}$]

$$y - 2 = -2(x - 3)$$

[one point slope form $y - y_1 = m(x - x_1)$]

$$\Rightarrow 2x + y - 8 = 0 \quad \ldots(ii)$$

By solving lines (i) and (ii), we find point P

$$P \equiv \left(\frac{13}{5}, \frac{14}{5}\right)$$

Now, a line passes through $A(3, 2)$ and having slope $\frac{1}{0}$.

[since, line is perpendicular to X-axis]

$$\therefore \quad y - 2 = \frac{1}{0}(x - 3) \Rightarrow x = 3$$

Coordinate at point $Q = (3, 0)$

Now, equation of line PQ is

$$y - \frac{14}{5} = \frac{0 - \frac{14}{5}}{3 - \frac{13}{5}}\left(x - \frac{13}{5}\right)$$

$$\therefore \quad 7x + y - 21 = 0$$

2015 (I)

Directions (Q. Nos. 54-56) *Consider the ΔABC with vertices $A(-2, 3)$, $B(2, 1)$ and $C(1, 2)$.*

54. What is the circumcentre of the ΔABC ?

(a) $(-2, -2)$ (b) $(2, 2)$
(c) $(-2, 2)$ (d) $(2, -2)$

(**a**) Let $P(x, y)$ be the circumcentre of ΔABC.

$$\therefore \quad AP^2 = PB^2$$

$$\Rightarrow (x+2)^2 + (y-3)^2 = (x-2)^2 + (y-1)^2$$

$$\Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y = x^2 + 4 - 4x + y^2 + 1 - 2y$$

$$\Rightarrow 4x - 6y + 13 = 4 - 4x + 1 - 2y$$

$$\Rightarrow 8x - 4y + 8 = 0$$

$$\Rightarrow 2x - y + 2 = 0 \quad \ldots(i)$$

Also, $AP^2 = PC^2$

$$\Rightarrow (x+2)^2 + (y-3)^2 = (x-1)^2 + (y-2)^2$$

$$\Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y = x^2 + 1 - 2x + y^2 + 4 - 4y$$

$$\Rightarrow 4x - 6y + 13 = -2x - 4y + 5$$

$$\Rightarrow 6x - 2y + 8 = 0$$

$$\Rightarrow 3x - y + 4 = 0 \quad \ldots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$-x - 2 = 0 \Rightarrow x = -2$$

$$\therefore \quad y = 3 \times (-2) + 4 = -2$$

Hence, the required circumcentre is $(-2, -2)$.

55. What is the centroid of ΔABC ?

(a) $\left(\frac{1}{3}, 1\right)$ (b) $\left(\frac{1}{3}, 2\right)$
(c) $\left(1, \frac{2}{3}\right)$ (d) $\left(\frac{1}{2}, 3\right)$

(**b**) Given, vertices of a triangle are $A(-2, 3)$, $B(2, 1)$ and $C(1, 2)$.

Then, centroid of ΔABC

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$\therefore$ Centroid of ΔABC

$$= \left(\frac{-2 + 2 + 1}{3}, \frac{3 + 1 + 2}{3}\right) = \left(\frac{1}{3}, 2\right)$$

56. What is the foot of the altitude from the vertex A of the ΔABC?

(a) (1, 4) (b) (–1, 3) (c) (–2, 4) (d) (–1, 4)

(**d**) Let D be the foot of altitude from A in ΔABC and $D \equiv (x_1, y_1)$

Slope of $BC = \frac{1-2}{2-1} = -1$

$$\left[\because \text{slope} = \frac{y_1 - y_2}{x_1 - x_2}\right]$$

$$\therefore \quad m = -1$$

Also, slope of AD is $\left(\frac{y_1 - 3}{x_1 + 2}\right)$.

But $\frac{y_1 - 3}{x_1 + 2} \cdot (-1) = -1 \quad [\because AD \perp BC]$

$\Rightarrow \quad y_1 - 3 = x_1 + 2 \Rightarrow y_1 - x_1 = 5$

From the given points, only $(-1, 4)$ satisfies this equation. Hence, the required foot of altitude is $(-1, 4)$.

57. The perpendicular distance between the straight lines $6x + 8y + 15 = 0$ and $3x + 4y + 18 = 0$ is

(a) $\frac{3}{2}$ units (b) $\frac{3}{10}$ unit

(c) $\frac{3}{4}$ unit (d) $\frac{2}{7}$ unit

(**b**) We have, $6x + 8y + 15 = 0$ and $6x + 8y + 18 = 0$

$\therefore$ Perpendicular distance between them

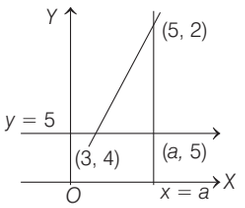
$$= \left|\frac{18 - 15}{\sqrt{36 + 64}}\right| = \frac{3}{10} \text{ unit}$$

$\left[\because \text{distance between two parallel lines} = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|\right]$

58. The area of a triangle, whose vertices are $(3, 4)$, $(5, 2)$ and the point of intersection of the lines $x = a$ and $y = 5$, is 3 sq units. What is the value of a?

(a) 2 (b) 3 (c) 4 (d) 5

(**d**)



We have, $\Delta = 3$ sq units

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 5 & 2 & 1 \\ 3 & 4 & 1 \\ a & 5 & 1 \end{vmatrix} = 3$$

$$\Rightarrow \frac{1}{2}[5(4 - 5) - 2(3 - a) + 1(15 - 4a)] = 3$$

$$\Rightarrow \quad \frac{1}{2}[-5 - 2(3 - a) + (15 - 4a)] = 3$$

$$\Rightarrow \quad \frac{1}{2}[-5 - 6 + 2a + 15 - 4a] = 3$$

$\Rightarrow 2 - a = \pm 3 \Rightarrow a = 5$ or -1

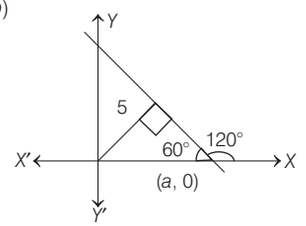
$\therefore \quad a = 5$

59. The length of perpendicular from the origin to a line is 5 units and the line makes an angle $120°$ with the positive direction of X-axis. The equation of the line is

(a) $x + \sqrt{3}y = 5$ (b) $\sqrt{3}x + y = 10$

(c) $\sqrt{3}x - y = 10$ (d) None of these

(**b**)



Let the required line intercept X-axis at a.

Slope of the line is

$$m = \tan 120° = -\sqrt{3}$$

Also, $\sin 60° = \frac{5}{a}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{5}{a} \Rightarrow a = \frac{10}{\sqrt{3}}$$

$\therefore$ Equation of line passing through $(a, 0)$ and having slope $-\sqrt{3}$ is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 0 = -\sqrt{3}\left(x - \frac{10}{\sqrt{3}}\right)$$

$$\Rightarrow \quad y = -\sqrt{3}x + 10$$

$$\therefore \quad \sqrt{3}x + y = 10$$

60. The equation of the line joining the origin to the point of intersection of the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ is

(a) $x - y = 0$ (b) $x + y = 0$

(c) $x = 0$ (d) $y = 0$

(**a**) We have, $\frac{x}{a} + \frac{y}{b} = 1 \quad \ldots(i)$

and $\frac{x}{b} + \frac{y}{a} = 1 \quad \ldots(ii)$

$$\Rightarrow \quad bx + ay = ab$$

and $\quad ax + by = ab$

On comparing Eqs. (i) and (ii), we get

$$bx + ay = ax + by$$

$$\Rightarrow \quad x(b - a) = y(b - a)$$

$$\Rightarrow \quad \frac{x}{y} = 1 \Rightarrow x = y$$

$\therefore$ Equation the line is $x - y = 0$

Now, from Eq. (i), we get

$\frac{y}{a} + \frac{y}{b} = 1$, $y = \frac{ab}{a + b}$ and $x = \frac{ab}{a + b}$

$\therefore$ Equation of line joining origin and $\left(\frac{ab}{a + b}, \frac{ab}{a + b}\right)$ is $y - 0 = 1(x - 0)$

$$\therefore \quad y = x$$

or $x - y = 0$

2015 (I)

61. A line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$, its y-intercept is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) 3

(**b**) Given equation of line is $y = 3 - 3x$

$$\Rightarrow \quad m = -3 \quad [\because y = mx + c]$$

$\therefore$ Slope of the line perpendicular to the line $y = 3 - 3x$, is

$$m' = \frac{-1}{-3} = \frac{1}{3}$$

$\therefore$ Required equation of the line is

$$y - 2 = \frac{1}{3}(x - 2)$$

$[\because y - y_1 = m(x - x_1) =$ one point slope form$]$

$$\Rightarrow \quad 3y - 6 = x - 2$$

$$\Rightarrow \quad 3y - x = 4$$

$$\Rightarrow \quad \frac{x}{-4} + \frac{y}{\frac{4}{3}} = 1$$

So, y-intercept is $\frac{4}{3}$.

2014 (II)

62. $A(3, 4)$ and $B(5, -2)$ are two points and P is a point such that $PA = PB$. If the area of ΔPAB is 10 sq units, then what are the coordinates of P?

(a) Only (1, 0)

(b) Only (7, 2)

(c) Either (1, 0) or (7, 2)

(d) Neither (1, 0) nor (7, 2)

(**c**) We have, $A(3, 4)$ and $B(5, -2)$

Let $P(x, y)$

Given that, $PA = PB$

$$\Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow (x - 3)^2 + (y - 4)^2 = (x - 5)^2 + (y + 2)^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2 + 4y + 4$$

$$\Rightarrow \quad 4x - 12y = 4$$

$$\Rightarrow \quad x - 3y = 1 \quad \ldots(i)$$

$\because$ Area of $\Delta PAB = 10$

$$\therefore \quad \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \pm 10$$

$$\Rightarrow x(4 + 2) - y(3 - 5) + 1(-6 - 20) = \pm 20$$

$$\Rightarrow \quad 6x + 2y - 26 = \pm 20$$

$$\Rightarrow \quad 6x + 2y - 26 = 20$$

or $\quad 6x + 2y - 26 = -20$

$$\Rightarrow \quad 6x + 2y = 46 \quad \ldots(ii)$$

or $\quad 6x + 2y = 6 \quad \ldots(iii)$

On solving Eqs. (i) and (ii), we get

$x = 7$, $y = 2$

Similarly, solving Eqs. (i) and (iii), we get

$x = 1$, $y = 0$

Hence, coordinates of P are

(7, 2) or (1, 0).

63. What is the product of the perpendiculars drawn from the points $(\pm\sqrt{a^2-b^2},0)$ upon the line $bx\cos\alpha + ay\sin\alpha = ab$?

(a) a^2 (b) b^2
(c) a^2+b^2 (d) $a+b$

(***b***) We have, equation of line is
$bx\cos\alpha + ay\sin\alpha = ab$
Perpendicular distance from point $(\sqrt{a^2-b^2}, 0)$ is

$$d_1 = \left|\frac{b\cos\alpha\sqrt{a^2-b^2}+0-ab}{\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha}}\right|$$

$$\left[\because \text{distance from } (x_1, y_1) \text{ to } ax+by+c=0 \text{ is } -\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|\right]$$

Similarly, perpendicular distance from point $(-\sqrt{a^2-b^2}, 0)$ is

$$d_2 = \left|\frac{-b\cos\alpha\sqrt{a^2-b^2}+0-ab}{\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha}}\right|$$

Now, $d_1 \times d_2$

$$= \frac{(b\cos\alpha\sqrt{a^2-b^2}-ab)}{(\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha})}\cdot\frac{(b\cos\alpha\sqrt{a^2-b^2}+ab)}{(\sqrt{b^2\cos^2\alpha+a^2\sin^2\alpha})}$$

$$= \frac{b^2\cos^2\alpha(a^2-b^2)-a^2b^2}{b^2\cos^2\alpha+a^2\sin^2\alpha}$$

$$= \frac{a^2b^2\cos^2\alpha-b^4\cos^2\alpha-a^2b^2}{b^2\cos^2\alpha+a^2\sin^2\alpha}$$

$$= \frac{a^2b^2(\cos^2\alpha-1)-b^4\cos^2\alpha}{b^2\cos^2\alpha+a^2\sin^2\alpha}$$

$$= \frac{-b^2[a^2\sin^2\alpha+b^2\cos^2\alpha]}{b^2\cos^2\alpha+a^2\sin^2\alpha}$$

$= -b^2 = b^2$ [since, distance is positive]

64. Which one of the following is correct in respect of the equations $\frac{x-1}{2}=\frac{y-2}{3}$ and $2x+3y=5$?

(a) They represent two lines which are parallel
(b) They represent two lines which are perpendicular
(c) They represent two lines which are neither parallel nor perpendicular
(d) The first equation does not represent a line

(***b***) We have, equation of line is

$$\frac{x-1}{2}=\frac{y-2}{3}$$

$\Rightarrow \quad 3x-3=2y-4$
$\Rightarrow \quad 3x-2y+1=0$
and equation of second line is
$2x+3y=5$
$\therefore$ Slope of first line, $m_1=\frac{3}{2}$
and slope of second line, $m_2=-\frac{2}{3}$
$\therefore \quad m_1m_2=-1$
Hence, two lines are perpendicular to each other.

2014 (I)

65. Consider the following points
I. $(0, 5)$ II. $(2, -1)$
III. $(3, -4)$
Which of the above lie on the line $3x+y=5$ and at a distance $\sqrt{10}$ from $(1, 2)$?

(a) Only I (b) Only II
(c) I and II (d) I, II and III

(***c***) Given, equation of line
$3x+y=5$
Let $S \equiv 3x+y-5=0$
I. At point (0, 5),
$S_{(0,5)} \equiv 3(0)+5-5$
$= 0+0=0$
So, the point (0, 5) lie on the given line.
and distance between (0, 5) and (1, 2)
$= \sqrt{(1-0)^2+(2-5)^2} = \sqrt{(1)^2+(-3)^2}$

$$\left[\because \text{by distance formula, } d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right]$$

$= \sqrt{1+9} = \sqrt{10}$
II. At point (2, – 1),
$S_{(2,-1)} \equiv 3(2)+(-1)-5$
$= 6-1-5=6-6=0$
So, the point (2, – 1) also lie on the given line and distance between (2, – 1) and (1, 2)
$= \sqrt{(1-2)^2+(2+1)^2}$
$= \sqrt{(-1)^2+(3)^2}$
$= \sqrt{1+9} = \sqrt{10}$
III. At point (3, – 4),
$S_{(3,-4)} = 3(3)+(-4)-5$
$= 9-4-5=9-9=0$
So, the point (3, – 4) also lie on the given line and distance between (3, – 4) and (1, 2)
$= \sqrt{(1-3)^2+(2+4)^2}$
$= \sqrt{(-2)^2+(6)^2}$
$= \sqrt{4+36}$
$= \sqrt{40} = 2\sqrt{10}$
Hence, points (0, 5) and (2, – 1) lie on the line $3x+y=5$ and at a distance $\sqrt{10}$ from (1, 2).

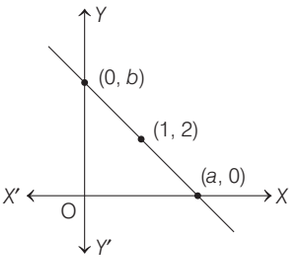
66. What is the equation of the line through $(1, 2)$, so that the segment of the line intercepted between the axes is bisected at this point?

(a) $2x-y=4$
(b) $2x-y+4=0$
(c) $2x+y=4$
(d) $2x+y+4=0$

(***c***) Let the required equation be

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \ldots(i)$$

Whose intercept on *X* and *Y*-axes are *a* and *b*, respectively.



Given that the line segment is bisected by the point (1, 2) *i.e.,* (1, 2) is the middle point of the line segment passes through the points $(a, 0)$ and $(0, b)$.

$\therefore \quad (1,2) = \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$

$\Rightarrow \quad \frac{a}{2}=1$

$\Rightarrow \quad a=2$ and $\frac{b}{2}=2$

$\Rightarrow \quad b=4$

Put the value of *a* and *b* in Eq. (i), we get

$$\frac{x}{2}+\frac{y}{4}=1$$

$\Rightarrow \quad 2x+y=4$

Which is the required equation.

67. What is the equation of straight line passing through the point (4, 3) and making equal intercepts on the coordinate axes?

(a) $x + y = 7$
(b) $3x + 4y = 7$
(c) $x - y = 1$
(d) None of the above

(*a*) Let the required equation in intercept form

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \ldots (i)$$

where a and b are the intercepts on X and Y-axes, respectively.

Given that, the straight line (i) making equal intercept on the coordinate axes.

$\therefore \quad a = b$

From Eq. (i),

$$\frac{x}{a} + \frac{y}{a} = 1$$

$\Rightarrow \quad x + y = a \quad \ldots(ii)$

Also, the straight line passing through the point (4, 3).

$\therefore \quad 4 + 3 = a \Rightarrow a = 7$

Now, put the value of a in Eq. (ii), we get

$$x + y = 7$$

Which is the required equation.

68. What is the equation of the line midway between the lines $3x - 4y + 12 = 0$ and $3x - 4y = 6$?

(a) $3x - 4y - 9 = 0$
(b) $3x - 4y + 9 = 0$
(c) $3x - 4y - 3 = 0$
(d) $3x - 4y + 3 = 0$

(*d*) The given equation of lines are

$$3x - 4y + 12 = 0 \quad \ldots(i)$$

and $\quad 3x - 4y = 6$

or $\quad 3x - 4y - 6 = 0 \quad \ldots(ii)$

On comparing with $ax + by + c = 0$, we get

$$a_1 = a_2 = 3,$$

$$b_1 = b_2 = -4$$

and $\quad c_1 = 12,\ c_2 = -6$

So, the equation of line mid way between the given lines is

$$3x - 4y + \frac{(c_1 + c_2)}{2} = 0$$

$\Rightarrow \quad 3x - 4y + \frac{(12 - 6)}{2} = 0$

$\Rightarrow \quad 3x - 4y + 3 = 0$

Which is the required equation.

16

CIRCLE

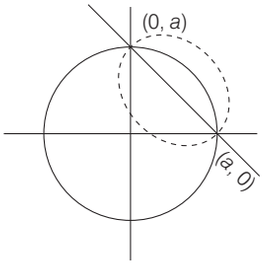
2019 (I)

1. A circle is drawn on the chord of a circle $x^2 + y^2 = a^2$ as diameter. The chord lies on the line $x + y = a$. What is the equation of the circle?

(a) $x^2 + y^2 - ax - ay + a^2 = 0$
(b) $x^2 + y^2 - ax - ay = 0$
(c) $x^2 + y^2 + ax + ay = 0$
(d) $x^2 + y^2 + ax + ay - 2a^2 = 0$

(**b**) Given, equation of circle is $x^2 + y^2 = a^2$.

$x + y = a$ is chord of a circle.



∴ End points of diameter of required circle is $(a,0)$ and $(0, a)$.

∴ Equation of circle is

$x(x - a) + y(y - a) = 0$

$\Rightarrow x^2 + y^2 - ax - ay = 0$

2. The circle $x^2 + y^2 + 4x - 7y + 12 = 0$, cuts an intercept on Y-axis equal to

(a) 1 (b) 3
(c) 4 (d) 7

(**a**) Given, $x^2 + y^2 + 4x - 7y + 12 = 0$

For intercept on Y-axis put $x = 0$, we get

$$y^2 - 7y + 12 = 0$$
$$(y - 4)(y - 3) = 0$$
$$y = 3,\ y = 4$$

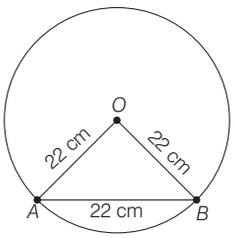
Length of intercept on Y-axis

$$= |y_2 - y_1| = |3 - 4| = 1$$

3. In a circle of diameter 44 cm, the length of a chord is 22 cm. What is the length of minor arc of the chord?

(a) $\frac{484}{21}$ cm
(b) $\frac{242}{21}$ cm
(c) $\frac{121}{21}$ cm
(d) $\frac{44}{7}$ cm

(**a**) Given, diameter of a circle be 44 cm.



$\Rightarrow \quad 2r = 44$

$\Rightarrow \quad r = 22$

$\Rightarrow \Delta OAB$ is an equilateral triangle.

$\Rightarrow \quad \angle AOB = 60°$

∴ Length of minor arc

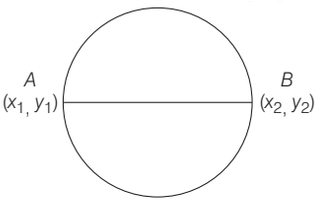
$$= \left(\frac{60°}{360°}\right) \times 2\pi \times 22$$
$$= \frac{1}{6} \times 2 \times \frac{22}{7} \times 22$$
$$= \frac{484}{21} \text{ cm}$$

2018 (II)

4. The equation of a circle whose end points of a diameter are (x_1, y_1) and (x_2, y_2) is

(a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = x^2 + y^2$
(b) $(x - x_1)^2 + (y - y_1)^2 = x_2 y_2$
(c) $x^2 + y^2 + 2x_1x_2 + 2y_1y_2 = 0$
(d) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(**d**) Equation of circle if end points of diameter are (x_1, y_1) and (x_2, y_2) are

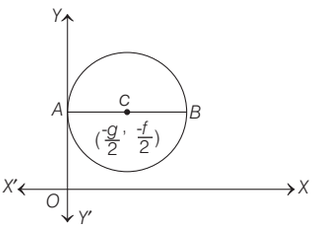


$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

5. If Y-axis touches the circle $x^2 + y^2 + gx + fy + \frac{c}{4} = 0$, then the normal at this point intersects the circle at the point.

(a) $\left(-\frac{g}{2}, \frac{f}{2}\right)$ (b) $\left(-g, -\frac{f}{2}\right)$
(c) $\left(-\frac{g}{2}, f\right)$ (d) $(-g, -f)$

(**b**)



The equation of circle is

$$x^2 + y^2 + gx + fy + \frac{c}{4} = 0$$

$\because$ Centre $= \left(-\frac{g}{2}, \frac{-f}{2}\right)$ and

Radius $= \sqrt{\frac{g^2}{4} + \frac{f^2}{4} - \frac{c}{4}}$

Since, circle touches Y-axis, then

AC = radius

$$\Rightarrow \left|\frac{g}{2}\right| = \sqrt{\frac{g^2}{4} + \frac{f^2}{4} - \frac{c}{4}} \Rightarrow \frac{f^2}{4} = \frac{c}{4}$$

∴ circle touches Y-axis at

$$y^2 + fy + \frac{f^2}{4} = 0 \Rightarrow \left(y + \frac{f}{2}\right)^2 = 0$$

$\Rightarrow \quad y = \frac{-f}{2}$

$\therefore \quad A\left(0, \frac{-f}{2}\right)$

$\therefore$ Normal at A will pass through centre C and intersect circle again at B.

$\therefore$ Coordinates of B are $\left(-g, \frac{-f}{2}\right)$.

2017 (II)

6. The equation of the circle which passes through the points $(1, 0)$, $(0, -6)$ and $(3, 4)$ is

(a) $4x^2 + 4y^2 + 142x + 47y + 140 = 0$
(b) $4x^2 + 4y^2 - 142x - 47y + 138 = 0$
(c) $4x^2 + 4y^2 - 142x + 47y + 138 = 0$
(d) $4x^2 + 4y^2 + 150x - 49y + 138 = 0$

(**c**) Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Since, above circle passes through $(1, 0)$, $(0, -6)$ and $(3, 4)$

$\therefore \quad 1 + 2g + c = 0 \quad ...(i)$

$36 - 12f + c = 0 \quad ...(ii)$

and $9 + 16 + 6g + 8f + c = 0$

or $6g + 8f + c + 25 = 0 \quad ...(iii)$

On solving (i), (ii) and (iii), we get

$$g = -\frac{71}{4}, f = \frac{47}{8}, c = \frac{69}{2}$$

$\therefore$ Equation of circle will be

$$x^2 + y^2 + 2\left(-\frac{71}{4}\right)x + 2\left(\frac{47}{8}\right)y + \frac{69}{2} = 0$$

$$\Rightarrow \quad 4x^2 + 4y^2 - 142x + 47y + 138 = 0$$

2017 (I)

7. The two circles $x^2 + y^2 = r^2$ and $x^2 + y^2 - 10x + 16 = 0$ intersect at two distinct points. Then which one of the following is correct?

(a) $2 < r < 8$
(b) $r = 2$ or $r = 8$
(c) $r < 2$
(d) $r > 2$

(**a**) Given, circle $x^2 + y^2 = r^2$

$C_1(0, 0), r_1 = r$

and $x^2 + y^2 - 10x + 16 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -10 \Rightarrow g = -5, f = 0, c = 16$

$C_2(-g, -f) = C_2(5, 0)$

$\therefore$ Radius $r_2 = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(5)^2 + (0)^2 - 16}$

$= \sqrt{25 - 16} = 3$

Now, $c_1c_2 = \sqrt{(5 - 0)^2 + (0 - 0)^2}$

$= \sqrt{(5)^2 + 0^2} = 5$ and $r_1 + r_2 = r + 3$

For intersect at two distinct point

$(r_1 - r_2) < c_1c_2 < (r_1 + r_2)$

$\Rightarrow \quad r - 3 < c_1c_2 < r + 3$

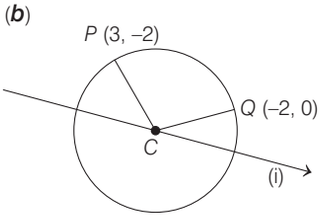
$\Rightarrow \quad r - 3 < 5 < r + 3$

$\therefore \quad 2 < r < 8$

8. What is the equation of the circle which passes through the points $(3, -2)$ and $(-2, 0)$ and having its centre on the line $2x - y - 3 = 0$?

(a) $x^2 + y^2 + 3x + 2 = 0$
(b) $x^2 + y^2 + 3x + 12y + 2 = 0$
(c) $x^2 + y^2 + 2x = 0$
(d) $x^2 + y^2 = 5$

(**b**)



$2x - y - 3 = 0 \quad ... (i)$

$\because$ Centre of the given circle lies on the line (i)

Let x-coordinate of centre $= t$

$\therefore$ y-coordinate of centre $= 2t - 3$ [$\because$ centre lies on line (i)]

$\therefore$ Coordinates of centre

$C \equiv (t, 2t - 3)$

$\therefore \quad CP^2 = CQ^2$

$\Rightarrow (t - 3)^2 + (2t - 3 - (-2))^2$
$= (t - (-2))^2 + (2t - 3 - 0)^2$

$\Rightarrow \quad (t^2 + 9 - 6t) + (4t^2 + 1 - 4t)$
$= t^2 + 4 + 4t + 4t^2 + 9 - 12t$

$\Rightarrow \quad t = \frac{-3}{2}$

$\therefore$ Coordinates of

$C \equiv (t, 2t - 3) \equiv \left(\frac{-3}{2}, -6\right)$

Now, radius of the circle,

$$r = \sqrt{\left[\frac{-3}{2} - (-2)\right]^2 + [-6 - 0]^2} = \sqrt{\frac{145}{4}}$$

$\therefore$ Equation of required circle is

$$\left[x - \left(\frac{-3}{2}\right)\right]^2 + [y - (-6)]^2 = \left(\sqrt{\frac{145}{4}}\right)^2$$

$\Rightarrow \quad x^2 + y^2 + 3x + 12y + 2 = 0$

2016 (II)

9. What is the radius of the circle passing through the point $(2, 4)$ and having centre at the intersection of the lines $x - y = 4$ and $2x + 3y + 7 = 0$?

(a) 3 units (b) 5 units
(c) $3\sqrt{3}$ units (d) $5\sqrt{2}$ units

(**d**) Given,

$x - y = 4 \quad ... (i)$

and $2x + 3y + 7 = 0 \quad ... (ii)$

On solving Eqs. (i) and (ii), we get

$x = 1$ and $y = -3$

$\therefore$ Centre of the circle is $(1, -3)$.

$\therefore$ Radius of the circle is

$= \sqrt{(2 - 1)^2 + (4 + 3)^2}$

$= \sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$ units

2016 (I)

Directions (Q. Nos. 10 and 11)

Consider the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ *and* $x^2 + y^2 - 8x + 2y + 8 = 0$

10. What is the distance between the centres of the two circles?

(a) 5 units (b) 6 units
(c) 8 units (d) 10 units

(**a**) Given, circle are

$(x - 1)^2 + (y - 3)^2 = r^2 \quad ...(i)$

and $x^2 + y^2 - 8x + 2y + 8 = 0 \quad ...(ii)$

Centre of circle (i) $= (1, 3)$ and radius $r_1 = r$

Centre of circle (ii) $= (4, -1)$

and radius $(r_2) = \sqrt{16 + 1 - 8} = \sqrt{9} = 3$

$\therefore$ Distance between centres

$= \sqrt{(1 - 4)^2 + (3 + 1)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ units

11. If the circles intersect at two distinct points, then which one of the following is correct?

(a) $r = 1$ (b) $1 < r < 2$
(c) $r = 2$ (d) $2 < r < 8$

(**d**) Condition for two circles intersect at two distinct points. Distance between centres $<$ Sum of radii

$5 < r + 3$

$\Rightarrow \quad r > 2$

Directions (Q. Nos. 12 and 13) *Consider a circle passing through the origin and the points* (a, b) *and* $(-b, -a)$.

12. On which line does the centre of the circle lie?

(a) $x + y = 0$ (b) $x - y = 0$
(c) $x + y = a + b$ (d) $x - y = a^2 - b^2$

13. What is the sum of the squares of the intercepts cut-off by the circle on the axes?

(a) $\left(\frac{a^2 + b^2}{a^2 - b^2}\right)^2$ (b) $2\left(\frac{a^2 + b^2}{a - b}\right)^2$

(c) $4\left(\frac{a^2 + b^2}{a - b}\right)^2$ (d) None of these

› **(Solutions for 12 and 13)** Given that circle passes through $(0, 0)$, (a, b) and $(-b, -a)$

Now, equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

When $x = 0, y = 0$

$$\Rightarrow \quad 0 + 0 + 0 + 0 + c = 0$$

$$\Rightarrow \quad c = 0 \qquad ...(i)$$

When $x = a, y = b$

$$a^2 + b^2 + 2ga + 2fb = 0 \quad [\because c = 0]$$

$$\Rightarrow \quad ga + fb = -\frac{1}{2}(a^2 + b^2) \qquad ...(ii)$$

When $x = -b, y = -a$

$$b^2 + a^2 - 2gb - 2fa = 0$$

$$\Rightarrow \quad gb + fa = \frac{1}{2}(a^2 + b^2) \qquad ...(iii)$$

On multiplying Eq. (ii) by a and Eq. (iii) by b, we get

$$a^2g + abf = -\frac{1}{2}(a^2 + b^2)\cdot a$$

$$b^2g + abf = \frac{1}{2}(a^2 + b^2)\cdot b$$

$$(a^2 - b^2)\,g = -\frac{1}{2}(a^2 + b^2)(a + b)$$

$$\Rightarrow \quad g = -\frac{1}{2}\frac{(a^2 + b^2)(a + b)}{(a^2 - b^2)}$$

$$\Rightarrow \quad g = -\frac{1}{2}\left(\frac{a^2 + b^2}{a - b}\right) \qquad ...(iv)$$

Again, on multiplying Eq. (ii) by b and Eq. (iii) by a, we get

$$abg + fb^2 = -\frac{1}{2}(a^2 + b^2)\cdot b$$

$$abg + fa^2 = \frac{1}{2}(a^2 + b^2)\cdot a$$

$$f(b^2 - a^2) = -\frac{1}{2}(a^2 + b^2)(b + a)$$

$$\Rightarrow \quad f = -\frac{1}{2}\frac{a^2 + b^2}{b^2 - a^2}(b + a)$$

$$f = -\frac{1}{2}\left(\frac{a^2 + b^2}{b - a}\right) \qquad ...(v)$$

12. (***a***) From Eqs. (iv) and (v), we get

$$f = -g \quad \Rightarrow \quad y = -x$$

$\therefore$ Required equation of line is $x + y = 0$

13. (***b***) The equation of circle is

$$x^2 + y^2 - \left(\frac{a^2 + b^2}{a - b}\right)x - \left(\frac{a^2 + b^2}{b - a}\right)y = 0$$

Now, for x-intercept, put $y = 0$

$$\therefore \quad x^2 - \left(\frac{a^2 + b^2}{a - b}\right)x = 0$$

$$\Rightarrow \quad x = \frac{a^2 + b^2}{a - b}$$

For y-intercept, put $x = 0$

$$y^2 - \left(\frac{a^2 + b^2}{b - a}\right)y = 0$$

$$\Rightarrow \quad y = \frac{a^2 + b^2}{b - a}$$

Sum of the square of intercepts

$$= (x\text{-intercept})^2 + (y\text{-intercept})^2$$

$$= \left(\frac{a^2 + b^2}{a - b}\right)^2 + \left(\frac{a^2 + b^2}{b - a}\right)^2$$

$$= 2\left(\frac{a^2 + b^2}{a - b}\right)^2$$

2015 (II)

14. A straight line $x = y + 2$ touches the circle $4(x^2 + y^2) = r^2$. The value of r is

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 2 (d) 1

› (***b***) Given, $x^2 + y^2 = \frac{r^2}{4}$

We know that the line $y = mx + c$ meets the circle in unique real point or touch the circle $x^2 + y^2 = r^2$, if $r = \left|\frac{c}{\sqrt{1 + m^2}}\right|$

Since, the straight line $x = y + 2$ touches the given circle.

Hence, $\left|\frac{2}{\sqrt{2}}\right| = \frac{r}{2} \quad \Rightarrow \quad r = 2\sqrt{2}$

15. If the centre of the circle passing through the origin is $(3, 4)$, then the intercepts cut-off by the circle on X-axis and Y-axis respectively, are

(a) 3 units and 4 units
(b) 6 units and 4 units
(c) 3 units and 8 units
(d) 6 units and 8 units

› (***d***) We have, centre $= (3, 4)$ and radius $= 5$

Equation of circle having centre (h, k) and radius a is $(x - h)^2 + (y - k)^2 = a^2$

Equations $(x - 3)^2 + (y - 4)^2 = 25$

For *X*-intercept

On putting $y = 0$, $(x - 3)^2 + 16 = 25$

$\Rightarrow \quad (x - 3)^2 = 9$

$\Rightarrow \quad x - 3 = 3$ and -3

$\Rightarrow \quad x = 6$ and 0

For *Y*-intercept

On putting $x = 0$, $9 + (y - 4)^2 = 25$

$\Rightarrow \quad y - 4 = 4$ and -4

$\therefore \quad y = 8$ and 0

x-intercept $= 6$ and y-intercept $= 8$

2014 (II)

Directions (Q. Nos. 16 and 17) *Read the following information carefully and answer the questions given below.*

Consider the circles

$x^2 + y^2 + 2ax + c = 0$ and
$x^2 + y^2 + 2by + c = 0$.

16. What is the distance between the centres of the two circles?

(a) $\sqrt{a^2 + b^2}$ (b) $a^2 + b^2$
(c) $a + b$ (d) $2(a + b)$

› (***a***) Equations of circles are

$$x^2 + y^2 + 2ax + c = 0$$

and $x^2 + y^2 + 2by + c = 0$

Since, the centres of two circles are $(-a, 0)$ and $(0, -b)$.

$\therefore$ Distance between two centres $= \sqrt{a^2 + b^2}$

17. The two circles touch each other, if

(a) $c = \sqrt{a^2 + b^2}$ (b) $\frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$

(c) $c = \frac{1}{a^2} + \frac{1}{b^2}$ (d) $c = \frac{1}{a^2 + b^2}$

› (***b***) Two circles touch each other iff Distance between two centres = Sum of radii of two circles

$$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{a^2 - c} + \sqrt{b^2 - c}$$

On squaring both sides, we get

$$a^2 + b^2 = a^2 - c + b^2 - c + 2\sqrt{(a^2 - c)(b^2 - c)}$$

$$\Rightarrow \quad c = \sqrt{(a^2 - c)(b^2 - c)}$$

Again, squaring both sides, we get

$$c^2 = a^2b^2 - a^2c - b^2c + c^2$$

$$\Rightarrow \quad a^2b^2 = (a^2 + b^2)c$$

$$\therefore \quad \frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$$

17

CONIC SECTION

2019 (II)

1. If the angle between the lines joining the end points of minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with one of the its foci is $\frac{\pi}{2}$, then what is the eccentricity of the ellipse?

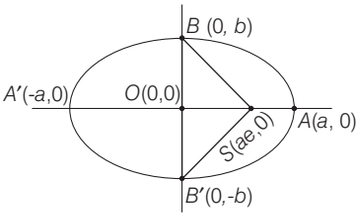
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$

(***b***) Equation of the ellipse,

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

End points of minor axis are $(0, b)$, $(0, -b)$ and one foci is $(ae, 0)$



Slope of line $BS = \frac{0-b}{ae-0} = -\frac{b}{ae}(m_1)$

Slope of line $B'S = \frac{0+b}{ae-0} = \frac{b}{ae}(m_2)$

According to the question, angle between BS and $B'S$ is $\frac{\pi}{2}$.

i.e. BS and $B'S$ are perpendicular,

$\therefore \quad m_1m_2 = -1$

$\frac{-b}{ae}\times\frac{b}{ae} = -1$

$\Rightarrow \quad b^2 = a^2e^2 \quad \ldots$ (i)

We know that, $e = \sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow \quad a^2e^2 = a^2 - b^2$

$\Rightarrow \quad a^2e^2 = a^2 - a^2e^2$ [from Eq. (i)]

$\Rightarrow \quad 2a^2e^2 = a^2 \Rightarrow e^2 = \frac{1}{2}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$

2019 (I)

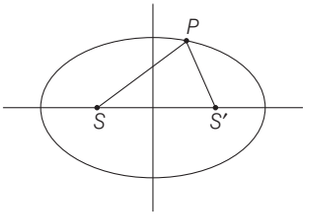
2. The sum of the focal distances of a point on an ellipse is constant and equal to

(a) length of minor axis
(b) length of major axis
(c) length of latusrectum
(d) sum of the lengths of semi major and semi minor axes

(***b***) The sum of the focal distance of a point on a ellipse is constant and equal to the length of major axis.

We know that,

$PS + PS' = 2a$



3. The equation $2x^2 - 3y^2 - 6 = 0$ represents

(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

(***d***) Given, $2x^2 - 3y^2 - 6 = 0$

$\Rightarrow \quad 2x^2 - 3y^2 = 6$

$\Rightarrow \quad \frac{x^2}{3}-\frac{y^2}{2}=1$

Which represents the equation of a hyperbola.

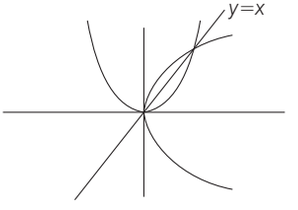
4. The two parabolas $y^2 = 4ax$ and $x^2 = 4ay$ intersect

(a) at two points on the line $y = x$
(b) only at the origin
(c) at three points one of which lies on $y + x = 0$
(d) only at $(4a, 4a)$

(***a***) Given, $y^2 = 4ax$

and $x^2 = 4ay$

The graph of given curve is clearly from graph the given curve is intersect at two points on the line $y = x$



2018 (II)

5. The second degree equation $x^2 + 4y^2 - 2x - 4y + 2 = 0$ represents

(a) a point
(b) an ellipse of semi-major axis 1
(c) an ellipse with eccentricity $\frac{\sqrt{3}}{2}$
(d) None of the above

(***a***) Given that,

$x^2 + 4y^2 - 2x - 4y + 2 = 0$

$(x^2 - 2x + 1) + 4\left(y^2 - y + \frac{1}{2}\right) = 0$

$(x-1)^2 + 4\left(y-\frac{1}{2}\right)^2 = 0$

$\therefore \quad x = 1, y = \frac{1}{2}$

Which represents a point

2018 (I)

6. What is the equation of the ellipse whose vertices are $(\pm 5, 0)$ and foci are at $(\pm 4, 0)$?

(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (b) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(c) $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (d) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

(**a**) We have,

Vertices = $(\pm\ 5, 0)$ and foci = $(\pm 4, 0)$

$\therefore$ $a = 5$ and $ae = 4$ [$\because$ vertex = $(\pm a, 0)$ and focus $(\pm ae, 0)$

$\Rightarrow$ $e = \frac{4}{5}$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow$ $\left(\frac{4}{5}\right)^2 = 1 - \frac{b^2}{(5)^2}$ [$\because a = 5$]

$\Rightarrow$ $\frac{16}{25} = 1 - \frac{b^2}{25}$

$\Rightarrow$ $16 = 25 - b^2$

$\Rightarrow$ $b^2 = 9$

$\Rightarrow$ $b = 3$

$\therefore$ Equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\Rightarrow$ $$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

2017 (II)

7. A man running round a racecourse notes that the sum of the distances of two flag-posts from him is always 10 m and the distance between the flag-posts is 8 m. The area of the path he encloses is

(a) 18π m^2

(b) 15π m^2

(c) 12π m^2

(d) 8π m^2

(**b**) Since the shape of a racecourse is of the form of ellipse.

Let the equation of racecourse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Now, according to the question

$PF_1 + PF_2 = 10$ and $F_1F_2 = 8$

$\therefore$ $2a = 10$ and $2ae = 8$

$\Rightarrow$ $a = 5$ and $e = \frac{4}{5}$

Again, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow$ $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow$ $\left(\frac{4}{5}\right)^2 = 1 - \frac{b^2}{(5)^2}$

$\Rightarrow$ $\frac{16}{25} = 1 - \frac{b^2}{25} \Rightarrow \frac{b^2}{25} = \frac{9}{25}$

$\Rightarrow$ $b^2 = 9 \Rightarrow b = 3$

Now, area of an ellipse = πab

$\therefore$ Required area = π (5) (3)

$= 15\ \pi$ m^2

8. The position of the point $(1, 2)$ relative to the ellipse $2x^2 + 7y^2 = 20$ is

(a) outside the ellipse

(b) inside the ellipse but not at the focus

(c) on the ellipse

(d) at the focus

(**a**) Let $S : 2x^2 + 7y^2 - 20$

$\therefore$ $S_1 = 2(1)^2 + 7(2)^2 - 20$ [Put $x = 1$, $y = 2$]

$\Rightarrow$ $S_1 = 2 + 28 - 20$

$\Rightarrow$ $S_1 = 10$

$\therefore$ $S_1 > 0$

Hence, point (1, 2) lies outside the ellipse.

9. The equation of the ellipse whose centre is at origin, major axis is along X-axis with eccentricity $\frac{3}{4}$ and latusrectum 4 units is

(a) $\frac{x^2}{1024} + \frac{7y^2}{64} = 1$ (b) $\frac{49x^2}{1024} + \frac{7y^2}{64} = 1$

(c) $\frac{7x^2}{1024} + \frac{49y^2}{64} = 1$ (d) $\frac{x^2}{1024} + \frac{y^2}{64} = 1$

(**b**) Given that,

Eccentricity of ellipse, $e = \frac{3}{4}$ ...(i)

It is given that major axis is along X-axis.

$\therefore$ $\frac{2b^2}{a} = 4$

$\Rightarrow$ $b^2 = 2a$ ...(ii)

Since, eccentricity of an ellipse,

$b^2 = a^2(1 - e^2)$

$\Rightarrow$ $2a = a^2(1 - (3/4)^2)$

$\Rightarrow$ $2a = a^2\left(1 - \frac{9}{16}\right)$

$\Rightarrow$ $2a = \frac{7}{16}a^2 \Rightarrow 32a = 7a^2$

$\Rightarrow$ $a = \frac{32}{7}$ [$\because a \neq 0$]

Put the value of a in Eq (ii), we get

$$b^2 = 2 \times \frac{32}{7} \Rightarrow b^2 = \frac{64}{7}$$

$\therefore$ Equation of the ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\left(\frac{32}{7}\right)^2} + \frac{y^2}{\frac{64}{7}} = 1$

$\Rightarrow$ $\frac{49x^2}{1024} + \frac{7y^2}{64} = 1$

2017 (I)

10. What is the equation of the ellipse having foci $(\pm 2, 0)$ and the eccentricity $\frac{1}{4}$?

(a) $\frac{x^2}{64} + \frac{y^2}{60} = 1$ (b) $\frac{x^2}{60} + \frac{y^2}{64} = 1$

(c) $\frac{x^2}{20} + \frac{y^2}{24} = 1$ (d) $\frac{x^2}{24} + \frac{y^2}{20} = 1$

(**a**) Given,

coordinates of foci $\equiv$ (2, 0) and (– 2, 0)

and eccentricity = $\frac{1}{4}$

We know that coordinates of foci $\equiv (ae, 0)$ and $(-ae, 0)$

$a = \frac{2}{e} = \frac{2}{(1/4)} = 8$ $\left[\because e = \frac{1}{4}\right]$

$\therefore$ $a^2 = 64$

$\because$ $b^2 = a^2(1 - e^2) = 64\left[1 - \left(\frac{1}{4}\right)^2\right] = 60$

$\therefore$ Required equation ellipse will be

$$\frac{x^2}{64} + \frac{y^2}{60} = 1$$

2016 (II)

11. What is the equation of the hyperbola having latusrectum and eccentricity 8 and $\frac{3}{\sqrt{5}}$ respectively?

(a) $\frac{x^2}{25} - \frac{y^2}{20} = 1$ (b) $\frac{x^2}{40} - \frac{y^2}{20} = 1$

(c) $\frac{x^2}{40} - \frac{y^2}{30} = 1$ (d) $\frac{x^2}{30} - \frac{y^2}{25} = 1$

(**a**) Given, length of latusrectum

$= \frac{2b^2}{a} = 8$ ...(i)

and eccentricity = $\sqrt{1 + \left(\frac{b}{a}\right)^2} = \frac{3}{\sqrt{5}}$

$\Rightarrow$ $1 + \left(\frac{b}{a}\right)^2 = \frac{9}{5}$

$\Rightarrow$ $\frac{b^2}{a^2} = \frac{4}{5}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 20$

∴ Equation of hyperbola will be

$$\frac{x^2}{25} - \frac{y^2}{20} = 1.$$

12. If the ellipse $9x^2 + 16y^2 = 144$ intercepts the line $3x + 4y = 12$, then what is the length of the chord so formed?

(a) 5 units (b) 6 units
(c) 8 units (d) 10 units

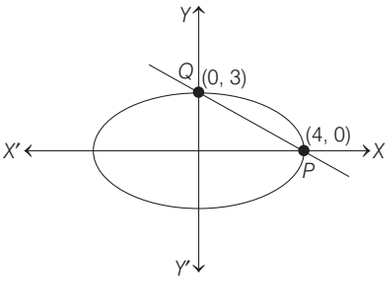
(**a**) Given,

$$9x^2 + 16y^2 = 144$$

$$\Rightarrow \quad \frac{x^2}{16} + \frac{y^2}{9} = 1 \qquad \ldots (i)$$

and $3x + 4y = 12$

$$\Rightarrow \quad \frac{x}{4} + \frac{y}{3} = 1 \qquad \ldots (ii)$$



From figure, given ellipse intercepts the line at point (4, 0) and (0, 3).

∴ Length of chord

$PQ = \sqrt{(4-0)^2 + (0-3)^2} = 5$ units

13. What is the eccentricity of rectangular hyperbola?

(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\sqrt{5}$ (d) $\sqrt{6}$

(**a**) We know that,

equation of rectangular hyperbola

$$x^2 - y^2 = p^2 \Rightarrow \frac{x^2}{p^2} - \frac{y^2}{p^2} = 1$$

Here, length of transverse axis and conjugate axis are equal.

$$\therefore \text{Eccentricity} = \sqrt{1 + \left(\frac{b}{a}\right)^2} = \sqrt{1 + \left(\frac{p}{p}\right)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

Directions (Q. Nos. 14 and 15)

Consider the parabola $y = x^2 + 7x + 2$ *and the straight line* $y = 3x - 3$.

14. What are the coordinates of the point on the parabola which is closest to the straight line?

(a) (0, 2) (b) (–2, – 8)
(c) (–7,2) (d) (1, 10)

15. What is the shortest distance from the above point on the parabola to the line?

(a) $\frac{\sqrt{10}}{2}$ (b) $\frac{\sqrt{10}}{5}$ (c) $\frac{1}{\sqrt{10}}$ (d) $\frac{\sqrt{5}}{4}$

Solutions (Q. Nos. 14 and 15)

Given, $y = x^2 + 7x + 2$

$$= \left(x + \frac{7}{2}\right)^2 - \frac{41}{4}$$

Let the coordinates of the point on this parabola be $\left(P - \frac{7}{2}, P^2 - \frac{41}{4}\right)$

The equation of the line is $y = 3x - 3$

$\Rightarrow y - 3x + 3 = 0$

Distance of the point from the line

$$z = \frac{P^2 - \frac{41}{4} - 3\left(P - \frac{7}{2}\right) + 3}{\sqrt{10}}$$

$$\therefore \quad \frac{dz}{dt} = \frac{1}{\sqrt{10}}(2P - 3) = 0$$

$$\Rightarrow \quad P = \frac{3}{2}$$

(**b**) Coordinates of the point on the parabola which is closest to the straight line

$$= \left(P - \frac{7}{2}, P^2 - \frac{41}{4}\right)$$

$$= \left(\frac{3}{2} - \frac{7}{2}, \left(\frac{3}{2}\right)^2 - \frac{41}{4}\right) = (-2, -8)$$

$$\left[\text{for } P = \frac{3}{2}\right]$$

(**c**) Shortest distance from the point to parabola

$$= \frac{P^2 - \frac{41}{4} - 3\left(P - \frac{7}{2}\right) + 3}{\sqrt{10}}$$

$$= \frac{\left(\frac{3}{2}\right)^2 - \frac{41}{4} - 3\left(\frac{3}{2} - \frac{7}{2}\right) + 3}{\sqrt{10}}$$

$$= \frac{\frac{9}{4} - \frac{41}{4} + 6 + 3}{\sqrt{10}}$$

$$= \frac{-8 + 6 + 3}{\sqrt{10}} = \frac{1}{\sqrt{10}} \text{ unit}$$

2015 (II)

16. Consider any point P on the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ in the first quadrant. Let r and s represent its distances from (4, 0) and (–4, 0) respectively, then $(r + s)$ is equal to

(a) 10 units (b) 9 units
(c) 8 units (d) 6 units

(**a**) We have an ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Clearly, its foci are (4, 0) and (– 4, 0).

[∵ S (ae,0), S' (– ae, 0)]

∴ $PS + PS' = 2a$ = major axis

$\Rightarrow \quad r + s = 2(5) = 10$ units

17. The eccentricity of the hyperbola $16x^2 - 9y^2 = 1$ is

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{4}{5}$ (d) $\frac{5}{4}$

(**b**) Given, $\frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{1}{9}} = 1$

∴ Eccentricity,

$$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{\frac{1}{9}}{\frac{1}{16}}$$

$$= 1 + \frac{16}{9} = \frac{25}{9}$$

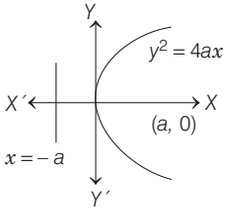
$$\Rightarrow \quad e = \frac{5}{3}$$

2015 (I)

18. The point on the parabola $y^2 = 4ax$ nearest to the focus has its abscissa

(a) $x = 0$ (b) $x = a$
(c) $x = \frac{a}{2}$ (d) $x = 2a$

(**a**)



∴ Required abscissa is $x = 0$.

19. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point $(3\sqrt{5}, 1)$ and the length of its latusrectum is $\frac{4}{3}$ units. The length of the conjugate axis is

(a) 2 units (b) 3 units
(c) 4 units (d) 5 units

(**c**) Since, hyperbola passes through $(3\sqrt{5}, 1)$.

$$\therefore \quad \frac{(3\sqrt{5})^2}{a^2} - \frac{1}{b^2} = 1 \quad \Rightarrow \frac{45}{a^2} - \frac{1}{b^2} = 1$$

$$\Rightarrow \quad 45b^2 - a^2 = a^2b^2 \qquad \ldots (i)$$

Since, length of latusrectum $= \frac{2b^2}{a}$

$$\frac{2b^2}{a} = \frac{4}{3}$$

$\Rightarrow \quad 6b^2 = 4a$

$\Rightarrow \quad a = \frac{6b^2}{4}$ …(ii)

On putting the value from Eq. (ii) in Eq. (i), we get

$$45b^2 - \left(\frac{6b^2}{4}\right)^2 = \left(\frac{6b^2}{4}\right)^2 b^2$$

$\Rightarrow \quad 45b^2 - \frac{36b^4}{16} = \frac{36b^4 \cdot b^2}{16}$

$\Rightarrow \quad 45b^2 = \frac{36b^4}{16}(b^2 + 1)$

$\Rightarrow \quad 45 \times 16 = 36b^2(b^2 + 1)$

$\Rightarrow \quad b^4 + b^2 = 20$

$\therefore \quad b = 2$

$\therefore$ Length of conjugate axis $= 2b = 4$

2014 (II)

20. Where does the line cut the parabola? Line $2y = 3x + 12$ cuts the parabola $4y = 3x^2$

(a) at (–2 , 3) only
(b) at (4, 12) only
(c) at both (–2 , 3) and (4, 12)
(d) Neither at (–2 , 3) nor at (4, 12)

(**c**) Equation of line is

$$2y = 3x + 12 \quad \ldots(i)$$

Equation of parabola is $4y = 3x^2$ ...(ii)

On putting the value of y from Eq. (i) to Eq. (ii), we get

$$2(3x + 12) = 3x^2$$

$\Rightarrow \quad 3x^2 - 6x - 24 = 0$

$\Rightarrow \quad x^2 - 2x - 8 = 0$

$\Rightarrow \quad (x - 4)(x + 2) = 0$

$\Rightarrow \quad x = 4$ and $x = -2$

On putting these values in Eq. (ii), we get

$$y = 12$$

and $\quad y = 3$

Hence, points are (–2, 3) and (4, 12).

21. What is the length of the latusrectum of an ellipse $25x^2 + 16y^2 = 400$?

(a) $\frac{25}{2}$ (b) $\frac{25}{4}$
(c) $\frac{16}{5}$ (d) $\frac{32}{5}$

(**d**) Equation of ellipse is

$$25x^2 + 16y^2 = 400$$

$\Rightarrow \quad \frac{x^2}{16} + \frac{y^2}{25} = 1$

Here, $\quad a^2 = 16$ and $b^2 = 25$

$\therefore$ Length of latusrectum

$$= \frac{2a^2}{b} = \frac{2 \times 16}{5} = \frac{32}{5}$$

2014 (I)

22. What is the sum of the major and minor axes of the ellipse whose eccentricity is 4/5 and length of latusrectum is 14.4 units?

(a) 32 units (b) 48 units
(c) 64 units (d) None of these

(**c**) We know that, length of major axes of an ellipse $= 2a$

and length of minor axes of an ellipse $= 2b$

Given that, eccentricity of an ellipse

$$= 4 / 5 = e \quad \ldots (i)$$

and length of latusrectum of an ellipse = 14.4 units

$\Rightarrow \quad \frac{2b^2}{a} = 14.4$

$\Rightarrow \quad \frac{b^2}{a} = 7.2$

$\Rightarrow \quad b^2 = 7.2\ a$ …(ii)

Since, eccentricity of an ellipse,

$$b^2 = a^2 (1 - e^2)$$

$\Rightarrow \quad 7.2a = a^2\left[1 - \left(\frac{4}{5}\right)^2\right]$

[from Eqs. (i) and (ii)]

$\Rightarrow \quad 7.2a = a^2\left(1 - \frac{16}{25}\right)$

$\Rightarrow \quad 7.2a = a^2 \times \frac{9}{25}$

$\Rightarrow \quad 9a^2 - 7.2 \times 25a = 0$

$\Rightarrow \quad 9a^2 - 36 \times 5a = 0$

$\Rightarrow \quad 9a\,(a - 20) = 0$

$\Rightarrow \quad a = 20 \quad [\because a \neq 0]$

Put the value of a in Eq. (ii), we get

$$b^2 = 7.2 \times 20$$

$\Rightarrow \quad b^2 = 72 \times 2 = 144$

$\Rightarrow \quad b^2 = (12)^2$

$\therefore \quad b = 12$

Hence, the sum of the major and minor axes

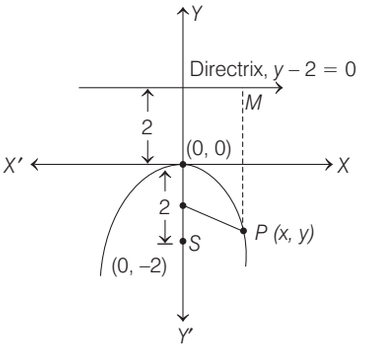
$= 2a + 2b = 2\,(a + b) = 2\,(20 + 12)$

$= 2 \times 32 = 64$ units

23. What is the equation of parabola whose vertex is at (0, 0) and focus is at (0, – 2)?

(a) $y^2 + 8x = 0$
(b) $y^2 - 8x = 0$
(c) $x^2 + 8y = 0$
(d) $x^2 - 8y = 0$

(**c**) Given vertex of the parabola = (0, 0) and focus of the parabola = (0, – 2)



Let P be any point on the parabola, then equation of directrix is $y - 2 = 0$

$\therefore$ Equation of parabola is $PS = PM$

$\Rightarrow \quad (x - 0)^2 + (y + 2)^2 = \frac{|y - 2|}{\sqrt{1}}$

$\Rightarrow \quad \left[\sqrt{x^2 + (y + 2)^2}\right]^2 = |y - 2|^2$

$\Rightarrow \quad x^2 + y^2 + 4 + 4y = y^2 + 4 - 4y$

$x^2 = -8y$, which is the required equation of parabola.

18

THREE DIMENSIONAL GEOMETRY

2019 (II)

1. If the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies on the plane $2x-4y+z=7$, then what is the value of k?

(a) 2 (b) 3
(c) 5 (d) 7

(**d**) Equation of line

$$=\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}=r$$

$\therefore (r+4, r+2, 2r+k)$ point lies on the line.

This line lies on the plane

$$2x-4y+z=7$$

Then, the point $(r+4, r+2, 2r+k)$ lies on the plane, we get

$$2(r+4)-4(r+2)+(2r+k)=7$$
$$\Rightarrow \quad 2r+8-4r-8+2r+k=7$$
$$\Rightarrow \quad k=7$$

Hence, the value of k is 7.

2. A point on the line

$$\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+2}{7}$$

has coordinates

(a) (3, 5, 4) (b) (2, 5, 5)
(c) (– 1, – 1, 5) (d) (2, – 1, 0)

(**b**) Equation of the line

$$\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+2}{7}$$

From option, point (2, 5, 5) is satisfying the given equation of line.

$$\left[\because \frac{2-1}{1}=\frac{5-3}{2}=\frac{5+2}{7}\Rightarrow 1=1=1\right]$$

Hence, the coordinates of required point (2, 5, 5).

3. If the points $(x, y, -3), (2, 0, -1)$ and $C(4, 2, 3)$ lie on a straight line, then what are the values of x and y respectively?

(a) 1, – 1 (b) –1, 1
(c) 0, 2 (d) 3, 4

(**a**) Points, $A(x, y, -3)$, $B(2, 0, -1)$ and $(4, 2, 3)$. These points lie on a straight line, then direction ratios of $AB = \lambda$ (direction ratios of BC)

$$\therefore (2-x, 0-y, -1+3)$$
$$=(4-2, 2-0, 3+1)$$
$$\Rightarrow (2-x, -y, 2)=(2, 2, 4)$$
$$\Rightarrow (2-x, -y, 2)=2(1, 1, 2)$$

Comparing both sides,

$$2-x=1\Rightarrow x=1$$

and $-y=1\Rightarrow y=-1$

4. A straight line passes through the point (1, 1, 1) makes an angle 60° with the positive direction of Z-axis, and the cosine of the angles made by it with the positive directions of the Y-axis and the X-axis are in the ratio $\sqrt{3}:1$. What is the acute angle between the two possible positions of the line?

(a) 90° (b) 60° (c) 45° (d) 30°

(**b**) Let the straight line makes the angle with X-axis, Y-axis and Z-axis be α, β and γ.

$$\therefore \gamma=60^\circ \text{ and } \frac{\cos\beta}{\cos\alpha}=\frac{\sqrt{3}}{1}$$

If l, m and n are the direction cosines of the lines, then

$$n=\cos\gamma=\cos 60^\circ=\frac{1}{2}$$

and $\frac{m}{l}=\frac{\cos\beta}{\cos\alpha}=\frac{\sqrt{3}}{1}$

$$\Rightarrow \quad \frac{m}{l}=\frac{\sqrt{3}}{1}=k \text{ (Let)}$$
$$\therefore \quad m=\sqrt{3}k, l=k$$

We know that, $l^2+m^2+n^2=1$

$$k^2+3k^2+\frac{1}{4}=1$$
$$\Rightarrow 4k^2=1-\frac{1}{4}=\frac{3}{4}$$
$$\Rightarrow \quad k^2=\frac{3}{16}\Rightarrow k=\pm\frac{\sqrt{3}}{4}$$
$$\therefore \quad l_1=\frac{\sqrt{3}}{4}, m_1=\frac{3}{4}, n_1=\frac{1}{2}$$

and $l_2=-\frac{\sqrt{3}}{4}, m_2=-\frac{3}{4}, n_2=\frac{1}{2}$

We know that,

$$\cos\theta=|l_1l_2+m_1m_2+n_1n_2|$$
$$\Rightarrow \cos\theta=\left|-\frac{3}{16}-\frac{9}{16}+\frac{1}{4}\right|$$
$$=\left|\frac{-3-9+4}{16}\right|=\left|\frac{-8}{16}\right|$$
$$\cos\theta=\frac{1}{2}=\cos 60^\circ$$
$$\therefore \quad \theta=60^\circ$$

5. A point on a line has coordinates $(p+1, p-3, \sqrt{2}p)$ where p is any real number. What are the direction cosines of the line?

(a) $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$
(b) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}, \frac{1}{2}, -\frac{1}{2}$
(d) Cannot be determined due to insufficient data

(**d**) Coordinate of a point on a line is $(p+1, p-3, \sqrt{2}p)$, p is any real number.

Equation of a line, whose direction ratios are a, b and c and passing through the point (x_1, y_1, z_1)

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r$

$\therefore (ar + x_1, br + y_1, cr + z_1)$ any point on the line.

According to the questions,

$(ar + x_1, br + y_1, cr + z_1)$

$= (p + 1, p - 3, \sqrt{2}p)$

$\therefore \quad ar = p + 1 - x_1 \quad \ldots$ (i)

$br = p - 3 - y_1 \quad \ldots$ (ii)

$cr = \sqrt{2}p - z_1 \quad \ldots$(iii)

Squaring and adding of (i), (ii) and (iii)

$(a^2 + b^2 + c^2)r^2 = (p + 1 - x_1)^2$

$+ (p - 3 - y_1)^2 + (\sqrt{2}p - z_1)^2$

We can not find the values of a, b and c.

Hence, the direction cosines of the line can not be determined due to insufficient data.

2019 (I)

6. The centroid of the triangle with vertices $A(2, -3, 3)$, $B(5, -3, -4)$ and $C(2, -3, -2)$ is the point

(a) (–3, 3, – 1) (b) (3, – 3, – 1)
(c) (3, 1, – 3) (d) (–3, – 1, – 3)

(**b**) Given vertices of triangle ABC are $A(2, -3, 3)$, $B(5, -3, -4)$ and $C(2, -3, -2)$

$\therefore$ Centroid of ΔABC

$= \left(\frac{2 + 5 + 2}{3}, \frac{-3 - 3 - 3}{3}, \frac{3 - 4 - 2}{3}\right)$

$= \left(\frac{9}{3}, \frac{-9}{3}, \frac{-3}{3}\right) = (3, -3, -1)$

7. What is the radius of the sphere $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$?

(a) 5 (b) 2 (c) 7 (d) 3

(**c**) Given, equation of sphere

$x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$

On comparing with

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$, we get

$2u = -6, 2v = 8, 2w = -10, d = 1$

$\Rightarrow u = -3, v = 4, w = -5, d = 1$

$\therefore$ Radius of sphere $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{(-3)^2 + (4)^2 + (-5)^2 - 1}$

$= \sqrt{9 + 16 + 25 - 1} = \sqrt{49} = 7$

8. The equation of the plane passing through the intersection of the planes $2x + y + 2z = 9$, $4x - 5y - 4z = 1$ and the point $(3, 2, 1)$ is

(a) $10x - 2y + 2z = 28$
(b) $10x + 2y + 2z = 28$
(c) $10x + 2y - 2z = 28$
(d) $10x - 2y - 2z = 24$

(**a**) Equation of the plane passing through the intersection of plane $2x + y + 2z = 9$, $4x - 5y - 4z = 1$ is

$(2x + y + 2z - 9) + \lambda(4x - 5y - 4z - 1) = 0$ ...(i)

Since, plane (i) passes through the point (3, 2, 1)

$\therefore (2 \times 3 + 2 + 2 \times 1 - 9) + \lambda(4 \times 3 - 5 \times 2 - 4 \times 1 - 1) = 0$

$\Rightarrow \quad 1 + \lambda(-3) = 0$

$\Rightarrow \quad \lambda = \frac{1}{3}$

On putting $\lambda = \frac{1}{3}$ in Eq. (i), we get

$(2x + y + 2z - 9) + \frac{1}{3}(4x - 5y - 4z - 1) = 0$

$\Rightarrow 6x + 3y + 6z - 27 + 4x - 5y - 4z - 1 = 0$

$\Rightarrow 10x - 2y + 2z - 28 = 0$

$\therefore \quad 10x - 2y + 2z = 28$

9. The distance between the parallel planes $4x - 2y + 4z + 9 = 0$ and $8x - 4y + 8z + 21 = 0$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{7}{4}$

(**a**) Given equation of planes

$4x - 2y + 4z + 9 = 0$ ...(i)

and $8x - 4y + 8z + 21 = 0$

$\Rightarrow \quad 4x - 2y + 4z + \frac{21}{2} = 0$ ...(ii)

Distance between parallel planes (i) and (ii)

$= \frac{\left|\frac{21}{2} - 9\right|}{\sqrt{(4)^2 + (-2)^2 + (4)^2}}$

$= \frac{\frac{3}{2}}{\sqrt{16 + 4 + 16}} = \frac{\frac{3}{2}}{6} = \frac{1}{4}$

10. What are the direction cosines of Z-axis?

(a) <1, 1, 1> (b) <1, 0, 0>
(c) <0, 1, 0> (d) <0, 0, 1>

(**d**) Direction cosines of Z-axis are $<\cos 90°, \cos 90°, \cos 0°> < 0, 0, 1>$

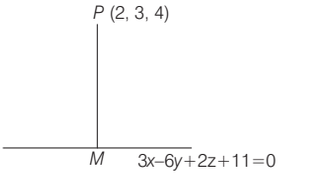
2018 (II)

11. What is the distance of the point (2, 3, 4) from the plane $3x - 6y + 2z + 11 = 0$?

(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units

(**a**) Distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is

$d = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$



$PM = \left|\frac{3(2) - 6(3) + 2(4) + 1}{\sqrt{(3)^2 + (-6)^2 + (2)^2}}\right|$

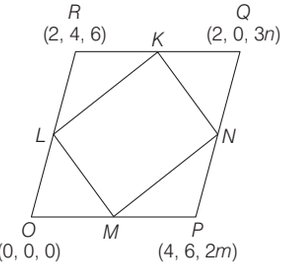
$= \left|\frac{6 - 18 + 8 + 11}{\sqrt{9 + 36 + 4}}\right| = \left|\frac{7}{\sqrt{49}}\right|$

$PM = 1$ unit

12. Coordinates of the points O, P, Q and R respectively $(0, 0, 0)$, $(4, 6, 2m)$, $(2, 0, 2n)$ and $(2, 4, 6)$ L, M, N and K OR, OP, PQ and QR respectively such that $LMNK$ is a parallelogram whose two adjacent sides LK and LM are each of length $\sqrt{2}$?

(a) 6, 2 (b) 1, 3
(c) 3, 1 (d) None of these

(**c**) $LMNK$ is a parallelogram we know that, if we join mid- point of any quadrilateral we get a parallelogram



So. M, N, K, L are mid- points of OP, PQ, QR and RO respectively

$\therefore$ Coordinate of M

$= \left(\frac{0 + 4}{2}, \frac{0 + 6}{2}, \frac{0 + 2m}{2}\right) = (2, 3, m)$

[$\because$ Coordinate of mid-point

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

Coordinate of $L = (1, 2, 3)$

Coordinate of $K = (2, 2, 3 + n)$

Now, $LM = \sqrt{2}$

$\Rightarrow \sqrt{(2 - 1)^2 + (3 - 2)^2 + (m - 3)^2} = \sqrt{2}$

$\Rightarrow \quad \sqrt{1 + 1 + (m - 3)^2} = \sqrt{2}$

On squaring both side,

$\Rightarrow \quad 2 + (m - 3)^2 = 2$

$\Rightarrow \quad (m - 3)^2 = 0$

$\Rightarrow \quad m = 3$

Again, $LK = \sqrt{2}$

$\Rightarrow \sqrt{(2-1)^2 + (2-2)^2 + (3+n-3)^2} = \sqrt{2}$

$\Rightarrow \sqrt{1+0+n^2} = \sqrt{2}$

Squaring both side,

$\Rightarrow 1 + n^2 = 2$

$\Rightarrow n^2 = 1$

$\Rightarrow n = 1$

13. The line $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}$ is given by

(a) $x + y + z = 6, x + 2y - 3z = -4$

(b) $x + 2y - 2x = -1, 4x + 4y - 5z - 3 = 0$

(c) $3x + 2y - 3z = 0, 3x - 6y + 3z = -2$

(d) $3x + 2y - 3z = -2, 3x - 6y + 3z = 0$

(**d**) Given lines,

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ [let]

$\therefore \frac{x-1}{2} = \lambda$

$\Rightarrow x = 2\lambda + 1$

$\frac{y-2}{3} = \lambda$

$\Rightarrow y = 3\lambda + 2$ and $\frac{z-3}{4} = \lambda$

$\Rightarrow z = 4\lambda + 3$

by checking options if $3x + 2y - 3z$, then

$3(2\lambda + 1) + 2(3\lambda + 2) - 3(4\lambda + 3) = 6\lambda + 3 + 6\lambda + 4 - 12\lambda - 9 = -2$

$3(2\lambda + 1) - 6(3\lambda + 2) + 3(4\lambda + 3)$

$\Rightarrow 6\lambda + 3 - 18\lambda - 12 + 12\lambda + 9 = 0$

So, option (d) is correct.

14. Consider the following statements

1. The angle between the planes $2x - y + z = 1$ and $x + y + 2z = 3$ is $\frac{\pi}{3}$
2. The distance between the planes $6x - 3y + 6z + 2 = 0$ and $2x - y + 2z + 4 = 0$ is $\frac{10}{9}$

Which of the above statement is/are correct?

(a) Only 1

(b) Only 2

(c) Both 1 and 2

(d) Neither 1 nor 2

(**c**) **Statement 1** Given,

$2x - y + z = 1$ and $x + y + 2z = 3$

Here, $a_1 = 2, b_1 = -1, c_1 = 1$

and $a_2 = 1, b_2 = 1, c_2 = 2$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} = \frac{2 \times 1 + (-1 \times 1) + 1 \times 2}{\sqrt{2^2 + (-1)^2 + (1)^2}\sqrt{1^2 + 1^2 + 2^2}}$$

$$= \frac{4-1}{\sqrt{4+1+1}\sqrt{4+1+1}} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$\therefore \theta = \frac{\pi}{3}$

So, Statement 1 is correct.

Statement 2 Distance between two planes

$ax + by + cz + d_1 = 0$

$ax + by + cz + d_2 = 0$

$$\text{distance } S = \left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$d_1 = \frac{2}{3}, d_2 = 4$

$$\text{distance} = \left|\frac{4 - \frac{2}{3}}{\sqrt{2^2 + 1^2 + 2^2}}\right| = \left|\frac{\frac{10}{3}}{\sqrt{9}}\right|$$

$$= \frac{10}{3 \times 3} = \frac{10}{9}$$

2018 (I)

15. Let the coordinates of the points A, B, C be $(1, 8, 4), (0, -11, 4)$ and $(2, -3, 1)$ respectively. What are the coordinates of the point D which is the foot of the perpendicular from A on BC ?

(a) (3, 4, –2) (b) (4, –2, 5)

(c) (4, 5, –2) (d) (2, 4, 5)

(**c**) We have,

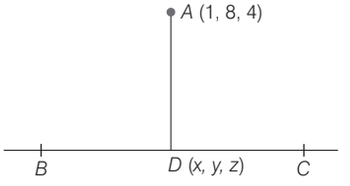
$A(1, 8, 4), B(0, -11, 4)$ and $C(2, -3, 1)$

$\therefore$ Equation of BC is

$\frac{x-0}{2-0} = \frac{y+11}{-3+11} = \frac{z-4}{1-4}$

$\Rightarrow \frac{x}{2} = \frac{y+11}{8} = \frac{z-4}{-3} = \lambda$ [say]

$\Rightarrow x = 2\lambda, y = 8\lambda - 11, z = -3\lambda + 4$



Now, DR's of

$AD = \langle 2\lambda - 1, 8\lambda - 11 - 8, -3\lambda + 4 - 4 \rangle$

$= \langle 2\lambda - 1, 8\lambda - 19, -3\lambda \rangle$

Since, $AD \perp BC$

$\therefore 2(2\lambda - 1) + 8(8\lambda - 19) - 3(-3\lambda) = 0$

$\Rightarrow 4\lambda - 2 + 64\lambda - 152 + 9\lambda = 0$

$\Rightarrow 77\lambda = 154$

$\Rightarrow \lambda = 2$

$\therefore$ Coordinates of

$D = (2 \times 2, 8 \times 2 - 11, -3 \times 2 + 4)$

$= (4, 5, -2)$

16. What is the equation of the plane passing through the points $(-2, 6, -6), (-3, 10, -9)$ and $(-5, 0, -6)$?

(a) $2x - y - 2z = 2$

(b) $2x + y + 3z = 3$

(c) $x + y + z = 6$

(d) $x - y - z = 3$

(**a**) Equation of the plane passing through three points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Equation of plane is

$$\begin{vmatrix} x - (-2) & y - 6 & z - (-6) \\ -3 - (-2) & 10 - 6 & -9 - (-6) \\ -5 - (-2) & 0 - 6 & -6 - (-6) \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x + 2 & y - 6 & z + 6 \\ -1 & 4 & -3 \\ -3 & -6 & 0 \end{vmatrix} = 0$$

$\Rightarrow (x + 2)[4 \times 0 - (-6)(-3)] - (y - 6)[(-1)(0) - (-3)(-3)] + (z + 6)[(-1)(-6) - (-3)(4)] = 0$

$\Rightarrow (x + 2)(-18) - (y - 6)(-9) + (z + 6)(18) = 0$

$\Rightarrow 2(x + 2) - (y - 6) - 2(z + 6) = 0$

$\Rightarrow 2x - y - 2z - 2 = 0$

$\Rightarrow 2x - y - 2z = 2$

17. A sphere of constant radius r through the origin intersects the coordinate axes in A, B and C. What is the locus of the centroid of the ΔABC?

(a) $x^2 + y^2 + z^2 = r^2$

(b) $x^2 + y^2 + z^2 = 4r^2$

(c) $9(x^2 + y^2 + z^2) = 4r^2$

(d) $3(x^2 + y^2 + z^2) = 2r^2$

(**c**) Let $A(a, 0, 0), B(0, b, 0)$ and $C(0, 0, c)$.

$\therefore$ Equation of sphere passing through A, B, C and origin is

$x^2 + y^2 + z^2 - ax - by - cz = 0$

$\therefore$ Radius $= r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4} + \frac{c^2}{4}}$

$\Rightarrow 4r^2 = a^2 + b^2 + c^2$ ...(i)

Let (α, β, γ) be the centroid of triangle.

$\therefore \alpha = \frac{a + 0 + 0}{3}, \beta = \frac{0 + b + 0}{3}, \gamma = \frac{0 + 0 + c}{3}$

$\Rightarrow a = 3\alpha, b = 3\beta, c = 3\gamma$ ...(ii)

From Eqs. (i) and (ii), we have

$(3\alpha)^2 + (3\beta)^2 + (3\gamma)^2 = 4r^2$

$\Rightarrow 9(\alpha^2 + \beta^2 + \gamma^2) = 4r^2$

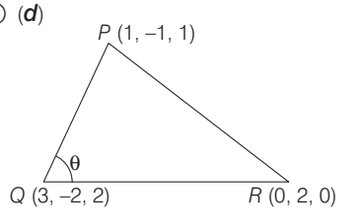
$\therefore$ Locus of the centroid of ΔABC is

$9(x^2 + y^2 + z^2) = 4r^2$

18. The coordinates of the vertices P, Q and R of a triangle PQR are $(1,-1,1), (3,-2,2)$ and $(0,2,6)$ respectively. If $\angle RQP = \theta$, then what is $\angle PRQ$ equal to ?

(a) $30° + \theta$ (b) $45° - \theta$
(c) $60° - \theta$ (d) $90° - \theta$

(*d*)



DR's of $PQ = < 3-1, -2-(-1), 2-1 >$
$< a_1, b_1, c_1 > = < 2, -1, 1 >$
and DR's of $PR = < 0-1, 2-(-1), 6-1 > < a_2, b_2, c_2 > = < -1, 3, 5 >$
Now, $a_1a_2 + b_1b_2 + c_1c_2$
$= 2 \times (-1) + (-1) \times 3 + 1 \times 5$
$= -2 - 3 + 5 = 0$
$\therefore \quad PQ \perp PR$
$\Rightarrow \quad \angle QPR = 90°$
Now, $\angle PQR + \angle QPR + \angle PRQ = 180°$
$\Rightarrow \quad \theta + 90° + \angle PRQ = 180°$
$\Rightarrow \quad \angle PRQ = 90° - \theta$

2017 (II)

19. The length of the normal from origin to the plane $x + 2y - 2z = 9$ is equal to

(a) 2 units (b) 3 units
(c) 4 units (d) 5 units

(*b*) Length of normal

$$= \left| \frac{0 + 2.0 - 2.0 - 9}{\sqrt{(1)^2 + (2)^2 + (-2)^2}} \right|$$

$$= \left| \frac{-9}{\sqrt{1 + 4 + 4}} \right|$$

$$= \frac{9}{3} = 3 \text{ units}$$

20. If α, β and γ are the angles which the vector OP (O being the origin) makes with positive direction of the coordinate axes, then which of the following are correct?

1. $\cos^2\alpha + \cos^2\beta = \sin^2\gamma$
2. $\sin^2\alpha + \sin^2\beta = \cos^2\gamma$
3. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(*c*) We know that,
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow \cos^2\alpha + \cos^2\beta = 1 - \cos^2\gamma$
$\Rightarrow \cos^2\alpha + \cos^2\beta = \sin^2\gamma$
So, Statement 1 is correct.
Again,
$1 - \sin^2\alpha + 1 - \sin^2\beta + \cos^2\gamma = 1$
$\Rightarrow 1 + \cos^2\gamma = \sin^2\alpha + \sin^2\beta$
So, Statement 2 is incorrect.
Again,
$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$
$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
$\therefore$ Statements 3 is correct.

21. The point of intersection of the line joining the points $(-3,4,-8)$ and $(5,-6,4)$ with XY-plane is

(a) $\left(\frac{7}{3}, -\frac{8}{3}, 0\right)$ (b) $\left(-\frac{7}{3}, -\frac{8}{3}, 0\right)$
(c) $\left(-\frac{7}{3}, \frac{8}{3}, 0\right)$ (d) $\left(\frac{7}{3}, \frac{8}{3}, 0\right)$

(*a*) Equation of line joining $(-3, 4, -8)$ and $(5, -6, 4)$ is

$$\frac{x+3}{5+3} = \frac{y-3}{-6-4} = \frac{z+8}{4+8} = \lambda$$

$$\Rightarrow \frac{x+3}{8} = \frac{y-3}{-10} = \frac{z+8}{12} = \lambda$$

$\Rightarrow \quad x = 8\lambda - 3, y = -10\lambda + 4,$
$z = 12\lambda - 8$
since line intersect XY-plane
$\therefore \quad z = 0$
$\Rightarrow \quad 12\lambda - 8 = 0$
$\Rightarrow \quad \lambda = \frac{2}{3}$
$\therefore$ Point of intersection

$$= \left(8 \times \frac{2}{3} - 3, -10 \times \frac{2}{3} + 4, 0\right)$$

$$= \left(\frac{7}{3}, \frac{-8}{3}, 0\right)$$

22. If the angle between the lines whose direction ratios are $(2,-1,2)$ and $\langle x, 3, 5 \rangle$ is $\frac{\pi}{4}$, then the smaller value of x is

(a) 52 (b) 4
(c) 2 (d) 1

(*b*) If two lines have direction ratios $< a_1, b_1, c_1 >$ and $< a_2, b_2, c_2 >$ then the angle between them i.e. θ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Now, according to the question $\cos\frac{\pi}{4}$

$$= \frac{2x - 3 + 10}{\sqrt{(2)^2 + (-1)^2 + (2)^2}\sqrt{x^2 + (3)^2 + (5)^2}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{2x + 7}{3\sqrt{34 + x^2}}$$

$$\Rightarrow \quad 3\sqrt{34 + x^2} = \sqrt{2}(2x + 7)$$

$$\Rightarrow \quad 9(34 + x^2) = 2(2x + 7)^2$$

$$\Rightarrow \quad 306 + 9x^2 = 8x^2 + 56x + 98$$

$$\Rightarrow \quad x^2 - 56x + 208 = 0$$

$$\Rightarrow \quad (x - 4)(x - 52) = 0$$

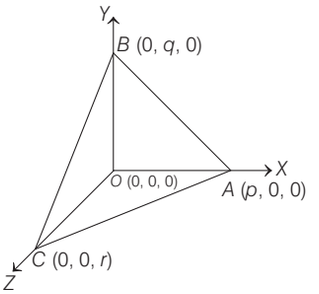
$$\Rightarrow \quad x = 4, 52$$

$\therefore$ Smaller value of x is 4.

23. A variable plane passes through a fixed point (a, b, c) and cuts the axes in A, B and C respectively. The locus of the centre of the sphere $OABC$, O being the origin, is

(a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
(b) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$
(c) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$
(d) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 2$

(*c*)



Equation of plane passing through A, B, C is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$

Since, above plane passes through (a,b,c)

$\therefore \quad \frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 1 \quad \ldots(i)$

Now, equation of sphere passing through A,B,C,O is

$$x^2 + y^2 + z^2 - px - qy - rz = 0$$

$\therefore \quad$ Centre $= \left(\frac{p}{2}, \frac{q}{2}, \frac{r}{2}\right)$

Let the coordinates of centre of sphere be (x, y, z)

$\therefore \quad x = \frac{p}{2}, y = \frac{q}{2}, z = \frac{r}{2}$

$\Rightarrow \quad P = 2x, q = 2y, r = 2z$

On putting these value in Eq. (i), we get the equation of locus of sphere

$$\frac{a}{2x} + \frac{b}{2y} + \frac{c}{2z} = 1$$

$$\Rightarrow \quad \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

24. The equation of the plane passing through the line of intersection of the planes $x + y + z = 1, 2x + 3y + 4z = 7$, and perpendicular to the plane $x - 5y + 3z = 5$ is given by

(a) $x + 2y + 3z - 6 = 0$
(b) $x + 2y + 3z + 6 = 0$
(c) $3x + 4y + 5z - 8 = 0$
(d) $3x + 4y + 5z + 8 = 0$

(**a**) Let $P_1: x + y + z - 1 = 0$

and $P_2: 2x + 3y + 4z - 7 = 0$

∴ Equation of plane passing through line of intersection of P_1 and P_2 is given by

$$P_1 + \lambda P_2 = 0$$
$$\Rightarrow (x + y + z - 1) + \lambda(2x + 3y + 4z - 7) = 0$$
$$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - 1 - 7\lambda = 0$$

This plane is perpendicular to the plane $x - 5y + 3z = 5$

$$\therefore \quad 1(1 + 2\lambda) - 5(1 + 3\lambda) + 3(1 + 4\lambda) = 0$$
$$\Rightarrow \quad 1 + 2\lambda - 5 - 15\lambda + 3 + 12\lambda = 0$$
$$\Rightarrow \quad -\lambda - 1 = 0$$
$$\Rightarrow \quad \lambda = -1$$

Equation of required plane is

$$(1 - 2)x + (1 - 3)y + (1 - 4)z - 1 + 7 = 0$$
$$\Rightarrow \quad -x - 2y - 3z + 6 = 0$$
$$\Rightarrow \quad x + 2y + 3z - 6 = 0$$

2017 (I)

25. A straight line with direction cosines $\langle 0, 1, 0 \rangle$ is

(a) parallel to *X*-axis
(b) parallel to *Y*-axis
(c) parallel to *Z*-axis
(d) equally inclined to all the axes

(**b**) We know that direction cosines of *Y*-axis are (0, 1, 0).

26. $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ are four distinct points. What are the coordinates of the point which is equidistant from the four points?

(a) $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}, \frac{a+b+c}{3}\right)$

(b) (a, b, c)

(c) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

(d) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

(**c**) Let the required point be $P(x, y, z)$

Also, let $O(0,0,0)$, $A(a,0,0)$

$B(0,b,0)$ and $C(0,0,c)$ be four distinct points.

According to the equations, we have

$$PO^2 = PA^2 = PB^2 = PC^2$$
$$\Rightarrow \quad x^2 + y^2 + z^2 = (x - a)^2 + y^2 + z^2$$
$$= x^2 + (y - b)^2 + z^2$$
$$= x^2 + y^2 + (z - c)^2$$
$$\Rightarrow \quad a^2 - 2ax = 0, b^2 - 2by = 0$$

and $\quad c^2 - 2cz = 0$

$$\Rightarrow \quad x = a/2, y = b/2, z = c/2$$

∴ The required point is

$$P(a/2, b/2, c/2)$$

27. The points $P(3,2,4)$, $Q(4,5,2)$, $R(5,8,0)$ and $S(2,-1,6)$ are

(a) vertices of a rhombus which is not a square
(b) non-coplanar
(c) collinear
(d) coplanar but not collinear

(**d**) $PQ = \sqrt{(4-3)^2 + (5-2)^2 (2-4)^2}$

$$= \sqrt{(1)^2 + (3)^2 + (-2)^2}$$
$$= \sqrt{1 + 9 + 4} = \sqrt{14}$$
$$QR = \sqrt{(5-4)^2 + (8-5)^2 + (0-2)^2}$$
$$= \sqrt{(1)^2 + (3)^2 + (-2)^2}$$
$$= \sqrt{1 + 9 + 4} = \sqrt{14}$$
$$RS = \sqrt{(2-5)^2 + (-1-8)^2 + (6-0)^2}$$
$$= \sqrt{(-3)^2 + (-9)^2 + (6)^2}$$
$$= \sqrt{9 + 81 + 36} = \sqrt{126}$$

and $PS = \sqrt{(2-3)^2 + (-1-2)^2 + (6-4)^2}$

$$= \sqrt{1 + 9 - 4} = \sqrt{14}$$

Since, $PQ \neq RS$

⇒ Given points are not vertices of a rhombus.

Now, direction ratios of PQ are

$4 - 3, 5 - 2, 2 - 4$ i.e. $1, 3, -2$

direction ratios of PR are

$5 - 3, 8 - 2, 0 - 4$ i.e. $2, 6, -4$

direction ratios of PS are

$2 - 3, -1 - 2, 6 - 4$ i.e. $-1, -3, 2$

$$|PQ| = \sqrt{1 + 9 + 4} = \sqrt{14}$$
$$|PR| = \sqrt{4 + 36 + 16} = \sqrt{56} = 2\sqrt{14}$$
$$|PS| = \sqrt{1 + 9 + 4} = \sqrt{14}$$

$\because \quad |PQ| = n|PR|$ but $|PQ| \neq m|PS|$

So, these points are not collinear.

Given, points P, Q, R and S are coplanar if PQ, PR and PS are coplanar

i.e. $\quad [PQ\ PR\ PS] = 0$

Here, $\begin{vmatrix} 1 & 3 & -2 \\ 2 & 6 & -4 \\ -1 & -3 & 2 \end{vmatrix}$

$$= (-1)\begin{vmatrix} -1 & -3 & 2 \\ 2 & 6 & -4 \\ -1 & -3 & 2 \end{vmatrix}$$

$= 0$ (since R_1 and R_3 are identical)

⇒ PR, PQ and PS are coplanar

⇒ P, Q, R and S are coplanar.

28. The line passing through the points $(1, 2, -1)$ and $(3, -1, 2)$ meets the yz-plane at which one of the following points?

(a) $\left(0, -\frac{7}{2}, \frac{5}{2}\right)$ (b) $\left(0, \frac{7}{2}, \frac{1}{2}\right)$

(c) $\left(0, -\frac{7}{2}, -\frac{5}{2}\right)$ (d) $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

(**d**) Equation of line passing through the points

$(1, 2, -1)$ and $(3, -1, 2)$ is

$$\frac{x-1}{3-1} = \frac{y-2}{-1-2} = \frac{z+1}{2+1}$$
$$\Rightarrow \frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+1}{3} = \lambda \quad \text{[say]} \ \ldots \text{(i)}$$

General point on line (i) is

$$x = 2\lambda + 1, y = -3\lambda + 2$$

and $\quad z = 3\lambda - 1$

Since, line (i) meets the yz-plane

∴ x-coordinate will be zero

$$\Rightarrow \quad 2\lambda + 1 = 0$$
$$\Rightarrow \quad \lambda = -1/2$$
$$\therefore \quad y = -3(-1/2) + 2$$
$$= 3/2 + 2 = 7/2$$

and $\quad z = 3(-1/2) - 1 = \frac{-3-2}{2}$

$$= -5/2$$

required point is $(0, 7/2, -5/2)$.

29. Under which one of the following conditions are the lines $x = ay + b; z = cy + d$ and $x = ey + f; z = gy + h$ perpendicular?

(a) $ae + cg - 1 = 0$
(b) $ae + bf - 1 = 0$
(c) $ae + cg + 1 = 0$
(d) $ag + ce + 1 = 0$

(**c**) For the first line,

$$x - b = ay$$
$$z - d = cy$$
$$\Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

and for the second line,

$$x - f = ey$$
$$z - h = gy$$
$$\Rightarrow \frac{x-f}{e} = \frac{y}{1}$$
$$= \frac{z-h}{g}$$

$\because$ these are perpendicular

So, $a \cdot e + 1 \cdot 1 + cg = 0$

$$\Rightarrow \quad ae + cg + 1 = 0$$

2016 (II)

Directions (Q.Nos. 30 and 31) *Let Q be the image of the point P* $(-2, 1, -5)$ *in the plane* $3x - 2y + 2z + 1 = 0$.

30. Consider the following statements.

I. The coordinates of Q are $(4, -3, -1)$.

II. PQ is of length more than 8 units.

III. The point $(1, -1, -3)$ is the mid-point of the line segment PQ and lies on the given plane.

Which of the above statements are correct?

(a) I and II (b) II and III
(c) I and III (d) I, II and III

(**d**) Given, point $P(-2, 1, -5)$ and plane $3x - 2y + 2z + 1 = 0$.

Let PM be the length of the perpendicular from point $P(-2, 1, -5)$ to the given plane.

Then, equation of PM is

$$\frac{x+2}{3} = \frac{y-1}{-2} = \frac{z+5}{2} = \lambda$$

Coordinates of point M are $(3\lambda - 2, -2\lambda + 1, 2\lambda - 5)$

$\because$ This point lies on the plane.

$\therefore \quad 3(3\lambda - 2) - 2(-2\lambda + 1) + 2(2\lambda - 5) + 1 = 0$

$\Rightarrow \quad 9\lambda - 6 + 4\lambda - 2 + 4\lambda - 10 + 1 = 0$

$\Rightarrow \quad 17\lambda - 17 = 0 \Rightarrow \lambda = 1$

Coordinates of M are $(1, -1, -3)$.

Since, Q is the image of point P.

Now, let coordinates of Q be (x_1, y_1, z_1) then

$$\frac{x_1 + (-2)}{2} = 1, \frac{y_1 + 1}{2} = -1, \frac{z_1 + (-5)}{2} = -3$$

$\Rightarrow \quad x_1 - 2 = 2, y_1 + 1 = -2, z_1 - 5 = -6$

$\Rightarrow \quad x_1 = 4, y_1 = -3, z_1 = -1$

$\therefore$ $Q(4, -3, -1)$ and

$$PQ = \sqrt{(4+2)^2 + (-3-1)^2 + (-1+5)^2} = \sqrt{36 + 16 + 16} = \sqrt{68}$$

31. Consider the following statements.

I. The direction ratios of the segment PQ are $< 3, -2, 2 >$.

II. The sum of the squares of direction cosines of the line segment PQ is unity.

Which of the above statements is/are correct?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

(**c**) We have, $P(-2, 1, -5)$ and $Q(4, -3, -1)$.

Direction ratios of line segment PQ are proportional to

$4 - (-2), -3 - 1, -1 - (-5)$

i.e. $6, -4, 4$ or $3, -2, 2$

Direction cosines of line segment PQ are $= \frac{6}{\sqrt{6^2 + (-4)^2 + (-4)^2}}, \frac{-4}{\sqrt{6^2 + (-4)^2 + (-4)^2}}, \frac{-4}{\sqrt{6^2 + (-4)^2 + (-4)^2}}$

$\Rightarrow \quad \frac{6}{\sqrt{68}}, -\frac{4}{\sqrt{68}}, -\frac{4}{\sqrt{68}}$

$\therefore$ Sum of the squares of direction cosines of the line segment PQ is

$$\frac{36}{68} + \frac{16}{68} + \frac{16}{68} = \frac{68}{68} = 1 \quad \text{[unity]}$$

Directions (Q. Nos. 32 and 33) *A line L passes through the point* $P(5, -6, 7)$ *and is parallel to the planes* $x + y + z = 1$ *and* $2x - y - 2z = 3$.

32. What are the direction ratios of the line of intersection of the given planes?

(a) $< 1, 4, 3 >$ (b) $< -1, -4, 3 >$
(c) $< 1, -4, 3 >$ (d) $< 1, -4, -3 >$

(**c**) Given point, $P(5, -6, 7)$

and planes $x + y + z = 1$

and $2x - y - 2z = 3$

Equation of planes in vector form are

$\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$

and $\mathbf{r} \cdot (2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 3$

Since, line is parallel to the given planes.

$\therefore$ Line is perpendicular to the vectors, $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$.

$\Rightarrow$ Line is parallel to the vector

$$(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ 2 & -1 & -2 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(-2 + 1) - \hat{\mathbf{j}}(-2 - 2) + \hat{\mathbf{k}}(-1 - 2)$$

$$= \hat{\mathbf{i}}(-1) - \hat{\mathbf{j}}(-4) + \hat{\mathbf{k}}(-3)$$

$$= -\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$\therefore$ Direction ratios of the line of intersection of the given plane are $-1, 4, -3$ or $1, -4, 3$.

33. What is the equation of the line L?

(a) $\frac{x-5}{-1} = \frac{y+6}{4} = \frac{z-7}{-3}$

(b) $\frac{x+5}{-1} = \frac{y-6}{4} = \frac{z+7}{-3}$

(c) $\frac{x-5}{-1} = \frac{y+6}{-4} = \frac{z-7}{3}$

(d) $\frac{x-5}{-1} = \frac{y+6}{-4} = \frac{z-7}{-3}$

(**a**) Since, line passes through point $P(5, -6, 7)$.

$\therefore$ Equation of line

$$\frac{x-5}{-1} = \frac{y+6}{4} = \frac{z-7}{-3}$$

2016 (I)

Directions (Q. Nos. 34 and 35) *A plane P passes through the line of intersection of the planes* $2x - y + 3z = 2$, $x + y - z = 1$ *and the point* $(1, 0, 1)$.

34. What are the direction ratios of the line of intersection of the given planes?

(a) $< 2, -5, -3 >$ (b) $< 1, -5, -3 >$
(c) $< 2, 5, 3 >$ (d) $< 1, 3, 5 >$

(**a**) Given equations of plane are

$2x - y + 3z = 2$ ...(i)

and $x + y - z = 1$ ...(ii)

Let the direction ratios of line of intersection be $< a, b, c >$.

$\therefore \quad 2a - b + 3c = 0$ ...(iii)

and $a + b - c = 0$ ...(iv)

$[\because a_1a_2 + b_1b_2 + c_1c_2 = 0]$

On solving Eqs. (iii) and (iv), we get

$$\frac{a}{-2} = \frac{-b}{-5} = \frac{c}{3}$$

$\Rightarrow \quad \frac{a}{-2} = \frac{b}{5} = \frac{c}{3}$ or $\frac{a}{2} = \frac{b}{-5} = \frac{c}{-3}$

$\Rightarrow \quad < a, b, c > = < 2, -5, -3 >$

35. What is the equation of the plane P ?

(a) $2x + 5y - 2 = 0$
(b) $5x + 2y - 5 = 0$
(c) $x + z - 2 = 0$
(d) $2x - y - 2z = 0$

(**b**) The equation of plane P is

$(2x - y + 3z - 2) + \lambda(x + y - z - 1) = 0$

It passes through $(1, 0, 1)$.

We get,

$(2 - 0 + 3 - 2) + \lambda(1 + 0 - 1 - 1) = 0$

$\Rightarrow \quad 3 - \lambda = 0 \quad \Rightarrow \quad \lambda = 3$

Hence, equation of plane P is

$(2x - y + 3z - 2) + 3(x + y - z - 1) = 0$

$\Rightarrow \quad 5x + 2y - 5 = 0$

36.

If the plane P touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is r equal to?

(a) $\frac{2}{\sqrt{29}}$ (b) $\frac{4}{\sqrt{29}}$ (c) $\frac{5}{\sqrt{29}}$ (d) 1

(**c**) Plane P touches the sphere $x^2 + y^2 + z^2 = r^2$

Here, the centre of sphere is (0, 0, 0).

$\therefore$ Radius of sphere, r = Perpendicular distance from centre of sphere to the tangent plane.

$$r = \left| \frac{0+0+0-5}{\sqrt{5^2+2^2}} \right| = \left| \frac{-5}{\sqrt{29}} \right|$$

$$\Rightarrow \quad r = \frac{5}{\sqrt{29}} \text{ units}$$

2015 (II)

37. The lines $2x = 3y = -z$ and $6x = -y = -4z$

(a) are perpendicular
(b) are parallel
(c) intersect at an angle 45°
(d) intersect at an angle 60°

(**a**) $\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$ and $\frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-\frac{1}{4}}$

DR's of the lines are

$< \frac{1}{2}, \frac{1}{3}, -1 >$ and $< \frac{1}{6}, -1, -\frac{1}{4} >$

$$\frac{1}{2} \times \frac{1}{6} + \frac{1}{3}(-1) + (-1)\left(-\frac{1}{4}\right) = \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = 0$$

Hence, both lines are perpendicular.

38. The radius of the sphere $3x^2 + 3y^2 + 3z^2 - 8x + 4y + 8z - 15 = 0$ is

(a) 2 (b) 3
(c) 4 (d) 5

(**b**) Given, $3x^2 + 3y^2 + 3z^2 - 8x + 4y + 8z - 15 = 0$

$$\Rightarrow x^2 + y^2 + z^2 - \frac{8}{3}x + \frac{4}{3}y + \frac{8}{3}z - 5 = 0$$

Compare it with equation of a sphere

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

we get $2u = -\frac{8}{3}, 2v = \frac{4}{3}$

$$2w = \frac{8}{3}, d = -5$$

$$\therefore \quad u = -\frac{4}{3}, v = \frac{2}{3}, w = \frac{4}{3}, d = -5$$

Now, radius of a sphere

$$= \sqrt{u^2 + v^2 + w^2 - d}$$

$$= \sqrt{\frac{16}{9} + \frac{4}{9} + \frac{16}{9} + 5}$$

$$= \sqrt{4+5} = \sqrt{9} = 3$$

39. The direction ratios of the line perpendicular to the lines with direction ratios $< 1, -2, -2 >$ and $< 0, 2, 1 >$ are

(a) $< 2, -1, 2 >$
(b) $< -2, 1, 2 >$
(c) $< 2, 1, -2 >$
(d) $< -2, -1, -2 >$

(**a**) Let the direction ratio of the line be a, b, c.

We have, $a - 2b - 2c = 0$

and $0 \cdot a + 2b + c = 0$

$$\therefore \quad \frac{a}{-2+4} = \frac{-b}{1-0} = \frac{c}{2-0}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{-1} = \frac{c}{2}$$

$$\Rightarrow \quad < 2, -1, 2 >$$

40. What are the coordinates of the foot of the perpendicular drawn from the point (3, 5, 4) on the plane $z = 0$?

(a) (0, 5, 4) (b) (3, 5, 0)
(c) (3, 0, 4) (d) (0, 0, 4)

(**b**) The foot of perpendicular drawn from (3, 5, 4) on the $z = 0$ is

$$\frac{x-3}{0} = \frac{y-5}{0} = \frac{z-4}{1} = -\frac{4}{1}$$

$$\Rightarrow \quad x = 3, y = 5, z = 0$$

Hence, $(x, y, z) = (3, 5, 0)$

41. The lengths of the intercepts on the coordinate axes made by the plane $5x + 2y + z - 13 = 0$ are

(a) (5, 2, 1) units
(b) $\left(\frac{13}{5}, \frac{13}{2}, 13\right)$ units
(c) $\left(\frac{5}{13}, \frac{2}{13}, \frac{1}{13}\right)$ units
(d) (1, 2, 5) units

(**b**) Given, $5x + 2y + z - 13 = 0$

or $$\frac{x}{\frac{13}{5}} + \frac{y}{\frac{13}{2}} + \frac{z}{13} = 1$$

$\therefore$ Lengths of intercepts are $\frac{13}{5}, \frac{13}{2}$ and 13.

2015 (I)

Directions (Q. Nos. 42 and 43) *The projections of a directed line segment on the coordinate axes are* 12, 4, 3, *respectively.*

42. What is the length of the line segment?

(a) 19 units (b) 17 units
(c) 15 units (d) 13 units

(**d**) Length of the line segment $= \sqrt{x^2 + y^2 + z^2}$

[$\because$ the distance of a point $P(x, y, z)$ from origin O is $OP = \sqrt{x^2 + y^2 + z^2}$]

$$= \sqrt{12^2 + 4^2 + 3^2}$$

$$= 13 \text{ units}$$

43. What are the direction cosines of the line segment?

(a) $\left(\pm\frac{12}{13}, \pm\frac{4}{13}, \pm\frac{3}{13}\right)$
(b) $\left(\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}\right)$
(c) $\left(\frac{12}{13}, -\frac{4}{13}, -\frac{3}{13}\right)$
(d) $\left(-\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}\right)$

(**a**) Direction cosines of the line segment

$$= \left(\pm\frac{12}{13}, \pm\frac{4}{13}, \pm\frac{3}{13}\right)$$

$$\left[\because \frac{\pm a}{\sqrt{a^2+b^2+c^2}}, \frac{\pm b}{\sqrt{a^2+b^2+c^2}}, \frac{\pm c}{\sqrt{a^2+b^2+c^2}}\right]$$

Directions (Q. Nos. 44 and 45) *From the point* $P(3, -1, 11)$, *a perpendicular is drawn on the line* L *given by the equation* $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. *Let* Q *be the foot of the perpendicular.*

44. What are the direction ratios of the line segment PQ ?

(a) (1, 6, 4)
(b) (–1, 6, – 4)
(c) (–1, – 6, 4)
(d) (2, – 6, 4)

(**b**) We have,

$$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$\Rightarrow x = 2\lambda, y = 3\lambda + 2, z = 4\lambda + 3$

Point $Q(2\lambda, 3\lambda + 2, 4\lambda + 3)$

$\therefore$ Direction ratios of the line passing through PQ,

$[(2\lambda - 3), (3\lambda + 2 + 1), (4\lambda + 3 - 11)]$

$= 2\lambda - 3, 3\lambda + 3, 4\lambda - 8$

$$\therefore \quad (2\lambda - 3) \cdot 2 + (3\lambda + 3) \cdot 3 + (4\lambda - 8) \cdot 4 = 0$$

$$\Rightarrow \quad 4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$$

$$\Rightarrow \quad 29\lambda = 29$$

$$\Rightarrow \quad \lambda = 1$$

Hence, direction ratios

$\equiv (2 - 3, 3 + 3, 4 - 8)$

$\equiv (-1, 6, -4)$

45. What is the length of the line segment PQ ?

(a) $\sqrt{47}$ units
(b) 7 units
(c) $\sqrt{53}$ units
(d) 8 units

(**c**) Length of the line segment PQ, when $Q \equiv (2, 5, 7)$

$\therefore PQ = \sqrt{(2-3)^2 + (5+1)^2 + (7-11)^2}$

[if $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two given points, then the distance between these points is $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$]

$= \sqrt{1 + 36 + 16}$
$= \sqrt{53}$ units

Directions (Q. Nos. 46 and 47) *A triangular plane ABC with centroid* $(1, 2, 3)$ *cuts the coordinate axes at A, B, C, respectively.*

46. What are the intercepts made by the plane ABC on the axes?

(a) 3, 6, 9 (b) 1, 2, 3
(c) 1, 4, 9 (d) 2, 4, 6

(**a**) We have, $\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$

Since, triangular plane ABC with centroid (1, 2, 3) cuts the coordinate axes.
$\therefore$ Intercepts made by X-axis, Y-axis and Z-axis are 3, 6, 9.

47. What is the equation of the plane ABC ?

(a) $x + 2y + 3z = 1$
(b) $3x + 2y + z = 3$
(c) $2x + 3y + 6z = 18$
(d) $6x + 3y + 2z = 18$

(**d**) We have, $\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$

Since, triangular plane ABC with centroid (1, 2, 3) cuts the coordinate axes.
$\therefore$ Intercepts made by X-axis, Y-axis and Z-axis are 3, 6, 9.
$\therefore$ Equation of the plane ABC is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$$

or $6x + 3y + 2z = 18$

Directions (Q. Nos. 48 and 49) *A point* $P(1, 2, 3)$ *is one vertex of a cuboid formed by the coordinate planes and the planes passing through P and parallel to the coordinate planes.*

48. What is the length of one of the diagonals of the cuboid?

(a) $\sqrt{10}$ units (b) $\sqrt{14}$ units
(c) 4 units (d) 5 units

(**b**) Length of diagonals of the cuboid

$= \sqrt{1^2 + 2^2 + 3^2}$
$= \sqrt{14}$ units

49. What is the equation of the plane passing through $P(1, 2, 3)$ and parallel to XY-plane?

(a) $x + y = 3$
(b) $x - y = -1$
(c) $z = 3$
(d) $x + 2y + 3z = 14$

(**c**) The equation of the plane passing through P (1, 2, 3) and parallel to XY-plane is $z = 3$.

2014 (II)

Directions (Q. Nos. 50 and 51) *Read the following information carefully and answer the questions given below. Consider a sphere passing through the origin and the points* $(2, 1, -1), (1, 5, -4), (-2, 4, -6)$.

50. What is the radius of the sphere?

(a) $\sqrt{12}$ (b) $\sqrt{14}$
(c) 12 (d) 14

(**b**) Equation of sphere passing through origin is

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$

which passes through the points (2, 1, – 1), (1, 5, – 4) and (–2, 4, – 6).

$\therefore \quad 4u + 2v - 2w = -6 \quad ...(i)$
$2u + 10v - 8w = -42 \quad ...(ii)$
and $-4u + 8v - 12w = -56 \quad ...(iii)$

On solving above equations, we get

$u = 1, v = -2$ and $w = 3$

$\therefore$ Radius of sphere $= \sqrt{u^2 + v^2 + w^2}$
$= \sqrt{1 + 4 + 9} = \sqrt{14}$

51. What is the centre of the sphere?

(a) (–1, 2 , – 3) (b) (1, – 2 , 3)
(c) (1, 2 , – 3) (d) (–1, – 2 , – 3)

(**a**) Centre of sphere,
$(-u, -v, -w) = (-1, 2, -3)$

52. Consider the following statements

1. The sphere passes through the point (0,4,0).
2. The point (1, 1, 1) is at a distance of 5 units from the centre of the sphere.

Which of the above statement(s) is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**a**) 1. Equation of sphere is

$x^2 + y^2 + z^2 + 2x - 4y + 6z = 0$

Put the value (0, 4, 0), we get

$0 + 16 + 0 + 0 - 16 + 0 = 0$

So, the sphere passes through the point (0, 4, 0).
Hence, Statement 1 is correct.

2. Distance between the point (1, 1, 1) and centre of sphere (–1, 2, – 3)

$= \sqrt{(1+1)^2 + (1-2)^2 + (1+3)^2}$
$= \sqrt{4 + 1 + 16} = \sqrt{21} \neq 5$

Hence, Statement 2 is not correct.

Directions (Q. Nos. 53 and 54) *Read the following information carefully and answer the questions given below.*

The line joining the points (2, 1, 3) and $(4, -2, 5)$ cuts the plane $2x + y - z = 3$.

53. Where does the line cut the plane?

(a) (0, – 4, – 1) (b) (0, – 4, 1)
(c) (1, 4, 0) (d) (0, 4, 1)

(**d**) Since, equation of a straight line joining two fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} = \lambda$$

$\therefore$ Equation of line passing through the points (2, 1, 3) and (4, – 2, 5) is

$$\frac{x-2}{4-2} = \frac{y-1}{-2-1} = \frac{z-3}{5-3} = \lambda$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-1}{-3} = \frac{z-3}{2} = \lambda$$

$\Rightarrow \quad x = 2\lambda + 2, \; y = -3\lambda + 1$

and $\quad z = 2\lambda + 3$

Since, this line cuts the plane $2x + y - z = 3$.

So, $(2\lambda + 2, -3\lambda + 1, 2\lambda + 3)$ satisfies the equation of plane.

$\therefore \quad 2(2\lambda + 2) - 3\lambda + 1 - 2\lambda - 3 = 3$

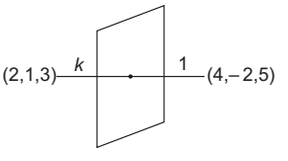
$\Rightarrow \quad -\lambda + 2 = 3 \Rightarrow \lambda = -1$

Hence, points are $[2(-1) + 2, -3(-1) + 1, 2(-1) + 3]$, i.e. (0, 4, 1).

54. What is the ratio in which the plane divides the line?

(a) 1 : 1 (b) 2 : 3
(c) 3 : 4 (d) None of these

(**d**) Let the ratio plane divides the line in k : 1.



Then, $0 = \frac{4k+2}{k+1}$

$\Rightarrow \quad 4k + 2 = 0 \Rightarrow k = -\frac{1}{2}$

and $\quad 4 = \frac{-2k+1}{k+1}$

$\Rightarrow \quad 4k + 4 = -2k + 1 \Rightarrow k = -\frac{1}{2}$

Hence, plane divides the line in ratio 1 : 2 externally.

Directions (Q. Nos. 55-56) *Read the following information carefully and answer the questions given below.*
Consider the plane passing through the points $A(2, 2, 1)$, $B(3, 4, 2)$ *and* $C(7, 0, 6)$.

55. Which one of the following points lies on the plane?

(a) (1, 0, 0) (b) (1, 0, 1)
(c) (0, 0, 1) (d) None of these

(*a*) We know that, equation of plane passing through three non-collinear points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Put the value of $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3), we get

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & 2 & 1 \\ 5 & -2 & 5 \end{vmatrix} = 0$$

$\Rightarrow \quad (x-2)(10+2) - (y-2)(5-5) + (z-1)(-2-10) = 0$

$\Rightarrow \quad 12x - 12z = 12$

$\Rightarrow \quad x - z = 1$

Hence, the equation of plane passes through (1, 0, 0).

56. What are the direction ratios of the normal to the plane?

(a) $<1, 0, 1>$
(b) $< 0, 1, 0 >$
(c) $< 1, 0, -1>$
(d) None of these

(*c*) Direction ratios of the normal to the plane $x - z = 1$ are $(1, 0, -1)$.

2014 (I)

Directions (Q. Nos. 57-59) *A straight line passes through* $(1, -2, 3)$ *and perpendicular to the plane* $2x + 3y - z = 7$.

57. What are the direction ratios of normal to plane?/

(a) $< 2, 3, -1>$ (b) $< 2, 3, 1>$
(c) $< -1, 2, 3>$ (d) None of these

58. Where does the line meet the plane?

(a) (2, 3, – 1) (b) (1, 2, 3)
(c) (2, 1, 3) (d) (3, 1, 2)

59. What is the image of the point $(1, -2, 3)$ in the plane?

(a) (2, – 1, 5) (b) (–1, 2, – 3)
(c) (5, 4, 1) (d) None of these

Solutions (Q. Nos. 57-59) Given that, a line passes through the point (1, – 2, 3) and perpendicular to the plane $2x + 3y - z = 7$ i.e., this straight line is a normal to the plane.

57. (*a*) Equation of the plane,

$$2x + 3y - z = 7$$

On compare with $ax + by + cz = d$, $a = 2$, $b = 3$ and $c = -1$

which is the required direction ratios of normal to the plane

i.e. $< a, b, c > = < 2, 3, -1>$

58. (*d*) The point, where the line meet the plane = Foot (x, y, z) of a point $(1, -2, 3)$ in a plane

$$2x + 3y - z = 7$$

$$\therefore \quad \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

$$\Rightarrow \quad \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-1} = \frac{-[2(1) + 3(-2) - 1(3) - 7]}{(2)^2 + (3)^2 + (-1)^2}$$

$$\Rightarrow \quad \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-1} = \frac{-(2 - 6 - 3 - 7)}{4 + 9 + 1} = \frac{-(-14)}{14} = 1$$

$$\Rightarrow \quad \frac{x-1}{2} = 1 \Rightarrow x - 1 = 2 \Rightarrow x = 3$$

$$\Rightarrow \quad \frac{y+2}{3} = 1 \Rightarrow y + 2 = 3 \Rightarrow y = 1$$

$$\Rightarrow \quad \frac{z-3}{-1} = 1 \Rightarrow z - 3 = -1 \Rightarrow z = 2$$

$\therefore$ Required point = $(x, y, z) = (3, 1, 2)$

59. (*c*) We know that, the image (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax + by + cz + d = 0$ is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

$\therefore$ Image of the point (1, – 2 , 3) in the plane $2x + 3y - z - 7 = 0$ is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-1} = \frac{-2[2(1) + 3(-2) + (-1)(3) - 7]}{(2)^2 + (3)^2 + (-1)^2}$$

$$= \frac{-2[2 + (-6) + (-3) - 7]}{4 + 9 + 1} = \frac{-2(-14)}{14} = 2$$

$$\Rightarrow \quad \frac{x-1}{2} = 2 \Rightarrow x - 1 = 4 \Rightarrow x = 5$$

$$\Rightarrow \quad \frac{y+2}{3} = 2 \Rightarrow y + 2 = 6 \Rightarrow y = 4$$

$$\Rightarrow \quad \frac{z-3}{-1} = 2 \Rightarrow z - 3 = -2 \Rightarrow z = 1$$

Hence, the required image is (5, 4, 1).

Directions (Q. Nos. 60 and 61)
Consider the spheres
$x^2 + y^2 + z^2 - 4y + 3 = 0$ *and*
$x^2 + y^2 + z^2 + 2x + 4z - 4 = 0$.

60. What is the distance between the centres of the two spheres?

(a) 5 units (b) 4 units
(c) 3 units (d) 2 units

61. Consider the following statements

I. The two spheres intersect each other.
II. The radius of first sphere is less than that of second sphere.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Solutions (Q. Nos. 60 and 61) Given equations of sphere,

$x^2 + y^2 + z^2 - 4y + 3 = 0$ …(i)

and $x^2 + y^2 + z^2 + 2x + 4z - 4 = 0$ …(ii)

Comparing with the standard equation of sphere,

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

we get

$u_1 = 0, v_1 = -2, w_1 = 0$ and $d_1 = 3$ [for first sphere]

and $u_2 = 1, v_2 = 0, w_2 = 2$ and $d_2 = -4$ [for second sphere]

60. (*c*) Centre of Ist sphere *i.e.,*

$c_1 \rightarrow (-u_1, -v_1, -w_1) \rightarrow (0, 2, 0)$

and centre of IInd sphere *i.e.,*

$c_2 \rightarrow (-u_2, -v_2, -w_2) \rightarrow (-1, 0, -2)$

Now, distance between two centres *i.e.,*

$c_1c_2 = \sqrt{(-1-0)^2 + (0-2)^2 + (-2-0)^2}$

$= \sqrt{1 + 4 + 4}$

$[\because d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}]$

$= \sqrt{9} = 3$ units

61. (*c*) **I.** Radius of Ist sphere,

$$r_1 = \sqrt{u_1^2 + v_1^2 + w_1^2 - d_1}$$

$$= \sqrt{(0)^2 + (2)^2 + (0)^2 - (3)}$$

$$= \sqrt{4 - 3} = \sqrt{1} = 1$$

Radius of IInd sphere,

$$r_2 = \sqrt{u_2^2 + v_2^2 + w_2^2 - d_2}$$

$= \sqrt{(-1)^2 + (0)^2 + (-2)^2 + (4)}$

$= \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

Now, $r_1 + r_2 = 1 + 3 = 4$

$\because \quad c_1c_1 < r_1 + r_2$

So, both sphere intersect each other.

II. We have, radius of Ist sphere $(r_1) = 1$

and radius of IInd sphere $(r_2) = 3$

i.e., $r_1 < r_2$

So, the radius of first sphere is less than that of second sphere.

Hence, statement I and II both correct.

Directions (Q. Nos. 63 and 64) *The vertices of a* ΔABC *are* $A(2, 3, 1)$, $B(-2, 2, 0)$ *and* $C(0, 1, -1)$.

62. What is the cosine of angle ABC?

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{\sqrt{6}}$ (d) None of these

63. What is the area of the triangle?

(a) $6\sqrt{2}$ sq units (b) $3\sqrt{2}$ sq units

(c) $10\sqrt{3}$ sq units (d) None of these

64. What is the magnitude of the line joining mid-points of the sides AC and BC?

(a) $\frac{1}{\sqrt{2}}$ unit (b) 1 unit

(c) $\frac{3}{\sqrt{2}}$ units (d) 2 units

Solutions (Q. Nos.62 and 64)

Given that vertices of a triangle are,

$(x_1, y_1, z_1) \equiv A\ (2, 3, 1)$,

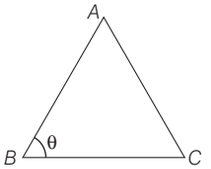
$(x_2\ y_2, z_2) \equiv B\ (-2, 2, 0)$,

and $(x_3\ y_3, z_3) \equiv C\ (0, 1, -1)$

62. (***a***) Now, DR's of $AB = < a_1, b_1, c_1 >$

$= < -2 - 2, 2 - 3, 0 - 1 >$

$= < -4, -1, -1 >$



and DR's of $BC = < a_2, b_2, c_2 >$

$= < 0 + 2, 1 - 2, -1 - 0 >$

$= < 2, -1, -1 >$

Let θ be the angle between AB and BC.

$$\therefore \cos\theta = \left|\frac{(a_1a_2 + b_1b_2 + c_1c_2)}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$= \left|\left\{\frac{-4 \times 2 + (-1)(-1) + (-1) \times (-1)}{\sqrt{16 + 1 + 1}\sqrt{4 + 1 + 1}}\right\}\right|$$

$$= \left|\left(\frac{-8 + 1 + 1}{\sqrt{18}\sqrt{6}}\right)\right|$$

$$= \left|\frac{-6}{6\sqrt{3}}\right| = \left|\frac{-1}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \cos\theta = \cos \angle ABC = \frac{1}{\sqrt{3}}$$

63. (***b***) Now, we find

$$\Delta x = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{3(0 + 1) - 1(2 - 1) + 1(-2 - 0)\}$$

$$= \frac{1}{2}(3 - 1 - 2) = \frac{1}{2}(3 - 3) = \frac{1}{2} \times 0 = 0$$

$$\Delta y = \frac{1}{2}\begin{vmatrix} z_1 & x_1 & 1 \\ z_2 & x_2 & 1 \\ z_3 & x_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 0 & -2 & 1 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{1(-2 - 0) - 2(0 + 1) + 1(0 - 2)\}$$

$$= \frac{1}{2}(-2 - 2 - 2) = \frac{1}{2} \times -(6) = -3$$

and $$\Delta z = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{2(2 - 1) - 3(-2 - 0) + 1(-2 - 0)\}$$

$$= \frac{1}{2}(2 + 6 - 2) = \frac{1}{2} \times 6 = 3$$

$\therefore$ Required area of ΔABC

$= \sqrt{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}$

$= \sqrt{(0)^2 + (-3)^2 + (3)^2}$

$= \sqrt{0 + 9 + 9} = \sqrt{18} = 3\sqrt{2}$

64. (***c***)Mid-point of AC

$$= \left(\frac{2 + 0}{2}, \frac{3 + 1}{2}, \frac{1 - 1}{2}\right) = (1, 2, 0) = P$$ [say]

and mid-point of

$$BC = \left(\frac{-2 + 0}{2}, \frac{2 + 1}{2}, \frac{0 - 1}{2}\right)$$

$= (-1, 3/2, -1/2) = Q$ [say]

Now, magnitude of the line joining P and $Q = |PQ|$

$= \sqrt{(1 + 1)^2 + (2 - 3/2)^2 + (0 + 1/2)^2}$

$= \sqrt{(2)^2 + (1/2)^2 + (1/2)^2} = \sqrt{4 + \frac{1}{4} + \frac{1}{4}}$

$= \sqrt{4 + \frac{1}{2}} = \sqrt{9/2} = \frac{3}{\sqrt{2}}$ units

Directions (Q. Nos. 65 and 66) *Let a vector* **r** *make angles* 60°, 30° *with X and Y-axis, respectively.*

65. What angle does **r** make the Z-axis?

(a) 30° (b) 60° (c) 90° (d) 120°

66. What are the direction cosines of **r**?

(a) $\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$

(b) $\left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \right\rangle$

(c) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$

(d) $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$

Solutions (Q. Nos. 65 and 66)

Given that,

A vector **r** make an angle 60° with X-axis i.e.

$$l = \cos 60° = \frac{1}{2}$$

and a vector **r** make an angle 30° with Y-axis

i.e. $m = \cos 30° = \frac{\sqrt{3}}{2}$

65. (***c***) Let the vector **r** make an angle θ with Z-axis i.e., $n = \cos\theta$

We know that,

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \left(\frac{1}{4} + \frac{3}{4}\right) + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - 1 = 0$$

$$\Rightarrow \cos\theta = 0 = \cos 90°$$

$$\therefore \theta = 90°$$

Hence, 90° angle does **r** make with Z-axis.

66. (***a***) The direction cosine of the vector $\mathbf{r} = < l, m, n >$

$\Rightarrow = < \cos 60°, \cos 30°, \cos 90° >$

$\Rightarrow = < \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$

67. If a line passes through the points $(6, -7, -1)$ and $(2, -3, 1)$, then what are the direction ratios of the line?

(a) $< 4, -4, 2 >$

(b) $< 4, 4, 2 >$

(c) $< -4, 4, 2 >$

(d) $< 2, 1, 1 >$

(***c***) If a line passes through the points (x_1, y_1, z_1) and (x_2, y_2, z_2), then its direction ratio is

$< x_2 - x_1, y_2 - y_1, z_2 - z_1 >$.

$\therefore$ The direction ratio of the line which passes through the points $(6, -7, -1)$ and $(2, -3, 1)$ is

$< 2 - 6, -3 + 7, 1 + 1 > = < -4, 4, 2 >$

19

FUNCTIONS

2019 (II)

Directions (Q. Nos. 1 and 2) *Read the following information and answer the two items that follow.*

Let $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$.

1. For $x = \dfrac{\sqrt{\pi}}{2}$, what is the value of $[ho(gof)](x)$?

(a) 0 (b) 1
(b) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{2}$

2. What is $[fo(fof)](2)$ equal to ?

(a) 2
(b) 8
(c) 16
(d) 256

Solutions (Q. Nos. 1 and 2) Given, $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$

1. (**a**) $(gof)(x) = g\{f(x)\} = \tan x^2$

Now, $[ho(gof)](x) = h\{(gof)(x)\}$
$= \log(\tan x^2)$

for $x = \dfrac{\sqrt{\pi}}{2}$

$[ho(gof)]\left(\dfrac{\sqrt{\pi}}{2}\right) = \log\tan\left(\dfrac{\pi}{4}\right)$

$= \log 1 = 0$

2. (**d**) $(fof)(x) = f\{f(x)\}$

$= (x^2)^2$

$= x^4$

Now, $[fo(fof)](x) = f\{(fof)(x)\}$

$= (x^4)^2 = x^8$

$\therefore [fo(fof)](2) = 2^8$

$= 256$

Directions (Q.Nos. 3-5) *Read the following information and answer the three items that follow.*

Consider the function

$f(x) = g(x) + h(x)$

where, $g(x) = \sin\left(\dfrac{x}{4}\right)$

and $h(x) = \cos\left(\dfrac{4x}{5}\right)$

3. What is the period of the function $g(x)$?

(a) π (b) 2π
(c) 4π (d) 8π

4. What is the period of the function $h(x)$?

(a) π (b) $\dfrac{4\pi}{5}$
(c) $\dfrac{5\pi}{2}$ (d) $\dfrac{3\pi}{2}$

5. What is the period of the function $f(x)$?

(a) 10π (b) 20π
(c) 40π (d) 80π

(Solutions Q. Nos. 3-5) Given, $f(x) = g(x) + h(x)$,

where, $g(x) = \sin\left(\dfrac{x}{4}\right)$ and

$h(x) = \cos\left(\dfrac{4x}{5}\right)$

3. (**d**) $g(x) = \sin\left(\dfrac{x}{4}\right)$

$g(x + 8\pi) = \sin\left(\dfrac{x + 8\pi}{4}\right)$

$= \sin\left(2\pi + \dfrac{x}{4}\right)$

$= \sin\left(\dfrac{x}{4}\right) = g(x)$

$\therefore$ Period of the function $g(x) = 8\pi$

4. (**c**) $h(x) = \cos\left(\dfrac{4x}{5}\right)$

$h\left(x + \dfrac{5\pi}{2}\right) = \cos\dfrac{4}{5}\left(x + \dfrac{5\pi}{2}\right)$

$= \cos\left(2\pi + \dfrac{4x}{5}\right)$

$= \cos\left(\dfrac{4x}{5}\right) = h(x)$

$\therefore$ Pperiod of the function $h(x) = \dfrac{5\pi}{2}$

5. (**c**) $f(x) = g(x) + h(x)$

$= \sin\left(\dfrac{x}{4}\right) + \cos\left(\dfrac{4x}{5}\right)$

$f(x + 40\pi) = \sin\left(\dfrac{x + 40\pi}{4}\right) + \cos\dfrac{4}{5}(x + 40\pi)$

$= \sin\left(10\pi + \dfrac{x}{4}\right) + \cos\left(32\pi + \dfrac{4x}{5}\right)$

$= \sin\left(5\times 2\pi + \dfrac{x}{4}\right) + \cos\left(16\times 2\pi + \dfrac{4x}{5}\right)$

$= \sin\left(\dfrac{x}{4}\right) + \cos\left(\dfrac{4x}{5}\right) = f(x)$

$\therefore$ Period of the function $f(x) = 40\pi$

Directions (Q.Nos. 6 and 7) *Read the following information and answer the two items that follow.*

Let $f(x) = x^2 + 2x - 5$

and $g(x) = 5x + 30$

6. What are the roots of the equation $g[(f(x)] = 0$?

(a) 1, –1 (b) –1, – 1
(c) 1, 1 (d) 0, 1

7. Consider the following statements.

1. $f[g(x)]$ is a polynomial of degree 3.
2. $g[g(x)]$ is a polynomial of degree 2.

Which of the above statements is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

Solutions (Q. Nos. 6 and 7) Given,
$f(x) = x^2 + 2x - 5,\ g(x) = 5x + 30$
$\therefore g[f(x)] = 5(x^2 + 2x - 5) + 30$
$= 5x^2 + 10x + 5$
$f[g(x)] = (5x + 30)^2 + 2(5x + 30) - 5$
$= 25x^2 + 900 + 300x + 10x + 60 - 5$
$= 25x^2 + 310x + 955$
and $g[g(x)] = 5(5x + 30) + 30$
$= 25x + 180$

6. (***b***) The equation, $g[f(x)] = 0$
$5x^2 + 10x + 5 = 0$
$\Rightarrow x^2 + 2x + 1 = 0$
$\Rightarrow (x + 1)^2 = 0$
$\therefore x = -1, -1$
Hence, the roots of this equation are $-1, -1$.

7. (***d***) 1. $f[g(x)] = 25x^2 + 310x + 955$
$f[g(x)]$ is a polynomial of degree 2.
So, Statement 1 is not correct.
2. $g[g(x)] = 25x + 180$
$g[g(x)]$ is a polynomial of degree 1.
So, Statement 2 is not correct.
Hence, the Statement neither 1 nor 2 correct.

2019 (I)

8. A function f defined by $f(x) = \ln(\sqrt{x^2 + 1} - x)$ is
(a) an even function
(b) an odd function
(c) both even and odd function
(d) neither even nor odd function

(***b***) We have, $f(x) = \log(\sqrt{x^2 + 1} - x)$
$\therefore f(-x) = \log(\sqrt{x^2 + 1} + x)$
$= \log\left(\frac{(\sqrt{x^2 + 1} + x)(\sqrt{x^2 + 1} - x)}{\sqrt{x^2 + 1} - x}\right)$
$= \log\left(\frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} - x}\right)$
$= \log\left(\frac{1}{\sqrt{x^2 + 1} - x}\right)$
$= -\log(\sqrt{x^2 + 1} - x) = -f(x)$

9. The domain of the function f defined by $f(x) = \log_x 10$ is
(a) $x > 10$
(b) $x > 0$ excluding $x = 10$
(c) $x \geq 10$
(d) $x > 0$ excluding $x = 1$

(***d***) We have, $f(x) = \log_x 10$
$= \frac{\log 10}{\log x} = \frac{1}{\log x}$
$\therefore f(x)$ is define when $x > 0$ and $x \neq 1$.

10. For $r > 0$, $f(r)$ is the ratio of perimeter to area of a circle of radius r. Then, $f(1) + f(2)$ is equal to
(a) 1 (b) 2 (c) 3 (d) 4

(***c***) We have,
$f(r) = \frac{\text{Perimeter of a circle with radius } r}{\text{Area of a circle with radius } r}$
$\Rightarrow f(r) = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$
$\therefore f(1) = \frac{2}{1} = 2 \Rightarrow f(2) = \frac{2}{2} = 1$
$\therefore f(1) + f(2) = 2 + 1 = 3$

11. If $f(x) = 3^{1+x}$, then $f(x)\, f(y)\, f(z)$ is equal to
(a) $f(x + y + z)$
(b) $f(x + y + z + 1)$
(c) $f(x + y + z + 2)$
(d) $f(x + y + z + 3)$

(***c***) We have, $f(x) = 3^{1+x}$
Similarly, $f(y) = 3^{1+y}$
and $f(z) = 3^{1+z}$
$\therefore f(x)\, f(y)\, f(z) = 3^{1+x+1+y+1+z}$
$= 3^{1+2+x+y+z}$
$= f(2 + x + y + z)$

12. The domain of the function $f(x) = \sqrt{(2 - x)(x - 3)}$ is
(a) $(0, \infty)$ (b) $[0, \infty)$ (c) $[2, 3]$ (d) $(2, 3)$

(***c***) We have, $f(x) = \sqrt{(2 - x)(x - 3)}$
$f(x)$ will be define if $(2 - x)(x - 3) \geq 0$
$\Rightarrow (x - 2)(x - 3) \leq 0$
$\therefore 2 \leq x \leq 3$

13. If $f(x) = \frac{x - 2}{x + 2}, x \neq -2$, then what is $f^{-1}(x)$ equal to ?
(a) $\frac{4(x + 2)}{x - 2}$ (b) $\frac{x + 2}{4(x - 2)}$
(c) $\frac{x + 2}{x - 2}$ (d) $\frac{2(1 + x)}{1 - x}$

(***d***) Given, $f(x) = \frac{x - 2}{x + 2} \Rightarrow y = \frac{x - 2}{x + 2}$
$\Rightarrow x - 2 = xy + 2y$
$\Rightarrow x - xy = 2y + 2$
$\Rightarrow x(1 - y) = 2y + 2$
$\Rightarrow x = \frac{2(y + 1)}{1 - y}$
$\Rightarrow f^{-1}(y) = \frac{2(y + 1)}{1 - y}$
$\therefore f^{-1}(x) = \frac{2(x + 1)}{1 - x}$

14. If $f(x) = \log_{10}(1 + x)$, then what is $4f(4) + 5f(1) - \log_{10} 2$ equal to?
(a) 0 (b) 1 (c) 2 (d) 4

(***d***) We have, $f(x) = \log_{10}(1 + x)$
$\therefore 4f(4) = 4\log_{10}(1 + 4) = 4\log_{10} 5$
$5f(1) = 5\log_{10}(1 + 1) = 5\log_{10} 2$
$\therefore 4f(4) + 5f(1) - \log_{10} 2$
$= 4\log_{10} 5 + 5\log_{10} 2 - \log_{10} 2$
$= 4\log_{10} 5 + 4\log_{10} 2$
$= 4(\log_{10} 5 \times \log_{10} 2)$
$= 4\log_{10}(5 \times 2)$
$= 4\log_{10} 10 = 4 \times 1 = 4$

2018 (II)

15. If $f(x) = \frac{\sqrt{x - 1}}{x - 4}$, defines a function on R, then what is its domain ?
(a) $(-\infty, 4) \cup (4, \infty)$ (b) $[4, \infty)$
(c) $(1, 4) \cup (4, \infty)$ (d) $[1, 4) \cup (4, \infty)$

(***d***) We have,
$f(x) = \frac{\sqrt{x - 1}}{x - 4}$
$\therefore x - 1 \geq 0$
and $x - 4 \neq 0 \Rightarrow x \geq 1$
$x \neq 4$
So, $x \in [1, 4) \cup (4, \infty)$
Domain $= [1, 4) \cup (4, \infty)$

2018 (I)

16. When one of the following is correct in respect of the function $f : R \to R^+$ defined as $f(x) = |x + 1|$?
(a) $f(x^2) = [f(x)]^2$
(b) $f(|x|) = |f(x)|$
(c) $f(x + y) = f(x) + f(y)$
(d) None of the above

(***d***) Given, $f(x) = |x + 1|$
By checking the options, we get
(a) $f(x^2) = |x^2 + 1|$
$\{f(x)\}^2 = (x + 1)^2$
Which implies that $f(x^2) \neq \{f(x)\}^2$
(b) $f(|x|) = ||x| + 1|$
$|f(x)| = ||x + 1| = |x + 1|$
which implies that $f(|x|) \neq |f(x)|$
(c) $f(x + y) = |x + y + 1|$
$f(x) + f(y) = |x + 1| + |y + 1|$
which implies that $f(x + y) \neq f(x) + f(y)$
So, option (d) is correct.

17. Suppose $f : R \to R$ is defined by $f(x) = \dfrac{x^2}{1+x^2}$. What is the range of the function?

(a) [0, 1) (b) [0, 1]
(c) (0, 1] (d) (0, 1)

(**a**) Let $f(x) = y$

Then, $y \geq 0$ and $f(x) = y$

$\therefore \quad \dfrac{x^2}{x^2+1} = y$

$\Rightarrow \quad \dfrac{x^2+1}{x^2} = \dfrac{1}{y}$ for $y > 0$

$\Rightarrow \quad \dfrac{1}{x^2} = \dfrac{1-y}{y}$

$\Rightarrow \quad x = \sqrt{\dfrac{y}{1-y}}$

Now, $\sqrt{\dfrac{y}{1-y}}$ is real

$\Rightarrow \quad \dfrac{y}{1-y} \geq 0$

$\Rightarrow \quad 0 \leq y < 1$

So, range of $f(x)$ is [0, 1).

18. For f to be a function, what is the domain of f, if $f(x) = \dfrac{1}{\sqrt{|x| - x}}$?

(a) $(-\infty, 0)$ (b) $(0, \infty)$
(c) $(-\infty, \infty)$ (d) $(-\infty, 0)$

(**a**) We have,

$$f(x) = \frac{1}{\sqrt{|x| - x}}$$

$f(x)$ is defined, if

$|x| - x > 0 \Rightarrow |x| > x$

Case I $x > 0$

$\therefore \quad x > x \; [\because |x| = x, x > 0]$

which is not possible

Case II $x < 0$

$\therefore \quad -x > x \qquad [\because |x| = -x, x < 0]$

$\Rightarrow \quad 0 > 2x \Rightarrow x < 0$

Which is possible

$\therefore$ Domain of $f(x) = (-\infty, 0)$

19. What is the period of the the function $f(x) = \sin x$?

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) π (d) 2π

(**d**) We have,

$f(x) = \sin x$

$f(x + 2\pi) = \sin(2\pi + x) = \sin x$

$\Rightarrow \quad f(x + 2\pi) = f(x)$

Hence, period of $f(x) = \sin x$ has 2π.

20. If $f(x)$ is an even function, where $f(x) \neq 0$, then which one of the following is correct?

(a) $f'(x)$ is an even function
(b) $f'(x)$ is an odd function
(c) $f'(x)$ may be an even or odd function depending on the type of function
(d) $f'(x)$ is a constant function

(**b**) We have, $f(x)$ is an even function.

$\therefore \quad f(-x) = f(x)$

On differentiating both the sides, we have

$-f'(-x) = f'(x)$

$\Rightarrow \quad f'(-x) = -f'(x)$

$\therefore f'(x)$ is an odd function.

21. If $f : R \to S$ defined by $f(x) = 4\sin x - 3\cos x + 1$ is onto, then what is S equal to ?

(a) [–5, 5] (b) (–5, 5)
(c) (–4, 6) (d) [–4, 6]

(**d**) We have

$f(x) = 4\sin x - 3\cos x + 1$

We know that,

$-\sqrt{4^2 + (-3)^2} \leq 4\sin x - 3\cos x$

$\leq \sqrt{4^2 + (-3)^2} \quad [\because -\sqrt{a^2 + b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2 + b^2}]$

$\Rightarrow -5 \leq 4\sin x - 3\cos x \leq 5$

$\Rightarrow -5 + 1 \leq 4\sin x - 3\cos x + 1 \leq 5 + 1$

$\Rightarrow -4 \leq f(x) \leq 6$

$\therefore \quad f(x) \in [-4, 6]$

Since, $f(x)$ is onto.

$\therefore \quad S$ = Range of f

$= [-4, 6]$

2017 (II)

22. The inverse of the function $y = 5^{\ln x}$ is

(a) $x = y^{\frac{1}{\ln 5}}, y > 0$ (b) $x = y^{\ln 5}, y > 0$
(c) $x = y^{\frac{1}{\ln 5}}, y < 0$ (d) $x = 5^{\ln x}, y > 0$

(**a**) We have,

$y = 5^{\ln x} \Rightarrow \ln y = \ln 5^{\ln x}$

$\Rightarrow \quad \ln y = \ln x \ln 5$

$\Rightarrow \quad \ln x = \dfrac{\ln y}{\ln 5}$

$\Rightarrow \quad \ln x = \ln y^{\frac{1}{\ln 5}}$

$\therefore \quad x = y^{\frac{1}{\ln 5}}, y > 0 \qquad [\because 5^{\ln x} > 0]$

23. The function $f(x) = |x| - x^3$ is

(a) odd (b) even
(c) both even and odd
(d) neither even nor odd

(**d**) We have,

$f(x) = |x| - x^3$

On putting, $x = -x$, we get

$f(-x) = |-x| - (-x)^3$

$= |x| + x^3$

Since, $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

$\therefore f(x)$ is neither even nor odd.

24. If x is any real number, then $\dfrac{x^2}{1+x^4}$ belongs to which one of the following intervals?

(a) (0, 1) (b) $\left[0, \dfrac{1}{2}\right]$ (c) $\left(0, \dfrac{1}{2}\right)$ (d) [0, 1]

(**b**) Let, $y = \dfrac{x^2}{1+x^4} \Rightarrow y \geq 0$

Also, $y = \dfrac{x^2}{1+x^4} = \dfrac{1}{x^2 + \frac{1}{x^2}} \Rightarrow y \leq \dfrac{1}{2}$

$\therefore \quad y \in \left[0, \dfrac{1}{2}\right]$

25. If $f(x) = \dfrac{x}{2} - 1$, then on the interval $[0, \pi]$ which one of the following is correct?

(a) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $\dfrac{1}{f(x)}$ are both continuous
(b) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $f^{-1}(x)$ are both continuous
(c) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $\dfrac{1}{f(x)}$ are both discontinuous
(d) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function is discontinuous but $\dfrac{1}{f(x)}$ is continuous

(**c**) We have, $f(x) = \dfrac{x}{2} - 1 \; [0, \pi]$

$\therefore \quad \tan[f(x)] = \tan\left[\dfrac{x}{2} - 1\right]$

since $[x]$ is discontinuous at integers.

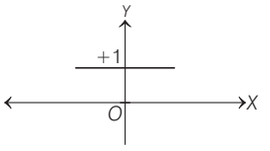
$\therefore \tan\left[\dfrac{x}{2} - 1\right]$ will be discontinuous at

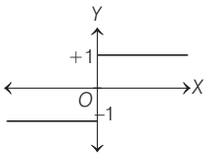
$x = 2 \in [0, \pi]$

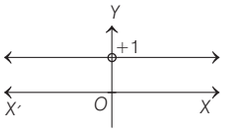
Again, $f^{-1}(x) = \dfrac{1}{f(x)} = \dfrac{1}{\frac{x}{2} - 1} = \dfrac{2}{x-2}$

it is clear that $f^{-1}(x)$ is discontinuous at $x = 2 \in [0, \pi]$.

26. Which one of the following graph represents the function $f(x)=\frac{x}{x},\ x\neq 0$?

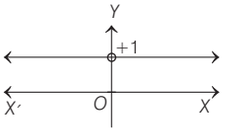
(a) 

(b) 

(c) 

(d) None of the above

(**c**) We have



$f(x)=\frac{x}{x},\ x\neq 0=1,\ x\neq 0$

So, this graph represents of the function $f(x)$.

27. Let $f(n)=\left[\frac{1}{4}+\frac{n}{1000}\right]$, where $[x]$ denote the integral part of x.

Then the value of $\sum_{n=1}^{1000} f(n)$ is

(a) 251 (b) 250 (c) 1 (d) 0

(**a**) We have, $f(n)=\left[\frac{1}{4}+\frac{n}{1000}\right]$

$$\therefore \sum_{n=1}^{1000} f(n)=\left[\frac{1}{4}+\frac{1}{1000}\right]+\left[\frac{1}{4}+\frac{2}{1000}\right]+\ldots+\left[\frac{1}{4}+\frac{749}{1000}\right]+\left[\frac{1}{4}+\frac{750}{1000}\right]+\left[\frac{1}{4}+\frac{751}{1000}\right]+\ldots+\ldots+\left[\frac{1}{4}+\frac{1000}{1000}\right]$$

$$0+0+\ldots+\ldots 0+\left[\frac{1}{4}+\frac{750}{1000}\right]+\left[\frac{1}{4}+\frac{751}{1000}\right]+\ldots+\left[\frac{1}{4}+\frac{1000}{1000}\right]$$

$= 1+1+1+\ldots+1$ (251 times) $= 251$

28. If $f(x)=\frac{4x+x^4}{1+4x^3}$ and $g(x)=\ln\left(\frac{1+x}{1-x}\right)$, then what is the value of $(fog)\left(\frac{e-1}{e+1}\right)$ equal to?

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$

(**b**) We have, $f(x)=\frac{4x+x^4}{1+4x^3}$

and $g(x)=\ln\left(\frac{1+x}{1-x}\right)$

Now, $g\left(\frac{e-1}{e+1}\right)=\ln\left(\frac{1+\frac{e-1}{e+1}}{1-\frac{e-1}{e+1}}\right)$

$$=\ln\left(\frac{e+1+e-1}{e+1-e+1}\right)$$

$$=\ln\left(\frac{2e}{2}\right)=\ln e=1 \quad \ldots(i)$$

$$\therefore \quad fog\left(\frac{e-1}{e+1}\right)=f\left(g\left(\frac{e-1}{e+1}\right)\right)=f(1)$$

$$=\frac{4\times 1+(1)^4}{1+4(1)^3}=\frac{4+1}{1+4}=\frac{5}{5}=1$$

2017 (I)

29. If $f(x)=\frac{x}{x-1}$, then what is $\frac{f(a)}{f(a+1)}$ equal to ?

(a) $f\left(-\frac{a}{a+1}\right)$ (b) $f(a^2)$

(c) $f\left(\frac{1}{a}\right)$ (d) $f(-a)$

(**b**) We have, $f(x)=\frac{x}{x-1}$

$$\therefore \quad f(a)=\frac{a}{a-1}$$

and $f(a+1)=\frac{(a+1)}{(a+1)-1}=\frac{a+1}{a}$

Now, $\frac{f(a)}{f(a+1)}=\frac{a/(a-1)}{(a+1)/a}$

$$=\frac{a^2}{a^2-1}=f(a^2)$$

30. Let $f:[-6,6]\to R$ be defined by $f(x)=x^2-3$. Considered the following :

1. $(f\circ f\circ f)(-1)=(f\circ f\circ f)(1)$
2. $(f\circ f\circ f)(-1)-4(f\circ f\circ f)(1)=(f\circ f)(0)$

Which of the above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) We have, $f(x)=x^2-3$

1. Now, $f(-1)=(-1)^2-3=-2$

$\Rightarrow f\circ f(-1)=f(-2)=(-2)^2-3=1$

$f\circ f\circ f(-1)=f(1)=1^2-3=-2$ ...(i)

Again, $f(1)=(1)^2-3=-2$

$\Rightarrow f\circ f(1)=f(-2)=(-2)^2-3=1$

$\Rightarrow f\circ f\circ f(1)=(1)^2-3=-2$ ... (ii)

From Eq. (i) and Eq. (ii)

$f\circ f\circ f(-1)=f\circ f\circ f(1)$

So, 1 part is correct

2. Now, $f(0)=(0)^2-3=-3$

$f\circ f(0)=f(-3)=(-3)^2-3$

$=9-3=6$

Now, $(f\circ f\circ f)(-1)-4(f\circ f\circ f)(1)$

$=-2-4(-2)=-2+8=6$

So, 2 part is correct.

31. Let $f(x)=px+q$ and $g(x)=mx+n$. Then, $f(g(x))=g(f(x))$ is equivalent to

(a) $f(p)=g(m)$
(b) $f(q)=g(n)$
(c) $f(n)=g(q)$
(d) $f(m)=g(p)$

(**c**) We have,

$f(x)=px+q$ and $g(x)=mx+n$

Now, $f(g(x))=f(mx+n)$

$=p(mx+n)+q$

and $g(f(x))=g(px+q)$

$=m(px+q)+n$

$=mpx+mq+n$

Since, $f(g(x))=g(f(x))$

$\Rightarrow pmx+pn+q=mpx+mq+n$

$\Rightarrow pn+q=mq+n$

$\therefore f(n)=g(q)$

32. Which one of the following functions is neither even nor odd?

(a) x^2-1 (b) $x+\frac{3}{x}$

(c) $|x|$ (d) $x^2(x-3)$

(**d**) For option (a)

$f(x)=x^2-1$

$f(-x)=(-x)^2-1=x^2-1$

Here, $f(x)=f(-x)$

$\therefore$ even function.

For option (b)

$f(x)=x+\frac{3}{x}$

$f(-x)=-x+\frac{3}{(-x)}=-\left(x+\frac{3}{x}\right)$

Here, $f(-x)=-f(x)$

$\therefore$ odd function.

For option (c)

$f(x)=|x|\ \ f(-x)=|-x|=|x|$

$\therefore f(x)=f(-x)$

$\therefore$ even function.

For option (d)

$f(x)=x^2(x-3)$

$f(-x)=(-x)^2[(-x)-3]$

$=-x^2(x+3)$

Here, $f(x)\neq f(-x)$ and $f(x)\neq -f(-x)$

$\therefore$ Neither even nor odd.

33. Let $f(a)=\dfrac{a-1}{a+1}$ Consider the following :

1. $f(2a)=f(a)+1$
2. $f\left(\dfrac{1}{a}\right)=-f(a)$

Which of the above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***b***) We have, $f(a)=\dfrac{a-1}{a+1}$

1. Now, $f(2a)=\dfrac{2a-1}{2a+1}$

$$f(a)+1=\frac{a-1}{a+1}+1$$

$$=\frac{(a-1)+(a+1)}{a+1}=\frac{2a}{a+1}$$

So, statement 1 is not true.

2. $f(1/a)=\dfrac{1/a-1}{1/a+1}=\dfrac{(1-a)/a}{(1+a)/a}$

$$=\frac{1-a}{1+a}$$

and $-f(a)=-\left(\dfrac{a-1}{a+1}\right)=\dfrac{1-a}{1+a}$

So, statement 2 is true.

34. Let $f(x):\begin{cases}x, & x \text{ is rational}\\ 0, & x \text{ is irrational}\end{cases}$

and $g(x):\begin{cases}0, & x \text{ is rational}\\ x, & x \text{ is irrational}\end{cases}$

if $f: R\to R$ and $g: R\to R$, then $(f-g)$ is

(a) one-one and into
(b) neither one-one nor onto
(c) many-one and onto
(d) one-one and onto

(***d***) Let $\phi(x)=f(x)-g(x)$

$$=\begin{cases}x, & x \text{ is rational number}\\ -x, & x \text{ is irrational number}\end{cases}$$

For one-one,

Take any straight line parallel to X-axis which will intersect $\phi(x)$ only at one point.

So, $\phi(x)$ is one -one.

For onto, as $\phi(x)$

$$=\begin{cases}x, & x \text{ is rational number}\\ -x, & x \text{ is irrational number}\end{cases}$$

Which shows $y=x$ and $y=-x$ for rational and irrational values

$\Rightarrow y\in$ Real number

$\therefore$ Range = codomain

so, $\phi(x)$ is onto

Thus, $(f-g)$ is one-one and onto.

2016 (I)

35. If $f(x_1)-f(x_2)=f\left(\dfrac{x_1-x_2}{1-x_1x_2}\right)$ for $x_1, x_2\in(-1,1)$, then what is $f(x)$ equal to?

(a) $\ln\left(\dfrac{1-x}{1+x}\right)$

(b) $\ln\left(\dfrac{2+x}{1-x}\right)$

(c) $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)$

(d) $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$

(***a***) Let $f(x)=\ln\left(\dfrac{1-x}{1+x}\right)$

Now,

$$f(x_1)-f(x_2)=\ln\left(\frac{1-x_1}{1+x_1}\right)-\ln\left(\frac{1-x_2}{1+x_2}\right)$$

$$=\ln\left(\frac{(1-x_1)(1+x_2)}{(1-x_2)(1+x_1)}\right)$$

$$=\ln\left(\frac{1+x_2-x_1-x_1x_2}{1+x_1-x_2-x_1x_2}\right)\ldots(i)$$

and $f\left(\dfrac{x_1-x_2}{1-x_1x_2}\right)=\ln\left(\dfrac{1-\dfrac{x_1-x_2}{1-x_1x_2}}{1+\dfrac{x_1-x_2}{1-x_1x_2}}\right)$

$$=\ln\left(\frac{1-x_1x_2-x_1+x_2}{1-x_1x_2+x_1-x_2}\right)\ldots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$f(x_1)-f(x_2)=f\left(\frac{x_1-x_2}{1-x_1x_2}\right)$$

36. What is the range of the function $y=\dfrac{x^2}{1+x^2}$ where $x\in R$?

(a) [0, 1) (b) [0, 1] (c) (0, 1) (d) (0, 1]

(***a***) Let $f(x)=\dfrac{x^2}{1+x^2}$

Clearly, domain $(f)=R$

Let $y=f(x)\Rightarrow y=\dfrac{x^2}{1+x^2}$

$\Rightarrow y+x^2y=x^2 \Rightarrow x^2-x^2y=y$

$\Rightarrow x^2(1-y)=y\Rightarrow x=\pm\sqrt{\dfrac{y}{1-y}}$

Clearly, x will take real values, if $\dfrac{y}{1-y}\geq 0$

$\Rightarrow \dfrac{y-0}{y-1}\leq 0$

$\Rightarrow 0\leq y<1 \Rightarrow y\in[0,1)$

Hence, range $f(x)$ is $[0, 1)$.

Directions (Q. Nos. 37 and 38)
Consider the function

$$f(x)=\frac{27(x^{2/3}-x)}{4}.$$

37. How many solutions does the function $f(x)=1$ have?

(a) One (b) Two (c) Three (d) Four

(***b***) Given function,

$$f(x)=\frac{27(x^{2/3}-x)}{4}$$

If $f(x)=1$

$$\Rightarrow 1=\frac{27(x^{2/3}-x)}{4}$$

$$\Rightarrow x^{2/3}-x=\frac{4}{27}$$

Let $x^{1/3}=\alpha\Rightarrow x=\alpha^3$

Then, $\alpha^2-\alpha^3=\dfrac{4}{27}$

$$\Rightarrow \alpha^2(1-\alpha)=\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$$

or $\alpha^2(1-\alpha)=\left(\dfrac{-1}{3}\right)\left(\dfrac{-1}{3}\right)\left(\dfrac{1+3}{3}\right)$

$$\Rightarrow \alpha\cdot\alpha\,(1-\alpha)=\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$$

or $\alpha\cdot\alpha\,(1-\alpha)=\left(\dfrac{-1}{3}\right)\left(\dfrac{-1}{3}\right)\left(\dfrac{4}{3}\right)$

$$\Rightarrow \alpha=\frac{2}{3} \text{ or } \alpha=\frac{-1}{3}$$

$$\Rightarrow x^{1/3}=\frac{2}{3} \text{ or } \frac{-1}{3}$$

$$\therefore x=\frac{8}{27} \text{ or } \frac{-1}{27}$$

Hence, two solution exists.

38. How many solutions does the function $f(x)=-1$ have?

(a) One (b) Two (c) Three (d) Four

(***a***) Given, $f(x)=\dfrac{27(x^{2/3}-x)}{4}$

We have, $f(x)=-1$

$$\Rightarrow \frac{27(x^{2/3}-x)}{4}=-1$$

$$\Rightarrow x^{2/3}-x=-\frac{4}{27}$$

Let $x^{1/3}=\alpha\Rightarrow x=\alpha^3$

Then, $\alpha^2-\alpha^3=-\dfrac{4}{27}$

$$\Rightarrow \alpha^2(1-\alpha)=\frac{-4}{27}$$

$$\Rightarrow \alpha\cdot\alpha(1-\alpha)=\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)$$

$$\Rightarrow \alpha=-\frac{1}{3}$$

$$\Rightarrow x^{1/3}=-\frac{1}{3}$$

$$\therefore x=-\frac{1}{27}$$

Hence, $f(x)=-1$ has one solution exists.

2015 (II)

39. If $g(x)=\frac{1}{f(x)}$ and $f(x)=x, x\neq 0$, then which one of the following is correct ?

(a) $f(f(f(g(g(f(x))))))=g(g(f(g(f(x)))))$
(b) $f(g(g(g(f(x)))))=g(g(f(g(f(x)))))$
(c) $f(g(f(g(g(f(g(x)))))))=g(g(f(g(f(x)))))$
(d) $f(f(f(g(g(f(x))))))=f(f(f(g(f(x)))))$

(b) Given, $g(x)=\frac{1}{f(x)}$ and $f(x)=x$

$\therefore \quad g(x)=\frac{1}{x}, x\neq 0$

Clearly, $g(f(x))=\frac{1}{f(x)}=\frac{1}{x}$;

$g(g\ (f(x)))=\frac{1}{\left(\frac{1}{x}\right)}=x$

$g(g(g(f(x))))=\frac{1}{(x)}=\frac{1}{x}$

$f(g(g(g(f(x)))))=\frac{1}{x}$ and $g(f(x))=\frac{1}{x}$

$f\ (g(f(x)))=\frac{1}{x}$

$g(f(g(f(x))))=\frac{1}{\left(\frac{1}{x}\right)}=x$;

$g(g(f(g(f(x)))))=\frac{1}{x}$

In option (b), LHS = RHS

40. The domain of the function $f(x)=\frac{1}{\sqrt{|x|-x}}$ is

(a) $[0,\infty)$ (b) $(-\infty,0)$
(c) $[1,\infty)$ (d) $(-\infty,0]$

(b) Given, $f(x)=\frac{1}{\sqrt{|x|-x}}$

Here, $f(x)$ is defined only when
$|x|-x>0$ or $|x|>x$
which is possible only for negative values of x.
$\therefore$ Domain of f is $(-\infty, 0)$.

41. If $f:R\to R, g:R\to R$ are two functions given by $f(x)=2x-3$ and $g(x)=x^3+5$, then $(fog)^{-1}(x)$ is equal to

(a) $\left(\frac{x+7}{3}\right)^{\frac{1}{3}}$ (b) $\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$
(c) $\left(x-\frac{7}{2}\right)^{\frac{1}{3}}$ (d) $\left(x+\frac{7}{2}\right)^{\frac{1}{3}}$

(b) Given, $f(x)=2x-3$
and $g(x)=x^3+5$
Now, $(fog)(x)=f(g(x))=2(x^3+5)-3$
$=2x^3+10-3$
$=2x^3+7$

Let $(fog)(x)=y$

$\therefore\ y=2x^3+7 \Rightarrow x=\left(\frac{y-7}{2}\right)^{1/3}$

$\therefore \quad (fog)^{-1}(x)=\left(\frac{x-7}{2}\right)^{1/3}$

42. $f(xy)=f(x)+f(y)$ is true for all

(a) polynomial functions f
(b) trigonometric functions f
(c) exponential functions f
(d) logarithmic functions f

(d) Let $f(x)=\log x$
$\therefore \quad f(xy)=\log(xy)$
and $f(x)=\log x, f(y)=\log y$
Now, $f(x)+f(y)=\log x+\log y$
$=\log(xy)=f(xy)$
Hence, $f(x)+f(y)=f(xy)$

43. Consider the following statements

Statement I The function $f:R\to R$ such that $f(x)=x^3$ for all $x\in R$ is one-one.

Statement II $f(a)=f(b)$ $\Rightarrow a=b$ for all $a,b\in R$, if the function f is one-one.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement II is the correct explanation of Statement I
(b) Both the statements are true and Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

(a) Statement I is correct, since $f(x)=x^3$ is one-one, $\forall\ x\in R$.
$\because \quad f(a)=f(b)$
$\Rightarrow \quad a^3=b^3$
$\Rightarrow \quad a=b$
Statement II is also correct and is the correct explanation of Statement I.

2015 (I)

44. For each non-zero real number x, let $f(x)=\frac{x}{|x|}$. The range of f is

(a) a null set
(b) a set consisting of only one element
(c) a set consisting of two elements
(d) a set consisting of infinitely many elements

(c) We have, $f(x)=\frac{x}{|x|}$ for $x\neq 0$

i.e., $f(x)=\begin{cases}\frac{x}{x}, \text{ if } x>0\\ \frac{x}{-x}, \text{ if } x<0\end{cases}=\begin{cases}1, \text{ if } x>0\\ -1, \text{ if } x<0\end{cases}$

Thus, range set of $f=\{1,-1\}$

45. If $f(x)=\log_e\left(\frac{1+x}{1-x}\right)$, $g(x)=\frac{3x+x^3}{1+3x^2}$ and $gof\ (t)=g(f(t))$, then what is $gof\left(\frac{e-1}{e+1}\right)$ equal to?

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$

(b) We have, $f(x)=\log_e\left(\frac{1+x}{1-x}\right)$ and
$g(x)=\frac{3x+x^3}{1+3x^2}$

To find $gof\left(\frac{e-1}{e+1}\right)$

Clearly, $gof\left(\frac{e-1}{e+1}\right)=g\left[f\left(\frac{e-1}{e+1}\right)\right]$ ...(i)

Let us first find $f\left(\frac{e-1}{e+1}\right)$, which is given

by $f\left(\frac{e-1}{e+1}\right)=\log_e\frac{\left(1+\frac{(e-1)}{(e+1)}\right)}{\left(1-\frac{(e-1)}{(e+1)}\right)}$

$=\log_e\left(\frac{e+1+e-1}{e+1-e+1}\right)$

$=\log_e\left(\frac{2e}{2}\right)=\log_e(e)=1$

Now, from Eq. (i), we have
$gof\left(\frac{e-1}{e+1}\right)=g(1)$

$=\frac{3(1)+(1)^3}{1+3(1)^2}=\frac{4}{4}=1$

46. Consider the following functions

I. $f(x)=x^3, x\in R$
II. $f(x)=\sin x, 0<x<2\pi$
III. $f(x)=e^x, x\in R$

Which of the above functions have inverse defined on their ranges?

(a) I and II (b) II and III
(c) I and III (d) I, II and III

(c) I. We have, $f(x)=x^3, x\in R$
Now, let $x_1, x_2\in R$ such that $f(x_1)=f(x_2)$
$\Rightarrow \quad x_1^3=x_2^3$
$\Rightarrow \quad (x_1-x_2)(x_1^2+x_2^2+x_1x_2)=0$

$$\Rightarrow (x_1 - x_2)\left[\begin{array}{l} x_1^2 + 2\cdot\frac{x_2}{2}\cdot x_1 + \left(\frac{x_2}{2}\right)^2 \\ \qquad -\left(\frac{x_2}{2}\right)^2 + x_2^2 \end{array}\right] = 0$$

$$\Rightarrow (x_1 - x_2)\left[\left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3x_2^2}{4}\right] = 0$$

$\Rightarrow \quad x_1 = x_2$

So, f is one-one.

Hence, f is invertible on their range.

II. We have, $f(x) = \sin x,\ 0 < x < 2\pi$

Clearly, $f\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

and $f\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

So, f is not one-one.

Hence, f is not invertible on their range.

III. We have, $f(x) = e^x,\ x \in R$

$\Rightarrow \quad f'(x) = e^x > 0$

So, f is strictly increasing function.

$\Rightarrow$ f is one-one.

Hence, f is invertible on their range.

2014 (II)

47. If $f(x) = ax + b$ and $g(x) = cx + d$ such that $f[g(x)] = g[f(x)]$, then which one of the following is correct?

(a) $f(c) = g(a)$ (b) $f(a) = g(c)$
(c) $f(c) = g(d)$ (d) $f(d) = g(b)$

(**d**) We have, $f(x) = ax + b$ and $g(x) = cx + d$

$f[(g(x)] = a(cx + d) + b$

$= acx + ad + b$

and $g[f(x)] = c(ax + b) + d$

$= acx + bc + d$

$\because \quad f[(gx)] = g[f(x)]$

$\therefore \quad ad + b = bc + d \Rightarrow f(d) = g(b)$

Directions (Q. Nos. 48 and 49) *Read the following information carefully and answer the questions given below.*

Consider the function $f(x) = \frac{x-1}{x+1}$.

48. What is $\frac{f(x)+1}{f(x)-1} + x$ equal to?

(a) 0 (b) 1
(c) $2x$ (d) $4x$

(**a**) We have, $f(x) = \frac{x-1}{x+1}$

On applying componendo and dividendo, we get

$$\frac{f(x)+1}{f(x)-1} = \frac{x-1+x+1}{x-1-x-1}$$

$\Rightarrow \quad \frac{f(x)+1}{f(x)-1} = -x$

$\therefore \frac{f(x)+1}{f(x)-1} + x = 0$

49. What is $f(2x)$ equal to?

(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{f(x)+1}{3f(x)+1}$
(c) $\frac{3f(x)+1}{f(x)+3}$ (d) $\frac{f(x)+3}{3f(x)+1}$

(**c**) We have,

$$f(x) = \frac{x-1}{x+1}$$

$$\Rightarrow f(2x) = \frac{2\left[\frac{[f(x)+1]}{1-f(x)}\right] - 1}{2\left[\frac{[f(x)+1]}{1-f(x)}\right] + 1}$$

$$\left[\because x = \frac{f(x)+1}{1-f(x)}\right]$$

$\therefore \quad f(2x) = \frac{3f(x)+1}{f(x)+3}$

50. What is $f[f(x)]$ equal to?

(a) x (b) $-x$
(c) $-\frac{1}{x}$ (d) None of these

(**c**) We have, $f(x) = \frac{x-1}{x+1}$

$\Rightarrow \quad f[f(x)] = \frac{f(x)-1}{f(x)+1}$

$\Rightarrow f[f(x)] = -\frac{1}{x} \quad \left[\because x = -\left\{\frac{f(x)+1}{f(x)-1}\right\}\right]$

51. The function $f : N \to N$, where N being the set of natural numbers, defined by $f(x) = 2x + 3$ is

(a) injective and surjective
(b) injective but not surjective
(c) not injective but surjective
(d) neither injective nor surjective

(**b**) We have, $f : N \to N$

$\therefore \quad f(x) = 2x + 3$

$\Rightarrow \quad f'(x) = 2 > 0$

So, $f(x)$ is increasing, $\forall\, x \in N$.

Hence, $f(x)$ is injective.

Let $f(x) = y$

$\therefore \quad y = 2x + 3$

$\Rightarrow \quad x = \frac{y-3}{2}$

Let $y = 4$

$\Rightarrow \quad x = \frac{1}{2}$

and $y \in N$ but $x \notin N$

Hence, $f(x)$ is not surjective.

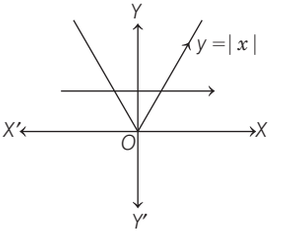
2014 (I)

52. Let N denotes the set of all non-negative integers and Z denotes the set of all integers. The function $f : Z \to N$ given by $f(x) = |x|$ is

(a) one-one but not onto
(b) onto but not one-one
(c) Both one-one and onto
(d) Neither one-one nor onto

(**d**) Given that, $f : Z \to N$ and $f(x) = |x|$

Graph of $f(x) = |x|$ is



We see that, if we draw a parallel line along X-axis, then it cuts the curve into more than one point. So, the function $f(x) = |x|$ is not one-one.

Here, codomain of $f = N$
[natural number]

and range of $f = Z^+$
[non-negative integers]

$\because$ Codomain of $f \neq$ Range of f

So, the function is not onto.

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LIMIT, CONTINUITY AND DIFFERENTIABILITY

2019 (II)

1. For what value of k is the function $f(x)=\begin{cases}2x+\frac{1}{4}, & x<0\\ k, & x=0\\ \left(x+\frac{1}{2}\right)^2, & x>0\end{cases}$ continuous?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

(*a*) Given,

$f(x)=\begin{cases}2x+\frac{1}{4}, & x<0\\ k, & x=0\\ \left(x+\frac{1}{2}\right)^2, & x>0\end{cases}$ continuous.

$\text{LHL}=\lim\limits_{x\to 0^-} f(x)=\lim\limits_{h\to 0^-}\left(2x+\frac{1}{4}\right)$

$=\lim\limits_{h\to 0}\left[2(0-h)+\frac{1}{4}\right]$

$=\lim\limits_{h\to 0}\left(-2h+\frac{1}{4}\right)$

$=\frac{1}{4}$

and $f(0)=k$

$\because$ The function $f(x)$ is continuous at $x=0$

$\therefore$ LHL = $f(0)$ = RHL

$\Rightarrow$ LHL = $f(0)$

$\Rightarrow \quad \frac{1}{4}=k$

Hence, $k=\frac{1}{4}$

2. What is the value of $\lim\limits_{x\to 0}\frac{\sin x^\circ}{\tan 3x^\circ}$?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

(*b*) $\lim\limits_{x\to 0}\frac{\sin x^\circ}{\tan 3x^\circ}=\lim\limits_{x\to 0}\left(\frac{x\times\frac{\sin x}{x}}{3x\times\frac{\tan 3x}{3x}}\right)$

$=\frac{1}{3}\lim\limits_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)}{\left(\frac{\tan 3x}{3x}\right)}=\frac{1}{3}$

$\left[\because \lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}=1 \text{ and } \lim\limits_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right]$

3. Consider the following statements in respect of the function $f(x)=\sin\left(\frac{1}{x}\right)$ for $x\neq 0$ and $f(0)=0$:

1. $\lim\limits_{x\to 0} f(x)$ exists

2. $f(x)$ is continuous at $x=0$

Which of the above statement is/are correct?

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

(*d*) Given, $f(x)=\begin{cases}\sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$

$\text{LHL}=\lim\limits_{x\to 0^-}\sin\left(\frac{1}{x}\right)$

$=\lim\limits_{h\to 0}\sin\left(\frac{1}{0-h}\right)$

$=\lim\limits_{h\to 0}-\sin\left(\frac{1}{h}\right)=-\sin\infty$

$=-$ (a rational number)

[$\because \sin\theta$ lies between –1 to 1]

$\text{RHL}=\lim\limits_{x\to 0^+}\sin\left(\frac{1}{x}\right)=\lim\limits_{h\to 0}\sin\left(\frac{1}{0+h}\right)$

$=\lim\limits_{h\to 0}\sin\left(\frac{1}{h}\right)=\sin\infty$

= a rational number

[$\because \sin\theta$ lies between – 1 to 1]

$\because$ LHL $\neq$ RHL

So, $f(x)$ does not exists.

$\because$ $f(x)=0$ at $x=0$

$\therefore$ LHL $\neq$ RHL $\neq f(0)$

So, $f(x)$ is not continuous.

Hence, the statements neither 1 nor 2 correct.

2019 (I)

4. $\lim\limits_{x\to 0}\frac{1-\cos^3 4x}{x^2}$ is equal to

(a) 0 (b) 12

(c) 24 (d) 36

(*c*) $\lim\limits_{x\to 0}\frac{1-\cos^3 4x}{x^2}$ $\left[\frac{0}{0}\text{ form}\right]$

On apply L′ Hospital rule we get

$\lim\limits_{x\to 0}\frac{-3\cos^2(4x)(-\sin 4x)(4)}{2x}$

$=\lim\limits_{x\to 0}\frac{12\cos^2 4x\sin 4x}{2x}\left[\frac{0}{0}\text{ form}\right]$

Again, apply's L′ Hospital rule, we get

$\lim\limits_{x\to 0}\frac{12[2\cos(4x)(-\sin 4x)(4)\ \sin 4x+\cos^2 4x(\cos 4x)(4)]}{2}$

$=\lim\limits_{x\to 0}\frac{12[-8\cos 4x\sin^2 4x+4\cos^3 4x]}{2}$

$=6(-8\times 0+4)=24$

5. The value of k which makes $f(x)=\begin{cases}\sin x, & x\neq 0\\ k, & x=0\end{cases}$ continuous at $x=0$, is

(a) 2 (b) 1
(c) –1 (d) 0

(*d*) Given, $f(x)$ is continuous at $x=0$.

$\therefore \quad \lim_{x\to 0} f(x)=f(0)$

$\Rightarrow \quad \lim_{x\to 0}\sin x = k$

$\therefore \quad k=0$

2018 (II)

6. Consider the function

$$f(x)=\begin{cases}\dfrac{\sin 2x}{5x}, & \text{if } x\neq 0\\ \dfrac{2}{15}, & \text{if } x=0\end{cases}$$

Which one of the following is correct in respect of the function?

(a) It is not continuous at $x=0$
(b) It is continuous at every x
(c) It is not continuous at $x=\pi$
(d) It is continuous at $x=0$

(*a*) We have,

$$f(x)=\begin{cases}\dfrac{\sin 2x}{5x}, & x\neq 0\\ \dfrac{2}{15}, & x=0\end{cases}$$

at $x=0$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\sin 2x\times 2}{5x\times 2}=\frac{2}{5}$$

$$f(0)=\frac{2}{15}$$

$\therefore \quad \lim_{x\to 0} f(x)\neq f(0)$

at $x=0$ function is discontinuous.

7. For the function $f(x)=|x-3|$, which one of the following is not correct?

(a) The function is not continuous at $x=-3$
(b) The function is continuous at $x=3$
(c) The function is differentiable at $x=0$
(d) The function is differentiable at $x=-3$

(*a*) We have,

$$f(x)=|x-3|$$

We know that, modulus function is continuous in R.

So, option (a) is incorrect.

8. If the function $f(x)=\dfrac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$ is continuous at each point in its domain, then what is the value of $f(0)$?

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

(*b*) We have,

$$f(x)=\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$$

Function is continuous at each point

$\therefore \quad f(0)=\lim_{x\to 0} f(x)$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\dfrac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$

$$=\lim_{x\to 0}\frac{2-\dfrac{1}{\sqrt{1-x^2}}}{2+\dfrac{1}{1+x^2}} \quad \text{[by L'Hospital]}$$

$$=\frac{2-1}{2+1}=\frac{1}{3}$$

$\therefore \quad f(0)=\dfrac{1}{3}$

9. If $f(x)=\sqrt{25-x^2}$, then what is $\lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1}$ equal to?

(a) $-\frac{1}{\sqrt{24}}$ (b) $\frac{1}{\sqrt{24}}$
(c) $-\frac{1}{4\sqrt{3}}$ (d) $\frac{1}{\sqrt{4\sqrt{3}}}$

(*a*) We have,

$$f(x)=\sqrt{25-x^2}$$

Now, $\lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1}$

$$=\lim_{x\to 1}\frac{\sqrt{25-x^2}-\sqrt{24}}{x-1}$$

$$=\lim_{x\to 1}\frac{\dfrac{-2x}{2\sqrt{25-x^2}}-0}{1-0} \quad \text{[by L'Hosptial]}$$

$$=-\frac{1}{\sqrt{24}}$$

10. What is $\lim_{\theta\to 0}\dfrac{\sqrt{1-\cos\theta}}{\theta}$ equal to?

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

(*c*) We have,

$$\lim_{\theta\to 0}\frac{\sqrt{1-\cos\theta}}{\theta}=\lim_{\theta\to 0}\frac{\sqrt{1-\left(1-2\sin^2\frac{\theta}{2}\right)}}{\theta}$$

$$=\lim_{\theta\to 0}\frac{\sqrt{2\sin^2\frac{\theta}{2}}}{\theta}=\lim_{\theta\to 0}\frac{\sqrt{2}\sin\frac{\theta}{2}}{\frac{\theta}{2}\times 2}=\frac{1}{\sqrt{2}}$$

11. Let $f(x+y)=f(x)\,f(y)$ and $f(x)=1+xg(x)\,\phi(x)$, where $\lim_{x\to 0} g(x)=a$ and $\lim_{x\to 0}\phi(x)=b$. Then, what is $f'(x)$ equal to?

(a) $1+abf(x)$
(b) $1+ab$
(c) ab
(d) $abf(x)$

(*d*) Let, $f(x+y)=f(x)\,f(y)$

and $f(x)=1+x\,g(x)\cdot\phi(x)$

Thus, $f(y)=1+yg(y)\phi(y)$

Where $\lim_{x\to 0} g(x)=a$ and $\lim_{x\to 0}\phi(x)=b$

using first principal

$$f'(x)=\lim_{y\to 0}\frac{f(x+y)-f(x)}{y}$$

$$=\lim_{y\to 0}\frac{f(x)\cdot f(y)-f(x)}{y}$$

$$=\lim_{y\to 0}\frac{f(x)[f(y)-1]}{y}$$

$$=\lim_{y\to 0} f(x)\left[\frac{1+yg(y)\phi(y)-1}{y}\right]$$

$$=f(x)\lim_{y\to 0} g(y)\lim_{y\to 0}\phi(y)$$

$$=f(x)\cdot a\cdot b$$

$$=ab\,f(x)$$

12. What is $\lim_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$ equal to?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$
(c) –2 (d) –3

(*d*) We have,

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+2\sin x-\sin x-1}{2\sin^2 x-2\sin x-\sin x+1}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x(\sin x+1)-(\sin x+1)}{2\sin x(\sin x-1)-1(\sin x-1)}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{(\sin x+1)(2\sin x-1)}{(\sin x-1)(2\sin x-1)}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\sin x+1}{\sin x-1}$$

$$=\frac{\frac{1}{2}+1}{\frac{1}{2}-1}$$

$$=\frac{\frac{3}{2}}{-\frac{1}{2}}=-3$$

2018 (I)

13. Consider the function $f(x)=\begin{cases}x^2\ \text{In}|x| & x\neq 0\\ 0 & x=0\end{cases}$ What is $f'(0)$ equal to ?

(a) 0 (b) 1
(c) – 1 (d) It does not exist

(*a*) Given function is

$$f(x)=\begin{cases}x^2\ln|x|, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\therefore\quad f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{h^2\log h}{h}$$

$$=\lim_{h\to 0}h\log h=0$$

14. If $f(x)=\dfrac{x^2-9}{x^2-2x-3}$, $x\neq 3$ is continuous at $x=3$, then which one of the following is correct?

(a) $f(3)=0$ (b) $f(3)=1.5$
(c) $f(3)=3$ (d) $f(3)=-1.5$

(*b*) Since, $f(x)$ is continuous at $x=3$

Therefore, $f(3)=\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2-2x-3}$

On applying L′ Hospital rule

$$f(3)=\lim_{x\to 3}\frac{\frac{d}{dx}(x^2-9)}{\frac{d}{dx}(x^2-2x-3)}$$

$$=\lim_{x\to 3}\frac{2x}{2x-2}$$

$$=\frac{2\cdot 3}{2\cdot 3-2}=\frac{6}{4}=1.5$$

15. What is $\lim\limits_{x\to 0}\dfrac{\tan x}{\sin 2x}$ equal to

(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) Limit does not exist

(*a*) We have,

$$\lim_{x\to 0}\frac{\tan x}{\sin 2x}$$

By using L' Hospital rule, we have

$$=\lim_{x\to 0}\frac{\sec^2 x}{2\cos 2x}$$

$$=\lim_{x\to 0}\frac{1}{2\cos 2x\cdot\cos^2 x}$$

$$=\frac{1}{2\cos 0^\circ\cos^2 0^\circ}=\frac{1}{2\times 1\times 1}$$

$$=\frac{1}{2}$$

16. What is $\lim\limits_{h\to 0}\dfrac{\sqrt{2x+3h}-\sqrt{2x}}{2h}$ equal to ?

(a) $\frac{1}{2\sqrt{2x}}$ (b) $\frac{1}{\sqrt{2x}}$
(c) $\frac{3}{2\sqrt{2x}}$ (d) $\frac{3}{4\sqrt{2x}}$

(*d*) $\lim\limits_{h\to 0}\dfrac{\sqrt{2x+3h}-\sqrt{2x}}{2h}$

By using, L' Hospital rule, we get

$$=\lim_{h\to 0}\frac{\frac{1}{2\sqrt{2x+3h}}\cdot 3-0}{2}$$

$$=\lim_{h\to 0}\frac{3}{4\sqrt{2x+3h}}$$

$$=\frac{3}{4}\cdot\frac{1}{\sqrt{2x+0}}=\frac{3}{4\sqrt{2x}}$$

2017 (II)

17. A function is defined as follows

$$f(x)=\begin{cases}-\dfrac{x}{\sqrt{x^2}}, & x\neq 0\\ 0, & x=0\end{cases}$$

Which one of the following is correct in respect of the above function?

(a) $f(x)$ is continuous at $x=0$ but not differentiable at $x=0$
(b) $f(x)$ is continuous as well as differentiable at $x=0$
(c) $f(x)$ is discontinuous at $x=0$
(d) None of the above

(*c*) We have,

$$f(x)=\begin{cases}\dfrac{-x}{\sqrt{x^2}}, x\neq 0\\ 0, x=0\end{cases}=\begin{cases}\dfrac{-x}{|x|}, x\neq 0\\ 0, x=0\end{cases}$$

$$=\begin{cases}\dfrac{-x}{x}, x>0\\ 0, x=0\\ \dfrac{-x}{-x}, x<0\end{cases}=\begin{cases}-1, x>0\\ 0, x=0\\ 1, x<0\end{cases}$$

Now, LHL $(x=0)=1$
and RHL $(x=0)=-1$
$\therefore$ LHL $\neq$ RHL

Hence, $f(x)$ is discontinuous at $x=0$

18. Consider the following statements

1. $x+x^2$ is continuous at $x=0$
2. $x+\cos\dfrac{1}{x}$ is discontinuous at $x=0$
3. $x^2+\cos\dfrac{1}{x}$ is continuous at $x=0$

Which of the above are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(*c*) Since $x+x^2$ is a polynomial function, therefore it will be continuous everywhere.

Also, $\cos\dfrac{1}{x}$ is continuous at $x=0$ as $\cos\left(\dfrac{1}{0}\right)\in[-1,1]$ i.e., an oscillating value.

$\therefore$ Statements 1 and 3 are correct.

19. A function is defined in $(0,\infty)$ by

$$f(x)=\begin{cases}1-x^2 & \text{for}\quad 0<x\leq 1\\ \ln x & \text{for}\quad 1<x\leq 2\\ \ln 2-1+0.5x & \text{for}\quad 2<x<\infty\end{cases}$$

Which one of the following is correct in respect of the derivative of the function, i.e., $f'(x)$?

(a) $f'(x)=2x$ for $0<x\leq 1$
(b) $f'(x)=-2x$ for $0<x\leq 1$
(c) $f'(x)=-2x$ for $0<x<1$
(d) $f'(x)=0$ for $0<x<\infty$

(*b*) We have,

$$f(x)=\begin{cases}1-x^2 & 0<x\leq 1\\ \ln x & 1<x\leq 2\\ \ln 2-1+0.5x & 2<x<\infty\end{cases}$$

It is clear that $f'(x)=-2x, 0<x\leq 1$

20. If $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin x}{x}=l$ and $\lim\limits_{x\to\infty}\dfrac{\cos x}{x}=m$, then which one of the following is correct?

(a) $l=1$, $m=1$ (b) $l=\frac{2}{\pi}$, $m=\infty$
(c) $l=\frac{2}{\pi}$, $m=0$ (d) $l=1$, $m=\infty$

(*c*) We have,

$$l=\lim_{x\to\frac{\pi}{2}}\frac{\sin x}{x}=\frac{\sin\pi/2}{\pi/2}=\frac{2}{\pi}$$

Again, $m=\lim\limits_{x\to\infty}\dfrac{\cos x}{x}$

$=\dfrac{\cos\infty}{\infty}=0\times$ an oscillating value in interval [-1, 1] $=0$

$\therefore\quad l=\dfrac{2}{\pi}$ and $m=0$

21. The left-hand derivative of $f(x)=[x]\sin(\pi x)$ at $x=k$ where k is an integer and $[x]$ is the greatest integer function, is

(a) $(-1)^k(k-1)\pi$
(b) $(-1)^{k-1}(k-1)\pi$
(c) $(-1)^k k\pi$
(d) $(-1)^{k-1}k\pi$

(**a**) $\underset{(\text{at } x=k)}{\text{LHD}} = \lim_{h\to 0}\frac{f(k-h)-f(k)}{h}$

$= \lim_{h\to 0}\frac{[k-h]\sin\pi(k-h)-[k]\sin k\pi}{h}$

$= \lim_{h\to 0}\frac{(k-1)\sin\pi(k-h)-0}{h}$

[$\because \sin k\pi = 0$, when $k \in$ integer]

$= \lim_{h\to 0}\frac{(k-1)\sin(\pi k-\pi h)}{h}$

$= \lim_{h\to 0}\frac{(k-1)(-1)^k\sin\pi h}{h}$

$= \lim_{h\to 0}(k-1)(-1)^k\pi\left(\frac{\sin\pi h}{\pi h}\right) = (-1)^k(k-1)\pi$

22. The set of all points, where the function $f(x) = \sqrt{1-e^{-x^2}}$ is differentiable, is

(a) $(0, \infty)$ (b) $(-\infty, \infty)$

(c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-1, \infty)$

(**c**) We have, $f(x) = \sqrt{1-e^{-x^2}}$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{1-e^{-x^2}}}(-e^{-x^2})(-2x)$

$= \frac{xe^{-x^2}}{\sqrt{1-e^{-x^2}}}$

which is defined $\forall\, x \in R$ except $x = 0$

$\therefore f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$.

23. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then $f(x)$ is

(a) continuous but not differentiable at $x = 0$

(b) differentiable at $x = 0$

(c) not continuous at $x = 0$

(d) None of the above

(**b**) We have,

$f(x) = x(\sqrt{x} - \sqrt{x+1})$

$\therefore$ LHL (at $x = 0$)

$= \lim_{x\to 0^-} x(\sqrt{x} - \sqrt{x+1})$

$= \lim_{h\to 0}(0-h)(\sqrt{0-h} - \sqrt{0-h+1})$

$= \lim_{h\to 0} -h(\sqrt{-h} - \sqrt{1-h}) = 0$

RHL (at $x = 0$) $= \lim_{x\to 0^+} x(\sqrt{x} - \sqrt{x+1})$

$= \lim_{h\to 0}(0+h)(\sqrt{0+h} - \sqrt{0+h+1})$

$= \lim_{h\to 0} h(\sqrt{h} - \sqrt{1+h}) = 0$

$f(0) = 0(\sqrt{0} - \sqrt{0+1}) = 0$

$\because$ LHL = RHL = $f(0)$

$\therefore f(x)$ is continuous at $x = 0$

Now, LHD, (at $x = 0$) $= \lim_{h\to 0}\frac{f(h)-f(0)}{-h}$

$= \lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$

$= \lim_{h\to 0}\frac{-h(\sqrt{-h} - \sqrt{h+1})-0}{-h}$

$= \lim_{h\to 0}(\sqrt{h} - \sqrt{h+1}) = -1$

RHD (at $x = 0$) $= \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{f(\sqrt{h} - \sqrt{h+1})-0}{h}$

$= \lim_{h\to 0}(\sqrt{h} - \sqrt{h+1}) = -1$

$\because$ LHD = RHD

$\therefore f(x)$ is differentiable at $x = 0$.

24. Let g be the greatest integer function. Then, the function $f(x) = (g(x))^2 - g(x)$ is discontinuous at

(a) all integers

(b) all integers except 0 and 1

(c) all integers except 0

(d) all integers except 1

(**d**) We have,

$g(x) = [x]$ and $f(x) = (g(x))^2 - g(x)$

$\therefore f(x) = [x]^2 - [x]$

since $[x]$ is discontinuous at all integers but $f(1) = [1]^2 - [1] = 0$

$\therefore f(x)$ is discontinuous at all integers except 1.

2017 (I)

25. What is $\lim_{x\to 0}\frac{e^x - (1+x)}{x^2}$ equal to?

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

(**b**) $\lim_{x\to 0}\frac{e^x - (1+x)}{x^2}$

$= \lim_{x\to 0}\frac{\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right) - (1+x)}{x^2}$

$= \lim_{x\to 0}\left(\frac{1}{2!} + \frac{x}{3!} + \ldots\right)$

$= \frac{1}{2!} = \frac{1}{2}$

26. If $F(x) = \sqrt{9-x^2}$, then what is $\lim_{x\to 1}\frac{F(x) - F(1)}{x-1}$ equal to?

(a) $-\frac{1}{4\sqrt{2}}$ (b) $\frac{1}{8}$

(c) $-\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

(**c**) We have, $F(x) = \sqrt{9-x^2}$

$\Rightarrow F(1) = \sqrt{9-(1)^2}$

$= \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$

Now, $\lim_{x\to 1}\frac{F(x) - F(1)}{x-1}$

$= \lim_{x\to 1}\frac{\sqrt{9-x^2} - 2\sqrt{2}}{x-1}$

$= \lim_{x\to 1}\frac{\frac{1}{2}\cdot\frac{1}{\sqrt{9-x^2}}(-2x)}{1}$

(Using L'Hospital rule)

$= -\frac{1}{\sqrt{9-1}} = -\frac{1}{2\sqrt{2}}$

27. Let $f(x+y) = f(x)f(y)$ for all x and y. Then, what is $f'(5)$ equal to [where $f'(x)$ is the derivative of $f(x)$]?

(a) $f(5)f'(0)$ (b) $f(5) - f'(0)$

(c) $f(5)f(0)$ (d) $f(5) + f'(0)$

(**a**) $\because$ $f(x+y) = f(x)\cdot f(y)$ for all x and y

$\Rightarrow f(0+0) = f(0)f(0) \Rightarrow f(0) = [f(0)]^2$

$\Rightarrow f(0) - [f(0)]^2 = 0$

$\Rightarrow f(0)[1 - f(0)] = 0$

$\Rightarrow f(0) = 0, f(0) = 1$

Now, we know

$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0}\frac{f(x)\cdot f(h) - f(x)}{h}$

[$\because f(x+y) = f(x)f(y)$]

$= f(x)\lim_{h\to 0}\frac{f(h) - 1}{h}$

$= f(x)\lim_{h\to 0}\frac{f(h) - f(0)}{h} = f(x)\cdot f'(0)$

$\therefore f'(5) = f(5)f'(0)$

28. Let $f(x)$ be defined as follows

$$f(x) = \begin{cases} 2x+1, & -3 < x < -2 \\ x-1, & -2 \le x < 0 \\ x+2, & 0 \le x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

(a) It is discontinuous at $x = -2$ but continuous at every other point.

(b) It is continuous only in the interval $(-3, -2)$.

(c) It is discontinuous at $x = 0$ but continuous at every other point.

(d) It is discontinuous at every point.

(**c**) Given,

$$f(x) = \begin{cases} 2x+1, & -3 < x < -2 \\ x-1, & -2 \le x < 0 \\ x+2, & 0 \le x < 1 \end{cases}$$

Here, $f(-2^-) = 2(-2) + 1 = -3$

$f(-2^+) = -2 - 1 = -3$

Here, $f(-2^-) = f(-2^+)$

$\therefore f(x)$ is continuous at $x = -2$

and, $f(0^-) = (0-1) = -1$

$f(0^+) = 0 + 2 = 0$

Here, $f(0^-) \ne f(0^+)$

$\therefore f(x)$ is discontinuous at $x = 0$

$\therefore f(x)$ is discontinuous at $x = 0$ but continuous at every point.

29. Consider the following statements

1. If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist, then $\lim_{x\to a}\{f(x)g(x)\}$ exists.
2. If $\lim_{x\to a}\{f(x)g(x)\}$ exists, then both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist.

Which of the above statement is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) We know,
When $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist, then $\lim_{x\to a}\{f(x)\,g(x)\}$ must exist. But when $\lim_{x\to a}\{f(x)\,g(x)\}$ exists, then it is not necessary that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist.

2016 (II)

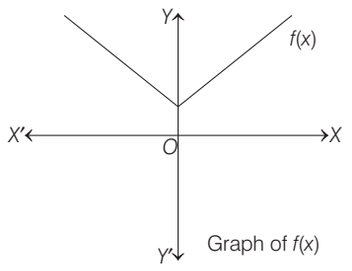
30. Consider the following in respect of the function
$f(x)=\begin{cases}2+x, x\ge 0\\ 2-x, x<0.\end{cases}$

I. $\lim_{x\to 1} f(x)$ does not exist.
II. $f(x)$ is differentiable at $x=0$.
III. $f(x)$ is continuous at $x=0$.

Which of the above statements is/are correct?

(a) I only
(b) III only
(c) II and III
(d) I and III

(**b**) Given, $f(x)=\begin{cases}2+x, & x\ge 0\\ 2-x, & x<0\end{cases}$.



Graph of $f(x)$

At $x=1$, LHL
$=\lim_{x\to 1^-}(2+x)=\lim_{h\to 0}(2+(1-h)=3$
RHL $=\lim_{x\to 1^+}(2+x)=\lim_{h\to 0}(2+(1+h))$
$=3$
Here, LHL = RHL
$\therefore \lim_{x\to 1} f(x)$ exists.
At $x=0$, graph of $f(x)$ makes sharp edge.
$\therefore f(x)$ is not differentiable at $x=0$.
At $x=0$,
LHL $=\lim_{x\to 0^-}(2-x)$
$=\lim_{h\to 0}[2-(0-h)]=2$
RHL $=\lim_{x\to 0^+}(2+x)$
$=\lim_{h\to 0}(2+(0+h)=2$
Value of $f(x)$ at $x=0$
i.e. $f(0)=2+0=2$
Here, LHL = RHL = $f(x)$
$\therefore f(x)$ is continuous at $x=0$.

31. Let $f: A\to R$, where $A=R\setminus\{0\}$ is such that $f(x)=\dfrac{x+|x|}{x}$. On which one of the following sets is $f(x)$ continuous?

(a) A
(b) $B=\{x\in R: x\ge 0\}$
(c) $C=\{x\in R: x\le 0\}$
(d) $D=R$

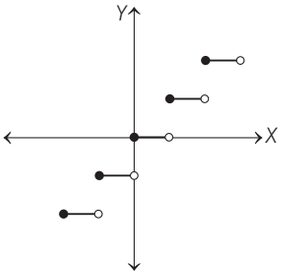
(**a**) We have,
$f: A\to R$,
where $A=R\setminus\{0\}$
such that, $f(x)=\dfrac{x+|x|}{x}$
$\Rightarrow f(x)=\begin{cases}2, & x>0\\ 0, & x<0\end{cases}$
Since, $f(x)$ is not defined for $x=0$.
$\therefore f(x)$ is discontinuous at $x=0$.
Also, $f(x)$ is constant function on $R-\{0\}$.
Hence, $f(x)$ is continuous on A, where $A=R-\{0\}$.

Directions (Q.Nos. 32-34) *Let $f(x)=[x]$, where $[.]$ is the greatest integer function and $g(x)=\sin x$ be two real valued functions over R.*

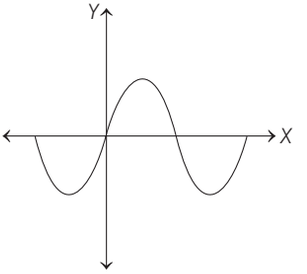
32. Which of the following statements is correct?

(a) Both $f(x)$ and $g(x)$ are continuous at $x=0$
(b) $f(x)$ is continuous at $x=0$ but $g(x)$ is not continuous at $x=0$
(c) $g(x)$ is continuous at $x=0$ but $f(x)$ is not continuous at $x=0$
(d) Both $f(x)$ and $g(x)$ are discontinuous at $x=0$

(**c**) Given, $f(x)=[x]$ and $g(x)=\sin x$



Graph of $f(x)=[x]$



Graph of $g(x)=\sin x$

$\because$ Graph of $f(x)$ breaks at $x=0$ and graph of $g(x)$ does not break at $x=0$.
$\therefore$ $g(x)$ is continuous at $x=0$ but $f(x)$ is not continuous at $x=0$.

33. Which one of the following statements is correct?

(a) $\lim_{x\to 0}(fog)(x)$ exists
(b) $\lim_{x\to 0}(gof)(x)$ exists
(c) $\lim_{x\to 0^-}(fog)(x)=\lim_{x\to 0^-}(gof)(x)$
(d) $\lim_{x\to 0^+}(fog)(x)=\lim_{x\to 0^+}(gof)(x)$

(**d**) $(fog)(x)=f(g(x))=f(\sin x)=[\sin x]$
LHL $=\lim_{x\to 0^-}[\sin x]=\lim_{h\to 0}[\sin(0-h)]$
$=-1$
RHL $=\lim_{x\to 0^+}[\sin x]=\lim_{h\to 0}[\sin(0+h)]$
$=0$
$\therefore \lim_{x\to 0}(fog)(x)$ does not exist.
and $\lim_{x\to 0}(gof)(x)$
$=\lim_{x\to 0} g(f(x))=\lim_{x\to 0} g([x])=\lim_{x\to 0}\sin[x]$
LHL $=\lim_{x\to 0^-}\sin[x]=\lim_{h\to 0}\sin[0-h]$
$=\lim_{h\to 0}\sin[-h]=-\sin 1$
RHL $=\lim_{x\to 0^+}\sin[x]=\lim_{h\to 0}\sin[0+h]$
$=\sin(0)=0$
$\therefore \lim_{x\to 0}(gof)(x)$ does not exist.
Clearly, we have
$\lim_{x\to 0^+}[\sin x]=\lim_{x\to 0^+}\sin[x]=0$
$\therefore \lim_{x\to 0^+}(fog)(x)=\lim_{x\to 0^+}(gof)(x)$

34. Which of the following statements are correct?

I. $(fof)(x) = f(x)$.

II. $(gog)(x) = g(x)$ only when $x = 0$.

III. $(go(fog))(x)$ can take only three values.

Select the correct answer using the code given below.

(a) I and II (b) II and III

(c) I and III (d) I, II and III

(d) I. $(fof)(x) = f(f(x)) = f([x]) = ([x]) = [x]$ $= f(x)$, which is correct

II. $(gog)(x) = g(g(x)) = g(\sin x)$ $= \sin(\sin x)$

when $x = 0$, $g(x) = \sin x = 0$ and $(gog)(x) = \sin(\sin x) = 0$,

which is correct,

III. $(go(fog))(x) = g([\sin x]) = \sin([\sin x])$,

which can take only three values .

Directions (Q. Nos. 35-37) Let

$$f(x) = \begin{cases} -2, & -3 \le x \le 0 \\ x - 2, & 0 < x \le 3 \end{cases} \text{ and}$$

$$g(x) = f(|x|) + |f(x)|.$$

35. Which of the following statements is/are correct?

I. $g(x)$ is differentiable at $x = 0$.

II. $g(x)$ is differentiable at $x = 2$.

Select the correct answer using the code given below.

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(d) We have,

$$f(x) = \begin{cases} -2, & -3 \le x \le 0 \\ x - 2, & 0 < x \le 3 \end{cases}$$

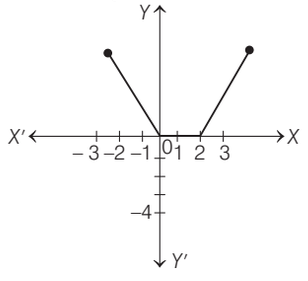
$$\Rightarrow f(|x|) = \begin{cases} -2, & -3 \le |x| \le 0 \\ |x| - 2, & 0 < |x| \le 3 \end{cases}$$

At $-3 \le |x| \le 0$, $f(|x|) = -2$ not possible.

$$\Rightarrow f(|x|) = \begin{cases} -x - 2, & -3 \le x \le 0 \\ x - 2, & 0 < x \le 3 \end{cases} \quad \ldots(i)$$

$$\text{and } |f(x)| = \begin{cases} |-2|, & -3 \le x \le 0 \\ |x - 2|, & 0 < x \le 3 \end{cases}$$

$$\Rightarrow |f(x)| = \begin{cases} 2 & -3 \le x \le 0 \\ (2 - x), & 0 < x < 2 \\ (x - 2), & 2 \le x \le 3 \end{cases} \quad \ldots(ii)$$



From Eqs. (i) and (ii), we get

$$g(x) = f(|x|) + |f(x)|$$

$$g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ 0, & 0 < x < 2 \\ 2x - 4 & 2 \le x \le 3 \end{cases}$$

$$g'(x) = \begin{cases} -1, & -3 \le x \le 0 \\ 0, & 0 < x < 2 \\ 2, & 2 \le x \le 3 \end{cases}$$

The graph of $g(x)$ is given.

Since, $g(x)$ has sharp edges at $x = 0$ and $x = 2$.

Hence, $g(x)$ is not differentiable at $x = 0$ and $x = 2$.

36. What is the value of the differential coefficient of $g(x)$ at $x = -2$?

(a) –1 (b) 0

(c) 1 (d) 2

(a) at $x = -2$

$g'(x) = -1$

37. Which of the following statements are correct?

I. $g(x)$ is continuous at $x = 0$.

II. $g(x)$ is continuous at $x = 2$.

III. $g(x)$ is continuous at $x = -1$.

Select the correct answer using the code given below.

(a) I and II (b) II and III

(c) I and III (d) I, II and III

(d) From the graph of $g(x)$, we can say that $g(x)$ is continuous at $x = 0$, $x = 2$ and $x = -1$.

38. Let $f(x) = \begin{cases} \dfrac{e^x - 1}{x}, & x > 0 \\ 0, & x = 0 \end{cases}$,

be a real valued function, then which of the following statements is/are correct?

I. $f(x)$ is right continuous at $x = 0$.

II. $f(x)$ is discontinuous at $x = 1$.

Select the correct answer using the code given below.

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(d) I. For right continuity at $x = 0$,

$\lim\limits_{x \to 0^+} f(x) = f(0)$

Now, $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h)$

$= \lim\limits_{h \to 0} f(h) = \lim\limits_{h \to 0} \dfrac{e^h - 1}{h} = 1$

and $f(0) = 0$

so, $\lim\limits_{x \to 0^+} f(x) \ne f(0)$

$\Rightarrow f(x)$ is not right continuous at $x = 0$.

II. At $x = 1$,

$\lim\limits_{x \to 1^-} = \lim\limits_{h \to 0} \dfrac{e^{(1-h)} - 1}{(1 - h)} = \dfrac{e - 1}{1} = e - 1$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1 + h)$

$= \lim\limits_{h \to 0} \dfrac{e^{1+h} - 1}{1 + h} = \dfrac{e - 1}{1} = e - 1$

and $f(1) = \dfrac{e^1 - 1}{1} = e - 1$

$\therefore \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1)$

$\Rightarrow f(x)$ is continuous at $x = 1$.

2016 (I)

Directions (Q. Nos. 39 and 40)

Consider the equation $x + |y| = 2y$.

39. Which of the following statements are not correct?

1. y as a function of x is not defined for all real x.
2. y as a function of x is not continuous at $x = 0$.
3. y as a function of x is differentiable for all x.

Select the correct answer using the codes given below.

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2 and 3

(c) Given, $x + |y| = 2y$

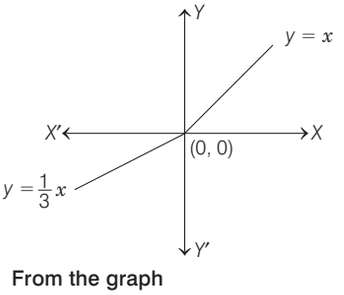
$x + y = 2y$, for $y > 0$

$\Rightarrow y = x$

and $x - y = 2y$, for $y < 0$

$\Rightarrow y = \dfrac{1}{3}x$

Graph of $x + |y| = 2y$



From the graph

y as a function x defined for all real x and y as function of x is not differentiable for all $x = 0$

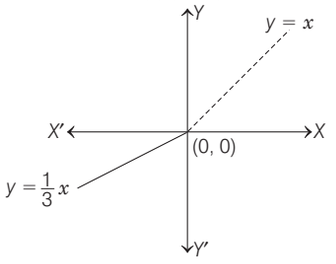
$\therefore$ Statements 1 and 3 are incorrect.

40. What is the derivative of y as a function of x with respect to x for $x < 0$?

(a) 2 (b) 1

(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{3}$

(**d**) Given, $x + |y| = 2y$



When $x < 0, y < 0$

$\therefore \quad y = \frac{1}{3}x$

$\therefore$ Derivative of y as a function of x w.r. t. x for $x < 0$ is $\frac{1}{3}$. $\left[\because \frac{dy}{dx} = \frac{1}{3}(1) = \frac{1}{3}\right]$

Directions (Q. Nos. 41 and 42)

Consider the function

$$f(x) = \frac{a^{[x]+x} - 1}{[x] + x} \text{ where}$$

$[\cdot]$ denotes the greatest integer function.

41. What is $\lim_{x \to 0^+} f(x)$ equal to

(a) 1 (b) $\ln a$
(c) $1 - a^{-1}$ (d) Limit does not exist

(**b**) Given, $f(x) = \frac{a^{[x]+x} - 1}{[x] + x}$

Now, RHL $= \lim_{x \to 0^+} \frac{a^{[x]+x} - 1}{[x] + x}$

As, $x \to 0^+$ (i.e. approaches 0 from the right), we get $[x] = 0$

$$\therefore \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{a^x - 1}{x}$$

$$= \lim_{h \to 0} \frac{a^{(0+h)} - 1}{(0 + h)}$$

$$= \lim_{h \to 0} \frac{a^h - 1}{h} = \ln a$$

42. What is $\lim_{x \to 0^-} f(x)$ equal to?

(a) 0
(b) $\ln a$
(c) $1 - a^{-1}$
(d) Limit does not exist

(**c**) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{a^{[x]+x} - 1}{[x] + x}$

As $x \to 0^-$ (i.e. approaches 0 from the left), we get $[x] = -1$

$$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{a^{-1+x} - 1}{-1 + x}$$

$$= \lim_{h \to 0} \frac{a^{-1+(0-h)} - 1}{-1 + (0 - h)}$$

$$= \frac{a^{-1} - 1}{-1} = 1 - a^{-1}$$

Directions (Q. Nos. 43 and 44)

A function $f(x)$ is defined as follows

$$f(x) = \begin{cases} x + \pi, & \text{for } x \in [-\pi, 0) \\ \pi \cos x, & \text{for } x \in \left[0, \frac{\pi}{2}\right] \\ \left(x - \frac{\pi}{2}\right)^2, & \text{for } x \in \left(\frac{\pi}{2}, \pi\right] \end{cases}$$

43. Consider the following statements

1. The function $f(x)$ is continuous at $x = 0$.
2. The function $f(x)$ is continuous at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) LHL $= \lim_{x \to 0^-} f(x) = \lim_{x \to 0} (x + \pi) = \pi$

RHL $= \lim_{x \to 0^+} f(x) = \lim_{x \to 0} \pi \cos x$
$= \pi \cos(0) = \pi$

Also, $f(0) = \pi \cos(0) = \pi$

Hence, $f(x)$ is continuous at $x = 0$.

Statement 1 is correct.

Now, for $x = \frac{\pi}{2}$

LHL $= \lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2} \pi \cos x$

$= \pi \cos \frac{\pi}{2} = 0$

RHL $= \lim_{x \to \pi/2^+} f(x) = \lim_{x \to \frac{\pi}{2}} \left(x - \frac{\pi}{2}\right)^2$

$= \left(\frac{\pi}{2} - \frac{\pi}{2}\right)^2 = 0$

Also, $f\left(\frac{\pi}{2}\right) = \pi \cos \frac{\pi}{2} = 0$

Hence, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

Statement 2 is correct.

44. Consider the following statements

1. The function $f(x)$ is differentiable at $x = 0$.
2. The function $f(x)$ is differentiable at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**d**) For $x = 0$,

$$\text{LHD} = \lim_{h \to 0} \frac{f(0 - h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{f(-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{(-h + \pi) - \pi}{-h} = \lim_{h \to 0} \frac{-h}{-h} = 1$$

$$\text{RHD} = \lim_{h \to 0^+} \frac{f(0 + h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\pi \cos h - \pi}{2}$$

$$= \lim_{h \to 0} \frac{\pi [\cos h - 1]}{2}$$

$$= \lim_{h \to 0} \frac{\pi [-2 \sin^2(h/2)]}{h}$$

$$= \lim_{h \to 0} \pi \left[-\sin\frac{(h/2)}{(h/2)} \times \sin(h/2)\right]$$

$$= \pi[-1 \times 0] = 0$$

LHD $\neq$ RHD

Hence, $f(x)$ is not differentiable at $x = 0$.

$\therefore$ Statement 1 is not correct.

For $x = \frac{\pi}{2}$,

$$\text{LHD} = \lim_{h \to 0} \frac{f(\pi/2 - h) - f(\pi/2)}{-h}$$

$$= \lim_{h \to 0} \frac{\pi \cos\left(\frac{\pi}{2} - h\right) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{\pi \sin h}{-h} = -\pi$$

$$\text{RHD} = \lim_{h \to 0} + \frac{f(\pi/2 + h) - f(\pi/2)}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{\pi}{2} + h - \frac{\pi}{2}\right)^2 - 0}{h}$$

$$= \lim_{h \to 0} \frac{h^2}{h} = 0$$

LHD $\neq$ RHD

Hence, $f(x)$ is not differentiable at $x = \frac{\pi}{2}$.

Statement 2 is not correct.

45. If $\lim_{x \to 0} \phi(x) = a^2$, where $a \neq 0$, then what is $\lim_{x \to 0} \phi\left(\frac{x}{a}\right)$ equal to?

(a) a^2 (b) a^{-2}
(c) $-a^2$ (d) $-a$

(**a**) Given, $\lim_{x \to 0} \phi(x) = a^2$

To find $\lim_{x \to 0} \phi\left(\frac{x}{a}\right)$

$\therefore \quad \lim_{x \to 0} \phi\left(\frac{x}{a}\right) = a^2$

46. What is $\lim_{x \to 0} e^{-\frac{1}{x^2}}$ equal to?

(a) 0 (b) 1
(c) –1 (d) Limit does not exist

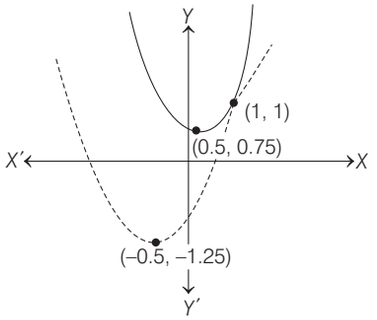
(a) $\lim_{x \to 0} e^{-1/x^2} = \lim_{h \to 0} e^{-1/(0+h)^2} = 0$

47. Consider the function $f(x) = |x - 1| + x^2$ where $x \in R$, then which one of the following statements is correct?

(a) $f(x)$ is continuous but not differentiable at $x = 0$
(b) $f(x)$ is continuous but not differentiable at $x = 1$
(c) $f(x)$ is differentiable at $x = 1$
(d) $f(x)$ is not differentiable at $x = 0$ and $x = 1$

(b) Given, $f(x) = |x - 1| + x^2$

$\therefore \quad f(x) = \begin{cases} x^2 + x - 1, & \text{for } x \geq 1 \\ x^2 - x + 1, & \text{for } x < 1 \end{cases}$



$\because$ There is a sharp edge at point (1, 1) in the graph of $y = |x - 1| + x^2$
$\therefore$ $f(x)$ is continuous but not differentiable at $x = 1$.

2015 (II)

48. If $f(x) = \sqrt{25 - x^2}$, then what is $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$ equal to?

(a) $\frac{1}{5}$ (b) $\frac{1}{\sqrt{24}}$ (c) $\sqrt{24}$ (d) $-\frac{1}{\sqrt{24}}$

(d) Consider, $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} \quad \left[\frac{0}{0} \text{form}\right]$

$= \lim_{x \to 1} \frac{f'(x)}{1} = \lim_{x \to 1} \frac{1(-2x)}{2\sqrt{25 - x^2}}$

$= -\frac{1}{\sqrt{24}}$ [by L'Hospital's rule]

49. Consider the function

$f(x) = \begin{cases} ax - 2, & \text{for } -2 < x < -1 \\ -1, & \text{for } -1 \leq x \leq 1 \\ a + 2(x-1)^2, & \text{for } 1 < x < 2 \end{cases}$

What is the value of a for which $f(x)$ is continuous at $x = -1$ and $x = 1$?

(a) – 1 (b) 1 (c) 0 (d) 2

(a) At $x = -1$,

LHL = $\lim_{h \to 0} f(-1 - h) = \lim_{h \to 0} a(-1 - h) - 2 = -a - 2$

RHL = $\lim_{h \to 0} f(-1 + h) = \lim_{h \to 0} (-1) = -1$

At $x = 1$, LHL = $\lim_{h \to 0} f(1 - h)$

$= \lim_{h \to 0} (-1) = -1$

RHL = $\lim_{h \to 0} f(1 + h) = \lim_{h \to 0} (a + 2h^2) = a$

Now, for f to be continuous at $x = -1$, we should have LHL = RHL = $f(-1)$

$\Rightarrow \quad -a - 2 = -1$

$\Rightarrow \quad a = -1$

Also for $a = -1$, f is continuous at $x = 1$.
Hence, $a = -1$

50. The function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x = \pi$, is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) –1 (d) 1

(c) Clearly, for f to be continuous at $x = \pi$,

$f(\pi) = \lim_{x \to \pi} \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x} \quad \left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to \pi} \frac{-\cos x - \sin x}{\cos x - \sin x}$ [by L'Hospital's rule]

$= \frac{-\cos \pi - \sin \pi}{\cos \pi - \sin \pi}$

$= \frac{-(-1) - 0}{-1 - 0} = \frac{1}{-1} = -1$

51. Consider the following functions

1. $f(x) = \begin{cases} \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

2. $f(x) = \begin{cases} 2x + 5, & \text{if } x > 0 \\ x^2 + 2x + 5, & \text{if } x \leq 0 \end{cases}$

Which of the above function(s) is/are derivable at $x = 0$?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) 1. Consider, $\lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$

$= \lim_{h \to 0} \frac{1/h - 0}{h}$

$= \lim_{h \to 0} \frac{1}{h^2} = \infty$ (not defined)

Hence, f is not differentiable at $x = 0$.

2. We have, $f(x) = \begin{cases} 2x + 5, & x > 0 \\ x^2 + 2x + 5, & x \leq 0 \end{cases}$

$\Rightarrow \quad f'(x) = \begin{cases} 2, & x > 0 \\ 2x + 2, & x \leq 0 \end{cases}$

$\because$ LHL of $f'(x)$ = RHL of $f'(x)$ at $x = 0$
$\therefore$ f is differentiable at $x = 0$.

52. If $f(x) = \frac{\sin(e^{x-2} - 1)}{\ln(x - 1)}$, then $\lim_{x \to 2} f(x)$ is equal to

(a) – 2 (b) – 1 (c) 0 (d) 1

(d) Consider, $\lim_{x \to 2} \frac{\sin(e^{x-2} - 1)}{\ln(x - 1)} \left[\frac{0}{0} \text{form}\right]$

$= \lim_{x \to 2} \frac{\cos(e^{x-2} - 1) \cdot e^{x-2}}{\frac{1}{x - 1}} = \frac{1 \times 1}{1} = 1$

[by L'Hospital's rule]

Directions (Q. Nos. 53 and 54)

Consider the function

$f(x) = \begin{cases} -2\sin x, & \text{if } x \leq -\frac{\pi}{2} \\ A \sin x + B, & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{if } x \geq \frac{\pi}{2} \end{cases}$

which is continuous everywhere.

53. The value of A is

(a) 1 (b) 0
(c) – 1 (d) – 2

54. The value of B is

(a) 1 (b) 0
(c) – 1 (d) – 2

Solutions (Q.Nos. 53 and 54) Given,

$f(x) = \begin{cases} -2\sin x, & \text{if } x \leq -\frac{\pi}{2} \\ A \sin x + B, & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{if } x \geq \frac{\pi}{2} \end{cases}$

Case I For continuity at $x = \frac{\pi}{2}$,

$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{h \to 0} A \sin\left(\frac{\pi}{2} - h\right) + B = \lim_{h \to 0} \cos\left(\frac{\pi}{2} + h\right) = \cos\left(\frac{\pi}{2}\right)$

$\Rightarrow \quad A \sin\frac{\pi}{2} + B = \cos\frac{\pi}{2} = 0$

$\Rightarrow \quad A + B = 0 \quad \ldots(i)$

Case II For continuity at $x = -\frac{\pi}{2}$,

$\lim_{x \to -\frac{\pi}{2}^-} f(x) = \lim_{x \to -\frac{\pi}{2}^+} f(x) = f\left(-\frac{\pi}{2}\right)$

$\Rightarrow \lim_{h\to 0} f\left(-\frac{\pi}{2}-h\right) = \lim_{h\to 0}\left(-\frac{\pi}{2}+h\right)$

$= f\left(-\frac{\pi}{2}\right)$

$\Rightarrow \lim_{h\to 0}\left[-2\sin\left(-\frac{\pi}{2}-h\right)\right]$

$= \lim_{h\to 0}\left[A\sin\left(-\frac{\pi}{2}+h\right)+B\right]$

$= -2\sin\left(-\frac{\pi}{2}\right)$

$\Rightarrow 2 = -A + B = 2$

$\Rightarrow A - B = -2$ ...(ii)

On solving Eqs. (i) and (ii), we get

$A = -1$ and $B = 1$

53. (**c**) The value of A is –1.

54. (**a**) The value of B is 1.

2015 (I)

55. If $G(x) = \sqrt{25 - x^2}$, then what is $\lim_{x\to 1}\frac{G(x) - G(1)}{x-1}$ equal to?

(a) $-\frac{1}{2\sqrt{6}}$ (b) $\frac{1}{5}$

(c) $-\frac{1}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{6}}$

(**a**) We have, $G(x) = \sqrt{25 - x^2}$

Clearly, $\lim_{x\to 1}\frac{G(x) - G(1)}{x-1} = G'(1)$ ...(i)

Now, let us first find $G'(x)$, which is given by

$G'(x) = \frac{1}{2}\times\frac{1}{\sqrt{25-x^2}}(-2x)$

$\Rightarrow G'(x) = \frac{-x}{\sqrt{25-x^2}}$

$\Rightarrow G'(1) = \frac{-1}{\sqrt{24}} = \frac{-1}{2\sqrt{6}}$

56. Consider the following statements

I. $f(x) = [x]$, where [ ] is the greatest integer function, is discontinuous at $x = n$, where $n \in Z$.

II. $f(x) = \cot x$ is discontinuous at $x = n\pi$, where $n \in Z$.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(**c**) I. We have, $f(x) = [x]$

Clearly, $\lim_{x\to n^+} f(x) = \lim_{h\to 0} f(n+h)$

$= \lim_{h\to 0}[n+h] = n$

and $\lim_{x\to n^-} f(x) = \lim_{h\to 0} f(n-h)$

$= \lim_{h\to 0}[n-h] = n-1$

$\because \lim_{x\to n^+} f(x) \neq \lim_{x\to n^-} f(x)$ for all $n \in Z$

So, f is discontinuous at $x = n \in Z$.

II. We have, $f(x) = \cot x$

Consider, $\lim_{x\to n\pi^+} f(x) = \lim_{h\to 0} f(n\pi + h)$

$= \lim_{h\to 0}\cot(n\pi + h)$

$= \lim_{h\to 0}\cot h$

which does not exist.

Thus, f is discontinuous at $x = n\pi$, $n \in Z$.

Directions (Q. Nos. 57 and 58) *Given, a function*

$$f(x) = \begin{cases} -1, & \text{if } x \le 0 \\ ax + b, & \text{if } 0 < x < 1, \text{where } a, \\ 1, & \text{if } x \ge 1 \end{cases}$$

b are constants. The function is continuous everywhere.

57. What is the value of b?

(a) –1 (b) 1 (c) 0 (d) 2

(**a**) We have,

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$

Now consider, $\lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{h\to 0} f(0+h) = -1$ [$\because f(0) = -1$]

$\Rightarrow \lim_{h\to 0} a(0+h) + b = -1$

$\Rightarrow b = -1$

58. What is the value of a?

(a) –1 (b) 0 (c) 1 (d) 2

(**d**) We have, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1)$

Now consider, $\lim_{x\to 1^-} f(x) = f(1)$

$\Rightarrow \lim_{h\to 0} f(1-h) = 1$ [$\because f(1) = 1$]

$\Rightarrow \lim_{h\to 0} a(1-h) + b = 1$

$\Rightarrow a + b = 1$

$\therefore a = 2$ [$\because b = -1$]

Directions (Q. Nos. 59 and 60)

Consider the function

$$f(x) = \begin{cases} \frac{\alpha\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases} \text{ which is}$$

continuous at $x = \frac{\pi}{2}$, *where* α *is a constant.*

59. What is the value of α?

(a) 6 (b) 3

(c) 2 (d) 1

(**a**) We have, $f(x) = \begin{cases} \frac{\alpha\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$

which is continuous at $x = \frac{\pi}{2}$, where α is a constant.

$\Rightarrow \lim_{x\to \pi/2} f(x) = f\left(\frac{\pi}{2}\right) = 3$ ...(i)

Now, consider

$\lim_{x\to \pi/2} f(x) = \lim_{x\to \pi/2}\frac{\alpha\cos x}{\pi - 2x}$

$= \alpha\lim_{x\to \pi/2}\frac{\cos x}{\pi - 2x}$

Now, put $y = x - \frac{\pi}{2}$, then as

$x \to \frac{\pi}{2} \Rightarrow y \to 0$

$\therefore \lim_{x\to \pi/2} f(x) = \alpha\lim_{y\to 0}\frac{\cos\left(y + \frac{\pi}{2}\right)}{\pi - 2\left(\frac{\pi}{2} + y\right)}$

$= \alpha\lim_{y\to 0}\frac{-\sin y}{-2y}$

$= \frac{\alpha}{2}\lim_{y\to 0}\frac{\sin y}{y} = \frac{\alpha}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{\alpha}{2} = 3 \Rightarrow \alpha = 6$

60. What is $\lim_{x\to 0} f(x)$ equal to?

(a) 0 (b) 3 (c) $\frac{3}{\pi}$ (d) $\frac{6}{\pi}$

(**d**) $\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{6\cos x}{\pi - 2x}$ ($\because \alpha = 6$)

$= 6\lim_{x\to 0}\frac{\cos x}{\pi - 2x}$

$= 6\left(\frac{\cos 0°}{\pi - 2(0)}\right) = \frac{6}{\pi}$ [$\because \cos 0° = 1$]

Directions (Q. Nos. 61 and 62) *Given that* $\lim_{x\to\infty}\left(\frac{2+x^2}{1+x} - Ax - B\right) = 3$.

61. What is the value of A?

(a) –1 (b) 1 (c) 2 (d) 3

(**b**) Given, $\lim_{x\to\infty}\left(\frac{2+x^2}{1+x} - Ax - B\right) = 3$

$\Rightarrow \lim_{x\to\infty}\left[\frac{2 + x^2 - (Ax + B)(1+x)}{1+x}\right] = 3$

$\Rightarrow \lim_{x\to\infty}\left[\frac{2 + x^2 - (Ax + Ax^2 + B + Bx)}{1+x}\right] = 3$

$\Rightarrow \lim_{x\to\infty}\left(\frac{2 + x^2 - Ax - Ax^2 - B - Bx}{1+x}\right) = 3$

$\Rightarrow \lim_{x\to\infty}\left(\frac{2x - A - 2Ax - B}{1}\right) = 3$

[by L' Hospital's rule]

$\Rightarrow \lim_{x\to\infty}[x(2 - 2A) - (A + B)] = 3$

For limit to be exist and equal to 3, we should have

$2 - 2A = 0$ and $A + B = -3$

$\Rightarrow A = 1$

62. What is the value of B?

(a) –1 (b) 3 (c) –4 (d) –3

(*c*) From above, we have $A + B = -3$

$\Rightarrow \quad 1 + B = -3 \Rightarrow B = -4$

2014 (II)

Directions (Q. Nos. 63-65) *Read the following information carefully and answer the questions given below.*

Consider the function

$$f(x) = \begin{cases} x^2 - 5, & x \le 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$$

63. What is $\lim_{x \to 3} f(x)$ equal to?

(a) 2 (b) 4 (c) 5 (d) 13

(*b*) We have, $f(x) = \begin{cases} x^2 - 5, & x \le 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$

To find $\lim_{x \to 3} f(x)$, consider LHL

$= \lim_{x \to 3^-} f(x) = \lim_{h \to 0} f(3-h)$

$= \lim_{h \to 0} [(3-h)^2 - 5]$

$= \lim_{h \to 0} (9 - 6h + h^2 - 5) = 4$

and RHL $= \lim_{x \to 3^+} f(x) = \lim_{x \to 0} f(3+h)$

$= \lim_{h \to 0} (\sqrt{3 + h + 13})$

$= \lim_{h \to 0} (\sqrt{16 + h}) = 4$

$\because \quad \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 4$

$\therefore \quad \lim_{x \to 3} f(x) = 4$

64. Consider the following statements

1. The function is discontinuous at $x = 3$.
2. The function is not differentiable at $x = 0$.

Which of the above statement(s) is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(*d*) 1. For continuous, we should have

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$\because \quad \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 4$

$\therefore$ $f(x)$ is continuous at $x = 3$.

2. We have, $f(x) = x^2 - 5$, if $x \le 3$

$\Rightarrow \quad f'(x) = 2x$, if $x < 3$

$\Rightarrow \quad f'(0) = 0$

Hence, $f(x)$ is differentiable at $x = 0$.

So, neither Statement 1 nor 2 is correct.

65. What is the differential coefficient of $f(x)$ at $x = 12$?

(a) 5/2 (b) 5
(c) 1/5 (d) 1/10

(*d*) We have, $f(x) = \sqrt{x + 13}$, for $x > 3$

$\Rightarrow \quad f'(x) = \dfrac{1}{2\sqrt{x+13}}$

$\therefore \quad f'(12) = \dfrac{1}{2\sqrt{12+13}} = \dfrac{1}{2 \times 5} = \dfrac{1}{10}$

66. Consider the function

$$f(x) = \begin{cases} \dfrac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2, & x \ge 0 \end{cases}$$

What is the non-zero value of k for which the function is continuous at $x = 0$?

(a) 1/4 (b) 1/2
(c) 1 (d) 2

(*b*) We have, $f(x) = \begin{cases} \dfrac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2, & x \ge 0 \end{cases}$

Since, $f(x)$ is continuous at $x = 0$.

$\therefore \quad \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

$\Rightarrow \quad \lim_{x \to 0^-} \left(\dfrac{\tan kx}{x}\right) = \lim_{x \to 0^+} (3x + 2k^2)$

$= 3(0) + 2k^2$

$\Rightarrow \quad \lim_{h \to 0} \left[\dfrac{\tan k(0-h)}{(0-h)}\right]$

$= \lim_{h \to 0} [3(0 + h) + 2k^2] = 2k^2$

$\Rightarrow \quad \lim_{h \to 0} \left(\dfrac{\tan kh}{h}\right) = 2k^2$

$\Rightarrow \quad \lim_{h \to 0} \left(\dfrac{k \tan kh}{kh}\right) = 2k^2$

$\Rightarrow \quad k = 2k^2 \qquad \left[\because \lim_{x \to 0} \dfrac{\tan x}{x} = 1\right]$

$\Rightarrow \quad k = \dfrac{1}{2} \qquad [\because k \ne 0]$

67. Consider the following statements

1. The function $f(x) = [x]$, where [] is the greatest integer function defined on R, is continuous at all points except at $x = 0$.
2. The function $f(x) = \sin|x|$ is continuous for all $x \in R$.

Which of the above statement(s) is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(*b*) 1. We know that, the greatest integer function is continuous at all points except integer. Therefore, Statement 1 is false.

2. Let $h(x) = \sin x$ and $g(x) = |x|$

$\Rightarrow \quad hog(x) = \sin|x|$

Now, let $f(x) = hog(x) = \sin|x|$

Since, $g(x)$ is continuous, $\forall x \in R$ and $h(x)$ is continuous, $\forall x \in R$.

$\therefore$ $hog(x)$ is continuous.

[$\because$ composition of two continuous functions is continuous]

Hence, $f(x)$ is continuous, $\forall x \in R$.

68. What is $\lim_{x \to 0} \dfrac{\log_5(1+x)}{x}$ equal to?

(a) 1 (b) $\log_5 e$
(c) $\log_e 5$ (d) 5

(*b*) Consider, $\lim_{x \to 0} \dfrac{\log_5(1+x)}{x}$

$= \lim_{x \to 0} \dfrac{\log_e(1+x)}{x \cdot \log_e 5} \quad \left[\because \log_a b = \dfrac{\log_e b}{\log_e a}\right]$

$= \dfrac{1}{\log_e 5} \lim_{x \to 0} \dfrac{\log_e(1+x)}{x}$

$= \log_5 e \qquad \left[\because \lim_{x \to 0} \dfrac{\log_e(1+x)}{x} = 1 \text{ and } \log_a b = \dfrac{1}{\log_b a}\right]$

69. What is $\lim_{x \to 0} \dfrac{5^x - 1}{x}$ equal to?

(a) $\log_e 5$ (b) $\log_5 e$
(c) 5 (d) 1

(*a*) $\lim_{x \to 0} \dfrac{5^x - 1}{x} = \log_e 5$

$\left[\because \lim_{x \to 0} \dfrac{a^x - 1}{x} = \log_e a\right]$

70. What is $\lim_{n \to \infty} \dfrac{1 + 2 + 3 + \ldots + n}{1^2 + 2^2 + 3^2 + \ldots + n^2}$ equal to?

(a) 5 (b) 2
(c) 1 (d) 0

(*d*) Consider,

$$\lim_{n \to \infty} \frac{1 + 2 + 3 + \ldots + n}{1^2 + 2^2 + 3^2 + \ldots + n^2}$$

$$= \lim_{n \to \infty} \frac{\frac{n(n+1)}{2}}{\frac{n(n+1)(2n+1)}{6}}$$

$$\left[\because 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \text{ and } = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n \to \infty} \frac{3}{2n+1} = 0$$

2014 (I)

71. If $f(9) = 9$ and $f'(9) = 4$, then what is $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ equal to?

(a) 36 (b) 9
(c) 4 (d) None of these

(c) Given that,

$f(9) = 9$ and $f'(9) = 4$

Now consider, $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to 9} \frac{\frac{1}{2\sqrt{f(x)}} \cdot f'(x)}{\frac{1}{2\sqrt{x}} \cdot 1}$

[by L' Hospital's rule]

$= \lim_{x \to 9} \frac{f'(x) \times \sqrt{x}}{\sqrt{f(x)}} = \frac{f'(9) \times \sqrt{9}}{\sqrt{f(9)}}$

[$\because f(9) = 9 \Rightarrow f(x) = x$ i.e. $x = 9$]

$= \frac{4 \times 3}{\sqrt{9}} = \frac{4 \times 3}{3} = 4$

72. Consider the following statements

I. The function $f(x) = \sqrt[3]{x}$ is continuous at all x except at $x = 0$.

II. The function $f(x) = [x]$ is continuous at $x = 2.99$, where [ ] is the bracket function.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(b) I. Given, $f(x) = \sqrt[3]{x}$

$\Rightarrow \quad f(x) = (x)^{1/3}$

Now, we check the continuity of the function at $x = 0$.

LHL $= f(0 - 0) = \lim_{h \to 0} f(0 - h)$

$= \lim_{h \to 0} (0 - h)^{1/3}$

$= (0 - 0)^{1/3} = 0$

RHL $= f(0 + 0) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} (0 + h)^{1/3}$

$= (0 + 0)^{1/3} = 0$

and $f(0) = (0)^{1/3} = 0$

$\because$ LHL = RHL = $f(0)$

So, function is continuous at $x = 0$.

II. Given, $f(x) = [x]$

which is the greatest integer function.

We know that, the greatest integer function is continuous for all x except integer values of x.

So, $f(x) = [x]$ is continuous at $x = 2.99$.

Alternate Method

LHL $= f(2.99 - 0) = \lim_{h \to 0} f(2.99 - h)$

$= \lim_{h \to 0} [2.99 - h] = \lim_{h \to 0} 2 = 2$

RHL $= f(2.99 + 0) = \lim_{h \to 0} f(2.99 + h)$

$= \lim_{h \to 0} [2.99 + h] = \lim_{h \to 0} 2 = 2$

and $f(2.99) = [2.99] = 2$

$\because$ LHL = RHL = $f(2.99)$

So, $f(x)$ is continuous at $x = 2.99$.

73. Consider the following statements

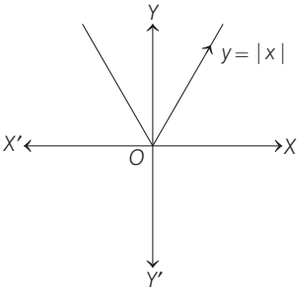
I. The function $f(x) = |x|$ is not differentiable at $x = 1$.

II. The function $f(x) = e^x$ is differentiable at $x = 0$.

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(b) I. From the graph of $f(x) = |x|$, we observe that, the curve has sharp turn at $x = 0$.



So, the function $f(x) = |x|$ is not differentiable only at $x = 0$.

i.e., $f(x) = |x|$ is differentiable at $x = 1$

II. Given function, $f(x) = e^x$

Now, we check the differentiability of $f(x)$ at $x = 0$.

Consider, $f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$

$= \lim_{h \to 0} \frac{e^{(0+h)} - e^0}{h} = \lim_{h \to 0} \frac{e^h - 1}{h}$ $\left[\frac{0}{0} \text{form}\right]$

$= \lim_{h \to 0} \frac{e^h - 0}{1} = e^0 = 1$

[by L' Hospital's rule]

Hence, $f(x) = e^x$ is differentiable at $x = 0$.

Directions (Q. Nos. 74-76) *Let $f(x)$ be a function defined in $1 \le x < \infty$ by*

$$f(x) = \begin{cases} 2 - x, & \text{for } 1 \le x \le 2 \\ 3x - x^2, & \text{for } x > 2 \end{cases}$$

74. Consider the following statements

I. The function is continuous at every point in the interval $[1, \infty)$.

II. The function is differentiable at $x = 1.5$.

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

75. What is the differentiable coefficient of $f(x)$ at $x = 3$?

(a) 1 (b) 2 (c) –1 (d) –3

76. Consider the following statements

I. $f'(2 + 0)$ does not exist.

II. $f'(2 - 0)$ does not exist.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Solutions (Q.Nos. 74-76) Given function,

$$f(x) = \begin{cases} 2 - x, & \text{for } 1 \le x \le 2 \\ 3x - x^2, & \text{for } x > 2 \end{cases}$$

and whole function defined in $1 \le x < \infty$.

74. (b) Since, the function is polynomial, so it is continuous as well as differentiable in the domain $[1, \infty) - \{2\}$.

Now, we check the continuity of the function at $x = 2$.

Consider, LHL $= f(2 - 0) = \lim_{h \to 0} f(2 - h)$

$= \lim_{h \to 0} 2 - (2 - h) = \lim_{h \to 0} h = 0$

and RHL $= f(2 + 0) = \lim_{h \to 0} f(2 + h)$

$= \lim_{h \to 0} 3(2 + h) - (2 + h)^2$

$= 3(2 + 0) - (2 + 0)^2 = 6 - 4 = 2$

Also, $f(2) = 2 - 2 = 0$

$\because \quad f(2) = \text{LHL} \ne \text{RHL}$

So, the function is not continuous at every point in the interval $[1, \infty)$ as it is not continuous at $x = 2$.

According to above discussion, we can say that f is differentiable at $x = 1.5$

75. (d) $\because f(x) = \begin{cases} 2 - x, & \text{for } 1 \le x \le 2 \\ 3x - x^2, & \text{for } x > 2 \end{cases}$

$$\Rightarrow \quad f'(x) = \begin{cases} -1, & \text{for } 1 < x < 2 \\ 3 - 2x, & \text{for } x > 2 \end{cases} \quad \ldots(i)$$

So, the differentiable coefficient of $f(x)$ at $x = 3$ is

$f'(3) = 3 - 2(3) = 3 - 6 = -3$

[$\because f'(x) = 3 - 2x$, for $x > 2$]

76. (***d***) Here, we check the existence of limit of $f'(x)$ at $x = 2$.

So, $f'(2 + 0) = \text{RHL} = \lim_{h \to 0} f'(2 + h)$

$= \lim_{h \to 0} 3 - 2\,(2 + h)$ [from Eq. (i)]

$= \lim_{h \to 0} 3 - 4 - 2h$

$= \lim_{h \to 0} -1 - 2h = -1$

and $f'(2 - 0) = \text{LHL}$

$= \lim_{h \to 0} f'(2 - h)$

$= \lim_{h \to 0} -1 = -1$

Since, both $f'(2 + 0)$ and $f'(2 - 0)$ exists and equal, therefore $f'(x)$ exist at $x = 2$.

77. What is $\lim_{x \to 0} \dfrac{(1 + x)^n - 1}{x}$ equal to?

(a) 0 (b) 1

(c) n (d) $n - 1$

(***c***) Consider, $\lim_{x \to 0} \left\{ \dfrac{(1 + x)^n - 1}{x} \right\}$

$= \lim_{x \to 0} \dfrac{(1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots) - 1}{x}$

[by binomial expansion of any index]

$= \lim_{x \to 0} \dfrac{nx + \dfrac{n(n-1)}{2!}x^2 + \ldots.}{x}$

$= \lim_{x \to 0} n + \dfrac{n(n-1)}{2!}x + \ldots$

$= n + 0 + 0 + \ldots = n$

78. What is $\lim_{x \to 0} \dfrac{x}{\sqrt{1 - \cos x}}$ equal to?

(a) $\sqrt{2}$

(b) $-\sqrt{2}$

(c) $\dfrac{1}{\sqrt{2}}$

(d) Does not exist

(***d***) Consider, $\lim_{x \to 0} \dfrac{x}{\sqrt{1 - \cos x}}$

$= \lim_{x \to 0} \dfrac{x}{\sqrt{1 - (1 - 2\sin^2 \frac{x}{2})}}$

$= \lim_{x \to 0} \dfrac{x}{\sqrt{2\sin^2 \frac{x}{2}}}$

$= \dfrac{1}{\sqrt{2}} \lim_{x \to 0} \dfrac{x}{|\sin \frac{x}{2}|}$

Now consider, LHL

$= f(0 - 0) = \lim_{h \to 0} f(0 - h)$

$= \dfrac{1}{\sqrt{2}} \lim_{h \to 0} \dfrac{-h}{\left|\sin\left(\frac{-h}{2}\right)\right|}$, where

$f(x) = \dfrac{1}{\sqrt{2}} \cdot \dfrac{x}{\left|\sin\left(\frac{x}{2}\right)\right|}$

$= -\dfrac{1}{\sqrt{2}} \lim_{h \to 0} \dfrac{h}{\left|-\sin\left(\frac{h}{2}\right)\right|}$

$= -\dfrac{1}{\sqrt{2}} \lim_{h \to 0} \dfrac{2 \cdot \left(\frac{h}{2}\right)}{\sin \frac{h}{2}}$

$= -\dfrac{1}{\sqrt{2}} \times 2 \times 1 = -\sqrt{2} \left[\because \lim_{\theta \to 0} \dfrac{\theta}{\sin\theta} = 1\right]$

and RHL $= f(0 + 0) = \lim_{h \to 0} f(0 + h)$

$= \dfrac{1}{\sqrt{2}} \lim_{h \to 0} \dfrac{h}{\left|\sin \frac{h}{2}\right|} = \dfrac{1}{\sqrt{2}} \lim_{h \to 0} \dfrac{2\left(\frac{h}{2}\right)}{\sin\left(\frac{h}{2}\right)}$

$= \dfrac{1}{\sqrt{2}} \times 2 \times 1 = \sqrt{2} \left[\because \lim_{\theta \to 0} \dfrac{\theta}{\sin\theta} = 1\right]$

$\because$ LHL $\neq$ RHL

So, limit does not exist.

Directions (Q. Nos. 79 and 80)

Consider the function

$f(x) = \dfrac{1 - \sin x}{(\pi - 2x)^2}$, *where* $x \neq \dfrac{\pi}{2}$ *and* $f\left(\dfrac{\pi}{2}\right) = \lambda$.

79. What is $\lim_{x \to \pi/2} f(x)$ equal to?

(a) 1 (b) 1/2

(c) 1/4 (d) 1/8

(***d***) Consider,

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \dfrac{1 - \sin x}{(\pi - 2x)^2}$ $\left[\dfrac{0}{0} \text{ form}\right]$

$= \lim_{x \to \pi/2} \dfrac{-\cos x}{2\,(\pi - 2x)\,(-2)}$

[by L' Hospital's rule]

$= \lim_{x \to \pi/2} \dfrac{\cos x}{4\,(\pi - 2x)}$ $\left[\dfrac{0}{0} \text{ form}\right]$

$= \lim_{x \to \pi/2} \dfrac{-\sin x}{4\,(-2)}$

$= \lim_{x \to \pi/2} \dfrac{\sin x}{8}$

[by L' Hospital's rule]

$= \dfrac{1}{8} \cdot \sin \dfrac{\pi}{2}$

$= \dfrac{1}{8} \times 1 = \dfrac{1}{8}$

80. What is the value of λ, if the function is continuous at $x = \dfrac{\pi}{2}$?

(a) 1/8 (b) 1/4

(c) 1/2 (d) 1

(***a***) Since, the given function is continuous at $x = \pi/2$.

$\therefore \quad f(\pi/2) = \lim_{x \to \pi/2} \dfrac{1 - \sin x}{(\pi - 2x)^2}$

$\Rightarrow \quad \lambda = \dfrac{1}{8}$

21

DIFFERENTIATION

2019 (II)

1. What is the derivative of $2^{(\sin x)^2}$ with respect to $\sin x$?

(a) $\sin x\, 2^{(\sin x)^2}\, ln\ 4$

(b) $2\sin x\, 2^{(\sin x)^2}\, ln\ 4$

(c) $ln\ (\sin x)\, 2^{(\sin x)^2}$

(d) $2\ \sin x \cos x\, 2^{(\sin x)^2}$

(a) Derivative of $2^{(\sin x)^2}$ with respect to $\sin x$

$$= \frac{\frac{d}{dx} 2^{(\sin x)^2}}{\frac{d}{dx}(\sin x)} = \frac{2^{(\sin x)^2} \log 2 \frac{d}{dx}(\sin x)^2}{\cos x}$$

$$= \frac{2^{(\sin x)^2} \log 2.\ 2 \sin x . \cos x}{\cos x}$$

$$= 2 \log 2.\ (\sin x).\ 2^{(\sin x)^2}$$

$$= \sin x.\ 2^{(\sin x)^2} .\log 4$$

2. Let $f(x) = x^2 + 2x - 5$ and $g(x) = 5x + 30$.
If $h(x) = 5f(x) - xg(x)$, then what is the derivative of $h(x)$?

(a) –40 (b) –20 (c) –10 (d) 0

(b) Given, $h(x) = 5f(x) - xg(x)$

$$= 5(x^2 + 2x - 5) - x\ (5x + 30)$$
$$= 5x^2 + 10x - 25 - 5x^2 - 30x$$
$$= -20x - 25$$

Differentiation w.r.t. x, we get

$$h'(x) = -20$$

Hence, derivative of $h(x)$ is –20.

Directions (Q. Nos. 3 and 4) *Read the following information and answer the two items that follow :*

Consider the equation $x^y = e^{x-y}$

3. What is $\frac{dy}{dx}$ at $x = 1$ equal to?

(a) 0 (b) 1 (c) 2 (d) 4

4. What is $\frac{d^2y}{dx^2}$ at $x = 1$ equal to?

(a) 0 (b) 1 (c) 2 (d) 4

Solutions (Q. Nos 3 and 4)

Given equation, $x^y = e^{x-y}$

Taking log both sides

$$y \log x = (x - y) \log e$$

$$\Rightarrow \quad y \log x = x - y \qquad [\because \log_e e = 1]$$

$$\Rightarrow (1 + \log x) y = x \Rightarrow y = \frac{x}{(1 + \log x)}$$

Differentiation w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(1 + \log x).1 - x\left(0 + \frac{1}{x}\right)}{(1 + \log x)^2}$$

$$\frac{dy}{dx} = \frac{1 + \log x - 1}{(1 + \log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$$

3. (a) $\because \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

At $x = 1$, $\frac{dy}{dx} = \frac{\log 1}{(1 + \log 1)^2}$

$$= \frac{0}{1} = 0 \qquad [\because \log 1 = 0]$$

4. (b) $\because \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

Differentiation. w.r.t. x, we get

$$(1 + \log x)^2 . \frac{1}{x} - (\log x).$$

$$\frac{d^2y}{dx^2} = \frac{2\ (1 + \log x)\left(0 + \frac{1}{x}\right)}{(1 + \log x)^4}$$

$$= \frac{\frac{1}{x}(1 + \log x)(1 + \log x - 2 \log x)}{(1 + \log x)^4}$$

$$= \frac{\frac{1}{x}(1 + \log x)(1 - \log x)}{(1 + \log x)^4}$$

At $x = 1$

$$\frac{d^2y}{dx^2} = \frac{1(1 + 0)(1 - 0)}{(1 + 0)^4} = 1$$

2019 (I)

5. What is the derivative of $\sec^2(\tan^{-1} x)$ with respect to x?

(a) $2x$ (b) $x^2 + 1$
(c) $x + 1$ (d) x^2

(a) Let $y = \sec^2 (\tan^{-1} x)$

On differentiating both sides w.r.t x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \sec^2 (\tan^{-1} x)$$

$$= 2 \sec (\tan^{-1} x) \cdot \sec(\tan^{-1} x)$$
$$\tan (\tan^{-1} x) \frac{d}{dx} (\tan^{-1} x)$$

$$= 2 \sec^2(\tan^{-1} x) \cdot x \cdot \frac{1}{1 + x^2}$$

$$= 2\ (1 + \tan^2 (\tan^{-1} x)) \cdot \frac{x}{1 + x^2}$$

$$= 2\ (1 + x^2) \cdot \frac{x}{1 + x^2}$$

$$= 2x$$

6. If $f(x) = \sin(\cos x)$, then $f'(x)$ is equal to

(a) $\cos(\cos x)$
(b) $\sin(-\sin x)$
(c) $(\sin x)\cos(\cos x)$
(d) $(-\sin x)\cos(\cos x)$

(d) Given, $f(x) = \sin (\cos x)$

$$\Rightarrow f'(x) = \cos (\cos x)\ (-\sin x)$$

2018 (II)

7. If $y = \tan^{-1}\left(\frac{5 - 2\tan\sqrt{x}}{2 + 5\tan\sqrt{x}}\right)$, then what is $\frac{dy}{dx}$ equal to?

(a) $-\frac{1}{2\sqrt{x}}$ (b) 1
(c) – 1 (d) $\frac{1}{2\sqrt{x}}$

(**a**) We have,

$$y = \tan^{-1}\left(\frac{5 - 2\tan\sqrt{x}}{2 + 5\tan\sqrt{x}}\right)$$

$$y = \tan^{-1}\left(\frac{\frac{5}{2} - \tan\sqrt{x}}{1 + \frac{5}{2}\tan\sqrt{x}}\right)$$

Let, $\frac{5}{2} = \tan A$

$$= \tan^{-1}\left(\frac{\tan A - \tan\sqrt{x}}{1 + \tan A\tan\sqrt{x}}\right)$$

$$= \tan^{-1}[\tan(A - \sqrt{x})]$$

$$= A - \sqrt{x}$$

$$y = \tan^{-1}\frac{5}{2} - \sqrt{x}$$

On differentiation w.r.t x, we get

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

8. If $u = e^{ax}\sin bx$ and $v = e^{ax}\cos bx$, then what is $u\frac{du}{dx} + v\frac{dv}{dx}$ equal to?

(a) ae^{2ax} (b) $(a^2 + b^2)e^{ax}$
(c) $ab\,e^{2ax}$ (d) $(a + b)e^{ax}$

(**a**) We have, $u = e^{ax}\sin bx$

On differentiation. w. r. t. x, we get

$$\frac{du}{dx} = e^{ax}(b\cos bx) + \sin bx\,(ae^{ax})$$

$$\frac{du}{dx} = e^{ax}(b\cos bx + a\sin bx) \quad ...(i)$$

Now, $v = e^{ax}\cos bx$

On differentiation w. r. t. x, we get

$$\frac{dv}{dx} = e^{ax}(-b\sin bx) + \cos bx\,(ae^{ax})$$

$$= e^{ax}(-b\sin bx + a\cos bx)$$

Now we have, $u\frac{du}{dx} + v\frac{dv}{dx}$

$$= e^{ax}\sin bx\,[e^{ax}(b\cos bx + a\sin bx)]$$
$$+ e^{ax}\cos bx[e^{ax}(-b\sin bx + a\cos bx)]$$
$$= e^{2ax}[b\sin bx\cos bx + a\sin^2 bx]$$
$$+ e^{2ax}(-b\cos bx\sin bx + a\cos^2 bx)$$
$$= e^{2ax}[a\sin^2 bx + a\cos^2 bx] = ae^{2ax}$$

9. If $y = \sin(\ln x)$, then which one of the following is correct?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dn^2} = 0$

(c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(d) $x\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

(**c**) We have, $y = \sin(\log_e x)$

Differentiation w. r. t. x, we get

$$\frac{dy}{dx} = \frac{\cos(\log_e x)}{x}$$

$$x\frac{dy}{dx} = \cos(\log_e x)$$

Again, differentiation. w. r. t. x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-\sin(\log_e x)}{x}$$

$$\Rightarrow\quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\sin(\log_e x)$$

$$\Rightarrow\quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow\quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

10. If $y = |\sin x|^{|x|}$, then what is the value of $\frac{dy}{dx}$ at $x = -\frac{\pi}{6}$?

(a) $\frac{2^{-\frac{\pi}{6}}(6\ln 2 - \sqrt{3}\pi)}{6}$

(b) $\frac{2^{\frac{\pi}{6}}(6\ln 2 + \sqrt{3}\pi)}{6}$

(c) $\frac{2^{-\frac{\pi}{6}}(6\ln 2 + \sqrt{3}\pi)}{6}$

(d) $\frac{2^{-\frac{\pi}{6}}(6\ln 2 - \sqrt{3}\pi)}{6}$

(**a**) We have,

$$y = |\sin x|^{|x|}$$

$$\therefore\quad x = -\frac{\pi}{6}, \sin x < 0, x < 0$$

$$y = (-\sin x)^{-x}$$

$\because\quad y = f(x)^{g(x)}$, So

$$\frac{dy}{dx} = f(x)^{g(x)}\left[\frac{g(x)}{f(x)}f'(x) + \log|f(x)|\cdot g'(x)\right]$$

$$\frac{dy}{dx} = (-\sin x)^{-x}\left[\frac{(-x)}{(-\sin x)}\cdot(-\cos x) + \log|(-\sin x)|(-1)|\right]$$

$$\left(\frac{dy}{dx}\right)_{x=-\pi/6} = \left[-\sin\left(\frac{-\pi}{6}\right)\right]^{\frac{\pi}{6}}\left\{\frac{\pi/6}{-\sin(-\pi/6)}\cdot-\cos\left(-\frac{\pi}{6}\right)\right\} - \log\left|-\sin\left(-\frac{\pi}{6}\right)\right|$$

$$= \left(\sin\frac{\pi}{6}\right)^{\frac{\pi}{6}}\left[\frac{-\frac{\pi}{6}}{\frac{1}{2}}\left(\frac{\sqrt{3}}{2}\right) - \log\left(\frac{1}{2}\right)\right]$$

$$= \left(\frac{1}{2}\right)^{\frac{\pi}{6}}\left[\frac{-\sqrt{3}\pi}{6} - \log\frac{1}{2}\right]$$

$$= 2^{-\pi/6}\left[\log 2 - \frac{\sqrt{3}\pi}{6}\right]$$

$$= 2^{-\frac{\pi}{6}}\left[\frac{6\log 2 - \sqrt{3}\pi}{6}\right]$$

11. What is $\frac{d}{dx}\sqrt{1 - \sin 2x}$ equal to, where $\frac{\pi}{4} < x < \frac{\pi}{2}$?

(a) $\cos x + \sin x$
(b) $-(\cos x + \sin x)$
(c) $\pm(\cos x + \sin x)$
(d) None of the above

(**a**) We have,

$$\frac{d}{dx}\sqrt{1 - \sin 2x},\ \frac{\pi}{4} < x < \frac{\pi}{2}$$

$$= \frac{d}{dx}\sqrt{\cos^2 x + \sin^2 x - 2\sin x\cos x}$$

$$\left[\because \sin^2 x + \cos^2 x = 1 \text{ and } \sin 2x = 2\sin x\cos x\right]$$

$$= \frac{d}{dx}|\cos x - \sin x|$$

$$= \frac{d}{dx}(\sin x - \cos x)$$

$$\left[\because \frac{\pi}{4} < x < \frac{\pi}{2} \therefore \sin x > \cos x\right]$$

$$= \cos x - (-\sin x) = \cos x + \sin x$$

2018 (I)

12. If $y = e^{x^2}\sin 2x$, then what is $\frac{dy}{dx}$ at $x = \pi$ equal to ?

(a) $(1 + \pi)e^{\pi^2}$ (b) $2\pi e^{\pi^2}$
(c) $2e^{\pi^2}$ (d) e^{π^2}

(**c**) We have, $y = e^{x^2}\sin 2x$

On differentiating both the sides, we get

$$\frac{dy}{dx} = 2\cos 2x\cdot e^{x^2} + 2xe^{x^2}\sin 2x$$

$$\therefore\quad \left(\frac{dy}{dx}\right)_{x=\pi} = 2\cos 2\pi\cdot e^{\pi^2} + 2\pi e^{\pi^2}\sin 2\pi$$

$$= 2(1)e^{\pi^2} + 2\pi e^{\pi^2}(0) = 2e^{\pi^2}$$

2017 (II)

13. If $y = (\cos x)^{(\cos x)^{(\cos x)^{\infty}}}$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y^2\tan x}{1 - y\ln(\cos x)}$ (b) $\frac{y^2\tan x}{1 + y\ln(\cos x)}$
(c) $\frac{y^2\tan x}{1 - y\ln(\sin x)}$ (d) $\frac{y^2\sin x}{1 + y\ln(\sin x)}$

(**a**) We have,

$$y = (\cos x)^{(\cos x)^{(\cos)\cdots\infty}}$$

Let $y = (\cos x)^{(\cos x)^{\infty}}$

$$\therefore\quad y = (\cos x)^y$$

Taking log both the sides, we get

$$\log y = y\log\cos x$$

Differentiating both sides, w.r.t x, we get

$$\frac{1}{y}\frac{dy}{dx}=\frac{dy}{dx}\log\cos x + y\cdot\frac{1}{\cos x}(-\sin x)$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1}{y}-\log\cos x\right]=-y\tan x$$

$$\Rightarrow \qquad \frac{dy}{dx}=\frac{-y^2\tan x}{1-y\log\cos x}$$

14. Consider the following statements

1. Derivative of $f(x)$ may not exist at some point.
2. Derivative of $f(x)$ may exist finitely at some point.
3. Derivative of $f(x)$ may be infinite (geometrically) at some point.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(***d***) All statement are true as all are fundamental statements.

15. If $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{2}{1+x^2}$ for all $|x|<1$

(b) $-\frac{2}{1+x^2}$ for all $|x|>1$

(c) $\frac{2}{1+x^2}$ for all $|x|<1$

(d) None of the above

(***a***) We have

$$y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)=\frac{\pi}{2}-\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$=\begin{cases}\left[\frac{\pi}{2}-(2\tan^{-1}x)\right], |x|<1\\ \left[\frac{\pi}{2}-(\pi-2\tan^{-1}x)\right], x>1\\ \left[\frac{\pi}{2}-(-\pi-2\tan^{-1}x)\right], x<-1\end{cases}$$

$$=\begin{cases}\frac{\pi}{2}-2\tan^{-1}x, |x|<1\\ -\frac{\pi}{2}+2\tan^{-1}x, x>1\\ \frac{3\pi}{2}+2\tan^{-1}x, x<-1\end{cases}$$

$$\therefore \quad \frac{dy}{dx}=\begin{cases}\frac{-2}{1+x^2}, |x|<1\\ \frac{2}{1+x^2}, x>1\\ \frac{2}{1+x^2}, x<-1\end{cases}$$

2017 (I)

16. What is $\frac{d^2x}{dy^2}$ equal to?

(a) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (b) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$

(c) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ (d) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

(***c***) $\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right)$

$$=\frac{-1}{\left(\frac{dy}{dx}\right)^2}\cdot\frac{d}{dy}\left(\frac{dy}{dx}\right) \quad \text{[by quotient rule]}$$

$$=\frac{-\frac{dx}{dy}\cdot\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\left(\frac{dy}{dx}\right)^2}=\frac{-\frac{dx}{dy}\cdot\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^2}$$

$$=\frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}=-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$$

17. What is the derivative of $\log_{10}(5x^2+3)$ with respect to x?

(a) $\frac{x\log_{10}e}{5x^2+3}$ (b) $\frac{2x\log_{10}e}{5x^2+3}$

(c) $\frac{10x\log_{10}e}{5x^2+3}$ (d) $\frac{x\log_e 10}{5x^2+3}$

(***c***) Let $y=\log_{10}(5x^2+3)$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{5x^2+3}(\log_{10}e)\frac{d}{dx}(5x^2+3)$$

[by chain rule]

$$=\frac{\log_{10}e}{5x^2+3}(10x)=\frac{10x\log_{10}e}{5x^2+3}$$

18. Suppose the function $f(x)=x^n$, $n\neq 0$ is differentiable for all x. Then n can be any element of the interval

(a) $[1,\infty)$ (b) $(0,\infty)$
(c) $\left(\frac{1}{2},\infty\right)$ (d) None of these

(***a***) Given, $f(x)=x^n$, $n\neq 0$

Here, $f'(x)=nx^{n-1}$

For $f(x)$ to be differentiable

$n-1\geq 0 \therefore n\geq 1$

$\therefore$ For $f(x)$ to be differentiable n can be any element of the interval $[1,\infty)$.

2016 (II)

Directions (Q. Nos. 19-21)

Let $f(x)=(|x|-|x-1|)^2$

19. What is $f'(x)$ equal to when $x>1$?

(a) 0 (b) $2x-1$
(c) $4x-2$ (d) $8x-4$

(***a***) We have, $f(x)=(|x|-|x-1|)^2$

When $x>1$,

$$f(x)=[x-(x-1)]^2=1$$

$\therefore \quad f'(x)=0$

20. What is $f'(x)$ equal to when $0<x<1$?

(a) 0 (b) $2x-1$
(c) $4x-2$ (d) $8x-4$

(***d***) $\because f(x)=(|x|-|x-1|)^2$

When $0<x<1$

$\therefore \quad f(x)=[x-\{-(x-1)\}]^2$

$$=(2x-1)^2$$

$\therefore \quad f'(x)=2(2x-1)(2)$

$$=8x-4$$

21. Which of the following equations is/are correct?

I. $f(-2)=f(5)$
II. $f''(-2)+f''(0.5)+f''(3)=4$

Select the correct answer using the code given below.

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(***a***) $\because$

$$|x|-|x-1|=\begin{cases}-1, & x<0\\ 2x-1, & 0\leq x<1\\ -1, & 1\leq x\end{cases}$$

$$\therefore f(x)=\begin{cases}1, & \text{when } 0>x \text{ or } x\geq 1\\ (2x-1)^2, & \text{when } 0\leq x<1\end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}0, & \text{when } 0>x \text{ or } x\geq 1\\ 4(2x-1), & \text{when } 0\leq x<1\end{cases}$$

$$\Rightarrow \quad f''(x)=\begin{cases}0, & \text{when } 0>x \text{ or } x\geq 1\\ 8, & \text{when } 0\leq x<1\end{cases}$$

I. $f(-2)=1$ and $f(5)=1$

so, $f(-2)=f(5)=1$

$\Rightarrow$ Statement I is correct.

II. $f''(-2)=0$

$f''(0.5)=8$ and $f''(3)=0$

Then, $f''(-2)+f''(0.5)+f''(3)$

$$=0+8+0$$

$$=8$$

So, statement II incorrect.

2016 (I)

Directions (Q. Nos. 22 and 23)

Consider the function

$$f(x)=\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix},$$

where p is a constant.

22. What is the value of $f'(0)$?

(a) p^3 (b) $3p^3$
(c) $6p^3$ (d) $-6p^3$

(d) Given, $f(x)=\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

where, p is a constant.

On differentiating both sides w.r.t. x, we get

$$f'(x)=\begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}+\begin{vmatrix} x^3 & \sin x & \cos x \\ 0 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix}+\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\Rightarrow f'(x)=\begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}+0+0$$

$$\Rightarrow f'(0)=\begin{vmatrix} 0 & \cos 0 & -\sin 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f'(0)=\begin{vmatrix} 0 & 1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$=(-1)(6p^3-0)$$
$$=-6p^3$$

23. What is the value of p for which $f''(0)=0$?

(a) $-\frac{1}{6}$ or 0 (b) -1 or 0
(c) $-\frac{1}{6}$ or 1 (d) -1 or 1

(a) From question 18,

$$f'(x)=\begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

Again, differentiating both sides, w.r.t. x, we get

$$\Rightarrow f''(x)=\begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}+\begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 0 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix}+\begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\Rightarrow f''(x)=\begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}+0+0$$

Since, we have $f''(0)=0$

$$\Rightarrow f''(0)=\begin{vmatrix} 0 & -\sin 0 & \cos 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}=0$$

$$\Rightarrow 6p^2+p=0$$

$$\Rightarrow p(6p+1)=0$$

$$\Rightarrow p=0 \text{ or } 6p+1=0$$

$$\Rightarrow p=0 \text{ or } p=-\frac{1}{6} \Rightarrow p=-\frac{1}{6} \text{ or } 0$$

24. If $y=\log_{10}x+\log_x 10+\log_x x+\log_{10}10$, then what is $\left(\frac{dy}{dx}\right)_{x=10}$ equal to?

(a) 10 (b) 2
(c) 1 (d) 0

(d) Given,

$$y=\log_{10}x+\log_x 10+\log_x x+\log_{10}10$$
$$=\frac{\log x}{\log 10}+\frac{\log 10}{\log x}+1+1$$
$$y=\frac{\log x}{\log 10}+\frac{\log 10}{\log x}+2$$

On differentiating both sides w. r. t. x, we get

$$\frac{dy}{dx}=\frac{1}{\log 10}\cdot\frac{1}{x}+\log 10\left(-\frac{1}{(\log x)^2}\cdot\frac{1}{x}\right)$$

At $x=10$,

$$\left(\frac{dy}{dx}\right)_{x=10}=\frac{1}{10\log 10}-\frac{\log 10}{10(\log 10)^2}$$
$$=\frac{1}{10\log 10}-\frac{1}{10\log 10}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=10}=0$$

Directions (Q. Nos. 25-27)

Let $f: R\to R$ be a function such that $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$ for $x\in R$.

25. What is $f(1)$ equal to?

(a) -2 (b) -1 (c) 0 (d) 4

(d) $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$ ...(i)

On differentiating Eq. (i) w.r.t. x thricely, we get

$f'(x)=3x^2+2xf'(1)+0+f''(2)+0+0$ ...(ii)

$f''(x)=6x+2f'(1)$ ...(iii)

and $f'''(x)=6$ ...(iv)

On putting $x=1, 2, 3$ in Eqs. (ii), (iii) and (iv), we get

$f'(1)=3+2f'(1)+f''(2)$

$f''(2)=12+2f'(1), f'''(3)=6$

On solving these equations, we get

$f'(1)=-5, f''(2)=2, f'''(3)=6$

$\therefore \quad f(x)=x^3+x^2(-5)+x(2)+6$

$\Rightarrow \quad f(x)=x^3-5x^2+2x+6$

$f(1)=(1)^3-5(1)^2+2(1)+6$
$=1-5+2+6=4$

26. What is $f'(1)$ equal to?

(a) -6 (b) -5 (c) 1 (d) 0

(b) We have, $f(x)=x^3-5x^2+2x+6$

On differentiating w.r.t. x, we get

$f'(x)=3x^2-10x+2$

At $x=1, f'(1)=3(1)^2-10(1)+2$
$=3-10+2=-5$

27. What is $f'''(10)$ equal to?

(a) 1 (b) 5 (c) 6 (d) 8

(c) We have, $f(x)=x^3-5x^2+2x+6$

On differetiating w.r.t. x, thricely we get

$f'(x)=3x^2-10x+2$

$\Rightarrow f''(x)=6x-10 \Rightarrow f'''(x)=6$

$\therefore f'''(10)=6$

Directions (Q. Nos. 28-30) *Consider the function $f(x)=|x^2-5x+6|$*

28. What is $f'(4)$ equal to?

(a) -4 (b) -3 (c) 3 (d) 2

(c) Given, $f(x)=|x^2-5x+6|$

$\Rightarrow f(x)=|(x-2)(x-3)|$

At $x=4$, we take

$f(x)=(x-2)(x-3)=x^2-5x+6$

On differentiating both sides w.r.t.x, we get

$$f'(x) = 2x - 5$$

At $x = 4$,

$$f'(4) = 2 \times 4 - 5 = 3$$

29. What is $f''(2.5)$ equal to?

(a) –3 (b) –2 (c) 0 (d) 2

(**b**) Given, $f(x) = |x^2 - 5x + 6|$

At $x = 2.5$,

$$f(x) = -(x-2)(x-3)$$
$$= -(x^2 - 5x + 6)$$

On differentiating both sides w.r.t. x, twicely, we get

$$f'(x) = -(2x - 5)$$
$$f''(x) = -2$$
$$\Rightarrow \quad f''(2.5) = -2$$

2015 (II)

30. The derivative of $\ln(x + \sin x)$ with respect to $(x + \cos x)$ is

(a) $\dfrac{1 + \cos x}{(x + \sin x)(1 - \sin x)}$

(b) $\dfrac{1 - \cos x}{(x + \sin x)(1 + \sin x)}$

(c) $\dfrac{1 - \cos x}{(x - \sin x)(1 + \cos x)}$

(d) $\dfrac{1 + \cos x}{(x - \sin x)(1 - \cos x)}$

(**a**) Let $u = \ln(x + \sin x)$

and $v = x + \cos x$

Now, $\dfrac{du}{dx} = \dfrac{1}{(x + \sin x)}(1 + \cos x)$

and $\dfrac{dv}{dx} = 1 - \sin x$

Now, we can find derivative of u w.r.t. v,

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{(1 + \cos x)/(x + \sin x)}{1 - \sin x}$$

$$\Rightarrow \quad \frac{du}{dv} = \frac{1 + \cos x}{(x + \sin x)(1 - \sin x)}$$

31. If

$$y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right],$$

where $0 < x < \dfrac{\pi}{2}$, then $\dfrac{dy}{dx}$ is equal to

(a) $-\dfrac{1}{2}$ (b) 2

(c) $\sin x + \cos x$ (d) $\sin x - \cos x$

(**a**) Given,

$$y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$$

$$= \cot^{-1}\left[\frac{\left|\sin\frac{x}{2} + \cos\frac{x}{2}\right| + \left|\sin\frac{x}{2} - \cos\frac{x}{2}\right|}{\left|\sin\frac{x}{2} + \cos\frac{x}{2}\right| - \left|\sin\frac{x}{2} - \cos\frac{x}{2}\right|}\right]$$

As $0 < x < \dfrac{\pi}{2}$

$$= \cot^{-1}\left[\frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + \left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) - \left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)}\right]$$

$$= \cot^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \cot^{-1}\cot\left(\frac{\pi}{2} - \frac{x}{2}\right) = \left(\frac{\pi}{2} - \frac{x}{2}\right)$$

$$\therefore \quad \frac{dy}{dx} = -\frac{1}{2}$$

32. The function $f(x) = \dfrac{x^2}{e^x}$ is monotonically increasing, if

(a) $x < 0$
(b) $x > 2$
(c) $0 < x < 2$
(d) $x \in (-\infty, 0) \cup (2, \infty)$

(**c**) We have, $f(x) = x^2 e^{-x}$

$$\Rightarrow \quad f'(x) = x^2 e^{-x}(-1) + e^{-x} 2x$$
$$= e^{-x}(-x^2 + 2x) = e^{-x}(2 - x)x$$

Since, $f(x)$ is monotonically increasing.

$$\therefore \quad f'(x) > 0$$
$$\Rightarrow \quad e^{-x} x(2 - x) > 0 \text{ or } x(x - 2) < 0$$

Hence, $0 < x < 2$

33. If $x^a y^b = (x - y)^{a+b}$, then the value of $\dfrac{dy}{dx} - \dfrac{y}{x}$ is equal to

(a) $\dfrac{a}{b}$ (b) $\dfrac{b}{a}$

(c) 1 (d) 0

(**d**) We have, $x^a y^b = (x - y)^{a+b}$

Taking log on both sides, we get

$a \log x + b \log y = (a + b) \log (x - y)$

On differentiating both sides, we get

$$\frac{a}{x} + \frac{b}{y}\frac{dy}{dx} = (a + b)\frac{1}{(x - y)}\left(1 - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left(\frac{b}{y} + \frac{a+b}{x-y}\right) = \frac{a+b}{x-y} - \frac{a}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\frac{(bx + ay)}{(bx + ay)} = \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = 0$$

34. If $s = \sqrt{t^2 + 1}$, then $\dfrac{d^2s}{dt^2}$ is equal to

(a) $\dfrac{1}{s}$ (b) $\dfrac{1}{s^2}$ (c) $\dfrac{1}{s^3}$ (d) $\dfrac{1}{s^4}$

(**c**) Gives, $s = \sqrt{t^2 + 1}$

Differentiating on both sides w.r.t. t, we get

$$\frac{ds}{dt} = \frac{1}{2\sqrt{t^2+1}} \times 2t = \frac{t}{\sqrt{t^2+1}}$$

Differentiating again both sides w.r.t. t, we get

$$\frac{d^2s}{dt^2} = \frac{\sqrt{t^2+1}(1) - t \times \frac{1}{2\sqrt{t^2+1}}(2t)}{(\sqrt{t^2+1})^2}$$

$$= \frac{t^2 + 1 - t^2}{(\sqrt{t^2+1})^2\sqrt{t^2+1}} = \frac{1}{(\sqrt{t^2+1})^3} = \frac{1}{s^3}$$

2015 (I)

35. What is the derivative of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2} - 1}{x}\right)$ with respect to $\tan^{-1} x$?

(a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) x

(**b**) Let $u = \tan^{-1}\left(\dfrac{\sqrt{1+x^2} - 1}{x}\right)$ and

$v = \tan^{-1} x$

Consider, $u = \tan^{-1}\left(\dfrac{\sqrt{1+x^2} - 1}{x}\right)$

On putting $x = \tan\theta$, we get

$$u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1 - \cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2}$$

$$\Rightarrow \quad u = \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow \quad \frac{du}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right) \qquad \ldots(i)$$

Now, consider $v = \tan^{-1} x$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{1}{1+x^2} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv}$$

$$= \frac{1}{2}\left(\frac{1}{1+x^2}\right) \cdot (1 + x^2) = \frac{1}{2}$$

Directions (Q. Nos. 36-38) *Consider the parametric equation*

$x = \dfrac{a(1-t^2)}{1+t^2}$, $y = \dfrac{2at}{1+t^2}$

36. What does the equation represent?

(a) It represents a circle of diameter a
(b) It represents a circle of radius a
(c) It represents a parabola
(d) None of the above

37. What is $\dfrac{dy}{dx}$ equal to?

(a) $\dfrac{y}{x}$ (b) $-\dfrac{y}{x}$ (c) $\dfrac{x}{y}$ (d) $-\dfrac{x}{y}$

38. What is $\dfrac{d^2y}{dx^2}$ equal to?

(a) $\dfrac{a^2}{y^2}$ (b) $\dfrac{a^2}{x^2}$ (c) $-\dfrac{a^2}{x^2}$ (d) $-\dfrac{a^2}{y^3}$

Solutions (Q. Nos. 36-38)

We have, $x = \dfrac{a(1-t^2)}{1+t^2}$ and $y = \dfrac{2at}{1+t^2}$

36. (***b***) On squaring and adding, we get

$$x^2 + y^2 = \frac{a^2(1-t^2)^2}{(1+t^2)^2} + \frac{4a^2t^2}{(1+t^2)^2}$$

$$= \frac{a^2}{(1+t^2)^2}[(1-t^2)^2 + 4t^2]$$

$$= \frac{a^2}{(1+t^2)^2}(1 + t^4 - 2t^2 + 4t^2)$$

$$= \frac{a^2}{(1+t^2)^2}(1+t^2)^2$$

$$\Rightarrow \quad x^2 + y^2 = a^2 \qquad \ldots(i)$$

which is an equation of circle with radius a.

37. (***d***) Consider $x = \dfrac{a(1-t^2)}{1+t^2}$

$$\Rightarrow \quad \frac{dx}{dt} = a\left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}\right]$$

$$= -2at\left[\frac{1+t^2+1-t^2}{(1+t^2)^2}\right] = \frac{-4at}{(1+t^2)^2}$$

and $y = \dfrac{2at}{1+t^2}$

$$\Rightarrow \quad \frac{dy}{dt} = 2a\left[\frac{(1+t^2)\cdot 1 - t(2t)}{(1+t^2)^2}\right]$$

$$= 2a\left[\frac{1-t^2}{1+t^2}\right]$$

Now, $\dfrac{dy}{dx} = \dfrac{dy}{dt}\cdot\dfrac{dt}{dx}$

$$= 2a\,\frac{(1-t^2)}{(1+t^2)^2}\cdot\frac{(1+t^2)^2}{-4at}$$

$$= -\frac{(1-t^2)}{2t} = \frac{-x}{y} \qquad \ldots(ii)$$

38. (***d***) From above, we have

$$y\frac{dy}{dx} = -x$$

[using Eq. (ii)]

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -1$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \frac{x^2}{y^2} = -1$$

[using Eq. (ii)]

$$\Rightarrow \quad y\frac{d^2y}{dx^2} = -1 - \frac{x^2}{y^2}$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} = -\frac{[y^2 + x^2]}{y^2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-a^2}{y^3}$$

[using Eq. (i)]

Directions (Q. Nos. 39 and 40) *Given that* $\dfrac{d}{dx}\left(\dfrac{1+x^2+x^4}{1+x+x^2}\right) = Ax + B.$

39. What is the value of A?

(a) –1 (b) 1
(c) 2 (d) 4

40. What is the value of B?

(a) –1 (b) 1
(c) 2 (d) 4

Solutions (Q. Nos. 39 and 40)

We have,

$$\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = Ax + B \qquad \ldots(i)$$

Let us first divide $x^4 + x^2 + 1$ by $x^2 + x + 1$.

$$\begin{array}{r} x^2 - x + 1 \\ x^2+x+1 \overline{)\,x^4 + x^2 + 1} \\ \underline{-x^4 - x^3 - x^2} \\ -x^3 + 1 \\ \underline{-x^3 - x^2 - x} \\ +x^2 + x + 1 \\ \underline{+x^2 + x + 1} \\ 0 \end{array}$$

Thus, $\dfrac{x^4 + x^2 + 1}{x^2 + x + 1} = x^2 - x + 1$

From Eq. (i), we have

$\dfrac{d}{dx}(x^2 - x + 1) = Ax + B$ [from Eq. (i)]

$$\Rightarrow \quad 2x - 1 = Ax + B \qquad \ldots(ii)$$

39. (***c***) On comparing the coefficient of x in Eq. (ii), we get

$$A = 2$$

40. (***a***) On comparing the coefficient of constant terms in Eq. (ii), we get

$$B = -1$$

2014 (II)

Directions (Q. Nos. 41 and 42) *Consider the curve* $x = a(\cos\theta + \theta\sin\theta)$ *and* $y = a(\sin\theta - \theta\cos\theta)$.

41. What is $\dfrac{dy}{dx}$ equal to?

(a) $\tan\theta$ (b) $\cot\theta$
(c) $\sin 2\theta$ (d) $\cos 2\theta$

(***a***) We have, $x = a(\cos\theta + \theta\sin\theta)$

and $y = a(\sin\theta - \theta\cos\theta)$

$$\Rightarrow \quad \frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta)$$

$$\Rightarrow \quad \frac{dx}{d\theta} = a\theta\cos\theta$$

and $\dfrac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta)$

$$\Rightarrow \quad \frac{dy}{d\theta} = a\theta\sin\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

42. What is $\dfrac{d^2y}{dx^2}$ equal to?

(a) $\sec^2\theta$ (b) $-\operatorname{cosec}^2\theta$
(c) $\dfrac{\sec^3\theta}{a\theta}$ (d) None of these

(***c***) We have, $\dfrac{dy}{dx} = \tan\theta$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \sec^2\theta\,\frac{d\theta}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \sec^2\theta\left(\frac{1}{a\theta\cos\theta}\right) \qquad \left[\because \frac{dx}{d\theta} = a\theta\cos\theta\right]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sec^3\theta}{a\theta}$$

43. If $y = x\ln x + xe^x$, then what is the value of $\dfrac{dy}{dx}$ at $x = 1$?

(a) $1 + e$ (b) $1 - e$
(c) $1 + 2e$ (d) None of these

(***c***) Given, $y = x\log x + xe^x$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = x\cdot\frac{1}{x} + \log x + xe^x + e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 + \log x + xe^x + e^x$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=1} = 1 + \log 1 + 1\cdot e^1 + e^1$$

$$= 1 + 2e \qquad [\because \log 1 = 0]$$

Directions (Q. Nos. 44-46) *Read the following information carefully and answer the questions given below.*
Let $f(x) = ax^2 + bx + c$ *such that* $f(1) = f(-1)$ *and a, b, c are in arithmetic progression.*

44. What is the value of b?
(a) –1
(b) 0
(c) 1
(d) Cannot be determined due to insufficient data

(b) We have, $f(x) = ax^2 + bx + c$
$\therefore \quad f(1) = a + b + c$
and $\quad f(-1) = a - b + c$
$\because \quad f(1) = f(-1)$
$\Rightarrow \quad a + b + c = a - b + c \Rightarrow b = 0$

45. $f'(a), f'(b)$ and $f'(c)$ are in
(a) AP
(b) GP
(c) HP
(d) arithmetic-geometric progression

(a) We have,
$f'(x) = 2ax + b$
$\therefore \quad f'(a) = 2a^2, f'(b) = 2ab = 0$
and $\quad f'(c) = 2ac \quad [\because b = 0]$
$\therefore \quad f'(a) = 2a^2$
$f'(b) = 0$
and $\quad f'(c) = -2a^2$
$[\because 2b = a + c \Rightarrow c = -a]$
Hence, $f'(a), f'(b)$ and $f'(c)$ are in AP.

46. $f''(a), f''(b)$ and $f''(c)$ are
(a) in AP only
(b) in GP only
(c) in both AP and GP
(d) Neither in AP nor in GP

(c) $f''(x) = 2a$
$\therefore \quad f''(a) = f''(b) = f''(c)$
Hence, $f''(a), f''(b)$ and $f''(c)$ are in both AP and GP.

2014 (I)

47. If $z = fof(x)$ where $f(x) = x^2$, then what is $\frac{dz}{dx}$ equal to?
(a) x^3 (b) $2x^3$
(c) $4x^3$ (d) $4x^2$

(c) Given that, $z = fof(x)$ and $f(x) = x^2$
$\Rightarrow \quad z = f\{f(x)\}$
$\Rightarrow \quad z = f(x^2)$
$\Rightarrow \quad z = (x^2)^2 = x^4$
On differentiating w.r.t. x, we get
$\frac{dz}{dx} = 4x^3$

48. What is the derivative of $\sqrt{\frac{1+\cos x}{1-\cos x}}$?
(a) $\frac{1}{2}\sec^2\frac{x}{2}$ (b) $-\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$
(c) $-\operatorname{cosec}^2\frac{x}{2}$ (d) None of these

(b) Let $y = \sqrt{\frac{1+\cos x}{1-\cos x}}$
$\Rightarrow \quad y = \sqrt{\frac{(1+\cos x)}{(1-\cos x)} \cdot \frac{(1+\cos x)}{(1+\cos x)}}$
[by rationalisation]
$\Rightarrow \quad y = \sqrt{\frac{(1+\cos x)^2}{(1-\cos^2 x)}}$
$\Rightarrow \quad y = \frac{1+\cos x}{\sqrt{\sin^2 x}} = \frac{1+\cos x}{\sin x}$
$\Rightarrow \quad y = \frac{1+2\cos^2\frac{x}{2}-1}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}} = \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}$
$\therefore \quad y = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \cot\frac{x}{2}$
On differentiating w.r.t. x, we get
$\frac{dy}{dx} = -\operatorname{cosec}^2\frac{x}{2}\cdot\frac{d}{dx}\left(\frac{x}{2}\right)$
$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$

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APPLICATION OF DERIVATIVES

2019 (II)

Directions (Q. Nos. (1-3) *Read the following information and answer the three items that follow :*

A curve $y = me^{mx}$ where $m > 0$ intersects Y-axis at a point P.

1. What is the slope of the curve at the point of intersection P?

(a) m (b) m^2 (c) $2m$ (d) $2m^2$

2. How much angle does the tangent at P make with y-axis?

(a) $\tan^{-1} m^2$ (b) $\cot^{-1}(1 + m^2)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{1+m^4}}\right)$ (d) $\sec^{-1}\sqrt{1+m^4}$

3. What is the equation of tangent to the curve at P?

(a) $y = mx + m$ (b) $y = -mx + 2m$

(c) $y = m^2x + 2m$ (d) $y = m^2x + m$

Solutions (Q. Nos. 1-3)

Given curve $y = me^{mx}$ where $m > 0$

$\because$ Curve intersects Y-axis at a point P, then $x = 0$

$\therefore \quad y = me^0 \Rightarrow y = m$

$\therefore$ Point $P(0, m)$

Now, differentiation w.r.t x of given curve,

$$\frac{dy}{dx} = m.e^{mx}.m$$

$$\frac{dy}{dx} = m^2e^{mx}$$

at point $P(0, m)$, $\frac{dy}{dx} = m^2e^0 = m^2$

1. (***b***) Slope of the curve at the point $P(0, m)$

$= \left(\frac{dy}{dx}\right)$ at point $P(0, m) = m^2$

2. (***c***) Let the tangent makes the angle with X-axis be θ, then

$\tan\theta = \left(\frac{dy}{dx}\right)$ at $P(0, m)$

$\Rightarrow \tan\theta = m^2 \Rightarrow \theta = \tan^{-1} m^2$

Now, the tangent will make the angle with Y-axis

$= \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1} m^2$

$= \cot^{-1} m^2 \left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$

$= \sin^{-1}\left(\frac{1}{\sqrt{1+m^4}}\right)$

$\left[\because \cot^{-1} x = \sin\left(\frac{1}{\sqrt{1+m^2}}\right)\right]$

3. (***d***) Equation of tangent to curve at P is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}(x - x_1)$$

$\Rightarrow y - m = m^2(x - 0)$

$\Rightarrow \quad y = m^2x + m$

4. If $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 7$ increases in the interval T and decreases in the interval S, then which one of the following is correct?

(a) $T = (-\infty, 2), \cup (3, \infty)$ and $S = (2, 3)$

(b) $T = \phi$ and $S = (-\infty, \infty)$

(c) $T = (-\infty, \infty)$ and $S = \phi$

(d) $T = (2, 3)$ and $S = (-\infty, 2) \cup (3, \infty)$

(***a***) Given, $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 7$

Differentiating w.r.t x, we get

$$f'(x) = \frac{3x^2}{3} - \frac{5}{2}.2x + 6 = x^2 - 5x + 6$$

$\because f(x)$ is increases in interval T,

$\therefore \quad f'(x) \geq 0$

$\Rightarrow \quad x^2 - 5x + 6 \geq 0$

$\Rightarrow \quad x^2 - 3x - 2x + 6 \geq 0$

$\Rightarrow \quad x(x - 3) - 2(x - 3) \geq 0$

$\Rightarrow (x - 3)(x - 2) \geq 0 \Rightarrow x \leq 2, x \geq 3$

$\therefore \quad T = (-\infty, 2) \cup (3, \infty)$

Again, $f(x)$ is decreases in interval S.

$\therefore \quad f'(x) \leq 0$

$\Rightarrow \quad x^2 - 5x + 6 \leq 0$

$\Rightarrow x^2 - 3x - 2x + 6 \leq 0$

$\Rightarrow x(x - 3) - 2(x - 3) \leq 0$

$\Rightarrow \quad (x - 3)(x - 2) \leq 0$

$\Rightarrow \quad 2 < x < 3 \Rightarrow x \in (2, 3)$

$\therefore \quad S = (2, 3)$

Directions Q. Nos. (5 and 6) *Read the following information and answer the two items that follow : consider the function*

$$f(x) = 3x^4 - 20x^3 - 12x^2 + 288x + 1$$

5. In which one of the following intervals is the function increasing?

(a) $(-2, 3)$ (b) $(3, 4)$

(c) $(-3, -2)$ (d) $(-4, -3)$

6. In which one of the following intervals is the function decreasing?

(a) $(-2, 3)$ (b) $(3, 4)$ (c) $(4, 6)$ (d) $(6, 9)$

Solutions (Q. Nos. 5 and 6)

Given function,

$f(x) = 3x^4 - 20x^3 - 12x^2 + 288x + 1$

Differentiation w.r.t. x, we get

$f'(x) = 12x^3 - 60x^2 - 24x + 288$

5. (***b***) $f(x)$ is increasing, if $f'(x) \geq 0$

$12x^3 - 60x^2 - 24x + 288 \geq 0$

$\Rightarrow \quad x^3 - 5x^2 - 2x + 24 \geq 0$

$\Rightarrow \quad (x + 2)(x^2 - 7x + 12) \geq 0$

$\Rightarrow \quad (x + 2)(x - 3)(x - 4) \geq 0$

$\therefore \quad x \leq -2, x \geq 3, x \geq 4$

Hence, $f(x)$ is increasing the interval $(3, 4)$.

6. (***a***) $f(x)$ is decreasing, if $f'(x) \le 0$

$12x^3 - 60x^2 - 24x + 288 \le 0$

$\Rightarrow \quad x^3 - 5x^2 - 2x + 24 \le 0$

$\Rightarrow \quad (x+2)(x^2 - 7x + 12) \le 0$

$\Rightarrow \quad (x+2)(x-3)(x-4) \le 0$

$\therefore \quad x \ge -2,\ x \le 3,\ x \le 4$

Hence $f(x)$ is decreasing the interval $(-2, 3)$.

2019 (I)

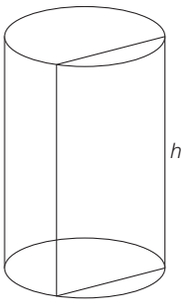
7. A given quantity of metal is to be cast into a half cylinder (i.e. with a rectangular base and semicircular ends). If the total surface area is to be minimum, then the ratio of the height of the half cylinder to the diameter of the semicircular ends is

(a) $\pi : (\pi + 2)$ (b) $(\pi + 2) : \pi$

(c) 1 : 1 (d) None of these

(***a***) Let r be the radius and h be the height of the half cylinder,

Then, surface area, $S = \pi rh + \pi r^2 + 2rh$



$\therefore \quad \frac{dS}{dr} = \pi h + 2\pi r + 2h$

On putting $\frac{dS}{dr} = 0$

$\Rightarrow \quad 2r = -\frac{(\pi h + 2h)}{\pi}$

$\Rightarrow \quad 2r = \frac{-h(\pi + 2)}{\pi}$

$\Rightarrow \quad \frac{2r}{h} = \frac{-(\pi+2)}{\pi} \Rightarrow \frac{h}{2r} = \frac{-\pi}{\pi + 2}$

Neglecting – sign as r and h can not be negative.

$\therefore \quad \frac{h}{2r} = \frac{\pi}{\pi + 2}$

8. The minimum distance from the point $(4, 2)$ to $y^2 = 8x$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 2 (d) $3\sqrt{2}$

(***b***) Let (x, y) be any point on the curve $y^2 = 8x$.

Then, the distance between (x, y) and $(4, 2)$ is

$D^2 = (x-4)^2 + (y-2)^2$

$\Rightarrow \quad D^2 = \left(\frac{y^2}{8} - 4\right)^2 + (y-2)^2 \quad ...(i)$

$[\because y^2 = 8x]$

$\Rightarrow \frac{dD^2}{dy} = 2\left(\frac{y^2}{8} - 4\right)\left(\frac{2y}{8}\right) + 2(y-2)$

$= 2\left(\frac{2y^3}{64} - 4 \times \frac{2y}{8}\right) + 2(y-2)$

$= \frac{y^3}{16} - 2y + 2y - 4 = \frac{y^3}{16} - 4$

$\Rightarrow \frac{d^2D^2}{d^2y} = \frac{3y^2}{16}$

On putting $\frac{dD^2}{dy} = 0 \Rightarrow \frac{y^3}{16} - 4 = 0$

$\Rightarrow \quad y^3 = 64 \Rightarrow y = 4$

At $y = 4, \frac{d^2D^2}{d^2y} > 0$

So, it is point of minima.

$\therefore$ Minimum $D = \sqrt{(2-4)^2 + (4-2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

9. What is the minimum value of $a^2x + b^2y$ where $xy = c^2$?

(a) abc (b) $2abc$

(c) $3abc$ (d) $4abc$

(***b***) Let $z = a^2x + b^2y \quad ...(i)$

Since, $xy = c^2 \Rightarrow y = \frac{c^2}{x}$

On putting $y = \frac{c^2}{x}$ in Eq. (i), we get

$\Rightarrow \quad z = a^2x + b^2\left(\frac{c^2}{x}\right) \quad ...(ii)$

On differentiability Eq. (ii) both sides, we get

$\frac{dz}{dx} = a^2 - \frac{b^2c^2}{x^2} \quad ...(iii)$

$\Rightarrow \quad \frac{d^2z}{dx^2} = \frac{2b^2c^2}{x^3} \quad ...(iv)$

For maxima and minima, we put $\frac{dz}{dx} = 0$

$\therefore \quad a^2 - \frac{b^2c^2}{x^2} = 0$

$\Rightarrow \quad \frac{b^2c^2}{x^2} = a^2$

$\Rightarrow \quad x = \pm\frac{bc}{a}$

At $x = \frac{bc}{a}, \frac{d^2z}{dx^2} = \frac{2a^3}{bc} > 0$

$\Rightarrow$ Gives minimum value

At $x = \frac{-bc}{a}, \frac{d^2z}{dx^2} = -\frac{2a^3}{bc} < 0$

Gives maximum value

$\therefore$ Minimum value of z at $x = \frac{bc}{a}$ is

$abc + abc = 2abc$.

2018 (II)

10. Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan\theta$. Then, the value of θ in the interval $(0, \pi)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(***b***) We have, $y = \cos^{-1}(\sin x)$

Differentiation w.r.t. x, we get

$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \sin^2 x}}\cos x$

$= \frac{-\cos x}{\cos x} \quad [\because \sin^2 x + \cos^2 x = 1]$

$\frac{dy}{dx} = -1$

Slope of the curve $= \tan\theta$

$\therefore \quad \tan\theta = -1$

$\Rightarrow \quad \tan\theta = -\tan\frac{\pi}{4}$

$\Rightarrow \quad \tan\theta = \tan\left(\pi - \frac{\pi}{4}\right) \quad [\because \theta \in (0, \pi)]$

$\therefore \quad \theta = \frac{3\pi}{4}$

11. Which one of the following is correct in respect of the function $f(x) = x\sin x + \cos x + \frac{1}{2}\cos^2 x$?

(a) It is increasing in the interval $\left(0, \frac{\pi}{2}\right)$

(b) It remains constant in the interval $\left(0, \frac{\pi}{2}\right)$

(c) It is decreasing in the interval $\left(0, \frac{\pi}{2}\right)$

(d) It is decreasing in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(***a***) We have,

$f(x) = x\sin x + \cos x + \frac{1}{2}\cos^2 x$

$f'(x) = x(\cos x) + \sin x - \sin x + \frac{1}{2}\cdot 2\cos x(-\sin x)$

$f'(x) = x\cos x - \sin x\cos x$

For checking, we put

$x = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$

$f'(x) = \frac{\pi}{4}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{4}$

$= \frac{\pi}{4}\cdot\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$= \frac{\pi}{4\sqrt{2}} - \frac{1}{2} > 0$

So, $f(x)$ is increasing in the interval $\left(0, \frac{\pi}{2}\right)$.

12. In which one of the following intervals is the function $f(x) = x^2 - 5x + 6$ decreasing?

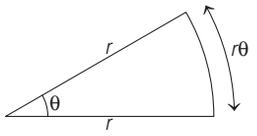
(a) $(-\infty, 2]$ (b) $[3, \infty)$
(c) $(-\infty, \infty)$ (d) $(2, 3)$

(**a**) We have,
$f(x) = x^2 - 5x + 6$
$f'(x) = 2x - 5$
For decreasing $f'(x) < 0$
$2x - 5 < 0 \Rightarrow x < \frac{5}{2} \Rightarrow x < 2.5$
$\therefore \quad x \in (-\infty, 2.5)$ or $x \in (-\infty, 2)$

13. A flower-bed in the form of a sector has been fenced by a wire of 40 m length. If the flower-bed has the greatest possible area, then what is the radius of the sector?

(a) 25 m (b) 20 m (c) 10 m (d) 5 m

(**c**) Let radius of sector be r and angle subtended at centre be θ.



$\therefore \quad r + r + r\theta = 40$
$\Rightarrow \quad 2r + r\theta = 40 \Rightarrow \theta = \frac{40 - 2r}{r}$
Area of the sector $= \frac{1}{2} r^2 \theta$
$A = \frac{1}{2} r^2 \left(\frac{40 - 2r}{r}\right)$
$A = \frac{1}{2} r (40 - 2r)$
$A = \frac{1}{2} (40r - 2r^2)$
On differentiating. w. r. t. r,
$\frac{dA}{dr} = \frac{1}{2} (40 - 4r)$
For maxima and minima
$\frac{dA}{dr} = 0$
$\Rightarrow \quad 40 - 4r = 0$
$\therefore \quad r = 10\text{m}$
Now, $\frac{d^2A}{dr^2} = -2$
So, A is greatest at $r = 10$ m

14. What is the minimum value of $[x(x-1)+1]^{\frac{1}{3}}$, where $0 \le x \le 1$?

(a) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$ (b) 1 (c) $\frac{1}{2}$ (d) $\left(\frac{3}{8}\right)^{1/3}$

(**a**) We have, $y = (x^2 - x + 1)^{\frac{1}{3}}$
$y^3 = x^2 - x + 1$
or $y' = x^2 - x + 1$
On differentiating w.r.t x
$\frac{dy'}{dx} = 2x - 1$
For maxima and minima
$\frac{dy'}{dx} = 0$
$\Rightarrow \quad 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$
Now, again differentiating w.r.t. x, we get
$\frac{d^2y'}{dx^2} = 2(+\text{ve})$
$\therefore y'$ is minimum at $x = \frac{1}{2}$
Minimum value of $y' = \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1$
$= \frac{1}{4} - \frac{1}{2} + 1$
$= \frac{1 - 2 + 4}{4} = \frac{3}{4}$
Hence, minimum value $= \left(\frac{3}{4}\right)^{1/3}$

2018 (I)

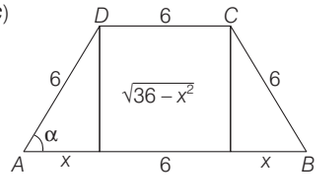
Directions (Q. Nos. 15-17) *Read the following information for the next three items that follow.*

Three sides of a trapezium are each equal to 6 cm. Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be the angle between a pair of adjacent sides.

15. If the area of the trapezium is the maximum possible, then what is α equal to ?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{5}$

(**c**)



$\therefore$ Area $= A = \frac{1}{2} (6 + 6 + 2x) \sqrt{36 - x^2}$
$= (6 + x) \sqrt{36 - x^2}$
$\therefore \quad \frac{d(A)}{dx} = \frac{d}{dx} [(6 + x) \sqrt{36 - x^2}]$
$= (6 + x) \left[\frac{-2x}{2\sqrt{36 - x^2}}\right] + \sqrt{36 - x^2}$
$= \sqrt{36 - x^2} - \frac{x(6 + x)}{\sqrt{36 - x^2}} = \frac{36 - 6x - 2x^2}{\sqrt{36 - x^2}}$
For maximum area,
$\frac{dA}{dx} = 0$
$\Rightarrow \quad 36 - 6x - 2x^2 = 0$
$\Rightarrow \quad 2x^2 + 6x - 36 = 0$
$\Rightarrow \quad x^2 + 3x - 18 = 0$
$\Rightarrow \quad x^2 + 6x - 3x - 18 = 0$
$\Rightarrow \quad x(x + 6) - 3(x + 6) = 0$
$\Rightarrow \quad (x + 6)(x - 3) = 0$
$\Rightarrow \quad x = 3, -6$
Again, on differentiating it $\frac{d^2A}{dx^2} =$
$\frac{(-6 - 4x)\sqrt{36 - x^2} - \frac{(36 - 6x - 2x^2)(-2x)}{2\sqrt{36 - x^2}}}{(36 - x^2)}$
At $x = 3$, $\frac{d^2A}{dx^2} = \frac{-18}{(27)^{1/2}}$
$\therefore \quad \frac{d^2A}{dx^2} < 0$
So, at $x = 3$, area is maximum.
Now, $\cos\alpha = \frac{x}{6} = \frac{3}{6} = \frac{1}{2}$
$\therefore \quad \cos\alpha = \cos\frac{\pi}{3} \Rightarrow \alpha = \frac{\pi}{3}$

16. If the area of the trapezium is maximum, what is the length of the fourth side?

(a) 8 cm (b) 9 cm (c) 10 cm (d) 12 cm

(**d**) So, fourth side $= x + 6 + x$
$= 3 + 6 + 3 = 12$

17. What is the maximum area of the trapezium?

(a) $36\sqrt{3}$ cm^2 (b) $30\sqrt{3}$ cm^2
(c) $27\sqrt{3}$ cm^2 (d) $24\sqrt{3}$ cm^2

(**c**) Maximum area
$= (6 + x) \sqrt{36 - x^2}$
$= (6 + 3) \sqrt{36 - 9}$
$= 9 \times \sqrt{27} = 9 \times \sqrt{27}$
$= 9 \times 3\sqrt{3} = 27\sqrt{3}$ cm^2

2017 (II)

18. Which one of the following is correct in respect of the function $f(x) = x(x-1)(x+1)$?

(a) The local maximum value is larger than local minimum value
(b) The local maximum value is smaller than local minimum value
(c) The function has no local maximum
(d) The function has no local minimum

(**a**) We have,
$f(x) = x(x-1)(x+1) = x^3 - x$
$\therefore \quad f'(x) = 3x^2 - 1$ and $f''(x) = 6x$
For maxima and minima, $f'(x) = 0$
$\Rightarrow \quad 3x^2 - 1 = 0 \Rightarrow x = \pm \frac{1}{\sqrt{3}}$
Now, $f''\left(\frac{1}{\sqrt{3}}\right) = \frac{6}{\sqrt{3}} > 0$

and $f''\left(\frac{-1}{\sqrt{3}}\right)=\frac{-6}{\sqrt{3}}<0$

$\therefore$ $f(x)$ is maxima at $x=\frac{-1}{\sqrt{3}}$ and minima at $x=\frac{1}{\sqrt{3}}$

$\therefore$ $f_{max}(x)=f\left(\frac{-1}{\sqrt{3}}\right)=\left(\frac{-1}{\sqrt{3}}\right)^3-\left(\frac{-1}{\sqrt{3}}\right)=\frac{2}{3\sqrt{3}}$

and $f_{min}(x)=f\left(\frac{1}{\sqrt{3}}\right)$

$=\left(\frac{1}{\sqrt{3}}\right)^3-\left(\frac{1}{\sqrt{3}}\right)=\frac{-2}{3\sqrt{3}}$

$\therefore$ $f_{max}>f_{min}$

19. The maximum value of $\frac{\ln x}{x}$ is

(a) e (b) $\frac{1}{e}$ (c) $\frac{2}{e}$ (d) 1

(**b**) Let, $f(x)=\frac{\ln x}{x}$

$\Rightarrow$ $f'(x)=\frac{\left(\frac{1}{x}\right)x-\ln x\cdot 1}{(x)^2}=\frac{1-\ln x}{x^2}$

Again, $f''(x)=\frac{(0-1/x)x^2-(1-\ln x)\cdot(2x)}{x^4}$

$=\frac{-x-2x+2x\ln x}{x^4}=\frac{-3x+2x\ln x}{x^4}$

$=\frac{2\ln x-3}{x^3}$

For maxima and minima,

$f'(x)=0 \Rightarrow \frac{1-\ln x}{x^2}=0$

$\Rightarrow$ $1-\ln x=0 \Rightarrow \ln x=1$

$\Rightarrow$ $x=e$

$\therefore$ $f''(e)=\frac{2\ln e-3}{e^3}=\frac{2-3}{e^3}$ [$\because \ln e=1$]

$=\frac{-1}{e^3}<0$

$\therefore$ At $x=e$, $f(x)$ has maximum value

$\therefore$ $f_{max}(x)=f(e)=\frac{\ln e}{e}=\frac{1}{e}$

20. A cylindrical jar without a lid has to be constructed using a given surface area of a metal sheet. If the capacity of the jar is to be maximum, then the diameter of the jar must be k times the height of the jar. The value of k is

(a) 1 (b) 2 (c) 3 (d) 4

(**b**) Let h and r be the height and radius of the cylindrical jar.

Let the area of the metal sheet be S and volume of jar be V. Then,

$S=2\pi rh+\pi r^2 \Rightarrow h=\frac{S-\pi r^2}{2\pi r}$ ...(i)

Again, $V=\pi r^2 h$

$=\pi r^2\left(\frac{s-\pi r^2}{2\pi r}\right)$ ...[from Eq. (i)]

$=\frac{1}{2}(Sr-\pi r^3)$

$\therefore$ $\frac{dV}{dr}=\frac{1}{2}(S-3\pi r^2)$

For maximum, $\frac{dV}{dr}=0$

$\therefore$ $\frac{1}{2}(S-3\pi r^2)=0 \Rightarrow S=3\pi r^2$

$\Rightarrow$ $2\pi rh+\pi r^2=3\pi r^2 \Rightarrow h=r$

Now, $\frac{d^2V}{dr^2}=-6\pi r$

So, V is maximum at $h=r$

$\therefore$ $2r=2h$

$\Rightarrow$ Diameter $=2\times$ Height

$\therefore$ $k=2$

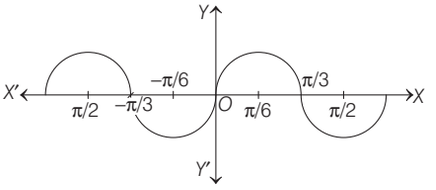
2017 (I)

21. What is the length of the longest interval in which the function $f(x)=3\sin x-4\sin^3 x$ is increasing?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

(**a**) Given,

$f(x)=3\sin x-4\sin^3 x=\sin 3x$



Graph of $f(x)=\sin 3x$

From above graph $f(x)=\sin 3x$ is increasing in the interval $\left[-\frac{\pi}{6},\frac{\pi}{6}\right]$

$\therefore$ Longest interval in which $y=\sin 3x$ is increasing $=\frac{\pi}{6}-\left(-\frac{\pi}{6}\right)=\frac{\pi}{3}$

22. Let $f(x)=x+\frac{1}{x}$, where $x\in(0,1)$. Then which one of the following is correct?

(a) $f(x)$ fluctuates in the interval
(b) $f(x)$ increases in the interval
(c) $f(x)$ decreases in the interval
(d) None of the above

(**c**) Given, $f(x)=x+\frac{1}{x}$

$\therefore f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}=\frac{(x+1)(x-1)}{x^2}$

[$\because a^2-b^2=(a+b)(a-b)$]

From question $x\in(0,1)$ and in the interval (0, 1)

$f'(x)<0$

$\therefore f(x)$ decreases when $x\in(0,1)$.

2016 (II)

23. Which one of the following statements is correct in respect of the function $f(x)=x^3\sin x$?

(a) It has local maximum at $x=0$
(b) It has local minimum at $x=0$
(c) It has neither maximum nor minimum at $x=0$
(d) It has maximum value 1

(**b**) Given, $f(x)=x^3\sin x$

$\Rightarrow$ $f'(x)=3x^2\sin x+x^3\cos x$

$=x^2(3\sin x+x\cos x)$

Now, $f'(x)=0$ gives

$x=0$, or $3\sin x+x\cos x=0$

$\Rightarrow$ $x=0$ is a critical point

$f''(x)=6x\sin x+3x^2\cos x+3x^2\cos x-x^3\sin x$

$=6x\sin x+6x^2\cos x-x^3\sin x$

Here, $f''(0)=0$

$\therefore$ We have to check for second derivative test

$\therefore f'''(x)=6\sin x+6x\cos x+12x\cos x-6x^2\sin x-3x^2\sin x-x^3\cos x$

$=6\sin x+18x\cos x-9x^2\sin x-x^3\cos x$

$f'''(0)=0$

$\therefore$ We have to check for higher order derivative test

$f^{iv}(x)=6\cos x+18\cos x-18x\sin x-18x\sin x-9x^2\cos x-3x^2\cos x+x^3\sin x$

$=24\cos x-36x\sin x-12x^2\cos x+x^3\sin x$

Here, $f^{iv}(0)=24$,

which is greater than zero.

[$\because f^{(n+1)}(c)>0$, when n is odd, then is local minimum]

$\therefore$ $f(x)$ has local minimum at $x=0$.

Directions (Q. Nos. 24 and 25)

$$\text{Let } f(x)=\begin{cases}3x^2+12x-1, & -1\le x\le 2\\ 37-x, & 2<x\le 3\end{cases}$$

24. Which of the following statements is/are correct?

I. $f(x)$ is increasing in the interval $\{-1,2\}$.

II. $f(x)$ is decreasing in the interval $\{2,3\}$.

Select the correct answer using the code given below.

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

25. Which of the following statements are correct?

I. $f(x)$ is continuous at, $x = 2$.

II. $f(x)$ attains greatest value at $x = 2$.

III. $f(x)$ is differentiable at $x = 2$.

Select the correct answer using the code given below.

(a) I and II (b) II and III

(c) I and III (d) I, II and III

Solutions (Q. Nos. 24 and 25)

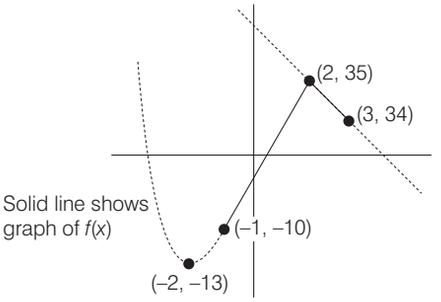
Given,

$$f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$$

$\because\ 3x^2 + 12x - 1 = 3(x^2 + 4x + 4) - 12 - 1$

$= 3(x + 2)^2 - 13$

Now, we can draw graph of $f(x)$ in which $(-2, -13)$ is vertex of $3x^2 + 12x - 1$



24. (**c**) From graph, $f(x) > 0$ for $x \in [-1, 2]$

$\therefore\ f(x)$ is increasing in the interval $[-1, 2]$ and $f(x) < 0$ for $x \in (2, 3]$

$\therefore f(x)$ is decreasing in the interval $(2, 3]$.

25. (**a**) $\because$ Graph of $f(x)$ does not break at $x = 2$.

$\therefore f(x)$ is continuous at $x = 2$.

$\because$ From graph, $f(x)$ attains maximum value at $x = 2$, which is 35.

Since, graph of $f(x)$ makes sharp edge at $x = 2$.

$\therefore f(x)$ is not differentiable at $x = 2$.

26. Let $f(x) = \begin{cases} \dfrac{e^x - 1}{x}, & x > 0 \\ 0, & x = 0 \end{cases}$ be a real valued function. Which one of the following statements is correct?

(a) $f(x)$ is a strictly decreasing function in $(0, x)$

(b) $f(x)$ is a strictly increasing function in $(0, x)$

(c) $f(x)$ is neither increasing nor decreasing in $(0, x)$

(d) $f(x)$ is not decreasing in $(0, x)$

(**b**) Given, $f(x) = \begin{cases} \dfrac{e^x - 1}{x}, & x > 0 \\ 0, & x = 0 \end{cases}$

$$\Rightarrow\ f'(x) = \frac{x\dfrac{d}{dx}(e^x - 1) - (e^x - 1)\dfrac{d}{dx}(x)}{x^2}$$

$$\Rightarrow\ f'(x) = \frac{x(e^x - 0) - (e^x - 1)}{x^2}$$

$$\Rightarrow f'(x) = \frac{xe^x - e^x + 1}{x^2}$$

$$\Rightarrow f'(x) = \frac{e^x (x - 1) + 1}{x^2}$$

Clearly, $f'(x) > 0, x > 0$

$\therefore f(x)$ is a strictly increasing function in $(0, x)$.

2016 (I)

Directions (Q. Nos. 27 and 28)

Consider the function $f(x) = |x - 1| + x^2$, *where* $x \in R$.

27. Which one of the following statements is correct?

(a) $f(x)$ is increasing in $\left(-\infty, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, \infty\right)$

(b) $f(x)$ is decreasing in $\left(-\infty, \frac{1}{2}\right)$ and increasing in $\left(\frac{1}{2}, \infty\right)$

(c) $f(x)$ is increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$

(d) $f(x)$ is decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$

28. Which one of the following statements is correct?

(a) $f(x)$ has local minima at more than one point in $(-\infty, \infty)$

(b) $f(x)$ has local maxima at more than one point in $(-\infty, \infty)$

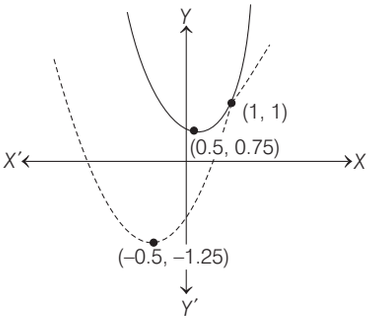
(c) $f(x)$ has local minima at one point only in $(-\infty, \infty)$

(d) $f(x)$ has neither maxima nor minima in $(-\infty, \infty)$

Solutions (Q. Nos. 27 and 28)

Given, $f(x) = |x - 1| + x^2$

$$\therefore\ f(x) = \begin{cases} x^2 + x - 1, & \text{for } x \ge 1 \\ x^2 - x + 1, & \text{for } x < 1 \end{cases}$$



27. (**b**) From the graph between $\left(-\infty, \frac{1}{2}\right)$.

$f(x)$ decreases when x increases.

$\therefore\ f(x)$ is decreasing in $\left(-\infty, \frac{1}{2}\right)$.

and from the graph between $\left(\frac{1}{2}, \infty\right)$.

$f(x)$ increases when x increases.

$\therefore\ f(x)$ is increasing in $\left(\frac{1}{2}, \infty\right)$.

28. (**c**) From the graph there is only one minimum point which is at point $\left(\frac{1}{2}, \frac{3}{4}\right)$.

$\left[\because \text{when } x = \frac{1}{2}, \text{ then } y = \left|\frac{1}{2} - 1\right| + \frac{1}{4} = \frac{3}{4}\right]$

$\therefore\ f(x)$ has local minima at one point only in $(-\infty, \infty)$.

2015 (II)

29. Consider the following statements

1. The function $f(x) = x^2 + 2\cos x$ is increasing in the interval $(0, \pi)$.
2. The function $f(x) = \ln(\sqrt{1 + x^2} - x)$ is decreasing in the interval $(-\infty, \infty)$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) 1. We have,

$f(x) = x^2 + 2\cos x$

On differentiating, we get

$f'(x) = 2x - 2\sin x$

Here, $f'(x) > 0$ in the interval $(0, \pi)$.

$\therefore$ $f(x)$ is increasing function.

2. We have,

$$f(x) = \ln(\sqrt{1+x^2} - x)$$

On differentiating, we get

$$f'(x) = \frac{1}{(\sqrt{1+x^2} - x)} \times \left(\frac{x}{\sqrt{1+x^2}} - 1\right)$$

Here, $f'(x) < 0$ in the interval $(-\infty, \infty)$.

$\therefore$ $f(x)$ is decreasing function.

30. Consider the following statements

1. $f(x) = \ln x$ is an increasing function on $(0, \infty)$.
2. $f(x) = e^x - x(\ln x)$ is an increasing function on $(1, \infty)$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) 1. $f(x) = \ln x$

$$\therefore \quad f'(x) = \frac{1}{x}$$

Here, $f'(x) > 0$ in the interval $(0, \infty)$.

$\therefore$ $f(x)$ is increasing function in $(0, \infty)$.

2. $f(x) = e^x - x(\ln x)$

$$f'(x) = e^x - \ln x - x \cdot \frac{1}{x} = e^x - \ln x - 1$$

Here, $f'(x) > 0$ in the interval $(1, \infty)$.

$\therefore$ $f(x)$ is increasing function in $(1, \infty)$.

Directions (Q. Nos. 31 and 32)

Consider the function $f(x) = \left(\frac{1}{x}\right)^{2x^2}$, *where* $x > 0$.

31. At what value of x does the function attain maximum value?

(a) e (b) $\sqrt{e}$ (c) $\frac{1}{\sqrt{e}}$ (d) $\frac{1}{e}$

(c) $f(x) = \left(\frac{1}{x}\right)^{2x^2} = x^{-2x^2} = e^{-2x^2 \log x}$

$$f'(x) = e^{-2x^2 \log x}\left[-2x^2 \cdot \frac{1}{x} + \log x\,(-4x)\right]$$

$\Rightarrow f'(x) = f(x)(-2x - 4x\log x)$

For maxima, $f'(x) = 0$

$\Rightarrow x^{-2x^2}(-2x - 4x\log x) = 0$

$\Rightarrow -2x(1 + 2\log x) = 0$

$\Rightarrow x = 0$ or $\log x = -\frac{1}{2}$

$\Rightarrow x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

Now, $f''(x) = f'(x)(-2x - 4x\log x) + f(x)\left(-2 - 4x\frac{1}{x} - 4\log x\right)$

$f''(x)$ is negative at $x = \frac{1}{\sqrt{e}}$

Hence, $x = \frac{1}{\sqrt{e}}$ is the point of maxima.

32. The maximum value of the function is

(a) e (b) $e^{\frac{2}{e}}$ (c) $e^{\frac{1}{e}}$ (d) $\frac{1}{e}$

(c) From question 25, $f(x)$ is maximum at $x = \frac{1}{\sqrt{e}}$

Maximum value of the function

$$= \left(\frac{1}{\frac{1}{\sqrt{e}}}\right)^{2\left(\frac{1}{\sqrt{e}}\right)^2}$$

$$= (\sqrt{e})^{2/e} = [(\sqrt{e})^2]^{1/e} = (e)^{1/e}$$

Directions (Q. Nos. 33 and 34)

Consider the function $f(x) = -2x^3 - 9x^2 - 12x + 1$.

33. The function $f(x)$ is an increasing function in the interval

(a) $(-2, -1)$ (b) $(-\infty, -2)$
(c) $(-1, 2)$ (d) $(-1, \infty)$

(a) Given, $f(x) = -2x^3 - 9x^2 - 12x + 1$

On differentiating both sides w.r.t. x, we get

$$f'(x) = -6x^2 - 18x - 12$$

For $f(x)$ to be increasing function,

$$f'(x) > 0$$

$\therefore \quad -6x^2 - 18x - 12 > 0$

$\Rightarrow \quad x^2 + 3x + 2 < 0$

$\Rightarrow \quad (x + 2)(x + 1) < 0$

$\therefore \quad -2 < x < -1$

Hence, $f(x)$ is increasing in $(-2, -1)$.

34. The function $f(x)$ is a decreasing function in the interval

(a) $(-2, -1)$
(b) $(-\infty, -2)$
(c) $(-1, \infty)$
(d) $(-\infty, -2) \cup (-1, \infty)$

(d) Given, $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow \quad f'(x) = -6x^2 - 18x - 12$

For $f(x)$ to be decreasing function, $f'(x) < 0$

$\therefore \quad -6x^2 - 18x - 12 < 0$

$\Rightarrow \quad x^2 + 3x + 2 > 0$

$\Rightarrow \quad (x + 2)(x + 1) > 0$

$\therefore \quad x \in (-\infty, -2) \cup (-1, \infty)$

2015 (I)

35. Consider the following statements

I. $y = \frac{e^x + e^{-x}}{2}$ is an increasing function on $[0, \infty)$.

II. $y = \frac{e^x - e^{-x}}{2}$ is an increasing function on $(-\infty, \infty)$.

Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(c) I. Let $f(x) = \frac{e^x + e^{-x}}{2}$

$$\Rightarrow f'(x) = \frac{e^x - e^{-x}}{2}$$

$$= \frac{1}{2}\left(e^x - \frac{1}{e^x}\right) = \frac{1}{2}\left(\frac{e^{2x} - 1}{e^x}\right) \quad \ldots(i)$$

Now, for $x \geq 0$, we have $2x \geq 0$

$$= e^{2x} \geq e^0 = e^{2x} \geq 1$$

[$\because e^x$ is an increasing function]

Also, for $x \geq 0$, $e^x \geq 1$

From Eq. (i), we have

$$f'(x) = \frac{1}{2}\left(\frac{e^{2x} - 1}{e^x}\right) \geq 0$$

So, $f(x)$ is increasing function on $[0, \infty)$.

II. Let $g(x) = \frac{e^x - e^{-x}}{2}$

$$\Rightarrow g'(x) = \frac{e^x + e^{-x}}{2} > 0$$

[$\because e^x$ and e^{-x} both are greater than zero in $(-\infty, \infty)$]

So, $g(x)$ is an increasing function on $(-\infty, \infty)$.

Hence, both statements are true.

Directions (Q. Nos. 36 and 37)

Consider the function $f(x) = \frac{x^2 - 1}{x^2 + 1}$, where $x \in R$.

36. At what value of x does $f(x)$ attain minimum value?

(a) –1 (b) 0 (c) 1 (d) 2

(b) We have, $f(x) = \frac{x^2 - 1}{x^2 + 1}$

$$\Rightarrow f'(x) = \frac{(x^2 + 1)(2x) - (x^2 - 1)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x(x^2 + 1 - x^2 + 1)}{(x^2 + 1)^2}$$

$$= \frac{2x(2)}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2}$$

For critical points, put $f'(x) = 0$

$\Rightarrow \frac{4x}{(x^2 + 1)^2} = 0 \Rightarrow 4x = 0 \therefore x = 0$

37. What is the minimum value of $f(x)$?

(a) 0 (b) $\frac{1}{2}$

(c) –1 (d) 2

(**c**) Here, $x = 0$ is the only critical point which could possible be the point of minima.

Note that for values close to 0 and the right of 0, $f'(x) > 0$ and for values close to 0 and the left of 0, $f'(x) < 0$.

Therefore, by first derivative test, $x = 1$ is a point of minima and the minimum value of $f(x)$ is given by

$$f(0) = \frac{0-1}{0+1} = -1$$

Directions (Q. Nos. 38 and 39)

Consider the function

$f(x) = 0.75x^4 - x^3 - 9x^2 + 7.$

38. What is the maximum value of the function?

(a) 1 (b) 3

(c) 7 (d) 9

39. Consider the following statements

I. The function attains local minima at $x = -2$ and $x = 3$.

II. The function increases in the interval $(-2, 0)$.

Which of the above statements is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

Solutions (Q. Nos. 38 and 39)

We have,

$f(x) = 0.75x^4 - x^3 - 9x^2 + 7$

$\Rightarrow \quad f'(x) = 3x^3 - 3x^2 - 18x$

$\Rightarrow \quad f'(x) = 3x(x^2 - x - 6)$

$= 3x(x-3)(x+2)$

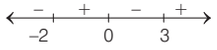
For critical points, put $f'(x) = 0$

$\Rightarrow \quad 3x(x-3)(x+2) = 0$

$\Rightarrow \quad x = 0,\ x = 3 \text{ or } x = -2$

Thus, there are only three critical points which could possible be the point of local maxima or local minima.

Now, the sign of $f'(x)$ is given by



Clearly, –2 and 3 are the points of local minima and 0 is the point of local maxima.

38. (*c*) The maximum value of the function is given by $f(0) = 7$

39. (*c*) Both statements are true.

2014 (II)

Directions (Q. Nos. 40 and 41) *Read the following information carefully and answer the questions given below.*

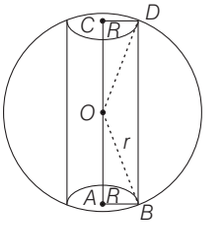
A cylinder is inscribed in a sphere of radius r.

40. What is the height of the cylinder of maximum volume?

(a) $\frac{2r}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $2r$ (d) $\sqrt{3}r$

(**a**) Let h be the height, R be the radius and V be the volume of cylinder.

In ΔOAB, we have



$$r^2 = R^2 + \left(\frac{h}{2}\right)^2 \quad \text{...(i)}$$

$$\left[\because OA = \frac{h}{2} \text{ as } \Delta OAB \cong \Delta OCD\right]$$

Clearly, $V = \pi R^2 h$

$\Rightarrow \quad V(h) = \pi\left(r^2 - \frac{h^2}{4}\right)h$ [using Eq. (i)]

$\Rightarrow \quad V(h) = \pi\left(r^2 h - \frac{h^3}{4}\right)$

$\Rightarrow \quad V'(h) = \pi\left(r^2 - \frac{3h^2}{4}\right)$ ...(ii)

For maximum, put $V'(h) = 0$

$\Rightarrow \quad r^2 = \frac{3h^2}{4} \quad \Rightarrow \quad h^2 = \frac{4r^2}{3}$

$\Rightarrow \quad h = \frac{2r}{\sqrt{3}} \quad [\because h > 0]$

Again, differentiating Eq. (ii) w.r.t. h, we get

$$V''(h) = \pi\left(\frac{-6h}{4}\right)$$

$\Rightarrow \quad V''\left(\frac{2r}{\sqrt{3}}\right) = \pi\left(\frac{-6}{4} \times \frac{2r}{\sqrt{3}}\right) < 0$

Thus, the volume is maximum when $h = \frac{2r}{\sqrt{3}}$.

41. What is the radius of the cylinder of maximum volume?

(a) $\frac{2r}{\sqrt{3}}$ (b) $\frac{\sqrt{2}r}{\sqrt{3}}$ (c) r (d) $\sqrt{3}r$

(**b**) Clearly, volume of cylinder is maximum when $h = \frac{2r}{\sqrt{3}}$.

By using the relation $r^2 = R^2 + \left(\frac{h}{2}\right)^2$, we have

$$R^2 = r^2 - \frac{h^2}{4} = r^2 - \frac{4r^2}{12} = \frac{8r^2}{12} = \frac{2r^2}{3}$$

$\therefore \quad R = \sqrt{\frac{2r^2}{3}} = \frac{\sqrt{2}r}{\sqrt{3}} \quad [\because R > 0]$

Directions (Q. Nos. 42 and 43) *Read the following information carefully and answer the questions given below.*

A rectangular box is to be made from a sheet of size $24'' \times 9''$ by cutting out identical squares of side length x from the four corners and turning up the sides.

42. What is the value of x for which the volume is maximum?

(a) 1 inch (b) 1.5 inch

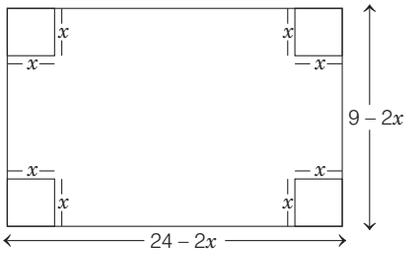
(c) 2 inch (d) 2.5 inch

43. What is the maximum volume of the box?

(a) 200 cu inch (b) 400 cu inch

(c) 100 cu inch (d) None of these

Solutions (Q. Nos. 42-43) Given, a rectangular box is to be made from a sheet of size $24'' \times 9''$ by cutting out identical square of side length x from the four corners.



Clearly, the length of rectangular box $= 24 - 2x$

and the width of rectangular box $= 9 - 2x$

42. (*c*) Let V be the volume of the box.

$\therefore \quad V = (24 - 2x)\cdot(9 - 2x)\cdot x$

$[\because \text{height of box} = x \text{ inch}]$

$\Rightarrow \quad = (216 - 48x - 18x + 4x^2)\cdot x$

$V(x) = 4x^3 - 66x^2 + 216x$

$\Rightarrow \quad V'(x) = 12x^2 - 132x + 216$

For maximum, put $V'(x) = 0$

$\Rightarrow \quad 12x^2 - 132x + 216 = 0$

$\Rightarrow \quad x^2 - 11x + 18 = 0$

$\Rightarrow \quad (x-9)(x-2) = 0 \Rightarrow x = 9 \text{ or } x = 2$

Now, $V''(x) = 24x - 132$

$\therefore \quad V''(9) = 216 - 132 = 84 > 0$

and $V''(2) = 48 - 132 = -84 < 0$

Thus, volume is maximum when $x = 2$ inch.

43. (***a***) Maximum volume of the box
$= (24 - 4)(9 - 4) \cdot 2$
$= 20 \times 5 \times 2 = 200$ cu inch

2014 (I)

Directions (Q. Nos. 44 and 45)
Consider the function

$$f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$$

44. What is the maximum value of the function?
(a) 1/2 (b) 1/3 (c) 2 (d) 3

45. What is the minimum value of the function?
(a) 1/2 (b) 1/3 (c) 2 (d) 3

Solutions (Q. Nos. 44-45)

Given function, $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$

Now,

$$f'(x) = \frac{\left[(x^2 + x + 1)\frac{d}{dx}(x^2 - x + 1) - (x^2 - x + 1)\frac{d}{dx}(x^2 + x + 1)\right]}{(x^2 + x + 1)^2}$$

$$\Rightarrow f'(x) = \frac{\left[(x^2 + x + 1)(2x - 1) - (x^2 - x + 1)(2x + 1)\right]}{(x^2 + x + 1)^2}$$

$$\Rightarrow f'(x) = \frac{\left[2x^3 + 2x^2 + 2x - x^2 - x - 1 - 2x^3 + 2x^2 - 2x - x^2 + x - 1\right]}{(x^2 + x + 1)^2}$$

$$\Rightarrow f'(x) = \frac{2x^2 - 2}{(x^2 + x + 1)^2}$$

For maximum or minimum value of $f(x)$, put $f'(x) = 0$

$$\Rightarrow \quad \frac{2(x^2 - 1)}{(x^2 + x + 1)^2} = 0 \Rightarrow x^2 - 1 = 0$$

$$\therefore \quad x = \pm 1$$

Again,

$$f''(x) = \frac{(x^2 + x + 1)(4x) - (2x^2 - 2)}{2(x^2 + x + 1)(2x + 1)}$$

$$= \frac{4(x^2 + x + 1)[x - (x^2 - 1)(2x + 1)]}{(x^2 + x + 1)^4}$$

$$= \frac{4(x - 2x^3 + 2x - x^2 + 1)}{(x^2 + x + 1)^3}$$

$$= \frac{4 \times (-2x^3 - x^2 + 3x + 1)}{(x^2 + x + 1)^3}$$

At $x = 1$,

$$f''(1) = \frac{4(-2 - 1 + 3 + 1)}{(1 + 1 + 1)^3} = \frac{4}{27} > 0$$

[minimum]

So, function $f(x)$ is minimum at $x = 1$.

At $x = -1$,

$$f''(-1) = \frac{4(2 - 1 - 3 + 1)}{(1 - 1 + 1)3}$$

$$= \frac{4 \times (-1)}{(1)^3} = -4 < 0 \text{ [maximum]}$$

So, function $f(x)$ is maximum at $x = -1$.

44. (***d***) Maximum value of the function at $x = -1$,is

$$f(-1) = \frac{1 + 1 + 1}{1 - 1 + 1} = 3$$

45. (***b***) Minimum value of the function at $x = 1$, is

$$f(1) = \frac{1 - 1 + 1}{1 + 1 + 1} = \frac{1}{3}$$

46. What is the slope of the tangent to the curve $y = \sin^{-1}(\sin^2 x)$ at $x = 0$?
(a) 0 (b) 1
(c) 2 (d) None of these

(***a***) Given curve,

$$y = \sin^{-1}(\sin^2 x)$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (\sin^2 x)^2}} \cdot \frac{d}{dx}(\sin^2 x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sin x \cdot \cos x}{\sqrt{1 - \sin^4 x}} \Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sqrt{1 - \sin^4 x}}$$

$$\left(\frac{dy}{dx}\right)_{x = 0} = \frac{\sin 0}{\sqrt{1 - \sin 0}} = \frac{0}{\sqrt{1 - 0}} = 0$$

So, slope of the tangent to the given curve $= \frac{dy}{dx} = 0$

Directions (Q. Nos. 47 and 48)
Consider the curve $y = e^{2x}$.

47. What is the slope of the tangent to the curve at (0, 1)?
(a) 0 (b) 1 (c) 2 (d) 4

(***c***) We know that,
Slope of the tangent to the given curve = Slope of the given curve
Given curve, $y = e^{2x}$
On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 2e^{2x}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(0, 1)} = 2 \cdot e^0 = 2 \times 1 = 2$$

$\therefore$ Slope of tangent of given curve

$$= \frac{dy}{dx} = 2$$

48. Where does the tangent to the curve at (0, 1) meet the X-axis?
(a) (1, 0) (b) (2, 0)
(c) (–1 / 2, 0) (d) (1/2, 0)

(***c***) Equation of tangent to the curve $y = e^{2x}$ at (0, 1) is

$$(y - 1) = \left(\frac{dy}{dx}\right)_{(0, 1)} (x - 0)$$

$$\Rightarrow \quad (y - 1) = 2(x - 0)$$

$$\Rightarrow \quad y - 1 = 2x$$

$$\therefore \quad y = 2x + 1$$

Since, the tangent meets X-axis.
So, put $y = 0$ in equation of tangent, we get

$$0 = 2x + 1$$

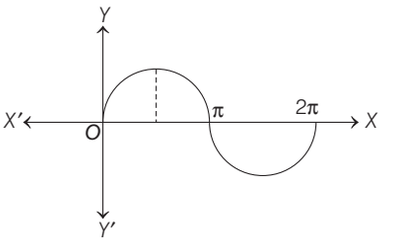
$$\Rightarrow \quad x = -1/2$$

Hence, at $\left(-\frac{1}{2}, 0\right)$, the tangent to the curve at (0, 1) meet the X-axis.

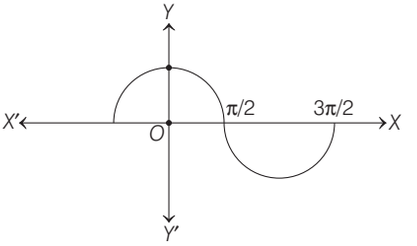
49. Consider the following statements
I. The function $f(x) = \sin x$ decreases on the interval $(0, \pi/2)$.
II. The function $f(x) = \cos x$ increases on the interval $(0, \pi/2)$.

Which of the above statement(s) is/are correct?
(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(***d***) I. Given function, $f(x) = \sin x$
Graph of $\sin x$



From the graph of $\sin x$, we observe that $f(x)$ increases on the interval $(0, \pi/2)$.
II. Given function, $f(x) = \cos x$
Graph of $\cos x$



From the graph of $\cos x$, we observe that $f(x)$ decreases on the interval $(0, \pi/2)$.

23

INDEFINITE INTEGRALS

2019 (II)

1. What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x-1)}{2} + c$

(b) $2\tan^{-1}(2x-1) + c$

(c) $\frac{\tan^{-1}(2x+1)}{2} + c$

(d) $\tan^{-1}(2x-1) + c$

(d) Let $I = \int \frac{dx}{2x^2 - 2x + 1}$

$= \frac{1}{2}\int \frac{dx}{x^2 - x + \frac{1}{2}}$

$= \frac{1}{2}\int \frac{dx}{x^2 - x + \frac{1}{4} - \frac{1}{4} + \frac{1}{2}}$

$= \frac{1}{2}\int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}}$

$= \frac{1}{2}\int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \frac{1}{2}.2\tan^{-1}\left[\frac{\left(x - \frac{1}{2}\right)}{\frac{1}{2}}\right] + c$

$\left[\because \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$

$= \tan^{-1}(2x - 1) + c$

2. What is $\int \frac{dx}{x(1 + \ln x)^n}$ equal to $(n \neq 1)$?

(a) $\frac{1}{(n-1)(1 + \ln x)^{n-1}} + c$

(b) $\frac{1-n}{(1 + \ln x)^{1-n}} + c$

(c) $\frac{n+1}{(1 + \ln x)^{n+1}} + c$

(d) $-\frac{1}{(n-1)(1 + \ln x)^{n-1}} + c$

(d) Suppose,

$I = \int \frac{dx}{x(1 + \ln x)^n}$ (where $n \neq 1$)

Let $1 + \ln x = t$

Diff. w.r.t. x, we get

$0 + \frac{1}{x} = \frac{dt}{dx}$

$\Rightarrow \quad \frac{dx}{x} = dt$

$\therefore \quad I = \int \frac{dt}{t^n} = \frac{t^{-n+1}}{-n+1} + c$

$= -\frac{1}{(n-1)t^{n-1}} + c$

$= -\frac{1}{(n-1)(1 + \log x)^{n-1}} + c$

2019 (I)

3. What is $\int \ln(x^2)dx$ equal to?

(a) $2x\ln(x) - 2x + C$

(b) $\frac{2}{x} + C$

(c) $2x\ln(x) + C$

(d) $\frac{2\ln(x)}{x} - 2x + C$

(a) Let $I = \int \ln(x^2)\,dx$

$= \int 2\ln x\,dx$

$= \ln x \int 2\,dx - \int\left(\frac{d}{dx}(\ln x)\int(2\,dx)\right)dx$

$= \ln x \cdot 2x - \int \frac{1}{x}\cdot 2x\,dx$

$= 2x\ln x - 2x + C$

4. What is $\int e^{x\ln(a)}dx$ equal to?

(a) $\frac{a^x}{\ln(a)} + C$ (b) $\frac{e^x}{\ln(a)} + C$

(c) $\frac{e^x}{\ln(ae)} + C$ (d) $\frac{ae^x}{\ln(a)} + C$

(a) Let $I = \int e^{x\ln a}\,dx$

$= \int e^{\ln a^x}\,dx$

$= \int a^x\,dx = \frac{a^x}{\ln a} + C$

2018 (II)

5. What is $\int \sin^3 x\cos x\,dx$ equal to?

(a) $\cos^4 x + C$

(b) $\sin^4 x + C$

(c) $\frac{(1 - \sin^2 x)^2}{4} + C$

(d) $\frac{(1 - \cos^2 x)^2}{4} + C$

(d) We have,

$\int \sin^3 x\cos x\,dx$

Let, $\sin x = t$

$\cos x\,dx = dt$

$= \int t^3\,dt$

$= \frac{t^4}{4} + C$

$= \frac{\sin^4 x}{4} + C$

$= \frac{(\sin^2 x)^2}{4} + C$

$= \frac{(1 - \cos^2 x)^2}{4} + C$

[$\because$ where, C is the constant of integratioins] [$\because \sin^2 x + \cos^2 x = 1$]

6. What is $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ equal to ?

(a) $\frac{1}{ab} \tan^{-1}\left(\frac{a \tan x}{b}\right) + C$

(b) $\frac{1}{ab} \tan^{-1}\left(\frac{a \tan x}{b}\right) + C$

(c) $\frac{1}{ab} \tan^{-1}\left(\frac{b \tan x}{b}\right) + C$

(d) None of the above

(**a**) We have,

$$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$

Divide numerator and denominator by $\cos^2 x$

$$I = \int \frac{\sec^2 x\, dx}{a^2 \tan^2 x + b^2}$$

Let $a \tan x = t$

$a \sec^2 x\, dx = dt$

$\sec^2 x\, dx = \frac{dt}{a}$

$$I = \frac{1}{a}\int \frac{dt}{t^2 + b^2}$$

$$= \frac{1}{a} \times \frac{1}{b} \tan^{-1}\left(\frac{t}{b}\right) + C$$

[$\because$ where, C is the constant of integration]

$$\left[\because \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C\right]$$

$$= \frac{1}{ab} \tan^{-1}\left(\frac{a \tan x}{b}\right) + C$$

2018 (I)

7. What is $\int \frac{dx}{2^x - 1}$ equal to ?

(a) $\ln(2^x - 1) + C$ (b) $\frac{\ln(1 - 2^{-x})}{\ln 2} + C$

(c) $\frac{\ln(2^{-x} - 1)}{2 \ln 2} + C$ (d) $\frac{\ln(1 + 2^{-x})}{\ln 2} + C$

(**b**) Let $I = \int \frac{dx}{2^x - 1} = \int \frac{2^{-x}}{1 - 2^{-x}} dx$

$$= \frac{1}{\ln 2} \int \frac{2^{-x} \ln 2}{1 - 2^{-x}} dx$$

On putting $1 - 2^{-x} = t$

$\Rightarrow 2^{-x} \ln 2\, dx = dt$

$\therefore I = \frac{1}{\ln 2} \int \frac{dt}{t}$

$= \frac{1}{\ln 2} . \ln t + C$

$= \frac{\ln(1 - 2^{-x})}{\ln 2} + C$

2017 (II)

8. What is $\int \tan^{-1}(\sec x + \tan x) dx$ equal to?

(a) $\frac{\pi x}{4} + \frac{x^2}{4} + C$

(b) $\frac{\pi x}{2} + \frac{x^2}{4} + C$

(c) $\frac{\pi x}{4} + \frac{\pi x^2}{4} + C$

(d) $\frac{\pi x}{4} - \frac{x^2}{4} + C$

(**a**) Let, $I = \int \tan^{-1}(\sec + \tan x) dx$

$$= \int \tan^{-1}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) dx$$

$$= \int \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) dx$$

$$= \int \tan^{-1}\left(\frac{1 + \cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)}\right) dx$$

$$= \int \tan^{-1}\left(\frac{2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right) dx$$

$$= \int \tan^{-1}\left(\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) dx$$

$$= \int \tan^{-1} \tan\left[\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2}\right)\right] dx$$

$$= \int \tan^{-1} \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

$$= \int \left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

$$= \frac{\pi x}{4} + \frac{x^2}{4} + C$$

9. $\int (\ln x)^{-1} dx - \int (\ln x)^{-2} dx$ is equal to

(a) $x(\ln x)^{-1} + C$

(b) $x(\ln x)^{-2} + C$

(c) $x(\ln x) + C$

(d) $x(\ln x)^2 + C$

(a) Let, $I = \int (\ln x)^{-1} dx - \int (\ln x)^{-2} dx$

$$= \int 1 \frac{1}{\ln x} dx - \int \frac{1}{(\ln x)^2} dx$$

$$= x \cdot \frac{1}{\ln x} - \int x \cdot \left(\frac{-1}{\ln x^2}\right) \cdot \frac{1}{x} dx - \int \frac{1}{(\ln x)^2} dx$$

$$= \frac{x}{\ln x} + \int \frac{1}{(\ln x)^2} dx - \int \frac{1}{\ln x^2} dx$$

$$= \frac{x}{\ln x} + c$$

$$= x(\ln x)^{-1} + C$$

2017 (I)

10. What is $\int \frac{dx}{x(x^7 + 1)}$ equal to?

(a) $\frac{1}{2} \ln\left|\frac{x^7 - 1}{x^7 + 1}\right| + C$ (b) $\frac{1}{7} \ln\left|\frac{x^7 + 1}{x^7}\right| + C$

(c) $\frac{1}{2} \ln\left|\frac{x^7 - 1}{7x}\right| + C$ (d) $\frac{1}{7} \ln\left|\frac{x^7}{x^7 + 1}\right| + C$

(**d**) Let, $I = \int \frac{dx}{x(x^7 + 1)} = \int \frac{x^6 dx}{x^7(x^7 + 1)}$

On putting $x^7 = t \Rightarrow 7x^6 dx = dt$

$\Rightarrow x^6 dx = dt / 7$

$$= \frac{1}{7} \int \frac{dt}{t(t + 1)} = \frac{1}{7}\left[\int \left(\frac{1}{t} - \frac{1}{t + 1}\right) dt\right]$$

$$= \frac{1}{7}[\ln t - \ln(t + 1)] + C$$

$$= \frac{1}{7} \ln\left(\frac{t}{t + 1}\right) + C$$

$$= \frac{1}{7} \ln\left(\frac{x^7}{x^7 + 1}\right) + C$$

11. What is $\int \frac{(x^{e-1} + e^{x-1}) dx}{x^e + e^x}$ equal to?

(a) $\frac{x^2}{2} + C$ (b) $\ln(x + e) + C$

(c) $\ln(x^e + e^x) + C$ (d) $\frac{1}{e} \ln(x^e + e^x) + C$

(**d**) $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx$

let $x^e + e^x = t$

$\Rightarrow (ex^{e-1} + e^x) dx = dt$

$\Rightarrow e(x^{e-1} + e^{x-1}) dx = dt$

$\Rightarrow (x^{e-1} + e^{x-1}) dx = \frac{dt}{e}$

Now, $\int \frac{dt / e}{t} = \frac{1}{e} \ln(t) + C$

$$= \frac{1}{e} \ln(x^e + e^x) + C$$

12. Let $f(x)$ be an indefinite integral of $\sin^2 x$.

Consider the following statements

Statement 1 The function $f(x)$ satisfies $f(x + \pi) = f(x)$ for real x.

Statement 2 $\sin^2(x + \pi) = \sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1
(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1
(c) Statement 1 is true but Statement 2 is false
(d) Statement 1 is false but Statement 2 is true

(d) From question

$$f(x) = \int \sin^2 x\, dx = \int \frac{1-\cos 2x}{2}\, dx$$

$$= \frac{1}{2}x - \frac{\sin 2x}{4} + C$$

Here, $f(x + \pi) = \frac{1}{2}(x + \pi) - \frac{\sin 2(x + \pi)}{4} + C$

$$= \frac{1}{2}(x + \pi) + \frac{\sin 2x}{4} + C$$

Here, $f(x + \pi) \neq f(x)$

∴ Statement 1 is false.

We know $\sin^2(x + \pi) = \sin^2 x$ is true for all real x.

∴ Statement 2 is true.

2016 (II)

13. What is $\int \frac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}}\, dx$ equal to?

(a) $\sqrt{\frac{x^4 + x^2 + 1}{x}} + C$
(b) $\sqrt{x^4 + 2 - \frac{1}{x^2}} + C$
(c) $\sqrt{x^2 + \frac{1}{x^2} + 1} + C$
(d) $\sqrt{\frac{x^4 - x^2 + 1}{x}} + C$

(c) Let $I = \int \frac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}}\, dx$

$$= \int \frac{x - \frac{1}{x^3}}{\sqrt{\frac{x^4 + x^2 + 1}{x^2}}}\, dx$$

$$= \int \frac{x - \frac{1}{x^3}}{\sqrt{x^2 + 1 + \frac{1}{x^2}}}\, dx$$

Again, let $x^2 + 1 + \frac{1}{x^2} = t$

$\Rightarrow \left(2x - \frac{2}{x^3}\right)dx = dt$

$\Rightarrow 2\left(x - \frac{1}{x^3}\right)dx = dt$

$\therefore \quad I = \int \frac{dt/2}{\sqrt{t}} = \frac{1}{2}\int t^{-1/2} dt$

$$= \frac{1}{2}\frac{t^{1/2}}{1/2} + C = t^{1/2} + C$$

$$= \left(x^2 + 1 + \frac{1}{x^2}\right)^{1/2} + C$$

$$= \sqrt{x^2 + \frac{1}{x^2} + 1} + C$$

14. What is $\int e^{\sin x} \frac{x\cos^3 x - \sin x}{\cos^2 x}\, dx$ equal to?

(a) $(x + \sec x)e^{\sin x} + C$
(b) $(x - \sec x)e^{\sin x} + C$
(c) $(x + \tan x)e^{\sin x} + C$
(d) $(x - \tan x)e^{\sin x} + C$

(b) Let $I = \int e^{\sin x} \frac{x\cos^3 x - \sin x}{\cos^2 x} dx$

$$= \int e^{\sin x}[x\cos x - \sec x \cdot \tan x]dx$$

$$= \int [xe^{\sin x} \cdot \cos x - e^{\sin x} \cdot \sec x \cdot \tan x]\, dx$$

$$= \int [xe^{\sin x} \cdot \cos x + e^{\sin x} - e^{\sin x} \sec x \cdot \tan x - \sec x \cdot e^{\sin x} \cdot \cos x]\, dx$$

$$= \int (xe^{\sin x} \cdot \cos x + e^{\sin x})\, dx$$

$$-\int [e^{\sin x} \sec x \cdot \tan x + \sec x \cdot e^{\sin x} \cos x]dx$$

$$= \int d(xe^{\sin x}) - \int d(\sec x \cdot e^{\sin x})$$

$$= xe^{\sin x} - \sec x\, e^{\sin x} + C$$

$$= e^{\sin x}(x - \sec x) + C$$

2015 (II)

15. $\int \frac{dx}{1 + e^{-x}}$ is equal to

(a) $1 + e^x + C$ (b) $\ln(1 + e^{-x}) + C$
(c) $\ln(1 + e^x) + C$ (d) $2\ln(1 + e^{-x}) + C$

where, C is the constant of integration.

(c) Let $I = \int \frac{dx}{1 + e^{-x}} = \int \frac{dx}{1 + \frac{1}{e^x}}$

$$= \int \frac{e^x\, dx}{1 + e^x}$$

Put $t = 1 + e^x$

$\Rightarrow \quad dt = e^x dx$

Now, $I = \int \frac{dt}{t} = \ln(t) + C$

$\Rightarrow \quad I = \ln(1 + e^x) + C$

2015 (I)

Directions (Q. Nos. 16 and 17) *Read the following information cerefully and answer these questions given below.*

The integral $\int \frac{dx}{a\cos x + b\sin x}$ is of the form $\frac{1}{r}\ln\left[\tan\left(\frac{x + \alpha}{2}\right)\right]$.

16. What is r equal to?

(a) $a^2 + b^2$ (b) $\sqrt{a^2 + b^2}$
(c) $a + b$ (d) $\sqrt{a^2 - b^2}$

17. What is α equal to?

(a) $\tan^{-1}\left(\frac{a}{b}\right)$ (b) $\tan^{-1}\left(\frac{b}{a}\right)$
(c) $\tan^{-1}\left(\frac{a + b}{a - b}\right)$ (d) $\tan^{-1}\left(\frac{a - b}{a + b}\right)$

Solutions (Q.Nos. 16 and 17)

Let $I = \int \frac{dx}{a\cos x + b\sin x}$

put $a = r\sin\alpha$ and $b = r\cos\alpha$

$$\therefore \quad I = \frac{1}{r}\int \frac{dx}{\sin\alpha\cos x + \cos\alpha\sin x}$$

$$= \frac{1}{r}\int \frac{dx}{\sin(x + \alpha)}$$

$$= \frac{1}{r}\int \text{cosec}\,(x + \alpha)\, dx$$

$$= \frac{1}{r}\ln[\text{cosec}\,(x + \alpha) - \cot(x + \alpha)] + C$$

$$= \frac{1}{r}\ln\left[\frac{1}{\sin(x + \alpha)} - \frac{\cos(x + \alpha)}{\sin(x + \alpha)}\right] + C$$

$$= \frac{1}{r}\ln\left[\frac{1 - \cos(x + \alpha)}{\sin(x + \alpha)}\right] + C$$

$$= \frac{1}{r}\ln\left[\frac{2\sin^2\left(\frac{x + \alpha}{2}\right)}{2\sin\left(\frac{x + \alpha}{2}\right)\cos\left(\frac{x + \alpha}{2}\right)}\right] + C$$

$$= \frac{1}{r}\ln\left[\tan\left(\frac{x + \alpha}{2}\right)\right] + C,$$

where, $r = \sqrt{a^2 + b^2}$ and $\alpha = \tan^{-1}\left(\frac{a}{b}\right)$

Hence, **16.** (*b*) and **17.** (*a*)

18. What is $\int \frac{dx}{\sqrt{x^2 + a^2}}$ equal to?

(a) $\ln\left|\frac{x + \sqrt{x^2 + a^2}}{a}\right| + C$
(b) $\ln\left|\frac{x - \sqrt{x^2 + a^2}}{a}\right| + C$
(c) $\ln\left|\frac{x^2 + \sqrt{x^2 + a^2}}{a}\right| + C$
(d) None of the above

where, C is the constant of integration.

(d) Let $I = \int \frac{dx}{\sqrt{x^2 + a^2}}$

$$= \ln|x + \sqrt{x^2 + a^2}| + C$$

Where, C is the constant of integration.

19. What is $\int \frac{xe^x dx}{(x + 1)^2}$ equal to?

(a) $(x + 1)^2 e^x + C$ (b) $(x + 1)e^x + C$
(c) $\frac{e^x}{x + 1} + C$ (d) $\frac{e^x}{(x + 1)^2} + C$

(**c**) Let $I = \int \frac{xe^x}{(x+1)^2}\,dx$

$= \int e^x \left(\frac{(x+1)-1}{(x+1)^2}\right)dx$

$= \int e^x \left(\frac{1}{x+1} + \left(\frac{-1}{(x+1)^2}\right)\right)dx$

$= e^x \left(\frac{1}{x+1}\right) + C$

Where, C is the constant of integration.

$[\because \int e^x\,[f(x) + f'(x)]\,dx = e^x\,f(x) + C]$

2014 (II)

Directions (Q. Nos. 20 and 21) *Read the following information carefully and answer the questions given below.*

Consider the function

$f''(x) = \sec^4 x + 4$ with $f(0) = 0$ and $f'(0) = 0$.

20. What is $f'(x)$ equal to?

(a) $\tan x - \frac{\tan^3 x}{3} + 4x$

(b) $\tan x + \frac{\tan^3 x}{3} + 4x$

(c) $\tan x + \frac{\sec^3 x}{3} + 4x$

(d) $-\tan x - \frac{\tan^3 x}{3} + 4x$

(**b**) Clearly, $f'(x) = \int f''(x)\,dx + C_1$

$= \int (\sec^4 x + 4)\,dx + C_1$

$= \int \sec^2 x \sec^2 x\,dx + \int 4\,dx + C_1$

$= \int (1 + \tan^2 x)\sec^2 x\,dx + 4x + C_1$

$= I_1 + 4x + C_1$

Put $\tan x = t$ in the integral I_1, then

$\sec^2 x\,dx = dt$

$\therefore I_1 = \int (1 + t^2)\,dt = t + \frac{t^3}{3} + C_2$

$= \tan x + \frac{\tan^3 x}{3} + C_2$

$\therefore f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x + C$

where, $C = C_1 + C_2$

$\because$ $f'(0) = 0 \Rightarrow C = 0$

Thus, $f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x$

21. What is $f(x)$ equal to?

(a) $\frac{2\ln\sec x}{3} + \frac{\tan^2 x}{6} + 2x^2$

(b) $\frac{3\ln\sec x}{2} + \frac{\cot^2 x}{6} + 2x^2$

(c) $\frac{4\ln\sec x}{3} + \frac{\sec^2 x}{6} + 2x^2$

(d) $\ln\sec x + \frac{\tan^4 x}{12} + 2x^2$

(**a**) Clearly, $f(x) = \int f'(x)\,dx + C_3$

$= \int \left(\tan x + \frac{\tan^3 x}{3} + 4x\right)dx + C_3$

$= \int \tan x\,dx + \frac{1}{3}\int \tan^3 x\,dx + 4\int x\,dx + C_3$

$= \int \tan x\,dx + \frac{1}{3}\int \tan x\,(\sec^2 x - 1)\,dx + 4\cdot\frac{x^2}{2} + C_3$

$= \frac{2}{3}\int \tan x\,dx + \frac{1}{3}\int \tan x\cdot\sec^2 x\,dx + 2x^2 + C_3$

$= \frac{2}{3}\ln(\sec x) + \frac{1}{3}I_2 + 2x^2 + C_3$

Consider, $I_2 = \int \tan x \sec^2 x\,dx$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$\Rightarrow I_2 = \int t\,dt = \frac{t^2}{2} + C_4 = \frac{\tan^2 x}{2} + C_4$

$\therefore f(x) = \frac{2}{3}\ln(\sec x) + \frac{1}{6}\tan^2 x + 2x^2 + C_5$

where, $C_5 = C_3 + \left(\frac{C_4}{3}\right)$

$\because$ $f(0) = 0$

$\therefore$ $0 = \frac{2}{3}\ln(1) + 0 + 0 + C_5$

$\Rightarrow$ $C_5 = 0$

$\therefore f(x) = \frac{2}{3}\ln(\sec x) + \frac{1}{6}\tan^2 x + 2x^2$

Directions (Q. Nos. 22 and 23) *Read the following information carefully and answer the questions given below.*

Consider $\int x\tan^{-1} x\,dx = A(x^2+1)\tan^{-1} x + Bx + C$, where C is the constant of integration.

22. What is the value of A?

(a) 1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) $\frac{1}{4}$

23. What is the value of B?

(a) 1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) $\frac{1}{4}$

Solutions (Q. Nos. 22 and 23)

Given, $\int x\tan^{-1} x\,dx = A(x^2+1)\tan^{-1} x + Bx + C$

where, C is the constant of integration.

Consider, $\int x\tan^{-1} x\,dx$

$= \tan^{-1} x\cdot\frac{x^2}{2} - \int \frac{d}{dx}(\tan^{-1} x)\cdot\frac{x^2}{2}\,dx$ [using integration by parts]

$= \frac{x^2\cdot\tan^{-1} x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$

$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}\left[\int \left(\frac{1+x^2-1}{1+x^2}\right)dx\right]$

$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}\left[\int dx - \int \frac{dx}{1+x^2}\right]$

$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}(x - \tan^{-1} x) + C$

$= \frac{x^2\tan^{-1} x}{2} - \frac{x}{2} + \frac{\tan^{-1} x}{2} + C$

$= \frac{1}{2}(x^2+1)\tan^{-1} x - \frac{x}{2} + C$

22. (**b**) Clearly, $A = \frac{1}{2}$

23. (**c**) Clearly, $B = -\frac{1}{2}$

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DEFINITE INTEGRALS

2019 (II)

Directions (Q.Nos. 1 and 2) *Read the following information and answer the questions given below.*

Consider the integrals

$$I_1 = \int_0^{\pi} \frac{x\,dx}{1+\sin x} \text{ and}$$

$$I_2 = \int_0^{\pi} \frac{(\pi - x)dx}{1-\sin(\pi + x)}$$

1. What is the value of I_1?

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

2. What is the value of $I_1 + I_2$?

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) 0

Solutions (Q.Nos. 1 and 2)

Given, $I_1 = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$,

$$I_2 = \int_0^{\pi} \frac{(\pi - x)\,dx}{1-\sin(\pi + x)}$$

$$I_1 = \int_0^{\pi} \frac{x\,dx}{1+\sin x} \quad \ldots(i)$$

$$= \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin(\pi - x)}$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I_1 = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin x} \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$2I_1 = \int_0^{\pi} \frac{(x + \pi - x)\,dx}{1+\sin x}$$

$$= \int_0^{\pi} \frac{\pi\,dx}{1+\sin x}$$

$$= \pi \int_0^{\pi} \frac{(1-\sin x)\,dx}{(1-\sin^2 x)}$$

$$= \pi \int_0^{\pi} \frac{(1-\sin x)\,dx}{\cos^2 x}$$

$$= \pi \int_0^{\pi} (\sec^2 x - \sec x \tan x)\,dx$$

$$= \pi\,[\tan x - \sec x]_0^{\pi}$$

$$= \pi\,[(\tan \pi - \sec \pi) - (\tan 0 - \sec 0)]$$

$$= \pi\,[(0+1) - (0-1)]$$

$$2I_1 = \pi\,(1+1) = 2\pi$$

$$\Rightarrow I_1 = \pi$$

Now, $I_2 = \int_0^{\pi} \frac{(\pi - x)\,dx}{1-\sin(\pi + x)}$

$$= \int_0^{\pi} \frac{(\pi - \pi + x)\,dx}{1-\sin(\pi - \pi - x)}$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\pi} \frac{x\,dx}{1-\sin(-x)}$$

$$= \int_0^{\pi} \frac{x\,dx}{1+\sin x} [\because \sin(-\theta) = \sin\theta]$$

$$= I_1 = \pi$$

1. (***c***) $I_1 = \int_0^{\pi} \frac{x\,dx}{1+\sin x} = \pi$

2. (***a***) $I_1 + I_2 = \pi + \pi = 2\pi$

2019 (I)

3. $\int_0^{\frac{\pi}{2}} |\sin x - \cos x|\,dx$ is equal to

(a) 0 (b) $2(\sqrt{2}-1)$
(c) $2\sqrt{2}$ (d) $2(\sqrt{2}+1)$

(***b***) $\int_0^{\frac{\pi}{2}} |\sin x - \cos x|\,dx$

$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1)\right] + \left[(-0-1) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{2}{\sqrt{2}} - 1 - 1 + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}} - 2$$

$$= 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$$

4. $\int_0^{\frac{\pi}{2}} e^{\sin x} \cos x\,dx$ is equal to

(a) $e+1$ (b) $e-1$
(c) $e+2$ (d) e

(***b***) Let $I = \int_0^{\frac{\pi}{2}} e^{\sin x} \cos x\,dx$

Let $\sin x = t$

$\Rightarrow \cos x\,dx = dt$

When $x = \frac{\pi}{2}, t = 1$

$x = 0, t = 0$

$$\therefore \quad I = \int_0^1 e^t\,dt$$

$$= [e^t]_0^1$$

$$= e^1 - e^0$$

$$= e - 1$$

2018 (II)

5. What is $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ equal to, where $[,]$ is the greatest integer function?

(a) $b-a$ (b) $a-b$
(c) 0 (d) $2(b-a)$

(***b***) We have,

$$\int_a^b [x]\,dx + \int_a^b [-x]\,dx$$

$$= \int_a^b ([x] + [-x])\,dx$$

$$[\because [x] + [-x] = -1, \text{ if } x \notin z]$$

$$= \int_a^b (-1)\,dx$$

$$= -(x)_a^b = -(b-a) = a-b$$

6. What is $\int_{-1}^{1}\left\{\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right\}dx$ equal to ?

(a) 0 (b) $-\frac{\pi}{4}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

(*d*) We have,

$$\int_{-1}^{1}\left[\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right]dx=\left[\tan^{-1}\left(\frac{1}{x}\right)\right]_{-1}^{1}$$

$$=\tan^{-1}(1)-\tan^{-1}(-1)$$

$$=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$$

2018 (I)

7. What is $\int_{0}^{\pi}e^{x}\sin x\,dx$ equal to ?

(a) $\frac{e^{\pi}+1}{2}$ (b) $\frac{e^{\pi}-1}{2}$

(c) $e^{\pi}+1$ (d) $\frac{e^{\pi}+1}{4}$

(*a*) Let $I=\int_{0}^{\pi}e^{x}\sin x\,dx$

$$=[\sin x\cdot e^{x}]_{0}^{\pi}-\int_{0}^{\pi}\left[\frac{d}{dx}\{\sin x\}\cdot e^{x}\right]dx$$

$$=[\sin x\cdot e^{x}]_{0}^{\pi}-\int_{0}^{\pi}\cos x\cdot e^{x}dx$$

$$=0-\left\{[\cos x\cdot e^{x}]_{0}^{\pi}+\int_{0}^{\pi}\sin x\cdot e^{x}dx\right\}$$

$$\Rightarrow \quad I=-[-e^{\pi}-1]-I$$

$$\Rightarrow \quad I+I=e^{\pi}+1$$

$$\Rightarrow \quad 2I=e^{\pi}+1$$

$$\Rightarrow \quad I=\frac{e^{\pi}+1}{2}$$

8. What is $\int_{1}^{e}x\ln x\,dx$ equal to?

(a) $\frac{e+1}{4}$ (b) $\frac{e^{2}+1}{4}$

(c) $\frac{e-1}{4}$ (d) $\frac{e^{2}-1}{4}$

(*b*) Let $I=\int_{1}^{e}x\log x dx$

$$=[\log x\cdot\int x dx]_{1}^{e}-\int_{1}^{e}\left[\frac{d}{dx}\{\log x\}\cdot\int x\,dx\right]dx$$

$$=\left[\log x\cdot\frac{x^{2}}{2}\right]_{1}^{e}-\int_{1}^{e}\frac{1}{x}\cdot\frac{x^{2}}{2}dx$$

$$=\frac{e^{2}}{2}-\frac{1}{2}\times\frac{1}{2}[x^{2}]_{1}^{e}=\frac{e^{2}}{2}-\frac{[e^{2}-1]}{4}$$

$$=\frac{2e^{2}-e^{2}+1}{4}=\frac{e^{2}+1}{4}$$

9. What is $\int_{0}^{\sqrt{2}}[x^{2}]\,dx$ equal to (where [.] is the greatest integer function) ?

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$

(c) $2(\sqrt{2}-1)$ (d) $\sqrt{3}-1$

(*a*) Let $I=\int_{0}^{\sqrt{2}}[x^{2}]dx$

$$=\int_{0}^{1}[x^{2}]dx+\int_{1}^{\sqrt{2}}[x^{2}]\,dx$$

$$=\int_{0}^{1}0\,dx+\int_{1}^{\sqrt{2}}1\,dx$$

$$\left[\because [x]=\begin{cases}0 & 0\le x<1\\ 1, & 1\le x<\sqrt{2}\end{cases}\right]$$

$$=0+[x]_{1}^{\sqrt{2}}=\sqrt{2}-1$$

10. What is the value of $\int_{-\pi/4}^{\pi/4}(\sin x-\tan x)\,dx$?

(a) $-\frac{1}{\sqrt{2}}+\ln\left(\frac{1}{\sqrt{2}}\right)$ (b) $\frac{1}{\sqrt{2}}$

(c) 0 (d) $\sqrt{2}$

(*c*) Let $I=\int_{-\pi/4}^{\pi/4}(\sin x-\tan x)\,dx$

Let $f(x)=\sin x-\tan x$

$\therefore \quad f(-x)=\sin(-x)-\tan(-x)$

$=-\sin x+\tan x$

$[\because \sin(-\theta)=-\sin\theta,\ \tan(-\theta)=-\tan\theta]$

$=-(\sin x-\tan x)=-f(x)$

$\therefore f(x)$ is odd function.

$\therefore \quad I=\int_{-\pi/4}^{\pi/4}(\sin x-\tan x)\,dx=0$

$[\because \int_{-a}^{a}f(x)\,dx=0$, if $f(x)$ is odd]

11. If $\int_{a}^{b}x^{3}\,dx=0$ and $\int_{a}^{b}x^{2}\,dx=\frac{2}{3}$, then what are the values of a and b respectively?

(a) $-1, 1$ (b) $1, 1$ (c) $0, 0$ (d) $2, -2$

(*a*) We have,

$$\int_{a}^{b}x^{3}dx=0\Rightarrow\left[\frac{x^{4}}{4}\right]_{a}^{b}=0$$

$$\Rightarrow \quad \frac{b^{4}-a^{4}}{4}=0\Rightarrow b^{4}=a^{4}$$

$$\Rightarrow \quad b=\pm a \qquad \ldots (i)$$

But $\int_{a}^{b}x^{3}\,dx=0$ and x^{3} is an odd function.

$$\therefore \quad a=-b \qquad \ldots (ii)$$

Again, $\int_{a}^{b}x^{2}dx=\frac{2}{3}\Rightarrow\left[\frac{x^{3}}{3}\right]_{a}^{b}=\frac{2}{3}$

$$\Rightarrow \quad \frac{b^{3}-a^{3}}{3}=\frac{2}{3}$$

$$\Rightarrow \quad b^{3}-a^{3}=2$$

$$\Rightarrow b^{3}-(-b)^{3}=2 \qquad \text{[from Eq. (ii)]}$$

$$\Rightarrow \quad 2b^{3}=2\ \Rightarrow b^{3}=1$$

$$\Rightarrow \quad b=1$$

$$\Rightarrow \quad a=-1 \qquad \text{[from Eq. (ii)]}$$

12. What is $\int_{0}^{1}x(1-x)^{9}\,dx$ equal to ?

(a) $\frac{1}{110}$ (b) $\frac{1}{132}$ (c) $\frac{1}{148}$ (d) $\frac{1}{240}$

(*a*) Let $I=\int_{0}^{1}x(1-x)^{9}\,dx$

$$=\int_{0}^{1}(1-x)\,x^{9}\,dx$$

$$\left[\because \int_{0}^{a}f(x)\,dx=\int_{0}^{a}f(a-x)\,dx\right]$$

$$=\int_{0}^{1}(x^{9}-x^{10})\,dx=\left[\frac{x^{10}}{10}-\frac{x^{11}}{11}\right]_{0}^{1}$$

$$=\left(\frac{1}{10}-\frac{1}{11}\right)=\frac{1}{110}$$

2017 (II)

13. What is $\int_{0}^{2\pi}\sqrt{1+\sin\frac{x}{2}}dx$ equal to?

(a) 8 (b) 4 (c) 2 (d) 0

(*a*) Let,

$$I=\int_{0}^{2\pi}\sqrt{1+\sin\frac{x}{2}}dx$$

$$=\int_{0}^{2\pi}\left|\sin\frac{x}{4}+\cos\frac{x}{4}\right|dx$$

$$=4\left[\sin\frac{x}{4}-\cos\frac{x}{4}\right]_{0}^{2\pi}$$

$$=4\left[\left(\sin\frac{\pi}{2}-\cos\frac{\pi}{2}\right)-(\sin 0^{\circ}-\cos 0^{\circ})\right]$$

$$=4[(1-0)-(0-1)]=8$$

14. The value of $\int_{0}^{\frac{\pi}{4}}\sqrt{\tan x}\,dx+\int_{0}^{\frac{\pi}{4}}\sqrt{\cot x}\,dx$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{\sqrt{2}}$

(*d*) Let,

$$I=\int_{0}^{\pi/4}\sqrt{\tan x}\,dx+\int_{0}^{\pi/4}\sqrt{\cot x}\,dx$$

$$=\int_{0}^{\pi/4}(\sqrt{\tan x}+\sqrt{\cot x})dx$$

$$=\int_{0}^{\pi/4}\left(\sqrt{\tan x}+\frac{1}{\sqrt{\tan x}}\right)dx$$

$$=\int_{0}^{\pi/4}\left(\frac{\tan x+1}{\sqrt{\tan x}}\right)dx$$

$$=\int_{0}^{\pi/4}\frac{\tan x+1}{\sqrt{\tan x}}\times\frac{\sec^{2}x}{1+\tan^{2}x}dx$$

Put $\tan x = t^2$

$\Rightarrow \quad \sec^2 x\,dx = 2t\,dt$

when, $\quad x = 0 \Rightarrow t = 0$

and $\quad x = \dfrac{\pi}{4} \Rightarrow t = 1$

$$\therefore \quad I = \int_0^1 \frac{t^2+1}{t} \cdot \frac{2t}{t^4+1} dt$$

$$= 2\int_0^1 \frac{t^2+1}{t^4+1} dt = 2\int_0^1 \frac{1+1/t^2}{t^2+1/t^2} dt$$

$$= 2\int_0^1 \frac{1+1/t^2}{(t-1/t)^2+2} dt$$

$$= \frac{2}{\sqrt{2}}\left[\tan^{-1}\left(\frac{t-1/t}{\sqrt{2}}\right)\right]_0^1$$

$$= \sqrt{2}\left[\tan^{-1}\frac{t^2-1}{\sqrt{2}t}\right]_0^1$$

$$= \sqrt{2}\,[\tan^{-1} 0 - \tan^{-1}(-\infty)] = \frac{\pi}{\sqrt{2}}$$

2017 (I)

15. What is $\int_0^{\frac{\pi}{2}} \frac{d\theta}{1+\cos\theta}$ equal to?

(a) $\frac{1}{2}$ (b) 1

(c) $\sqrt{3}$ (d) None of these

(**b**) $\int_0^{\pi/2} \frac{d\theta}{1+\cos\theta} = \frac{1}{2}\int_0^{\pi/2} \sec^2\theta/2\, d\theta$

$$= \frac{1}{2}[2\tan\theta/2]_0^{\pi/2} = [\tan\theta/2]_0^{\pi/2}$$

$$= \tan\left(\frac{\pi/2}{2}\right) - \tan 0$$

$$= \tan \pi/4 - 0 = 1$$

16. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then what is $\int_0^a f(x)\,g(x)dx$ equal to?

(a) $\int_0^a g(x)dx$ (b) $\int_0^a f(x)dx$

(c) $2\int_0^a f(x)dx$ (d) 0

(**b**) Let $I = \int_0^a f(x)\,g(x)\,dx$

$$= \int_0^a f(a-x)\,g(a-x)\,dx$$

$$\left[\because \int_0^k f(x)\,dx = \int^k f(k-x)\,dx\right]$$

$$= \int_0^a f(x)\,[2 - g(x)]\,dx$$

$$[\because g(x) + g(a-x) = 2]$$

$$= \int_0^a 2f(x)\,dx - \int_0^a f(x)\,g(x)\,dx$$

$$= 2\int_0^a f(x)\,dx - I$$

$\Rightarrow \quad 2I = 2\int_0^a f(x)\,dx \Rightarrow \quad I = \int_0^a f(x)\,dx$

17. What is $\int_{e^{-1}}^{e^2} \left|\frac{\ln x}{x}\right| dx$ equal to?

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 4

(**b**) Let $I = \int_{e^{-1}}^{e^2} \left|\frac{\ln x}{x}\right| dx$

$$= \int_{e^{-1}}^{e^0} \left(\frac{-\ln x}{x}\right) dx + \int_{e^0}^{e^2} \frac{\ln x}{x}\, dx$$

$$= -\int_{e^{-1}}^{e^0} \ln x\, d(\ln x) + \int_{e^0}^{e^2} \ln x\, d(\ln x)$$

$$= -\left[\frac{(\ln x)^2}{2}\right]_{e^{-1}}^{e^0} + \left[\frac{(\ln x)^2}{2}\right]_{e^0}^{e^2}$$

$$= \frac{-1}{2}\,[(\ln e^0)^2 - (\ln e^{-1})^2]$$

$$+ \frac{1}{2}[(\ln e^2)^2 - (\ln e^0)^2]$$

$$= \frac{-1}{2}\,[(0)^2 - (-1)^2] + \frac{1}{2}\,[(2)^2 - (0)^2]$$

$$= \frac{1}{2} + 2 = \frac{5}{2}$$

2016 (II)

18. Let $f(x)$ be a function such that $f'\left(\frac{1}{x}\right) + x^3 f'(x) = 0$. What is $\int_{-1}^{1} f(x)\,dx$ equal to?

(a) $2f(1)$ (b) 0 (c) $2f(-1)$ (d) $4f(1)$

(**c**) We have, $f'\left(\frac{1}{x}\right) + x^3 f'(x) = 0$

Let $I = \int_{-1}^{1} f(x)dx$

$\Rightarrow \quad I = [xf(x)]_{-1}^{1} - \int_{-1}^{1} xf'(x)dx$

$\Rightarrow \quad I = [f(1) + f(-1)] + \int_{-1}^{1} \frac{1}{x^2} f'\left(\frac{1}{x}\right)dx$

$$\left[\because f'(x) = \frac{-1}{x^3} f'\left(\frac{1}{x}\right)\right]$$

Again, let $I_1 = \int_{-1}^{1} \frac{1}{x^2} f'\left(\frac{1}{x}\right)dx$

Put $\frac{1}{x} = t$

$\Rightarrow \quad -\frac{1}{x^2}dx = dt$

$\therefore\ I_1 = -\int_{-1}^{1} f'(t)dt = -[f(t)]_{-1}^{1} = -f(1) + f(-1)$

$\therefore\ I = f(1) + f(-1) - f(1) + f(-1) = 2f(-1)$

19. If $\int_0^{\pi/2} \frac{dx}{3\cos x + 5} = k\cot^{-1} 2$, then what is the value of k?

(a) 1/4 (b) 1/2 (c) 1 (d) 2

(**b**) We have,

$$\int_0^{\pi/2} \frac{dx}{3\cos x + 5} = k\cot^{-1} 2 \quad \text{...(i)}$$

By LHS, let $\quad I = \int_0^{\pi/2} \frac{dx}{3\cos x + 5}$

$$= \int_0^{\pi/2} \frac{dx}{3\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + 5}$$

$$= \int_0^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)}{3\left(1-\tan^2\frac{x}{2}\right) + 5\left(1+\tan^2\frac{x}{2}\right)} dx$$

$$= \int_0^{\pi/2} \frac{\sec^2\frac{x}{2}}{8 + 2\tan^2\frac{x}{2}} dx$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{\sec^2\frac{x}{2}}{4 + \tan^2\frac{x}{2}} dx$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{\sec^2\frac{x}{2}}{(2)^2 + \tan^2\frac{x}{2}} dx$$

Again, let $\tan\frac{x}{2} = t$, then

$\sec^2\frac{x}{2}\,dx = 2\,dt$

$$\therefore\ I = \frac{1}{2}\int_0^1 \frac{2dt}{(2)^2 + t^2} = \frac{1}{2}\left[\tan^{-1}\left(\frac{t}{2}\right)\right]_0^1$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\cot^{-1}(2) \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), we get

$\therefore \quad k = \frac{1}{2}$

20. What is $\int_1^3 |1 - x^4|\,dx$ equal to?

(a) –232 / 5 (b) –116 / 5

(c) 116/5 (d) 232 / 5

(**d**) We have, $\int_1^3 \left|1 - x^4\right| dx$

Let $f(x) = 1 - x^4 = (1 + x^2)(1 - x^2)$

$= (1 + x^2)(1 + x)(1 - x)$

$\Rightarrow \quad \left|1 - x^4\right| = -(1 - x^4)$, if $x > 1$

$$\therefore \quad \int_1^3 \left|1 - x^4\right| dx = -\int_1^3 (1 - x^4)\,dx$$

$$= -\left[x - \frac{x^5}{5}\right]_1^3 = -\left[\left(3 - \frac{3^5}{5}\right) - \left(1 - \frac{1}{5}\right)\right]$$

$$= -\left[\left(3 - \frac{243}{5}\right) - \left(\frac{4}{5}\right)\right]$$

$$= -\left[\frac{15 - 243}{5} - \frac{4}{5}\right]$$

$$= -\left[\frac{15 - 243 - 4}{5}\right] = -\left[\frac{15 - 247}{5}\right]$$

$$= -\left[-\frac{232}{5}\right] = \frac{232}{5}$$

2016 (I)

Directions (Q. Nos. 21 and 22) *Read the following informations carefully and answer the questions given below.*

Consider the functions $f(x) = xg(x)$ and $g(x) = \left[\frac{1}{x}\right]$, where [·] is the greatest integer function.

21. What is $\int_{1/3}^{1/2} g(x)\,dx$ equal to?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{18}$ (d) $\frac{5}{36}$

(**b**) Given, $g(x) = \left[\frac{1}{x}\right]$

When $\frac{1}{3} < x < \frac{1}{2} \Rightarrow 2 < \frac{1}{x} < 3$

$\therefore \quad \left[\frac{1}{x}\right] = 2$

Now, we can find $\int_{1/3}^{1/2} g(x)\,dx$

$\therefore \int_{1/3}^{1/2} g(x)\,dx = \int_{1/3}^{1/2} \left[\frac{1}{x}\right] dx = \int_{1/3}^{1/2} 2\,dx$

$= 2\,[x]_{1/3}^{1/2} = 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 2 \times \frac{3-2}{6} = 2 \times \frac{1}{6} = \frac{1}{3}$

22. What is $\int_{1/3}^{1} f(x)\,dx$ equal to?

(a) $\frac{37}{72}$ (b) $\frac{2}{3}$ (c) $\frac{17}{72}$ (d) $\frac{37}{144}$

(**a**) Given, $f(x) = x\,g(x)$ and

$g(x) = \left[\frac{1}{x}\right]$

We have to split given limit into two parts.

When $\frac{1}{3} < x < \frac{1}{2} \Rightarrow 2 < \frac{1}{x} < 3 \Rightarrow \left[\frac{1}{x}\right] = 2$

$\therefore \quad f(x) = x\,g(x) = 2x$

When $\frac{1}{2} < x < 1 \Rightarrow 1 < \frac{1}{x} < 2 \Rightarrow \left[\frac{1}{x}\right] = 1$

$\therefore \quad f(x) = x\,g(x) = x$

Now,

$\int_{1/3}^{1} f(x)\,dx = \int_{1/3}^{1/2} 2x\,dx + \int_{1/2}^{1} x\,dx$

$= 2\left[\frac{x^2}{2}\right]_{1/3}^{1/2} + \left[\frac{x^2}{2}\right]_{1/2}^{1}$

$= \frac{2}{2}\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2\right] + \frac{1}{2}\left[(1)^2 - \left(\frac{1}{2}\right)^2\right]$

$= \left[\frac{1}{4} - \frac{1}{9}\right] + \frac{1}{2}\left[1 - \frac{1}{4}\right]$

$= \frac{9-4}{36} + \frac{1}{2}\left[\frac{4-1}{4}\right] = \frac{5}{36} + \frac{3}{8}$

$= \frac{10+27}{72} = \frac{37}{72}$

Directions (Q. Nos. 23 and 24) *Read the following informations carefully and answer the questions given below.*

Given that, $a_n = \int_0^{\pi} \frac{\sin^2\{(n+1)x\}}{\sin 2x}\,dx$

23. Consider the following statements

1. The sequence $\{a_{2n}\}$ is in AP with common difference zero.
2. The sequence $\{a_{2n+1}\}$ is in AP with common difference zero.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) $\because a_{2n} = \int_0^{\pi} \frac{\sin^2\{(2n+1)x\}}{\sin 2x}\,dx$ ...(i)

By applying property

$\int_0^{\pi} f(x)\,dx = \int_0^{\pi} f(\pi - x)\,dx$

$\therefore \quad a_{2n} = \int_0^{\pi} \frac{\sin^2\{(2n+1)(\pi - x)\}}{\sin 2(\pi - x)}\,dx$

$= \int_0^{\pi} \frac{\sin^2\{(2n+1)x\}}{-\sin 2x}\,dx$ ...(ii)

[$\because$ for any value of n, $\sin^2\{(2n+1)(\pi - x)\} = \sin^2\{(2n+1)x\}$]

On adding Eqs. (i) and (ii), we get

$2a_{2n} = \int_0^{\pi} \frac{\sin^2\{(2n+1)x\}}{\sin 2x}\,dx - \int_0^{\pi} \frac{\sin^2\{(2n+1)x\}}{\sin 2x}\,dx$

$\Rightarrow \quad 2a_{2n} = 0 \quad \Rightarrow \quad a_{2n} = 0$

Similarly, value $a_{2n+1} = 0$

$\therefore$ The sequence $\{a_{2n}\}$ is in AP with common difference zero and the sequence $\{a_{2n+1}\}$ is in AP with common difference zero.

24. What is $a_{n-1} - a_{n-4}$ equal to?

(a) –1 (b) 0 (c) 1 (d) 2

(**b**) $\because$

$a_{n-1} = \int_0^{\pi} \frac{\sin^2\{(n-1+1)\}x}{\sin 2x}\,dx$

$= \int_0^{\pi} \frac{\sin^2\{nx\}}{\sin 2x}\,dx$ ...(i)

By applying property

$\int_0^{\pi} f(x)\,dx = \int_0^{\pi} f(\pi - x)\,dx$

$\therefore \quad a_{n-1} = \int_0^{\pi} \frac{\sin^2\{n(\pi - x)\}}{\sin 2(\pi - x)}\,dx$

$= \int_0^{\pi} \frac{\sin^2\{nx\}}{-\sin 2x}\,dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2a_{n-1} = 0$

$a_{n-1} = 0$

Similarly, $a_{n-4} = 0$

$\therefore \quad a_{n-1} - a_{n-4} = 0 - 0 = 0$

25. What is $\int_{-2}^{2} x\,dx - \int_{-2}^{2} [x]\,dx$ equal to, where [·] is the greatest integer function?

(a) 0 (b) 1 (c) 2 (d) 4

(**c**) $\int_{-2}^{2} x\,dx - \int_{-2}^{2} [x]\,dx$

We have, $[x] = \begin{cases} -2, & -2 \le x < -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$

$\therefore \quad \int_{-2}^{2} x\,dx - \left[\int_{-2}^{-1} (-2)\,dx + \int_{-1}^{0} (-1)\,dx + \int_{0}^{1} 0\,dx + \int_{1}^{2} 1\,dx\right]$

$= 0 - [-2(x)_{-2}^{-1} + (-x)_{-1}^{0} + (x)_{1}^{2}]$

[$\because \int_{-a}^{a} f(x)\,dx = 0$, if $f(x)$ is odd]

$= -[-2(-1+2) + (0-1) + (2-1)]$

$= -[-2 - 1 + 1] = 2$

26. If $\int_{-2}^{5} f(x)\,dx = 4$ and $\int_{0}^{5} \{1 + f(x)\}\,dx = 7$, then what is $\int_{-2}^{0} f(x)\,dx$ equal to?

(a) –3 (b) 2 (c) 3 (d) 5

(**b**) Given,

$\int_{-2}^{5} f(x)\,dx = 4$, $\int_{0}^{5} \{1 + f(x)\}\,dx = 7$

Let

$I = \int_{-2}^{5} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx$

$\Rightarrow 4 = \int_{-2}^{0} f(x)\,dx + \int_{0}^{5} [(1 + f(x)) - 1]\,dx$

$\Rightarrow 4 = \int_{-2}^{0} f(x)\,dx + \int_{0}^{5} [1 + f(x)]\,dx - \int_{0}^{5} 1\,dx$

$\Rightarrow \quad 4 = \int_{-2}^{0} f(x)\,dx + 7 - (x)_0^5$

$\Rightarrow \quad 4 = \int_{-2}^{0} f(x)\,dx + 7 - 5$

$\Rightarrow \quad 4 = \int_{-2}^{0} f(x)\,dx + 2$

$\Rightarrow \quad \int_{-2}^{0} f(x)\,dx = 2$

27. What is $\int_0^{4\pi} |\cos x|\,dx$ equal to?

(a) 0 (b) 2 (c) 4 (d) 8

(**d**) Let $I = \int_0^{4\pi} |\cos x|\,dx$

$= 2\int_0^{2\pi} |\cos x|\,dx$

$[\because \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if $f(2a-x) = f(x)$, here $|\cos(4\pi - x)| = |\cos x|]$

$= 2\cdot 2\int_0^{\pi} |\cos x|\,dx$ $[\because |\cos(2\pi - x)| = |\cos x|]$

$= 2\cdot 2\cdot 2\int_0^{\pi/2} |\cos x|\,dx$

$[\because |\cos(\pi - x)| = |-\cos x| = |\cos x|]$

$= 8\int_0^{\pi/2} \cos x\,dx$

$\left[\because \cos x > 0,\ \forall\ x \in \left(0, \frac{\pi}{2}\right)\right]$

$= 8\,[\sin x]_0^{\pi/2} = 8\left[\sin\frac{\pi}{2} - \sin 0\right] = 8$

2015 (II)

28. The value of $\int_a^b \frac{x^7 + \sin x}{\cos x}\,dx$, where $a + b = 0$, is

(a) $2b - a\sin(b-a)$
(b) $a + 3b\cos(b-a)$
(c) $\sin a - (b-a)\cos b$
(d) 0

(d) We have, $a + b = 0 \Rightarrow a = -b$

$\int_a^b \frac{x^7 + \sin x}{\cos x}\,dx$

$= \int_{-b}^{b}\left(\frac{x^7}{\cos x} + \frac{\sin x}{\cos x}\right)dx$

$= \int_{-b}^{b}(x^7 \cdot \sec x + \tan x)\,dx$

The given integrand is odd function.
Hence, its answer is 0.

29. If $0 < a < b$, then $\int_a^b \frac{|x|}{x}\,dx$ is equal to

(a) $|b| - |a|$ (b) $|a| - |b|$
(c) $\frac{|b|}{|a|}$ (d) 0

(a) $\int_a^b \frac{|x|}{x}\,dx = \int_a^b 1\,dx$ $[\because 0 < a < b]$

$= [x]_a^b = |b| - |a|$

30. $\int_0^{2\pi} \sin^5\left(\frac{x}{4}\right)dx$ is equal to

(a) $\frac{8}{15}$ (b) $\frac{16}{15}$
(c) $\frac{32}{15}$ (d) 0

(c) We have, $\int_0^{2\pi} \sin^5\left(\frac{x}{4}\right)dx$

Let $\frac{x}{4} = t \Rightarrow dx = 4\,dt$

$\therefore \int_0^{\pi/2} \sin^5 t \cdot 4\,dt = 4\int_0^{\pi/2} \sin^5 t\,dt$

By Walli's formula,

$\int_0^{\pi/2} \sin^n dx = \int_0^{\pi/2} \cos^n x\,dx$

$= \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{2}{3}, & n \text{ is odd} \\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}, & \text{when } n \text{ is even} \end{cases}$

$= 4\cdot\frac{4\cdot 2}{5\cdot 3} = \frac{32}{15}$

31. $\int_{-1}^{1} x|x|\,dx$ is equal to

(a) 0 (b) $\frac{2}{3}$
(c) 2 (d) -2

(b) $\int_{-1}^{1} x|x|\,dx$

$= \int_{-1}^{0} x(-x)\,dx + \int_0^1 x(x)\,dx$

[when $a < c < b$, $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$]

$= \int_{-1}^{0} -x^2 dx + \int_0^1 x^2\,dx$

$= \left[-\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_0^1$

$= 0 - \left(-\frac{1}{3}\right) + \frac{1}{3} - 0 = \frac{2}{3}$

Directions (Q. Nos. 32 and 33) *Read the following informations carefully and answer the questions given below.*

Consider the integrals

$A = \int_0^{\pi} \frac{\sin x\,dx}{\sin x + \cos x}$ and

$B = \int_0^{\pi} \frac{\sin x\,dx}{\sin x - \cos x}$

32. Which one of the following is correct?

(a) $A = 2B$ (b) $B = 2A$
(c) $A = B$ (d) $A = 3B$

(c) Given, $A = \int_0^{\pi} \frac{\sin x\,dx}{\sin x + \cos x}dx$

and $B = \int_0^{\pi} \frac{\sin x}{\sin x - \cos x}\,dx$

Now,

$A = \int_0^{\pi} \frac{\sin(\pi - x)\,dx}{\sin(\pi - x) + \cos(\pi - x)}$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$= \int_0^{\pi} \frac{\sin x\,dx}{\sin x - \cos x} = B$

33. What is the value of B?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{4}$ (d) π

(b) $I = A = \int_0^{\pi} \frac{\sin x\,dx}{\sin x + \cos x}$ ...(i)

$I = B = \int_0^{\pi} \frac{\sin x}{\sin x - \cos x}\,dx$ ...(ii)

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

On adding Eqs. (i) and (ii), we get

$2I = \int_0^{\pi}\left(\frac{\sin x}{\sin x + \cos x} + \frac{\sin x}{\sin x - \cos x}\right)dx$

$\sin x(\sin x - \cos x)$

$\Rightarrow 2I = \int_0^{\pi} \frac{+\sin x + \cos x)}{\sin^2 x - \cos^2 x}\,dx$

$\Rightarrow 2I = \int_0^{\pi} \frac{2\sin^2 x}{\sin^2 x - \cos^2 x}\,dx$

$\Rightarrow 2I = 4\int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x - \cos^2 x}$ ...(iii)

$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx\right.$ if $\left.f(2a-x) = f(x)\right]$

$\Rightarrow 2I = 4\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x - \sin^2 x}$ ...(iv)

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$\Rightarrow 4I = 4\int_0^{\pi/2}\left(\frac{\sin^2 x - \cos^2 x}{\sin^2 x - \cos^2 x}\right)dx$

[adding Eqs. (iii) and (iv)]

$\Rightarrow 4I = 4\,(x)_0^{\pi/2}$

$\Rightarrow 4I = 4\times\frac{\pi}{2} \Rightarrow I = \frac{\pi}{2}$

Directions (Q. Nos. 34-37) *Read the following informations carefully and answer the questions given below.*

Consider the integral

$I_m = \int_0^{\pi} \frac{\sin 2mx}{\sin x}\,dx$, where m is a positive integer.

34. What is I_1 equal to?

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

35. What $I_2 + I_3$ equal to?

(a) 4 (b) 2 (c) 1 (d) 0

36. What is I_m equal to?

(a) 0 (b) 1
(c) m (d) $2m$

37. Consider the following

I. $I_m - I_{m-1}$ is equal to 0
II. $I_{2m} > I_m$

Which of the above is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Solutions (Q.Nos. 34-37)

We have,

$I_m = \int_0^{\pi} \frac{\sin 2mx}{\sin x}\,dx$, where m is a positive integer.

We know that , if m is a positive integer, then

$$\frac{\sin 2mx}{\sin x} = 2\,[\cos x + \cos 3x + \ldots + \cos(2m-1)\,x]$$

$$\therefore\ I_m = \int_0^{\pi} 2\,[\cos x + \cos 3x + \ldots + \cos(2m-1)\,x]\,dx$$

$$= 2\left[\sin x + \frac{\sin 3x}{3} + \ldots + \frac{\sin(2m-1)\,x}{(2m-1)}\right]_0^{\pi}$$

$$= 2\left(\sin\pi + \frac{\sin 3\pi}{3} + \ldots + \frac{\sin(2m-1)\,\pi}{(2m-1)}\right) - \left(\sin 0 + \frac{\sin 0}{3} + \ldots + \frac{\sin 0}{(2m-1)}\right)$$

$= 2\,[0-0]$ $[\because \sin n\pi = 0,\ \forall n \in Z']$

$\Rightarrow\quad I_m = 0$

34. (*a*) $\because I_m = 0 \quad \therefore\ I_1 = 0$

35. (*d*) $\because I_m = 0 \quad \therefore\ I_2 + I_3 = 0 + 0 = 0$

36. (*a*) $\because I_m = 0 \quad \therefore I_m = 0$

37. (*a*) I. $\because\ I_m = 0$

$\therefore\ I_m - I_{m-1} = 0 - 0 = 0$

II. $\because\quad I_m = 0 \ \Rightarrow\ I_{2m} = 0$

$\therefore\quad I_{2m} \neq I_m$

$\Rightarrow I_m - I_{m-1} = 0$

and $\quad I_{2m} = I_m$

2014 (II)

38. What is $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ equal to?

(a) $2ab$ (b) $2\pi ab$ (c) $\frac{\pi}{2ab}$ (d) $\frac{\pi}{ab}$

(*c*) Let $I = \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$$= \int_0^{\pi/2} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$$

[$\because$ divide numerator and denominator by $\cos^2 x$]

Let $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

When $x = 0$, then $t = 0$

and when $x = \frac{\pi}{2}$, then $t = \infty$

$$\therefore\ I = \int_0^{\infty} \frac{dt}{a^2 + b^2t^2} = \frac{1}{b^2}\int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$$

$$= \frac{1}{b^2}\cdot\frac{1}{\left(\frac{a}{b}\right)}\left[\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^{\infty}$$

$$\left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C\right]$$

$$= \frac{1}{ab}\ [\tan^{-1}(\infty) - \tan^{-1}(0)]$$

$$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2ab}$$

Directions (Q. Nos. 39-41) *Read the following information carefully and answer the questions given below.*

For the next three solutions consider

$$I = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

39. What is I equal to?

(a) $-\pi$ (b) 0

(c) π (d) 2π

40. What is $\int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin x}$ equal to?

(a) π (b) $\frac{\pi}{2}$

(c) 0 (d) 2π

41. What is $\int_0^{\pi} \frac{dx}{1+\sin x}$ equal to?

(a) 1 (b) 2

(c) 4 (d) –2

Solutions (Q.Nos. 39-41)

Given, $I = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$ ...(i)

$$= \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)}\,dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\pi} \frac{(\pi - x)}{1+\sin x}\,dx \quad \ldots(ii)$$

[$\because \sin(\pi - x) = \sin x$]

On adding Eqs. (i) and (ii), we get

$$2I = \pi\int_0^{\pi} \frac{dx}{1+\sin x} \quad \ldots(iii)$$

$$\Rightarrow\quad 2I = 2\pi\int_0^{\pi/2} \frac{dx}{1+\sin x}$$

$$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx, \text{ if } f(2a-x) = f(x)\right]$$

$$\Rightarrow\quad I = \pi\int_0^{\pi/2} \frac{dx}{1 + \left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow\quad I = \pi\int_0^{\pi/2} \frac{\sec^2\frac{x}{2}\,dx}{\tan^2\frac{x}{2} + 1 + 2\tan\frac{x}{2}}$$

$$\Rightarrow\quad I = \pi\int_0^{\pi/2} \frac{\left(\sec^2\frac{x}{2}\right)dx}{\left(\tan\frac{x}{2}+1\right)^2}$$

[$\because (a+b)^2 = a^2 + b^2 + 2ab$]

Let, $\tan\frac{x}{2} + 1 = t$

$\Rightarrow \sec^2\frac{x}{2}\cdot\frac{1}{2}\,dx = dt$

$\Rightarrow\quad \sec^2\frac{x}{2}\,dx = 2\,dt$

When $x = 0$, then $t = 1$ and when $x = \frac{\pi}{2}$, then $t = 2$

$$\therefore\ I = 2\pi\int_1^2 \frac{dt}{t^2} = -2\pi\left[\frac{1}{t}\right]_1^2$$

$$= -2\pi\left[\frac{1}{2} - 1\right] = -2\pi\left(-\frac{1}{2}\right) = \pi$$

39. (*c*) Same as above explanation, $I = \pi$

40. (*a*) Let $I_1 = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin x}$

$$= \int_0^{\pi} \frac{[\pi - (\pi - x)]\,dx}{1+\sin(\pi - x)}$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I_1 = \int_0^{\pi} \frac{x\,dx}{1+\sin x} = I = \pi$$

[$\because \sin(\pi - x) = \sin x$]

41. (*b*) From Eq. (iii),

$$2I = \pi\int_0^{\pi} \frac{dx}{1+\sin x}$$

$$\Rightarrow\quad \int_0^{\pi} \frac{dx}{1+\sin x} = \frac{2}{\pi}I$$

$$\Rightarrow\quad \int_0^{\pi} \frac{dx}{1+\sin x} = \frac{2}{\pi}\times\pi = 2 \quad [\because I = \pi]$$

Directions (Q. Nos. 42 and 43) *Read the following information carefully and answer the questions given below.*

Consider the integral

$$I = \int_0^{\pi} \ln(\sin\ x)\,dx$$

42. What is $\int_0^{\pi/2} \ln(\sin x)\,dx$ equal to?

(a) $4I$ (b) $2I$ (c) I (d) $\frac{I}{2}$

43. What is $\int_0^{\pi/2} \ln(\cos x)\,dx$ equal to?

(a) $\frac{I}{2}$ (b) I (c) $2I$ (d) $4I$

Solutions (Q.Nos. 42 and 43)

Consider $I = \int_0^{\pi} \ln(\sin x)\,dx$

$I = \int_0^{\pi} \ln(\sin x)\,dx$

$= 2\int_0^{\pi/2} \ln(\sin x)\,dx$ ...(i)

$$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx, \text{ if } f(2a-x)=f(x)\right]$$

$$= 2\int_0^{\pi/2} \ln\left[\sin\left(\frac{\pi}{2}-x\right)\right]dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$= 2\int_0^{\pi/2} \ln(\cos x)\,dx \quad \ldots(ii)$$

42. (***d***) From Eq. (i),

$$I = 2\int_0^{\pi/2} \ln(\sin x)\,dx$$

$$\Rightarrow \int_0^{\pi/2} \ln(\sin x)\,dx = \frac{1}{2}I$$

43. (***a***) From Eq. (ii), we have

$$I = 2\int_0^{\pi/2} \ln(\cos x)\,dx$$

$$\Rightarrow \int_0^{\pi/2} \ln(\cos x)\,dx = \frac{1}{2}I$$

2014 (I)

44. What is $\int_{-\pi/2}^{\pi/2} x\sin x\,dx$ equal to?

(a) 0 (b) 2
(c) –2 (d) π

(***b***) Let $I = \int_{-\pi/2}^{\pi/2} x\sin x\,dx$

$$I = 2\int_0^{\pi/2} x\sin x\,dx$$

$[\because f(x) = x\sin x$ is an even function.
$\Rightarrow f(-x) = (-x)\sin(-x) =$
$(-x)(-\sin x) = x\sin x = f(x)]$

$$\left[\because \int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function} \end{cases}\right]$$

$$I = 2[x(-\cos x) + \int \cos x\,dx]_0^{\pi/2} \quad \text{[by parts]}$$

$$= 2(-x\cos x + \sin x)_0^{\pi/2}$$

$$= 2(-\pi/2\cdot\cos\pi/2 + \sin\pi/2 + 0 - 0]$$

$$= 2(-\pi/2\times 0 + 1) = 2$$

45. What is $\int_0^{\pi/2} \ln(\tan x)\,dx$ equal to?

(a) ln 2 (b) – ln 2
(c) 0 (d) None of these

(***c***) Let $I = \int_0^{\pi/2} \ln(\tan x)dx \quad \ldots(i)$

$$= \int_0^{\pi/2} \ln\tan\left(\frac{\pi}{2}-x\right)dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi/2} \ln\cot x\,dx$$

$$= \int_0^{\pi/2} \ln(\tan x)^{-1}dx$$

$$= \int_0^{\frac{\pi}{2}} -\ln(\tan x)dx$$

$$I = -\int_0^{\pi/2} \ln(\tan x)dx = -I \text{ [from Eq. (i)]}$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow I = 0 \text{ or } \int_0^{\pi/2} I_n(\tan x)dx = 0$$

46. What is $\int_0^1 \frac{e^{\tan^{-1}x}\,dx}{1+x^2}$ equal to?

(a) $e^{\frac{\pi}{4}} - 1$ (b) $e^{\frac{\pi}{4}} + 1$ (c) $e - 1$ (d) e

(***a***) Let $I = \int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2}dx$

Let $t = \tan^{-1}x \Rightarrow dt = \frac{1}{1+x^2}dx$

Lower Limit $\rightarrow t = \tan^{-1}0 = 0$

Upper Limit $\rightarrow t = \tan^{-1}1 = \frac{\pi}{4}$

$$\therefore I = \int_0^{\pi/4} e^t\,dt = [e^t]_0^{\pi/4}$$

$$= [e^{\pi/4} - e^0] = e^{\pi/4} - 1$$

Directions (Q. Nos. 47 and 48) *Read the following informations carefully and answer the questions given below.*

Consider the integrals

$$I_1 = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

and $I_2 = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}}$.

47. What is $I_1 - I_2$ equal to?

(a) 0 (b) $2I_1$
(c) π (d) None of these

48. What is I_1 equal to?

(a) $\pi/24$ (b) $\pi/18$ (c) $\pi/12$ (d) $\pi/6$

Solutions (Q.Nos. 47 and 48)

Given that, $I_1 = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

and $I_2 = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad \ldots(i)$

I_1 can be written as

$$I_1 = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}$$

$$I_1 = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad \ldots(ii)$$

47. (***a***) We have,

$$I_1 - I_2 = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx - \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$= \int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}-\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right)dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$- \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}-\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}-\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$[\because \cos(\pi/2 - A) = \sin A,$
$\sin(\pi/2 - B) = \cos B]$

$$= -\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}-\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$= -\int_{\pi/6}^{\pi/3} \left\{\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} - \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right\}dx$$

$$= -(I_1 - I_2)$$

$$\Rightarrow 2(I_1 - I_2) = 0 \Rightarrow I_1 - I_2 = 0$$

48. (***c***) From Eq. (i),

$$I_1 = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad \ldots(iii)$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$\left[\because \int_a^b f(x) = \int_a^b f(a+b-x)\,dx\right]$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \quad \ldots(iv)$$

On adding Eqs. (iii) and (iv), we get

$$2I_1 = \int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right)dx$$

$$\Rightarrow 2I_1 = \int_{\pi/6}^{\pi/3} (1)\,dx = [x]_{\pi/6}^{\pi/3}$$

$$= \left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{6}$$

$$\Rightarrow I_1 = \frac{\pi}{12}$$

25

AREA UNDER THE CURVES

2019 (II)

1. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

(a) $\frac{2}{3}$ sq unit

(b) $\frac{4}{3}$ sq unit

(c) $\frac{1}{3}$ sq unit

(d) 1 sq unit

(**a**) Equations of curves,

$y^2 = 2x$ … (i)

and $y = x$ … (ii)

From Eqs. (i) and (ii),

$y^2 = 2y$

$\Rightarrow \quad y(y-2) = 0$

$\therefore \quad y = 0, 2$

$\therefore$ Required area

$$= \int_0^2 \left(\frac{y^2}{2} - y\right) dy$$

$$= \left[\frac{1}{2}\cdot\frac{y^3}{3} - \frac{y^2}{2}\right]_0^2$$

$$= \left[\frac{1}{6}(2)^3 - \frac{(2)^2}{2}\right] - 0$$

$$= \frac{8}{6} - \frac{4}{2} = \frac{8-12}{6}$$

$$= -\frac{4}{6} = \frac{-2}{3} = \frac{2}{3} \text{ sq unit}$$

($\because$ area will not be negative)

2. What is the area of the region bounded by $|x| < 5$, $y = 0$ and $y = 8$?

(a) 40 sq units

(b) 80 sq units

(c) 120 sq units

(d) 160 sq units

(**b**) Given curve $y = 0$ and $y = 8$ and $|x| < 5$

Case I When $x < 0$, then

area of the region bounded

$$= \int_{-5}^{0} 0\, dx - \int_{-5}^{0} 8\, dx = 0 - 8\,[x]_{-5}^{0}$$

$$= -8\,[0 + 5] = -40$$

$= 40$ sq units

[$\because$ area will not be negative]

Case II when $x > 0$, then

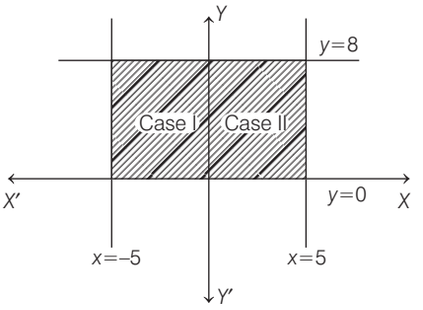
Area of the region bounded

$$= \int_0^5 0\, dx - \int_0^5 8\, dx = 0 - 8\,[x]_0^5$$

$$= -8[5 - 0] = -40$$

$= 40$ sq units

$\therefore$ Required area $= 40 + 40 = 80$ sq units

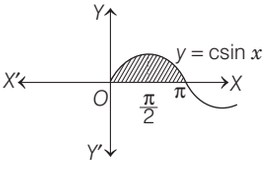


2019 (I)

3. What is the area of one of the loops between the curve $y = c \sin x$ and X-axis?

(a) c (b) $2c$ (c) $3c$ (d) $4c$

(**d**) $\therefore$ Required area $= 2\int_0^{\pi} c \sin x\, dx$



$$= 2c \int_0^{\pi} \sin x\, dx$$

$$= 2c[-\cos x]_0^{\pi}$$

$$= 2c\,[-(\cos \pi - \cos 0)]$$

$$= 2c\,(2)$$

$$= 4c \text{ sq units}$$

2018 (I)

4. What is the area of the region bounded by the parabolas $y^2 = 6(x - 1)$ and $y^2 = 3x$?

(a) $\frac{\sqrt{6}}{3}$ (b) $\frac{2\sqrt{6}}{3}$ (c) $\frac{4\sqrt{6}}{3}$ (d) $\frac{5\sqrt{6}}{3}$

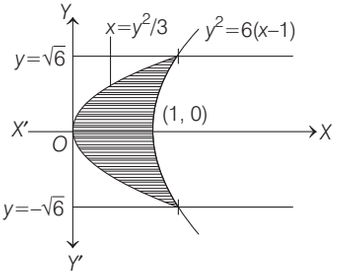
(**c**) Given,

$y^2 = 6(x - 1)$ … (i)

and $y^2 = 3x$ … (ii)

On solving Eqs. (i) and (ii), we get

$x = 2$ and $y = \pm\sqrt{6}$



$\therefore$ Required area $= \int_{-\sqrt{6}}^{\sqrt{6}} \left(1 + \frac{y^2}{6} - \frac{y^2}{3}\right) dy$

$$= 2\int_0^{\sqrt{6}} \left(1 - \frac{y^2}{6}\right) dy = 2\left[y - \frac{y^3}{18}\right]_0^{\sqrt{6}}$$

$$= 2 \times \left[\frac{18y - y^3}{18}\right]_0^{\sqrt{6}}$$

$$= 2 \times \left[\frac{18\sqrt{6} - 6\sqrt{6}}{18}\right]$$

$$= \frac{12\sqrt{6}}{9} = \frac{4\sqrt{6}}{3}$$

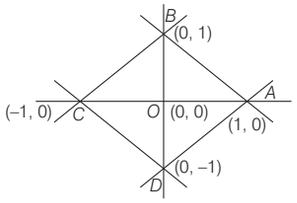
2017 (II)

5. The area bounded by the curve $|x|+|y|=1$

(a) 1 sq unit
(b) $2\sqrt{2}$ sq units
(c) 2 sq units
(d) $2\sqrt{3}$ sq units

(***c***) We have, $|x|+|y|=1$

$$= \begin{cases} x+y=1, \text{ I quadrant} \\ -x+y=1, \text{ II quadrant} \\ -x-y=1, \text{ III quadrant} \\ x-y=1, \text{ IV quadrant} \end{cases}$$



∴ Required area = 4×area (ΔOAB)

$$= 4\int_0^1 (1-x)\,dx = 4\left[x - \frac{x^2}{2}\right]_0^1$$

$$= 4\left[\left(1-\frac{1}{2}\right)-(0-0)\right]$$

$$= 4\times\frac{1}{2} = 2 \text{ sq units}$$

2016 (II)

6. What is the area bounded by the curves $|y| = 1 - x^2$?

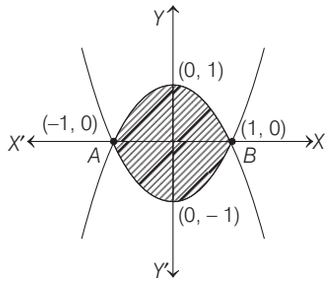
(a) 4/3 sq units
(b) 8/3 sq units
(c) 4 sq units
(d) 16/3 sq units

(***b***) We have,

$$|y| = 1 - x^2$$

We have to find area between the curves

$$y = 1 - x^2 \text{ and } y = x^2 - 1$$



∴ Required area

$$= 2\int_{-1}^{1}(1-x^2)\,dx$$

$$= 2\left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= 2\left[\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right]$$

$$= 2\left[\frac{2}{3}+\frac{2}{3}\right] = \frac{8}{3} \text{ sq units}$$

Directions (Q. Nos. 7 and 8) *Read the following information carefully and answer the questions given below.*

Consider the function
$f(x) = |x-1| + x^2$, where $x \in R$.

7. What is the area of the region bounded by X-axis, the curve $y = f(x)$ and the two ordinates $x = \frac{1}{2}$ and $x = 1$?

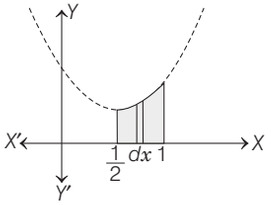
(a) $\frac{5}{12}$ sq unit (b) $\frac{5}{6}$ sq unit
(c) $\frac{7}{6}$ sq units (d) 2 sq units

(***a***) $f(x) = |x-1| + x^2$

When $\frac{1}{2} < x < 1$

$$f(x) = x^2 - x + 1$$

∴ Area of required region will be



$$\int_{1/2}^{1} f(x)\,dx = \int_{1/2}^{1}(x^2 - x + 1)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_{1/2}^{1}$$

$$= \left[\frac{(1)^3}{3} - \frac{(1)^2}{2} + (1)\right] - \left[\frac{\left(\frac{1}{2}\right)^3}{3} - \frac{\left(\frac{1}{2}\right)^2}{2} + \frac{1}{2}\right]$$

$$= \left(\frac{1}{3} - \frac{1}{2} + 1\right) - \left(\frac{1}{24} - \frac{1}{8} + \frac{1}{2}\right)$$

$$= \left(\frac{2-3+6}{6}\right) - \left(\frac{1-3+12}{24}\right)$$

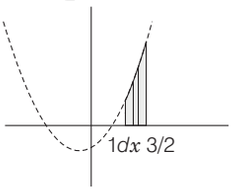
$$= \frac{5}{6} - \frac{10}{24} = \frac{20-10}{24} = \frac{10}{24} = \frac{5}{12} \text{ sq unit}$$

8. What is the area of the region bounded by X-axis, the curve $y = f(x)$ and the two ordinates $x = 1$ and $x = \frac{3}{2}$?

(a) $\frac{5}{12}$ sq unit (b) $\frac{7}{12}$ sq unit
(c) $\frac{2}{3}$ sq unit (d) $\frac{11}{12}$ sq unit

(***d***) $f(x) = |x-1| + x^2$

When $1 < x < \frac{3}{2}$, $f(x) = x^2 + x - 1$



∴ Area of required region will be

$$\int_1^{\frac{3}{2}} f(x)\,dx = \int_1^{\frac{3}{2}}(x^2 + x - 1)\,dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} - x\right]_1^{3/2}$$

$$= \left[\frac{\left(\frac{3}{2}\right)^3}{3} + \frac{\left(\frac{3}{2}\right)^2}{2} - \frac{3}{2}\right] - \left[\frac{(1)^3}{3} + \frac{(1)^2}{2} - 1\right]$$

$$= \left(\frac{27}{24} + \frac{9}{8} - \frac{3}{2}\right) - \left(\frac{1}{3} + \frac{1}{2} - 1\right)$$

$$= \left(\frac{27+27-36}{24}\right) - \left(\frac{2+3-6}{6}\right)$$

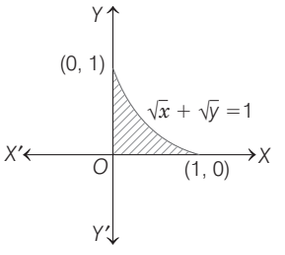
$$= \frac{18}{24} + \frac{1}{6} = \frac{18+4}{24} = \frac{22}{24} = \frac{11}{12} \text{ sq unit}$$

2015 (II)

9. The area bounded by the coordinate axes and the curve $\sqrt{x} + \sqrt{y} = 1$ is

(a) 1 sq unit (b) $\frac{1}{2}$ sq unit
(c) $\frac{1}{3}$ sq unit (d) $\frac{1}{6}$ sq unit

(***d***)



∴ Required area

$$= \int_0^1 (1-\sqrt{x})^2\,dx$$

$$= \int_0^1 (1 + x - 2\sqrt{x})\,dx$$

$$= \left[x + \frac{x^2}{2} - 2\cdot\frac{x^{3/2}}{3/2}\right]_0^1$$

$$= \left[1 + \frac{1}{2} - 2\times\frac{2}{3}(1)\right] - 0$$

$$= 1 + \frac{1}{2} - \frac{4}{3} = \frac{6+3-8}{6}$$

$$= \frac{1}{6} \text{ sq unit}$$

2015 (I)

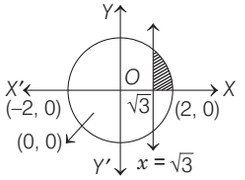
Directions (Q. Nos. 10 and 11) *Read the following information carefully and answer the questions given below.* Consider the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

10. What is the area of the region in the first quadrant enclosed by the X-axis, the line $x = \sqrt{3}$ and the circle?

(a) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (b) $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$

(c) $\frac{\pi}{3} - \frac{1}{2}$ (d) None of these

(a) Required area is shaded area represented in the following figure



$\therefore$ Required area $= \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$

$= \frac{1}{2}\left[x\sqrt{4 - x^2} + 4\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$

$= \frac{1}{2}\left[4\sin^{-1}(1) - \sqrt{3}\sqrt{1} - 4\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{1}{2}\left[4 \cdot \frac{\pi}{2} - \sqrt{3} - 4 \times \frac{\pi}{3}\right]$

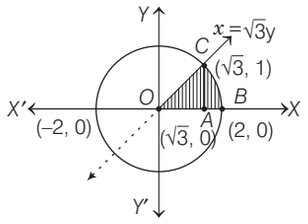
$= \pi - \frac{2\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$

11. What is the area of the region in the first quadrant enclosed by the X-axis, the line $x = \sqrt{3}y$ and the circle?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (d) None of these

(a) Required area is the shaded area represented in the following figure



Let us first find the intersection point of line $x = \sqrt{3}\,y$ and circle $x^2 + y^2 = 4$

On solving these two, we get

$x = \pm\sqrt{3}$ and $y = \pm 1$

$\therefore$ Required area = Area of region $OBCO$

= Area of ΔOAC + Area of region $ABCA$

$= \frac{1}{2} \times \sqrt{3} \times 1 + \frac{\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

Directions (Q. Nos. 12 and 13) *Read the following information carefully and answer the questions given below.* Consider the curves $y = \sin x$ and $y = \cos x$.

12. What is the area of the region bounded by the above two curves and the lines $x = 0$ and $x = \frac{\pi}{4}$?

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$ (c) $\sqrt{2}$ (d) 2

13. What is the area of the region bounded by the above two curves and the lines

$x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$?

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$ (c) $2\sqrt{2}$ (d) 2

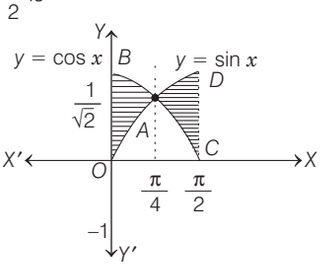
Solutions (Q. Nos. 12 and 13)

Given equation of curves are

$y = \sin x$ …(i)

and $y = \cos x$ …(ii)

The graph of above curves between 0 to $\frac{\pi}{2}$ is



12. (a) Required area = Area of region $OABO$

$= \int_0^{\pi/4} (\cos x - \sin x)\, dx$

$= [\sin x + \cos x]_0^{\pi/4}$

$= \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0\right]$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 = \sqrt{2} - 1$

13. (a) Required area = Area of region $ACDA$

$= \int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx$

$= [-\cos x - \sin x]_{\pi/4}^{\pi/2}$

$= -[\cos x + \sin x]_{\pi/4}^{\pi/2}$

$= -\left[0 + 1 - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$

$= \sqrt{2} - 1$

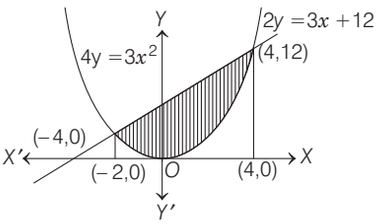
2014 (II)

Directions (Q. Nos. 14 and 15) *Read the following information carefully and answer the question given below.* The line $2y = 3x + 12$ cuts the parabola $4y = 3x^2$.

14. What is the area enclosed by the parabola and the line?

(a) 27 sq units
(b) 36 sq units
(c) 48 sq units
(d) 54 sq units

(a) Area enclosed by the parabola and the line



$= \int_{-2}^{4} \left[\frac{(3x + 12)}{2} - \frac{3x^2}{4}\right] dx$

$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$

$= \frac{1}{2}\left[\left\{\frac{3(4)^2}{2} + 12(4)\right\} - \left\{\frac{3(-2)^2}{2} + 12(-2)\right\}\right] - \frac{3}{4}\left(\frac{4^3}{3} - \frac{(-2)^3}{3}\right)$

$= \frac{1}{2}(24 + 48 - 6 + 24) - \frac{3}{4}\left(\frac{64 + 8}{3}\right)$

$= \frac{1}{2}(90) - 18$

$= 45 - 18 = 27$ sq units

15. What is the area enclosed by the parabola, the line and the Y-axis in the first quadrant?

(a) 7 sq units
(b) 14 sq units
(c) 20 sq units
(d) 21 sq units

(c) Area enclosed by the parabola, the line and the Y-axis in the first quadrant

$= \int_0^4 \left(\frac{3x + 12}{2} - \frac{3x^2}{4}\right) dx$

$= \left[\frac{3}{4}x^2 + 6x - \frac{x^3}{4}\right]_0^4$

$= 3 \times 4 + 24 - 16$

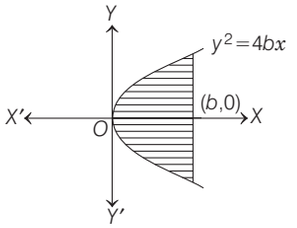
$= 12 + 24 - 16$

$= 20$ sq units

16. What is the area of the parabola $y^2 = 4bx$ bounded by its latusrectum?

(a) $2b^2/3$ sq units
(b) $4b^2/3$ sq units
(c) b^2 sq units
(d) $8b^2/3$ sq units

(**d**) Equation of parabola is $y^2 = 4bx$.



Area of parabola bounded by its latusrectum

$= 2\int_0^b \sqrt{4bx}\, dx = 4\sqrt{b} \times \frac{2}{3}[x^{3/2}]_0^b$

$= \frac{8\sqrt{b}}{3}[b^{3/2} - 0] = \frac{8b^2}{3}$ sq units

2014 (I)

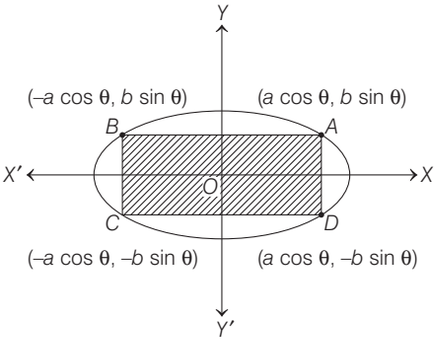
Directions (Q. Nos. 17 and 18) *Read the following information carefully and answer the question given below.*

Consider an ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

17. What is the area of the greatest rectangle that can be inscribed in the ellipse?

(a) ab (b) $2ab$ (c) $ab/2$ (d) $\sqrt{ab}$

(**b**) Let A ($a\cos\theta$, $b\sin\theta$) be any point on an ellipse. [$\because$ Ist quadrant]



Coordinate of B
$= \{a\cos(\pi - \theta), b\sin(\pi - \theta)\}$
$= (-a\cos\theta, b\sin\theta)$ [$\because$ IInd quadrant]
and coordinate of C
$= \{a\cos(\pi + \theta), b\sin(\pi + \theta)\}$
$= (-a\cos\theta, -b\sin\theta)$ [$\because$ IIIrd quadrant]
and coordinate of D
$= \{a\cos(2\pi - \theta), b\sin(2\pi - \theta)\}$
$= (a\cos\theta, -b\sin\theta)$ [$\because$ IV quadrant]
Now, area of rectangle $ABCD$,
$\Delta = CD \times AD$
$= (a\cos\theta + a\cos\theta) \times (b\sin\theta + b\sin\theta)$
$= 2a\cos\theta \times 2b\sin\theta$
$= 2ab\sin 2\theta$

Here, area of rectangle will be greatest, when $\sin 2\theta$ have its maximum value i.e. $\sin 2\theta = 1$.

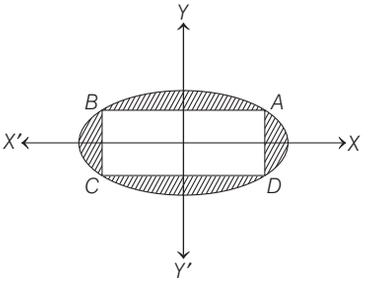
$\therefore \quad \Delta = 2ab \times 1 = 2ab$

Hence, area of greatest rectangle is equal to $2ab$, when $\sin 2\theta = 1$.

18. What is the area included between the ellipse and the greatest rectangle inscribed in the ellipse?

(a) $ab(\pi - 1)$ (b) $2ab(\pi - 1)$
(c) $ab(\pi - 2)$ (d) None of these

(**c**) We know that,
area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\Delta' = \pi ab$



$\therefore$ Required area = Area of shaded region = Area of an ellipse – Area of greatest rectangle
$= ab(\pi - 2)$

26

DIFFERENTIAL EQUATION

2019 (II)

1. What is the degree of the differential equation $\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2-x^2\left(\frac{d^4y}{dx^4}\right)=0$?

(a) 1 (b) 2
(c) 3 (d) 4

(*a*) Given differential equation,

$$\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2-x^2\left(\frac{d^4y}{dx^4}\right)=0$$

$$\Rightarrow \frac{d^4y}{dx^4}-\frac{1}{x^2}\left(\frac{d^3y}{dx^3}\right)-\frac{1}{x^2}\left(\frac{dy}{dx}\right)^2=0$$

We know that power of the heighest order of differentiation is the degree of differential equation.

So, the degree of it equation is 1.

2. Which one of the following is the differential equation that represents the family of curves $y=\frac{1}{2x^2-C}$, where C is an arbitrary constant?

(a) $\frac{dy}{dx}=4xy^2$ (b) $\frac{dy}{dx}=\frac{1}{y}$
(c) $\frac{dy}{dx}=x^2y$ (d) $\frac{dy}{dx}=-4xy^2$

(*d*) The differential equation of family of curves $y=\frac{1}{2x^2-C}$ … (i)

(where, C is any arbitrary constant)

Differentiation w.r.t. x of Eq. (i)

$$\frac{dy}{dx}=-\frac{1}{(2x^2-C)^2}\frac{d}{dx}(2x^2-C)$$

$$\Rightarrow \frac{dy}{dx}=-y^2.(4x-0)$$

$\Rightarrow \frac{dy}{dx}=-4xy^2$, it is required differential equation.

3. The differential equation which represents the family of curves given by $\tan y=C(1-e^x)$ is

(a) $e^x \tan y dx+(1-e^x)dy=0$
(b) $e^x \tan y dx+(1-e^x)\sec^2 y dy=0$
(c) $e^x (1-e^x)dx+\tan y dy=0$
(d) $e^x \tan y dy+(1-e^x)dx=0$

(*b*) The equation of the family of curves.

$$\tan y=C(1-e^x) \quad \text{… (i)}$$

Differentiation w.r.t. x, we get

$$\sec^2 y.\frac{dy}{dx}=C(0-e^x)$$

$$\Rightarrow \sec^2 y.\frac{dy}{dx}=-Ce^x \quad \text{… (ii)}$$

Eleminating C from Eqs. (i) and (ii), we get

$$\sec^2 y.\frac{dy}{dx}=-e^x\frac{\tan y}{(1-e^x)}$$

$$\Rightarrow (1-e^x)\sec^2 y\,dy=-e^x \tan y.dx$$

$$\Rightarrow e^x \tan y\,dx+(1-e^x)\sec^2 y\,dy=0$$

2019 (I)

4. The solution of the differential equation $\frac{dy}{dx}=\cos(y-x)+1$ is

(a) $e^x[\sec(y-x)-\tan(y-x)]=c$
(b) $e^x[\sec(y-x)+\tan(y-x)]=c$
(c) $e^x \sec(y-x)\tan(y-x)=c$
(d) $e^x=c\sec(y-x)\tan(y-x)$

(*a*) Given, $\frac{dy}{dx}=\cos(y-x)+1$ ...(i)

Let $y-x=t$

$$\Rightarrow \frac{dy}{dx}-1=\frac{dt}{dx}$$

$$\Rightarrow \frac{dy}{dx}=1+\frac{dt}{dx}$$

From Eq. (i), $1+\frac{dt}{dx}=\cos t+1$

$$\Rightarrow \frac{dt}{dx}=\cos t$$

$$\Rightarrow \sec t\,dt=dx$$

On integrating both sides, we get

$$\int \sec t\,dt=\int dx$$

$$\log(\sec t+\tan t)=x+a$$

$$\Rightarrow \sec t+\tan t=e^{x+a}$$

$$\Rightarrow \sec t+\tan t=e^x\cdot e^a$$

$$\Rightarrow \frac{e^x}{\sec t+\tan t}=e^{-a}$$

$$\Rightarrow \frac{e^x(\sec t-\tan t)}{(\sec t+\tan t)(\sec t-\tan t)}=e^{-a}$$

$$\Rightarrow \frac{e^x(\sec t-\tan t)}{\sec^2 t-\tan^2 t}=e^{-a}$$

$$\Rightarrow e^x(\sec t-\tan t)=e^{-a}$$

$$\Rightarrow e^x[\sec(y-x)-\tan(y-x)]=e^{-a}$$

$$\therefore e^x[\sec(y-x)-\tan(y-x)]=c, \quad [\text{where, } c=e^{-a}]$$

5. If $y=a\cos 2x+b\sin 2x$, then

(a) $\frac{d^2y}{dx^2}+y=0$ (b) $\frac{d^2y}{dx^2}+2y=0$
(c) $\frac{d^2y}{dx^2}-4y=0$ (d) $\frac{d^2y}{dx^2}+4y=0$

(*d*) Given, $y=a\cos 2x+b\sin 2x$ ...(i)

$$\Rightarrow \frac{dy}{dx}=-2a\sin 2x+2b\cos 2x$$

$$\Rightarrow \frac{d^2y}{dx^2}=-4a\cos 2x-4b\sin 2x$$

$$\Rightarrow \frac{d^2y}{dx^2}=-4(a\cos 2x+b\sin 2x)$$

$$\Rightarrow \frac{d^2y}{dx^2}=-4y \quad [\text{using Eq. (i)}]$$

$$\therefore \frac{d^2y}{dx^2}+4y=0$$

6. The differential equation of the system of circles touching the Y-axis at the origin is

(a) $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$

(b) $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$

(c) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

(d) $x^2 - y^2 - 2xy\frac{dy}{dx} = 0$

(c) (***c***) The system of circles touching the Y-axis at the origin is

$$(x-a)^2 + y^2 = a^2$$
$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = a^2$$
$$\Rightarrow \quad x^2 - 2ax + y^2 = 0 \qquad ...(i)$$

On differentiating Eq. (i) w.r.t. x, we get

$$2x - 2a + 2yy' = 0$$
$$\Rightarrow \quad x + yy' = a$$

Put value of a in Eq. (i), we get

$$x^2 - 2(x + yy')x + y^2 = 0$$
$$\Rightarrow \quad x^2 - 2x^2 - 2xyy' + y^2 = 0$$
$$\Rightarrow \quad -x^2 - 2xyy' + y^2 = 0$$
$$\therefore \quad x^2 - y^2 + 2xy\frac{dy}{dx} = 0$$

7. Consider the following in respect of the differential equation :

$$\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 9x = x$$

1. The degree of the differential equation is 1.
2. The order of the differential equation is 2.

Which of the above statements is/are correct ?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) (***c***) The order of highest order derivative occuring in the differential equation is 2 and its degree is 1.

8. What is the general solution of the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$?

(a) $x^2 + y^2 = C$ (b) $x^2 - y^2 = C$
(c) $x^2 + y^2 = Cxy$ (d) $x + y = C$

(a) (***a***) Given differential equation,

$$\frac{dy}{dx} + \frac{x}{y} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y}$$
$$\Rightarrow \quad ydy = -xdx$$

Integrating both sides, we get

$$\frac{y^2}{2} = \frac{-x^2}{2} + C_1$$
$$\therefore \quad x^2 + y^2 = C \qquad [\text{where, } C = 2C_1]$$

2018 (II)

9. The differential equation of the family of curves $y = p\cos(ax) + q\sin(ax)$, where p, q are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} - a^2y = 0$ (b) $\frac{d^2y}{dx^2} - ay = 0$
(c) $\frac{d^2y}{dx^2} + ay = 0$ (d) $\frac{d^2y}{dx^2} + a^2y = 0$

(d) (***d***) We have,

$$y = p\cos(ax) + q\sin(ax)$$

differentiation. w. r. t. x, we get

$$\frac{dy}{dx} = -pa\sin ax + qa\cos ax$$

Again, differentiation. w. r. t. x, we get

$$\frac{d^2y}{dx^2} = -pa^2\cos ax - qa^2\sin ax$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} = -a^2(p\cos ax + q\sin ax)$$
$$\therefore \quad \frac{d^2y}{dx^2} = -a^2y \Rightarrow \frac{d^2y}{dx^2} + a^2y = 0$$

10. The equation of the curve passing through the point $(-1, -2)$, which satisfies $\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$, is

(a) $17x^2y - 6x^2 + 3x^5 - 2 = 0$
(b) $6x^2y + 17x^2 + 2x^5 - 3 = 0$
(c) $6xy - 2x^2 + 17x^5 + 3 = 0$
(d) $17x^2y + 6xy - 3x^5 + 5 = 0$

(b) (***b***) We have,

$$\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$$
$$\Rightarrow \quad dy = \left(-x^2 - \frac{1}{x^3}\right)dx$$

On integrating both sides, we get

$$\int dy = \int\left(-x^2 - \frac{1}{x^3}\right)dx$$
$$\Rightarrow \quad y = -\frac{x^3}{3} - \left(\frac{x^{-2}}{-2}\right) + C$$
$$\Rightarrow \quad y = \frac{-x^3}{3} + \frac{1}{2x^2} + C \qquad ...(i)$$

given curve is passing through $(-1, -2)$

$$-2 = \frac{1}{3} + \frac{1}{2} + C$$
$$\Rightarrow \quad -2 = \frac{5}{6} + C$$
$$\Rightarrow \quad C = -2\frac{-5}{6} \Rightarrow C = \frac{-17}{6}$$

On putting in Eq. (i)

$$y = \frac{-x^3}{3} + \frac{1}{2x^2} - \frac{17}{6}$$
$$\Rightarrow \quad y = \frac{-2x^5 + 3 - 17x^2}{6x^2}$$
$$\Rightarrow \quad 6x^2y = -2x^5 + 3 - 17x^2$$
$$\therefore \quad 6x^2y + 2x^5 + 17x^2 - 3 = 0$$

11. What is the order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x} + d$, where a, b, c and d are arbitrary constants?

(a) 1 (b) 2 (c) 3 (d) 4

(d) (***d***) We have,

$y = a\cos x + b\sin x + ce^{-x} + d$,

a, b, c and d are arbitrary constants.

We know that,

order = number of arbitrary constant

So, order = 4

12. What is the solution of the differential equation

$$\ln\left(\frac{dy}{dx}\right) = ax + by \ ?$$

(a) $ae^{ax} + be^{by} = C$
(b) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{by} = C$
(c) $ae^{ax} + be^{-by} = C$
(d) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{-by} = C$

(d) (***d***) We have,

$$\ln\left(\frac{dy}{dx}\right) = ax + by$$
$$\frac{dy}{dx} = e^{ax+by}$$
$$\frac{dy}{dx} = e^{ax}\cdot e^{by}$$
$$\frac{dy}{e^{by}} = e^{ax}dx$$

Integrating both sides, we get

$$\int e^{-by}dy = \int e^{ax}dx \Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + C$$
$$\therefore \quad \frac{e^{ax}}{a} + \frac{e^{-by}}{b} + C = 0$$

13. What is the solution of the differential equation

$$\frac{dx}{dy} = \frac{x+y+1}{x+y-1} \ ?$$

(a) $y - x + 4\ln(x + y) = C$
(b) $y + x + 2\ln(x + y) = C$
(c) $y - x + \ln(x + y) = C$
(d) $y + x + 2\ln(x + y) = C$

Where, C is an arbitrary constant.

(**c**) We have, $\frac{dx}{dy} = \frac{x + y + 1}{x + y - 1}$

Let $x + y = u$

$$\frac{dx}{dy} + 1 = \frac{du}{dy} \Rightarrow \frac{dx}{dy} = \frac{du}{dy} - 1$$

$$\therefore \quad \frac{du}{dy} - 1 = \frac{u + 1}{u - 1} \Rightarrow \frac{du}{dy} = \frac{u + 1}{u - 1} + 1$$

$$\Rightarrow \quad \frac{du}{dy} = \frac{u + 1 + u - 1}{u - 1}$$

$$\Rightarrow \quad \frac{du}{dy} = \frac{2u}{u - 1} \Rightarrow \left(\frac{u - 1}{u}\right) du = 2\, dy$$

Integrating both sides, we get

$$\int \left(1 - \frac{1}{u}\right) du = \int 2\, dy$$

$\Rightarrow u - \log u = 2y + C \quad [\because u = x + y]$

$\Rightarrow x + y - \log(x + y) = 2y + C$

$\Rightarrow x - \log(x + y) = y + C$

$\therefore \quad y - x + \log(x + y) = C$

2018 (I)

14. What is the solution of the differential equation $x\,dy - y\,dx = 0$?

(a) $xy = C$ (b) $y = Cx$
(c) $x + y = C$ (d) $x - y = C$

(**b**) Given differential equation

$$x\,dy - y\,dx = 0$$

$$\Rightarrow \quad x\,dy = y\,dx$$

Variable seprate on both sides

$$\Rightarrow \quad \frac{dy}{y} = \frac{dx}{x}$$

On integration both sides, we get

$$\Rightarrow \quad \int \frac{dy}{y} = \int \frac{dx}{x}$$

$\Rightarrow \log y = \log x + \log C$

[where $\log C$ is integrating constant]

$\Rightarrow \quad y = xC$

$\therefore \quad y = Cx$

15. Which one of the following differential equations has a periodic solution?

(a) $\frac{d^2x}{dt^2} + \mu x = 0$

(b) $\frac{d^2x}{dt^2} - \mu x = 0$

(c) $x\frac{dx}{dt} + \mu t = 0$

(d) $\frac{dx}{dt} + \mu x t = 0$

(**a**) $\frac{d^2x}{dt^2} + \mu x = 0$ is the differential equation of simple harmonic motion, which has a periodic solution.

16. The order and degree of the differential equation $y^2 = 4a(x - a)$, where 'a' is an arbitrary constant, are respectively

(a) 1, 2 (b) 2, 1 (c) 2, 2 (d) 1, 1

(**a**) We have,

$$y^2 = 4a(x - a) \quad \ldots (i)$$

$$\Rightarrow \quad y^2 = 4ax - 4a^2$$

On differentiating both sides, we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad a = \frac{1}{2} y \frac{dy}{dx} \quad \ldots (ii)$$

On putting the values of a from Eq. (ii) in Eq. (i), we get

$$y^2 = 4 \times \frac{1}{2} y \frac{dy}{dx}\left(x - \frac{1}{2} y \frac{dy}{dx}\right)$$

$$\Rightarrow \quad y^2 = 2xy\frac{dy}{dx} - y^2\left(\frac{dy}{dx}\right)^2$$

$\therefore$ Order = 1 and degree = 2

17. What is the solution of $(1 + 2x)\,dy - (1 - 2y)\,dx = 0$?

(a) $x - y - 2xy = C$
(b) $y - x - 2xy = C$
(c) $y + x - 2xy = C$
(d) $x + y + 2xy = C$

(**a**) We have,

$$(1 + 2x)\,dy - (1 - 2y)\,dx = 0$$

$$\Rightarrow \quad (1 + 2x)\,dy = (1 - 2y)\,dx$$

$$\Rightarrow \quad \frac{dy}{1 - 2y} = \frac{dx}{1 + 2x}$$

On integrating both the sides, we get

$$\int \frac{dy}{1 - 2y} = \int \frac{dy}{1 + 2x}$$

$$\Rightarrow \quad -\frac{1}{2}\log(1 - 2y) = \frac{1}{2}\log(1 + 2x) + C'$$

$$\Rightarrow \quad -\log(1 - 2y) = \log(1 + 2x) + 2C'$$

$$\Rightarrow \log(1 + 2x) + \log(1 - 2y) = -2C'$$

$$\Rightarrow \quad \log(1 + 2x)(1 - 2y) = -2C'$$

$$\Rightarrow \quad (1 + 2x)(1 - 2y) = e^{-2C'}$$

$$\Rightarrow \quad 1 - 2y + 2x - 4xy = e^{-2C'}$$

$$\Rightarrow \quad 2x - 2y - 4xy = e^{-2C'} - 1$$

$$\Rightarrow \quad x - y - 2xy = \frac{1}{2}(e^{-2C'} - 1)$$

$$\therefore \quad x - y - 2xy = C$$

[where, $C = \frac{1}{2}(e^{-2C'} - 1)$]

18. What are the order and degree, respectively, of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 = y^4 + \left(\frac{dy}{dx}\right)^5$?

(a) 4, 5 (b) 2, 3 (c) 3, 2 (d) 5, 4

(**c**) Given differential equation is

$$\left(\frac{d^3y}{dx^3}\right)^2 = y^4 + \left(\frac{dy}{dx}\right)^5$$

Here, highest order derivative is $\frac{d^3y}{dx^3}$.

So, order = 3, and degree = 2

2017 (II)

19. The general solution of

$$\frac{dy}{dx} = \frac{ax + h}{by + k}$$

represents a circle only when

(a) $a = b = 0$ (b) $a = -b \neq 0$
(c) $a = b \neq 0, h = k$ (d) $a = b \neq 0$

(b) We have,

$$\frac{dy}{dx} = \frac{ax + h}{by + k}$$

$$\Rightarrow \quad \int (by + k)dy = \int (ax + h)dx$$

$$\Rightarrow \quad \frac{by^2}{2} + ky = \frac{ax^2}{2} + hx + C$$

$$\Rightarrow \quad \frac{a}{2}x^2 - \frac{b}{2}y^2 + hx - ky + C = 0$$

The above equation will represent a circle, when coefficient of x^2 = coefficient of y^2

$$\therefore \quad \frac{a}{2} = -\frac{b}{2} \Rightarrow \quad a = -b \neq 0$$

20. The order and degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \rho^2\left[\frac{d^2y}{dx^2}\right]^2$$

are respectively

(a) 3 and 2 (b) 2 and 2
(c) 2 and 3 (d) 1 and 3

(**b**) We have,

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \rho^2\left[\frac{d^2y}{dx^2}\right]^2$$

$\therefore$ Order = 2 and degree = 2

2017 (I)

21. If $xdy = y(dx + ydy)$; $y(1) = 1$ and $y(x) > 0$, then what is $y(-3)$ equal to?

(a) 3 (b) 2 (c) 1 (d) 0

(**a**) Given,

$$xdy = y(dx + ydy)$$

$$\Rightarrow \quad xdy = ydx + y^2dy$$

$$\Rightarrow \quad -y^2dy = ydx - xdy$$

$$\Rightarrow \quad -\int dy = \int \frac{ydx - xdy}{y^2}$$

$$\Rightarrow \quad -\int dy = \int \frac{d(xy)}{y^2}$$

$$\Rightarrow \quad -y = \frac{x}{y} + C \qquad \ldots (i)$$

Given, $y(1) = 1$

$\therefore$ From Eq. (i)

$$-1 = \frac{1}{1} + C \Rightarrow C = -2$$

$$\therefore \quad -y = \frac{x}{y} - 2 \qquad \ldots (ii)$$

On putting $x = -3$ in Eq. (ii)

$$-y = -\frac{3}{y} - 2$$

$$\Rightarrow \quad y^2 - 2y - 3 = 0$$

$$\Rightarrow (y-3)(y+1) = 0$$

$$\Rightarrow \quad y = 3 \text{ or } -1$$

But given $y(x) > 0$

$$\therefore \quad y(-3) = 3$$

22. What are the degree and order respectively of the differential equation

$$y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2 ?$$

(a) 1, 2 (b) 2, 1 (c) 1, 4 (d) 4, 1

(d) $y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2$

$$\Rightarrow \quad y = x\left(\frac{dy}{dx}\right)^2 + \frac{1}{\left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \quad y\left(\frac{dy}{dx}\right)^2 = x\left(\frac{dy}{dx}\right)^4 + 1$$

$\therefore$ Order = 1 and degree = 4

23. What is the differential equation corresponding to $y^2 - 2ay + x^2 = a^2$ by eliminating a?

(a) $(x^2 - 2y^2)p^2 - 4pxy - x^2 = 0$

(b) $(x^2 - 2y^2)p^2 + 4pxy - x^2 = 0$

(c) $(x^2 + 2y^2)p^2 - 4pxy - x^2 = 0$

(d) $(x^2 + 2y^2)p^2 - 4pxy + x^2 = 0$

(a) Given, $y^2 - 2ay + x^2 = a^2 \qquad \ldots (i)$

On differentiating both sides w.r.t 'x', we get

$$2y\frac{dy}{dx} - 2a\frac{dy}{dx} + 2x = 0$$

$$\Rightarrow 2yp - 2ap + 2x = 0 \qquad \left[\because p = \frac{dy}{dx}\right]$$

$$\Rightarrow \quad 2ap = 2yp + 2x$$

$$\Rightarrow \quad a = \frac{yp + x}{p}$$

Now, from Eq. (i)

$$y^2 - 2\left(\frac{yp+x}{p}\right)y + x^2 = \left(\frac{yp+x}{p}\right)^2$$

$$\Rightarrow \quad \frac{py^2 - 2y^2p - 2xy + px^2}{p} = \frac{y^2p^2 + x^2 + 2xyp}{p^2}$$

$$\Rightarrow p^2y^2 - 2p^2y^2 - 2pxy + p^2x^2 = y^2p^2 + x^2 + 2xyp$$

$$\Rightarrow -2p^2y^2 - 4xyp + p^2x^2 - x^2 = 0$$

$$\therefore \quad p^2(x^2 - 2y^2) - 4xyp - x^2 = 0$$

24. What is the general solution of the differential equation

$$ydx - (x + 2y^2)dy = 0?$$

(a) $x = y^2 + Cy$ (b) $x = 2Cy^2$

(c) $x = 2y^2 + Cy$ (d) None of these

(c) We have, $ydx - (x + 2y^2)dy = 0$

$$\Rightarrow \quad ydx = (x + 2y^2)dy$$

$$\Rightarrow \quad y\frac{dx}{dy} = x + 2y^2$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{x}{y} + 2y \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y$$

which is in the form of linear differential equation $\frac{dx}{dy} + Px = Q$

$$\therefore \quad \text{IF} = e^{\int \frac{-1}{y} dy} = \frac{1}{y}$$

$\therefore$ Solution of given differential equation is

$$x \times \left(\frac{1}{y}\right) = \int 2y\left(\frac{1}{y}\right)dy + C$$

$$\Rightarrow \quad \frac{x}{y} = \int 2dy + C$$

$$\Rightarrow \quad \frac{x}{y} = 2y + C$$

$$\therefore \quad x = 2y^2 + Cy$$

25. What is the solution of the differential equation

$$\ln\left(\frac{dy}{dx}\right) - a = 0 \ ?$$

(a) $y = xe^a + C$

(b) $x = ye^a + C$

(c) $y = \ln x + C$

(d) $x = \ln y + C$

(a) $\because \quad \ln\left(\frac{dy}{dx}\right) - a = 0$

$$\Rightarrow \quad \ln\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \quad \frac{dy}{dx} = e^a \Rightarrow dy = e^a dx$$

On integrating both sides, we get

$$\Rightarrow \quad \int dy = \int e^a dx$$

$$\therefore \quad y = e^a x + C$$

2016 (II)

26. What are the degree and order respectively of the differential equation satisfying $e^{y\sqrt{1-x^2} + x\sqrt{1-y^2}} = Ce^x$, (where $c > 0, |x| < 1, |y| < 1$)?

(a) 1, 1 (b) 1, 2 (c) 2, 1 (d) 2, 2

(a) We have, $e^{y\sqrt{1-x^2} + x\sqrt{1-y^2}} = Ce^x$

On taking log both sides, we get

$$(y\sqrt{1-x^2} + x\sqrt{1-y^2})\log e = (\log C + x \log e)$$

$$\Rightarrow \quad y\sqrt{1-x^2} + x\sqrt{1-y^2} = \log C + x$$

$$\Rightarrow \quad y\sqrt{1-x^2} + x\sqrt{1-y^2} - x = \log C$$

On differentiating w.r.t. x, we get

$$y\frac{1}{2\sqrt{1-x^2}}(-2x) + \frac{dy}{dx}\sqrt{1-x^2} + x\frac{1}{2\sqrt{1-y^2}}(-2y)\frac{dy}{dx} + \sqrt{1-y^2} - 1 = 0$$

$$\Rightarrow \frac{-xy}{\sqrt{1-x^2}} + \frac{dy}{dx}\left(\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right) + \sqrt{1-y^2} - 1 = 0$$

$$\Rightarrow \left(\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right)\frac{dy}{dx} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - 1 = 0,$$

which is a polynomial in $\frac{dy}{dx}$.

We observe here highest order derivative is one so, order = 1 and highest power raised to $\frac{dy}{dx}$ is one, so degree = 1.

27. What is the curve which passes through the point (1, 1) and whose slope is $\frac{2y}{x}$?

(a) Circle (b) Parabola

(c) Ellipse (d) Hyperbola

(b) Given, slope

$$\frac{dy}{dx} = \frac{2y}{x} \Rightarrow \int \frac{dy}{2y} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\log|y| = \log|x| + C \qquad \ldots (i)$$

Since, curve passing through the point (1, 1).

$$\therefore \quad \frac{1}{2}\log|1| = \log|1| + C \Rightarrow C = 0$$

From Eq. (i), $\frac{1}{2}\log|y| = \log|x|$

$$\Rightarrow \log|y| = 2\log|x|$$

$$\Rightarrow \log|y| = \log|x|^2$$

$$\therefore \quad y = x^2,$$

which is equation of parabola.

28. If $x\,dy = ydx + y^2dy$, $y > 0$ and $y(1) = 1$, then what is $y(-3)$ equal to?

(a) Only 3
(b) Only –1
(c) Both –1 and 3
(d) Neither –1 nor 3

(**a**) Given equation is $ydx + y^2dy = x\,dy$

$$\Rightarrow \quad \frac{ydx - xdy}{y^2} + dy = 0$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{x}{y}\right) + dy = 0$$

On integrating we get

$$\frac{x}{y} + y = C$$

Now, $y(1) = 1 \Rightarrow C = 2$

$$\therefore \quad \frac{x}{y} + y = 2$$

Now, to find $y(-3)$, putting $x = -3$ in above equation, we get

$$\frac{-3}{y} + y = 2$$

$$\Rightarrow \quad y^2 - 2y - 3 = 0$$

$$\Rightarrow \quad y = -1, 3$$

But given that $y > 0$.

$$\therefore \quad y = 3$$

29. What is the order of the differential equation $\frac{dx}{dy} + \int y\,dx = x^3$?

(a) 1
(b) 2
(c) 3
(d) Cannot be determined

(**b**) We have, $\frac{dx}{dy} + \int ydx = x^3$

$$\Rightarrow \quad \int ydx = x^3 - \frac{dx}{dy}$$

On differentiating w.r.t. x, we get

$$\frac{d}{dx}\left(\int ydx\right) = \frac{d}{dx}\left(x^3 - \frac{dx}{dy}\right)$$

$$\Rightarrow \quad y = 3x^2 - \frac{d}{dx}\left(\frac{dx}{dy}\right)$$

$$\Rightarrow \quad y = 3x^2 - \frac{d}{dx}\left(\frac{1}{dy/dx}\right)$$

$$\Rightarrow \quad y = 3x^2 - \frac{d}{dx}\left(\frac{1}{f'(x)}\right)$$

$$\Rightarrow \quad y = 3x^2 + \frac{1}{(f'(x))^2} \cdot f''(x)$$

$$\Rightarrow \quad y(f'(x))^2 = 3x^2\,(f'(x))^2 + f''(x)$$

$$\therefore \quad y\left(\frac{dy}{dx}\right)^2 = 3x^2\left(\frac{dy}{dx}\right)^2 + \frac{d^2y}{dx^2}$$

Hence, order of this differential equation is 2.

30. Which one of the following differential equations represents the family of straight lines, which are at unit distance from the origin?

(a) $\left(y - x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$

(b) $\left(y + x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

(c) $\left(y - x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

(d) $\left(y + x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$

(**c**) Let the equation of line be

$$y = mx + c \qquad \ldots(i)$$

$\because$ Distance of this line from origin is 1 unit.

$$\therefore \quad \left|\frac{c}{\sqrt{m^2+1}}\right| = 1 \Rightarrow c = \pm\sqrt{m^2+1}$$

$\therefore$ Equation of line which is at unit distance from origin is

$$y = mx \pm \sqrt{m^2+1}$$

By differentiating this equation w.r.t x, we get

$$\frac{dy}{dx} = m$$

Now, substitute this value of m in Eq. (i), we get

$$y = x\,\frac{dy}{dx} \pm \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}$$

$$\therefore \quad \left(y - x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

2016 (I)

31. What are the order and degree, respectively of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter?

(a) 1, 2 (b) 2, 2
(c) 1, 3 (d) 1, 4

(**d**) Given,

$$y = cx + c^2 - 3c^{3/2} + 2 \qquad \ldots(i)$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = c \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$y = \frac{dy}{dx} \times x + \left(\frac{dy}{dx}\right)^2 - 3\left(\frac{dy}{dx}\right)^{3/2} + 2$$

$$\Rightarrow y - x\,\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^2 - 2 = -3\left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \left[y - x\left(\frac{dy}{dx}\right) - \left(\frac{dy}{dx}\right)^2 - 2\right]^2 = 9\left(\frac{dy}{dx}\right)^3$$

Hence, order is 1 and degree is 4.

2015 (II)

32. The degree of the differential equation $\frac{dy}{dx} - x = \left(y - x\frac{dy}{dx}\right)^{-4}$ is

(a) 2 (b) 3 (c) 4 (d) 5

(**d**) Given, $\frac{dy}{dx} - x = \left(y - x\frac{dy}{dx}\right)^{-4}$

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^4\left(\frac{dy}{dx} - x\right) = 1$$

Hence, the degree of differential equation is 5.

33. The solution of $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$ is

(a) $\sin^{-1} y = \sin^{-1} x + C$
(b) $2\sin^{-1} y = \sqrt{1-x^2} + \sin^{-1} x + C$
(c) $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$
(d) $2\sin^{-1} y = x\sqrt{1-x^2} + \cos^{-1} x + C$

where, C is an arbitrary constant.

(**c**) Given, $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$

$$= \sqrt{(1-x^2) - y^2(1-x^2)}$$

$$= \sqrt{(1-x^2)\,(1-y^2)}$$

$$= \left(\sqrt{1-x^2}\right)\left(\sqrt{1-y^2}\right)$$

$$\Rightarrow \quad \frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\,dx$$

On integrating both sides, we get

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \sqrt{1-x^2}\,dx$$

$$\Rightarrow \sin^{-1} y = \frac{1}{2}\left[x\sqrt{1-x^2} + \sin^{-1} x\right] + A$$

$$\Rightarrow 2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C \qquad [\because C = 2A]$$

34. The differential equation of the family of circles passing through the origin and having centres on the X-axis is

(a) $2xy\frac{dy}{dx} = x^2 - y^2$

(b) $2xy\frac{dy}{dx} = y^2 - x^2$

(c) $2xy\frac{dy}{dx} = x^2 + y^2$

(d) $2xy\frac{dy}{dx} + x^2 + y^2 = 0$

(***b***) Let equation of family of circles passing through origin and having centre $(a, 0)$ be

$$(x-a)^2 + (y-0)^2 = a^2$$

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = a^2$$

$$\Rightarrow \quad x^2 + y^2 - 2ax = 0 \quad \ldots(i)$$

On differentiating Eq. (i), we get

$$2x + 2y\frac{dy}{dx} - 2a = 0$$

$$\Rightarrow \quad x + y\frac{dy}{dx} - a = 0$$

$$\Rightarrow \quad x + y\frac{dy}{dx} - \left[\frac{x^2 + y^2}{2x}\right] = 0 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad 2x^2 + 2xy\frac{dy}{dx} - x^2 - y^2 = 0$$

$$\Rightarrow \quad 2xy\frac{dy}{dx} + x^2 - y^2 = 0$$

$$\therefore \quad 2xy\frac{dy}{dx} = y^2 - x^2$$

35. The order and degree of the differential equation of parabolas having vertex at the origin and focus at $(a, 0)$, where $a > 0$, are respectively

(a) 1, 1 (b) 2, 1 (c) 1, 2 (d) 2, 2

(***a***) Equation of parabola having vertex at the origin and focus at $(a, 0)$, where $a > 0$ is $y^2 = 4ax$ ...(i)

On differentiating the above equation, we get

$$2y\frac{dy}{dx} = 4a \quad \text{[from Eq. (i)]}$$

$$y^2 = \left(2y.\frac{dy}{dx}\right)x$$

$$\Rightarrow \quad 2x\frac{dy}{dx} - y = 0$$

Hence, order and degree of differential equation are 1 and 1, respectively.

2015 (I)

36. Consider the following statements

I. The general solution of $\frac{dy}{dx} = f(x) + x$ is of the form $y = g(x) + C$, where C is an arbitrary constant.

II. The degree of $\left(\frac{dy}{dx}\right)^2 = f(x)$ is 2.

Which of the above statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(***c***) I. We have, $\frac{dy}{dx} = f(x) + x$

$$\Rightarrow \quad dy = [f(x) + x]\,dx$$

On integrating both sides, we get

$$\int dy = \int [f(x) + x]\,dx$$

$$\Rightarrow \quad y = \int f(x)\,dx + \frac{x^2}{2} + C$$

Let $\quad g(x) = \int f(x)\,dx + \frac{x^2}{2}$

Thus, general solution is of the form $y = g(x) + C$.

II. Consider the given differential equation, $\left(\frac{dy}{dx}\right)^2 = f(x)$

Clearly, the highest order derivative occurring in the differential equation is $\frac{dy}{dx}$ and its highest power is 2. Also, given equation is polynomial in derivative. So, degree of differential equation is 2.

37. What is the solution of the differential equation

$$\frac{ydx - xdy}{y^2} = 0 \ ?$$

(a) $xy = C$ (b) $y = Cx$
(c) $x + y = C$ (d) $x - y = C$

where, C is an arbitrary constant.

(***b***) Consider the given differential equation,

$$\frac{ydx - xdy}{y^2} = 0$$

$$\Rightarrow \quad d\left(\frac{x}{y}\right) = 0$$

$$\left[\because d\left(\frac{u}{v}\right) = \frac{v \cdot du - u \cdot dv}{v^2}\right]$$

On integrating both sides, we get

$$\int d\left(\frac{x}{y}\right) = C_1$$

$$\Rightarrow \quad \frac{x}{y} = C_1$$

$$\Rightarrow \quad x = C_1 y$$

$$\Rightarrow \quad y = \frac{1}{C_1}x$$

$$\therefore \quad y = Cx \quad \left[\text{where, } C = \frac{1}{C_1}\right]$$

38. What is the solution of the differential equation

$$\sin\left(\frac{dy}{dx}\right) - a = 0 \ ?$$

(a) $y = x\sin^{-1}a + C$
(b) $x = y\sin^{-1}a + C$
(c) $y = x + x\sin^{-1}a + C$
(d) $y = \sin^{-1}a + C$

where, C is an arbitrary constant.

(***a***) Consider the given differential equation,

$$\sin\left(\frac{dy}{dx}\right) - a = 0 \Rightarrow \sin\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \quad \frac{dy}{dx} = \sin^{-1}a$$

$$\Rightarrow \quad dy = (\sin^{-1}a)\,dx$$

On integrating both sides w.r.t. x, we get

$$\int dy = \int (\sin^{-1}a)\,dx$$

$$\Rightarrow \quad y = (\sin^{-1}a)\int 1dx$$

$$\therefore \quad y = (\sin^{-1}a)x + C$$

39. What is the solution of the differential equation

$$\frac{dx}{dy} + \frac{x}{y} - y^2 = 0?$$

(a) $xy = x^4 + C$ (b) $xy = y^4 + C$
(c) $4xy = y^4 + C$ (d) $3xy = y^3 + C$

where, C is an arbitrary constant.

(***c***) Consider the given differential equation, $\frac{dx}{dy} + \frac{x}{y} = y^2$

Which is linear differential equation of the form.

$\frac{dx}{dy} + Px = Q$, where, P and Q are constant or function of y only

Here, $P = \frac{1}{y}$ and $Q = y^2$

$$\therefore \quad \text{IF} = e^{\int P\,dy} = e^{\int \frac{1}{y}dy} = e^{\int \ln(y)} = y$$

Now, the solution of given differential equation is

$$x \cdot y = \int (y \cdot y^2)\,dy + C_1$$

$$[\because y \cdot (\text{IF}) = \int Q\,(\text{IF})\,dx + C]$$

$$\Rightarrow \quad x \cdot y = \int y^3\,dy + C_1$$

$$\Rightarrow \quad x \cdot y = \frac{y^4}{4} + C_1$$

$$\therefore \quad 4xy = y^4 + C \quad [\text{where, } C = 4C_1]$$

2014 (II)

40. What is the degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^2y}{dx^2}\right)^2 ?$$

(a) 1 (b) 2
(c) 3 (d) 4

(***c***) Consider the given differential equation,

$$\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^2y}{dx^2}\right)^2 \quad \ldots(i)$$

We know that, in order to find degree, differential equation should be free from fractional indices, therefore on squaring both sides, we get

$$\left(\frac{d^3y}{dx^3}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^4$$

Since, the power of highest order derivative is 3, therefore degree = 3.

41. What is the solution of the equation $\ln\left(\frac{dy}{dx}\right) + x = 0$?

(a) $y + e^x = C$
(b) $y - e^{-x} = C$
(c) $y + e^{-x} = C$
(d) $y - e^x = C$
where, C is an arbitrary constant.

(***c***) Consider the given differential equation

$$\ln\left(\frac{dy}{dx}\right) + x = 0$$

$$\Rightarrow \quad \ln\left(\frac{dy}{dx}\right) = -x \quad \Rightarrow \frac{dy}{dx} = e^{-x}$$

On separating the variables, we get

$$dy = e^{-x}dx$$

On integrating both sides, we get

$$\int dy = \int e^{-x}dx$$

$$\Rightarrow \quad y = \frac{e^{-x}}{-1} + C \Rightarrow y = -e^{-x} + C$$

$$\therefore \quad y + e^{-x} = C$$

42. Eliminating the arbitrary constants B and C in the expression $y = \frac{2}{3C}(Cx - 1)^{3/2} + B$, we get

(a) $x\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$
(b) $2x\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 1 + \left(\frac{dy}{dx}\right)^2$
(c) $\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 1$
(d) $\left(\frac{dy}{dx}\right)^2 + 1 = \frac{d^2y}{dx^2}$

(***b***) Consider the given expression is

$$y = \frac{2}{3C}(Cx - 1)^{3/2} + B$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{2}{3C}\cdot\frac{3}{2}(Cx - 1)^{1/2}\cdot C + 0 = (Cx - 1)^{1/2}$$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^2 = Cx - 1 \Rightarrow \left(\frac{dy}{dx}\right)^2 + 1 = Cx \quad \ldots(i)$$

Now, on differentiating w.r.t. x, we get

$$2\left(\frac{dy}{dx}\right)\cdot\frac{d^2y}{dx^2} = C$$

From Eq. (i),

$$\left(\frac{dy}{dx}\right)^2 + 1 = 2x\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$$

2014 (I)

43. What is the number of arbitrary constants in the particular solution of differential equation of third order?

(a) 0 (b) 1 (c) 2 (d) 3

(***a***) We know that, the solution obtained by giving particular values to the arbitrary constants in the general solution is called a particular solution i.e. in particular solution of differential of any order, there is no arbitrary constant at present.

Hence, the number of arbitrary constants in the particular solution of differential equation of third order is zero.

44. What is the general solution of the differential equation $x\,dy - y\,dx = y^2$?

(a) $x = Cy$ (b) $y^2 = Cx$
(c) $x + xy - Cy = 0$ (d) None of these
where, C is an arbitrary constant.

(***c***) Given differential equation is

$$x dy - y dx = y^2$$

$$\Rightarrow \quad -(y dx - x dy) = y^2$$

$$\Rightarrow \quad -\left(\frac{y dx - x dy}{y^2}\right) = 1$$

$$\Rightarrow \quad -d\left(\frac{x}{y}\right) = 1$$

$$\Rightarrow \quad d\left(\frac{x}{y}\right) = -1$$

On integrating both sides, we get

$$\frac{x}{y} = -x + C$$

$$\Rightarrow \quad x = -xy + Cy$$

$$\Rightarrow \quad x + xy - Cy = 0$$

which is the required general solution.

45. The solution of $\frac{dy}{dx} = |x|$ is

(a) $y = \frac{x|x|}{2} + C$
(b) $y = \frac{|x|}{2} + C$
(c) $y = \frac{x^2}{2} + C$
(d) $y = \frac{x^3}{2} + C$
where, C is an arbitrary constant.

(***a***) Given differential equation is

$$\frac{dy}{dx} = |x|$$

$$dy = |x|dx \quad \ldots(i)$$

Case I If $x > 0$

$$\int dy = \int x\,dx$$

[integrating both sides]

$$\Rightarrow \quad y = \frac{x^2}{2} + C$$

$$\Rightarrow \quad y = \frac{x\cdot(x)}{2} + C \quad \ldots(ii)$$

Case II If $x < 0$

$$\int dy = -\int x\,dx$$

[integrating both sides]

$$\Rightarrow y = \frac{-x^2}{2} + C \Rightarrow y = \frac{x\cdot(-x)}{2} + C \ldots(iii)$$

When we combined both cases, we get the required solution

$$y = \frac{x|x|}{2} + C$$

46. What is the solution of $\frac{dy}{dx} + 2y = 1$ satisfying $y(0) = 0$?

(a) $y = \frac{1 - e^{-2x}}{2}$ (b) $y = \frac{1 + e^{-2x}}{2}$
(c) $y = 1 + e^x$ (d) $y = \frac{1 + e^x}{2}$

(***a***) Given differential equation is

$$\frac{dy}{dx} + 2y = 1$$

On comparing with $\frac{dy}{dx} + Py = Q$, we get

$P = 2$ and $Q = 1$

Now, IF $= e^{\int P dx} = e^{\int 2dx} = e^{2x}$

Then, complete solution

$$y\cdot(\text{IF}) = \int Q(\text{IF})dx + C = \int 1\cdot e^{2x}dx + C$$

$$\Rightarrow \quad y\cdot e^{2x} = \frac{1}{2}e^{2x} + C$$

$$\Rightarrow \quad y = \frac{1}{2} + Ce^{-2x} \quad \ldots(i)$$

Now, at $x = 0 \Rightarrow y = 0$

From Eq. (i), we get

$$0 = \frac{1}{2} + Ce^0 \Rightarrow \frac{1}{2} + C = 0 \Rightarrow C = -\frac{1}{2}$$

$\therefore$ Required solution is

$$y = \frac{1}{2} - \frac{1}{2}\cdot e^{-2x} \Rightarrow y = \frac{1 - e^{-2x}}{2}$$

47. What is the equation of a curve passing through (0, 1) and whose differential equation is given by $dy = y \tan x\,dx$?

(a) $y = \cos x$ (b) $y = \sin x$
(c) $y = \sec x$ (d) $y = \text{cosec } x$

(***c***) The given differential equation of the curve is

$$dy = y \tan x dx$$

$$\Rightarrow \quad \int\frac{dy}{y} = \int \tan x\cdot dx$$

[on integrating both sides]

$$\Rightarrow \quad \log y = \log \sec x + \log C$$

$$\Rightarrow \quad \log y = \log C\cdot\sec x$$

$\Rightarrow \quad y = C \cdot \sec x \quad \ldots(i)$

Since, the curve passes through the origin (0, 1), then

$1 = C \cdot \sec 0 \Rightarrow C = 1$

$\therefore$ Required equation of curve is, $y = \sec x$

48. Consider the following statements in respect of the differential equation

$$\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$$

I. The degree of the differential equation is not defined.

II. The order of the differential equation is 2.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(c) Given differential equation is

$$\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$$

I. Since, the given differential equation is not a polynomial equation in its derivatives, then its degree is not defined.

II. Since, the highest order derivative in the given polynomial is 2.

Hence, the order of the differential equation is 2.

Directions (Q. Nos. 49-51) *The general solution of the differential equation* $(x^2 + x + 1)\,dy + (y^2 + y + 1)\,dx = 0$ *is* $(x + y + 1) = A(1 + Bx + Cy + Dxy)$*, where* B, C *and* D *are constants and* A *is parameter.*

49. What is B equal to?

(a) –1 (b) 1
(c) 2 (d) None of these

50. What is C equal to?

(a) 1 (b) –1
(c) 2 (d) None of these

51. What is D equal to?

(a) –1 (b) 1
(c) –2 (d) None of these

Solutions (Q. Nos. 49-51) Given differential equation is

$(x^2 + x + 1)dy + (y^2 + y + 1)dx = 0$

$\Rightarrow \quad (x^2 + x + 1)dy = -(y^2 + y + 1)dx$

$\Rightarrow \quad \frac{dy}{(1 + y + y^2)} = -\frac{dx}{(1 + x + x^2)}$

$\Rightarrow \quad \frac{dx}{(1 + x + x^2)} + \frac{dy}{(1 + y + y^2)} = 0$

$\Rightarrow \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} + \int \frac{dy}{\left(y + \frac{1}{2}\right)^2 + \frac{3}{4}} = 0$

$\Rightarrow \quad \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \int \frac{dy}{\left(y + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 0$

$\Rightarrow \quad \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left\{\frac{\left(x + \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right\} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left\{\frac{\left(y + \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right\}$

$= \frac{2}{\sqrt{3}} \tan^{-1} C_1$

$\left[\because \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$

$\Rightarrow \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2y + 1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \tan^{-1} C_1$

$\Rightarrow \quad \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2y + 1}{\sqrt{3}}\right) = \tan^{-1} C_1$

$\Rightarrow \quad \tan^{-1}\left\{\frac{\left(\frac{2x + 1}{\sqrt{3}}\right) + \left(\frac{2y + 1}{\sqrt{3}}\right)}{1 - \left(\frac{2x + 1}{\sqrt{3}}\right)\left(\frac{2y + 1}{\sqrt{3}}\right)}\right\} = \tan^{-1} C_1$

$\left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)\right]$

$\Rightarrow \quad \frac{\sqrt{3}[(2x + 1) + (2y + 1)]}{3 - (2x + 1)\cdot(2y + 1)} = C_1$

$\Rightarrow \quad \frac{\sqrt{3}(2x + 2y + 2)}{3 - (4xy + 2y + 2x + 1)} = C_1$

$\Rightarrow \quad \frac{2\sqrt{3}(x + y + 1)}{-4xy - 2y - 2x + 2} = C_1$

$\Rightarrow \quad 2\sqrt{3}(x + y + 1) = C_1(2 - 2x - 2y - 4xy)$

$\Rightarrow 2\sqrt{3}(x + y + 1) = 2C_1(1 - x - y - 2xy)$

$\Rightarrow \quad (x + y + 1) = \frac{C_1}{\sqrt{3}}(1 - x - y - 2xy)$

On comparing with $(x + y + 1) = A(1 + Bx + Cy + Dxy)$, here A is parameter and B, C and D are constants, we get

49. (a) The value of $B = -1$

50. (b) The value of $C = -1$

51. (c) The value of $D = -2$

27

VECTOR ALGEBRA

2019 (II)

1. Consider the following statements.

1. The magnitude of $\mathbf{a} \times \mathbf{b}$ is same as the area of a triangle with sides $\mathbf{a}$ and $\mathbf{b}$
2. If $\mathbf{a} \times \mathbf{b} = 0$, where $\mathbf{a} \neq 0, \mathbf{b} \neq 0$, then $\mathbf{a} = \lambda\, \mathbf{b}$.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(*b*) 1. We know that,

Area of triangle with sides $\mathbf{a}$ and $\mathbf{b}$

$$= \frac{1}{2}|\mathbf{a} \times \mathbf{b}|$$

So, Statement 1 is not correct.

2. $\mathbf{a} \times \mathbf{b} = 0$, where $\mathbf{a} \neq 0, \mathbf{b} = 0$,

So, $\mathbf{a}$ and $\mathbf{b}$ are parallel.

$$\Rightarrow \quad \mathbf{a} = \lambda\, \mathbf{b}$$

So, Statement 2 is correct.

Hence, only Statement 2 is correct.

2. If $\mathbf{a}$ and $\mathbf{b}$ are unit vectors and θ is the angle between them, then what is $\sin^2\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\mathbf{a} + \mathbf{b}|^2}{4}$ (b) $\frac{|\mathbf{a} - \mathbf{b}|^2}{4}$

(c) $\frac{|\mathbf{a} + \mathbf{b}|^2}{2}$ (d) $\frac{|\mathbf{a} - \mathbf{b}|^2}{2}$

(*b*) Given, $|\mathbf{a}| = 1, |\mathbf{b}| = 1$

We know that,

$$|\mathbf{a} - \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a} \cdot \mathbf{b}$$

$$\Rightarrow |\mathbf{a} - \mathbf{b}|^2 = 1 + 1 - 2\,|\mathbf{a}||\mathbf{b}|\cos\theta$$

$$\Rightarrow |\mathbf{a} - \mathbf{b}|^2 = 2 - 2\cos\theta$$

$$\Rightarrow |\mathbf{a} - \mathbf{b}|^2 = 2\,(1 - \cos\theta)$$

$$\Rightarrow |\mathbf{a} - \mathbf{b}|^2 = 2\left(1 - 1 + 2\sin^2\frac{\theta}{2}\right)$$

$$\Rightarrow |\mathbf{a} - \mathbf{b}|^2 = 2 \cdot \left(2\sin^2\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \sin^2\frac{\theta}{2} = \frac{|\mathbf{a} - \mathbf{b}|^2}{4}$$

3. Consider the following equations for two vectors $\mathbf{a}$ and $\mathbf{b}$.

1. $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = |\mathbf{a}|^2 - |\mathbf{b}|^2$
2. $(|\mathbf{a} + \mathbf{b}|)\,(|\mathbf{a} - \mathbf{b}|) = |\mathbf{a}|^2 - |\mathbf{b}|^2$
3. $|\mathbf{a} \cdot \mathbf{b}| + |\mathbf{a} \times \mathbf{b}| = |\mathbf{a}^2\,|\mathbf{b}|^2$

Which of the above statements are correct?

(a) 1, 2 and 3 (b) Only 1 and 2
(c) Only 1 and 3 (d) Only 2 and 3

(*c*) I : $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$

$$= \mathbf{a} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{b}$$

$$|\mathbf{a}|^2 - \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{b} - |\mathbf{b}|^2$$

$$[\because \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}]$$

$$= |\mathbf{a}|^2 - |\mathbf{b}|^2$$

So, Statement 1 is correct.

2. $(|\mathbf{a} + \mathbf{b}|)\,(|\mathbf{a} - \mathbf{b}|) = |\mathbf{a} + \mathbf{b}||\mathbf{a} - \mathbf{b}|$

$$\neq |\mathbf{a}|^2 - |\mathbf{b}|^2$$

So, Statement 2 is not correct.

3. $|\mathbf{a} \cdot \mathbf{b}|^2 + |\mathbf{a} \times \mathbf{b}|^2 = |\,|\mathbf{a}|\,|\mathbf{b}|\cos\theta|^2 + ||\mathbf{a}|\,|\mathbf{b}|\sin\theta|^2$

$$= |\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta + |\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta$$

$$= |\mathbf{a}|^2|\mathbf{b}|^2(\cos^2\theta + \sin^2\theta)$$

$$= |\mathbf{a}|^2|\mathbf{b}|^2 \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

So, Statement 3 is correct.

Hence, only Statements 1 and 3 are correct.

4. If the magnitude of the sum of two non-zero vectors is equal to the magnitude of their difference, then which one of the following is correct?

(a) The vectors are parallel
(b) The vectors are perpendicular
(c) The vectors are anti-parallel
(d) The vectors must be unit vectors

(*b*) Let $\mathbf{a}$ and $\mathbf{b}$ are the two non-zero vectors.

According to the question,

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$$

$$\Rightarrow \mathbf{a}^2 + \mathbf{b}^2 + 2\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^2 + \mathbf{b}^2 - 2\mathbf{a} \cdot \mathbf{b}$$

$$\Rightarrow \quad 4\mathbf{a} \cdot \mathbf{b} = 0$$

$$\Rightarrow \quad \mathbf{a} \cdot \mathbf{b} = 0$$

So, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

5. What is the scalar projection of

$\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ on

$\mathbf{b} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$?

(a) $\frac{\sqrt{6}}{9}$ (b) $\frac{19}{9}$

(c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

(*b*) $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

Projection of $\mathbf{a}$ on $\mathbf{b}$

$$= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$$

$$= \frac{(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}})}{\sqrt{16 + 16 + 49}}$$

$$= \frac{4 + 8 + 7}{\sqrt{81}} = \frac{19}{9}$$

2019 (I)

6. If $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, then what is $(\mathbf{b} - \mathbf{a}) \cdot (3\mathbf{a} + \mathbf{b})$ equal to ?

(a) 106 (b) –106
(c) 53 (d) –53

(*b*) We have, $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$$\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$$\therefore \mathbf{b} - \mathbf{a} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$$

$$= \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$$

and $3\mathbf{a} + \mathbf{b} = 3(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$
$= 5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$
$\therefore (\mathbf{b} - \mathbf{a}) \cdot (3\mathbf{a} + \mathbf{b})$
$= (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}) \cdot (5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$
$= 5 - 15 - 96 = -106$

7. If the position vectors of points **A** and **B** are $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ respectively, then what is the length of **AB** ?

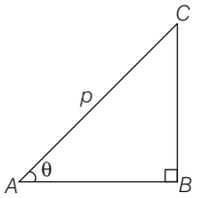
(a) $\sqrt{14}$ (b) $\sqrt{29}$
(c) $\sqrt{43}$ (d) $\sqrt{53}$

(*d*) We have , $\mathbf{OA} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$
$\mathbf{OB} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
$\therefore |\mathbf{AB}| = |\mathbf{OB} - \mathbf{OA}|$
$= |(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})|$
$= |(-\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}|$
$= \sqrt{(-1)^2 + (6)^2 + (-4)^2}$
$= \sqrt{1 + 36 + 16} = \sqrt{53}$

8. If in a right angled triangle *ABC*, hypotenuse $AC = p$, then what is $\mathbf{AB} \cdot \mathbf{AC} + \mathbf{BC} \cdot \mathbf{BA} + \mathbf{CA} \cdot \mathbf{CB}$ equal to?

(a) p^2 (b) $2p^2$
(c) $\frac{p^2}{2}$ (d) p

(*a*) In right angled ΔABC, we have $\angle ABC = 90°$



Let $\angle BAC = \theta$
Then, $\angle ACB = (90° - \theta)$
$\therefore \mathbf{AB} \cdot \mathbf{AC} + \mathbf{BC} \cdot \mathbf{BA} + \mathbf{CA} \cdot \mathbf{CB}$
$= |\mathbf{AB}||\mathbf{AC}| \cos\theta + |\mathbf{BC}||\mathbf{BA}| \cos 90° + |\mathbf{CA}||\mathbf{CB}| \cos(90° - \theta)$
$= |\mathbf{AB}||\mathbf{AB}| + 0 + |\mathbf{CB}||\mathbf{CB}|$
$= |\mathbf{AB}|^2 + |\mathbf{CB}|^2$
$= |\mathbf{AC}|^2 = p^2$

9. The sine of the angle between vectors $\mathbf{a} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ is

(a) $\frac{1}{\sqrt{26}}$ (b) $\frac{5}{\sqrt{26}}$ (c) $\frac{5}{26}$ (d) $\frac{1}{26}$

(*b*) Let θ be the angle between vectors **a** and **b**

$\therefore \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

Since,

$\mathbf{a} \cdot \mathbf{b} = (2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$
$= 8 - 18 + 3 = -7$
$|\mathbf{a}| = \sqrt{2^2 + (-6)^2 + (-3)^2}$
$= \sqrt{49} = 7$
$|\mathbf{b}| = \sqrt{4^2 + 3^2 + (-1)^2} = \sqrt{26}$
$\therefore \cos\theta = \frac{-7}{7 \times \sqrt{26}} = -\frac{1}{\sqrt{26}}$
$\therefore \sin\theta = \sqrt{1 - \cos^2\theta} = \frac{5}{\sqrt{26}}$

10. What is the value of λ for which the vectors $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $-2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ are perpendicular?

(a) 1 (b) 2 (c) 3 (d) 4

(*d*) Since, given vectors are perpendicular.
$\therefore (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 10\hat{\mathbf{k}}) = 0$
$\Rightarrow -6 + 4\lambda - 10 = 0$
$\Rightarrow 4\lambda - 16 = 0$
$\therefore \lambda = 4$

2018 (II)

11. Let $|\mathbf{a}| \neq 0, |\mathbf{b}| \neq 0$.
$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = |\mathbf{a}|^2 + |\mathbf{b}|^2$
holds if and only if

(a) **a** and **b** are perpendicular
(b) **a** and **b** are parallel
(c) **a** and **b** are inclined at an angle of 45°
(d) **a** and **b** are anti-parallel

(*a*) Given,
$|\mathbf{a}| \neq 0, |\mathbf{b}| \neq 0$
$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a} \cdot \mathbf{b}$
$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
$\mathbf{a} \cdot \mathbf{b} = 0$
So, **a** and **b** are perpendicular.

12. If $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, then what is $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ equal to ?

(a) x (b) $x + y$
(c) $-(x + y + z)$ (d) $(x + y + z)$

(*d*) $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$
Now, $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ $[\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1]$
$= (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
$= (x + y + z)$

13. A unit vector perpendicular to each of the vectors $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$ is

(a) $\frac{1}{\sqrt{3}}\hat{\mathbf{i}} + \frac{1}{\sqrt{3}}\hat{\mathbf{j}} - \frac{1}{\sqrt{3}}\hat{\mathbf{k}}$
(b) $\frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}} + \frac{1}{2}\hat{\mathbf{k}}$
(c) $\frac{1}{\sqrt{3}}\hat{\mathbf{i}} - \frac{1}{\sqrt{3}}\hat{\mathbf{j}} - \frac{1}{\sqrt{3}}\hat{\mathbf{k}}$
(d) $\frac{1}{\sqrt{3}}\hat{\mathbf{i}} + \frac{1}{\sqrt{3}}\hat{\mathbf{j}} + \frac{1}{\sqrt{3}}\hat{\mathbf{k}}$

(*a*) We know that,
$\hat{\mathbf{n}} = \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}$
$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -1 & 1 \\ 3 & -4 & -1 \end{vmatrix}$
$\mathbf{a} \times \mathbf{b} = 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$
$|\mathbf{a} \times \mathbf{b}| = 5\sqrt{3}$
$\therefore \hat{\mathbf{n}} = \frac{5(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})}{5\sqrt{3}}$
$= \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{3}}$

14. If $|\mathbf{a}| = 3, |\mathbf{b}| = 4$ and $|\mathbf{a} - \mathbf{b}| = 5$, then what is the value of $|\mathbf{a} + \mathbf{b}|$?

(a) 8 (b) 6
(c) $5\sqrt{2}$ (d) 5

(*d*) Given that,
$|\mathbf{a}| = 3$
$|\mathbf{b}| = 4$
and $|\mathbf{a} - \mathbf{b}| = 5$
$\therefore \mathbf{a} \cdot \mathbf{b} = 0$
$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 0 = 25$
$|\mathbf{a} + \mathbf{b}| = 5$

15. Let **a**, **b** and **c** be three mutually perpendicular vectors each of unit magnitude. If $\mathbf{A} = \mathbf{a} + \mathbf{b} + \mathbf{c}, \mathbf{B} = \mathbf{a} - \mathbf{b} + \mathbf{c}$ and $\mathbf{C} = \mathbf{a} - \mathbf{b} - \mathbf{c}$, then which one of the following is correct?

(a) $|\mathbf{A}| > |\mathbf{B}| > |\mathbf{C}|$
(b) $|\mathbf{A}| = |\mathbf{B}| \neq |\mathbf{C}|$
(c) $|\mathbf{A}| = |\mathbf{B}| = |\mathbf{C}|$
(d) $|\mathbf{A}| \neq |\mathbf{B}| \neq |\mathbf{C}|$

(*c*) Given that,
$\mathbf{A} = \mathbf{a} + \mathbf{b} + \mathbf{c}$
$\mathbf{B} = \mathbf{a} - \mathbf{b} + \mathbf{c}$
$\mathbf{C} = \mathbf{a} - \mathbf{b} - \mathbf{c}$
$|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$
$|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{b} \cdot \mathbf{c}| = \mathbf{c} \cdot \mathbf{a} = 0$
Now, $|\mathbf{A}| = \sqrt{a^2 + b^2 + c^2}$
$= \sqrt{1 + 1 + 1} = \sqrt{3}$
$|\mathbf{B}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$
$|\mathbf{C}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$
$\Rightarrow |\mathbf{A}| = |\mathbf{B}| = |\mathbf{C}|$

16. What is $(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b})$ equal to?

(a) $\mathbf{0}$ (b) $\mathbf{a} \times \mathbf{b}$
(c) $2(\mathbf{a} \times \mathbf{b})$ (d) $|\mathbf{a}|^2 - |\mathbf{b}|^2$

(**c**) We have,
$(\mathbf{a}-\mathbf{b})\times(\mathbf{a}+\mathbf{b})$
$=\mathbf{a}\times\mathbf{a}+\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{a}-\mathbf{b}\times\mathbf{b}$
$=\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{a}$ $[\because \mathbf{a}\times\mathbf{a}=0, \mathbf{b}\times\mathbf{b}=0]$
$=\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{b}$ $[\because \mathbf{a}\times\mathbf{b}=-\mathbf{b}\times\mathbf{a}]$
$=2(\mathbf{a}\times\mathbf{b})$

17. A spacecraft located at $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ is subjected to a force $\lambda\hat{\mathbf{k}}$ by firing a rocket. The spacecraft is subjected to a moment of magnitude
(a) λ (b) $\sqrt{3}\lambda$
(c) $\sqrt{5}\lambda$ (d) None of these

(**c**) We have,
$\mathbf{r}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\mathbf{F}=\lambda\hat{\mathbf{k}}$
We know that,
Moment $=\mathbf{r}\times\mathbf{F}$
$=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})\times(\lambda\hat{\mathbf{k}})$
$=-\lambda\hat{\mathbf{j}}+2\lambda\hat{\mathbf{i}}$ $\left[\begin{array}{c}\because \hat{\mathbf{i}}\times\hat{\mathbf{k}}=-\hat{\mathbf{j}}\\ \hat{\mathbf{j}}\times\hat{\mathbf{k}}=\hat{\mathbf{i}}\\ \hat{\mathbf{k}}\times\hat{\mathbf{k}}=0\end{array}\right]$
Magnitude of moment $=\sqrt{(-\lambda)^2+(2\lambda)^2}$
$=\sqrt{\lambda^2+4\lambda^2}=\sqrt{5\lambda^2}=\sqrt{5}\lambda$

18. In a triangle ABC, if taken in order, consider the following statements
1. $\mathbf{AB}+\mathbf{BC}+\mathbf{CA}=\mathbf{0}$
2. $\mathbf{AB}+\mathbf{BC}-\mathbf{CA}=\mathbf{0}$
3. $\mathbf{AB}-\mathbf{BC}+\mathbf{CA}=\mathbf{0}$
4. $\mathbf{BA}-\mathbf{BC}+\mathbf{CA}=\mathbf{0}$
How many of the above statement are correct?
(a) One (b) Two (c) Three (d) Four

(***a***) We know that,
In a triangle **ABC**
$\mathbf{AB}+\mathbf{BC}+\mathbf{CA}=\mathbf{0}$ [$\because$ by triangle law]
So, only first statement is correct.

2018 (I)

19. If $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $|\mathbf{a}|=2, |\mathbf{b}|=7$ and $\mathbf{a}\times\mathbf{b}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$, then what is the acute angle between $\mathbf{a}$ and $\mathbf{b}$?
(a) 30° (b) 45° (c) 60° (d) 90°

(***a***) Given, $|\mathbf{a}|=2$; $|\mathbf{b}|=7$
and $\mathbf{a}\times\mathbf{b}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$
$\because$ $|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}||\mathbf{b}|\sin\theta$
$\Rightarrow \sin\theta=\dfrac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}||\mathbf{b}|}=\dfrac{|3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}|}{2\times7}$
$=\dfrac{\sqrt{3^2+2^2+6^2}}{14}=\dfrac{\sqrt{49}}{14}$
$\Rightarrow \sin\theta=\dfrac{7}{14}=\dfrac{1}{2}\Rightarrow\sin\theta=\sin30°$
$\therefore$ $\theta=30°$

20. Let $\mathbf{p}$ and $\mathbf{q}$ be the position vectors of the points P and Q respectively with respect to origin O. The points R and S divide PQ internally and externally respectively in the ratio $2:3$. If OR and OS are perpendicular, then which one of the following is correct?
(a) $9p^2=4q^2$ (b) $4p^2=9q^2$
(c) $9p=4q$ (d) $4p=9q$

(***a***) The points R and S divider PQ internally and externally respectively in the ratio 2 : 3. The position vectors of R and S are
$\dfrac{3\mathbf{p}+2\mathbf{q}}{5}$ and $3\mathbf{p}-2\mathbf{q}$ respectively.
$OR=\dfrac{3\mathbf{p}+2\mathbf{q}}{5}$
$OS=3\mathbf{p}-2\mathbf{q}$
Now, $OR\perp OS$
$\Rightarrow$ $OR\cdot OS=0$
$\Rightarrow$ $\left(\dfrac{3\mathbf{p}+2\mathbf{q}}{5}\right).(3\mathbf{p}-2\mathbf{q})=0$
$\Rightarrow$ $(3\mathbf{p}+2\mathbf{q})\cdot(3\mathbf{p}-2\mathbf{q})=0$
$\Rightarrow$ $9\mathbf{p}\cdot\mathbf{p}-6\mathbf{p}\cdot\mathbf{q}+6\mathbf{q}\cdot\mathbf{p}-4\mathbf{q}.\mathbf{q}=0$
$\Rightarrow$ $9|\mathbf{p}|^2-4|\mathbf{q}|^2=0$
$[\because \mathbf{a}\cdot\mathbf{a}=|\mathbf{a}|^2$ and $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}]$
$\Rightarrow$ $9|\mathbf{p}|^2=4|\mathbf{q}|^2\Rightarrow 9\mathbf{p}^2=4\mathbf{q}^2$

21. What is the moment about the point $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ of a force represented by $3\hat{\mathbf{i}}+\hat{\mathbf{k}}$ acting through the point $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$?
(a) $-3\hat{\mathbf{i}}+11\hat{\mathbf{j}}+9\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+9\hat{\mathbf{k}}$
(c) $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+9\hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

(***a***) Given that,
$\mathbf{r}=(2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}})-(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})$
$=\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $\mathbf{F}=3\hat{\mathbf{i}}+\hat{\mathbf{k}}$
$\therefore$ Moment $\tau=\mathbf{r}\times\mathbf{F}$
$=(\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}})\times(3\hat{\mathbf{i}}+\hat{\mathbf{k}})$
$=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & -3 & 4\\ 3 & 0 & 1\end{vmatrix}$
$=\hat{\mathbf{i}}(-3-0)-\hat{\mathbf{j}}(1-12)+\hat{\mathbf{k}}(0+9)$
$=-3\hat{\mathbf{i}}+11\hat{\mathbf{j}}+9\hat{\mathbf{k}}$

22. If $\mathbf{a}+2\mathbf{b}+3\mathbf{c}=\mathbf{0}$ and $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}=\lambda(\mathbf{b}\times\mathbf{c})$, then what is the value of λ ?
(a) 2 (b) 3 (c) 4 (d) 6

(***d***) Given that,
$\mathbf{a}+2\mathbf{b}+3\mathbf{c}=0$
$\Rightarrow$ $\mathbf{a}+2\mathbf{b}=-3\mathbf{c}$
$\Rightarrow$ $(\mathbf{a}+2\mathbf{b})\times\mathbf{b}=-3\mathbf{c}\times\mathbf{b}$
$\Rightarrow$ $\mathbf{a}\times\mathbf{b}+2\mathbf{b}\times\mathbf{b}=3(\mathbf{b}\times\mathbf{c})$
$[\because \mathbf{c}\times\mathbf{b}=-\mathbf{b}\times\mathbf{c}]$
$\Rightarrow$ $\mathbf{a}\times\mathbf{b}=3(\mathbf{b}\times\mathbf{c})$ $[\mathbf{b}\times\mathbf{b}=0]$ … (i)
Again $3\mathbf{c}+\mathbf{a}=-2\mathbf{b}$
$\Rightarrow$ $(3\mathbf{c}+\mathbf{a})\times\mathbf{a}=-2\mathbf{b}\times\mathbf{a}$
$\Rightarrow$ $3\mathbf{c}\times\mathbf{a}+\mathbf{a}\times\mathbf{a}=2(\mathbf{a}\times\mathbf{b})$
$\Rightarrow$ $3(\mathbf{c}\times\mathbf{a})=2(\mathbf{a}\times\mathbf{b})$
$\Rightarrow$ $3(\mathbf{c}\times\mathbf{a})=6(\mathbf{b}\times\mathbf{c})$ [from Eq. (i)]
$\Rightarrow$ $\mathbf{c}\times\mathbf{a}=2(\mathbf{b}\times\mathbf{c})$ … (ii)
Now, $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}=\lambda(\mathbf{b}\times\mathbf{c})$
$=3(\mathbf{b}\times\mathbf{c})+(\mathbf{b}\times\mathbf{c})+2(\mathbf{b}\times\mathbf{c})$
$=6(\mathbf{b}\times\mathbf{c})$ [from Eqs. (i) and (ii)]
On comparing, we get
$\lambda=6$

23. If the vectors $\mathbf{K}$ and $\mathbf{A}$ are parallel to each other, then what is $k\mathbf{K}\times\mathbf{A}$ equal to ?
(a) $k^2\mathbf{A}$ (b) 0 (c) $-k^2\mathbf{A}$ (d) $\mathbf{A}$

(b) Since, $\mathbf{a}\times\mathbf{b}=0$, if $\mathbf{a}$ and $\mathbf{b}$ are parallel.
So, $k\,\mathbf{K}\times\mathbf{A}=0$ if $\mathbf{K}$ and $\mathbf{A}$ are parallel to each other.

2017 (II)

24. Let $\alpha=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$, $\beta=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\gamma=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ be three vectors. If α and β are both perpendicular to the vector δ and $\delta\cdot\gamma=10$, then what is the magnitude of δ?
(a) $\sqrt{3}$ units (b) $2\sqrt{3}$ units
(c) $\dfrac{\sqrt{3}}{2}$ unit (d) $\dfrac{1}{\sqrt{3}}$ unit

(***b***) Let $\delta=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$
It is given that $\delta\perp\alpha$ and $\delta\perp\beta$
$\therefore$ $\delta\cdot\alpha=\delta\cdot\beta=0$
$\therefore$ $x+2y-z=0$ …(i)
$[\because \alpha=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}]$
and $2x-y+3z=0$ …(ii)
$[\because \beta=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}]$
Again, $\delta\cdot\gamma=10$
$\therefore$ $2x+y+6z=10$ …(iii)
$[\because \gamma=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+6\hat{\mathbf{k}}]$
On solving Eqs. (i), (ii) and (iii), we get
$x=-2, y=2, z=2$
$\therefore$ $\delta=2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$
$\Rightarrow$ $|\delta|=\sqrt{(-2)^2+(2)^2+(2)^2}$
$=\sqrt{4+4+4}=\sqrt{12}=2\sqrt{3}$

25. If $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are two unit vectors, then the vector $(\hat{\mathbf{a}}+\hat{\mathbf{b}})\times(\hat{\mathbf{a}}\times\hat{\mathbf{b}})$ is parallel to
(a) $(\hat{\mathbf{a}}-\hat{\mathbf{b}})$ (b) $(\hat{\mathbf{a}}+\hat{\mathbf{b}})$
(c) $(2\hat{\mathbf{a}}-\hat{\mathbf{b}})$ (d) $(2\hat{\mathbf{a}}+\hat{\mathbf{b}})$

(*a*) We have,

$$(\hat{a}+\hat{b})\times(\hat{a}\times\hat{b})$$
$$=\hat{a}\times(\hat{a}\times\hat{b})+\hat{b}\times(\hat{a}\times\hat{b})$$
$$=(\hat{a}\cdot\hat{b})\hat{a}-(\hat{a}\cdot\hat{a})\hat{b}+(\hat{b}\cdot\hat{b})\hat{a}-(\hat{b}\cdot\hat{a})\hat{b}$$
$$=(\hat{a}\cdot\hat{b})\hat{a}-\hat{b}+\hat{a}-(\hat{b}\cdot\hat{a})\hat{b} \quad [\because \hat{a}\cdot\hat{a}=\hat{b}\cdot\hat{b}=1]$$
$$=(\hat{a}\cdot\hat{b})\hat{a}-(\hat{b}\cdot\hat{a})\hat{b}+\hat{a}-\hat{b}$$
$$=(\hat{a}\cdot\hat{b})\hat{a}-(\hat{a}\cdot\hat{b})\hat{b}+\hat{a}-\hat{b} \quad [\because \hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{a}]$$
$$=(\hat{a}\cdot\hat{b})(\hat{a}-\hat{b})+(\hat{a}-\hat{b})$$
$$=(\hat{a}\cdot\hat{b}+1)(\hat{a}-\hat{b})$$
$$\therefore(\hat{a}+\hat{b})\times(\hat{a}\times\hat{b})\,||\,(\hat{a}-\hat{b})$$

because $(\hat{a}\cdot\hat{b}+1)$ is a scalar quantity.

26. A force $\mathbf{F}=\hat{i}+3\hat{j}+2\hat{k}$ acts on a particle to displace it from the point $\mathbf{A}(\hat{i}+2\hat{j}-3\hat{k})$ to the point $\mathbf{B}(3\hat{i}-\hat{j}+5\hat{k})$. The work done by the force will be

(a) 5 units (b) 7 units
(c) 9 units (d) 10 units

(*c*) We have,

$\mathbf{F}=\hat{i}+3\hat{j}+2\hat{k}$, $\mathbf{A}(\hat{i}+2\hat{j}-3\hat{k})$

and $\mathbf{B}(3\hat{i}-\hat{j}+5\hat{k})$

$\therefore$ $\mathbf{AB}=(3\hat{i}-\hat{j}+5\hat{k})-(\hat{i}+2\hat{j}-3\hat{k})$
$=2\hat{i}-3\hat{j}+8\hat{k}$

Now, work done $=\mathbf{F}\cdot\mathbf{AB}$
$=(\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}-3\hat{j}+8\hat{k})$
$=1\times2+3\times(-3)+2\times8$
$=2-9+16=9$ units

27. For any vector $\mathbf{a}$

$$|\mathbf{a}\times\hat{i}|^2+|\mathbf{a}\times\hat{j}|^2+|\mathbf{a}\times\hat{k}|^2$$

is equal to

(a) $|\mathbf{a}|^2$ (b) $2|\mathbf{a}|^2$
(c) $3|\mathbf{a}|^2$ (d) $4|\mathbf{a}|^2$

(*b*) Let $\mathbf{a}=x\hat{i}+y\hat{j}+z\hat{k}$

Now, $\mathbf{a}\times\hat{i}=(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i}$
$=-y\hat{k}+z\hat{j}$

$\therefore$ $|\mathbf{a}\times\hat{i}|^2=(\sqrt{(z)^2+(-y)})^2=z^2+y^2$

Similarly, $|\mathbf{a}\times\hat{j}|^2=x^2+z^2$

and $|\mathbf{a}\times k|^2=x^2+y^2$

$\therefore$ $|\mathbf{a}\times\hat{i}|^2+|\mathbf{a}\times\hat{j}|^2+|\mathbf{a}\times\hat{k}|^2$
$=(z^2+y^2)+(x^2+z^2)+(x^2+y^2)$
$=2(x^2+y^2+z^2)$
$=2|\mathbf{a}|^2$ $[\because|\mathbf{a}|=\sqrt{x^2+y^2+z^2}]$

28. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a,b,c\neq1)$ are coplanar, then the value of

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$$ is equal to

(a) 0 (b) 1
(c) $a+b+c$ (d) abc

(*b*) Since, $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar

$$\therefore\quad\begin{vmatrix}a&1&1\\1&b&1\\1&1&c\end{vmatrix}=0$$

On applying $C_2\to C_2-C_1$, $C_3\to C_3-C_1$, we get

$$\begin{vmatrix}a&1-a&1-a\\1&b-1&0\\1&0&c-1\end{vmatrix}=0$$

$$\Rightarrow\quad a(b-1)(c-1)-(1-a)(c-1)-(1-a)(b-1)=0$$

on dividing by $(1-a)(1-b)(1-c)$, we get

$$\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$
$$\Rightarrow\quad\frac{1-1+a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$
$$\Rightarrow\quad\frac{1}{1-a}-1+\frac{1}{1-b}+\frac{1}{1-c}=0$$
$$\Rightarrow\quad\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

2017 (I)

29. If $\mathbf{a}=\hat{i}-\hat{j}+\hat{k}$, $\mathbf{b}=2\hat{i}+3\hat{j}+2\hat{k}$ and $\mathbf{c}=\hat{i}+m\hat{j}+n\hat{k}$ are three coplanar vectors and $|\mathbf{c}|=\sqrt{6}$, then which one of the following is correct?

(a) $m=2$ and $n=\pm1$
(b) $m=\pm2$ and $n=-1$
(c) $m=2$ and $n=-1$
(d) $m=\pm2$ and $n=1$

(*d*) $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar if $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=0$

$$\Rightarrow\quad\begin{vmatrix}1&-1&1\\2&3&2\\1&m&n\end{vmatrix}=0$$

$$\Rightarrow 1(3n-2m)+1(2n-2)+1(2m-3)=0$$
$$\Rightarrow\quad5n-5=0$$
$$\therefore\quad n=1$$

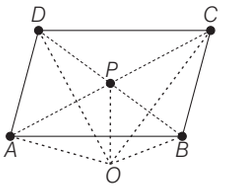
Now, $\mathbf{c}=\hat{i}+m\hat{j}+\hat{k}$

$$\Rightarrow\quad|\mathbf{c}|=\sqrt{1^2+m^2+(1)^2}=\sqrt{6}$$
$$\Rightarrow\quad\sqrt{m^2+2}=\sqrt{6}\Rightarrow\quad m^2=6-2=4$$
$$\therefore\quad m=\pm2$$

30. Let $ABCD$ be a parallelogram whose diagonals intersect at P and let O be the origin. What is $\mathbf{OA}+\mathbf{OB}+\mathbf{OC}+\mathbf{OD}$ equal to?

(a) $2\mathbf{OP}$ (b) $4\mathbf{OP}$ (c) $6\mathbf{OP}$ (d) $8\mathbf{OP}$

(*b*) Given, $ABCD$ is a parallelogram.



We known that, the diagonals of a parallelogram bisects each other,

therefore P is midpoint of AC and BD both

$\therefore$ $\mathbf{OA}+\mathbf{OC}=2\mathbf{OP}$ … (i)

$\mathbf{OB}+\mathbf{OD}=2\mathbf{OP}$ … (ii)

From Eqs. (i) and (ii), we get

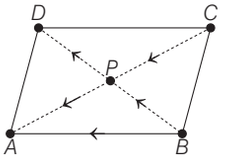
$\Rightarrow$ $\mathbf{OA}+\mathbf{OB}+\mathbf{OC}+\mathbf{OD}=4\mathbf{OP}$

31. $ABCD$ is a quadrilateral whose diagonals are AC and BD. Which one of the following is correct?

(a) $\mathbf{BA}+\mathbf{CD}=\mathbf{AC}+\mathbf{DB}$
(b) $\mathbf{BA}+\mathbf{CD}=\mathbf{BD}+\mathbf{CA}$
(c) $\mathbf{BA}+\mathbf{CD}=\mathbf{AC}+\mathbf{BD}$
(d) $\mathbf{BA}+\mathbf{CD}=\mathbf{BC}+\mathbf{AD}$

(*b*) We have,

In quadrilateral $ABCD$, AC and BD are diagonals.



Let diagonals AC and BD are intersects each other at P.

then $\mathbf{BA}=\mathbf{BP}+\mathbf{PA}$ … (i)

and $\mathbf{CD}=\mathbf{CP}+\mathbf{PD}$ … (ii)

On adding Eqs. (i) and (ii), we get

$\mathbf{BA}+\mathbf{CD}=(\mathbf{BA}+\mathbf{PA})+(\mathbf{CP}+\mathbf{PD})$
$=(\mathbf{BP}+\mathbf{PD})+(\mathbf{CP}+\mathbf{PA})$

$\Rightarrow$ $\mathbf{BA}+\mathbf{CD}=\mathbf{BD}+\mathbf{CA}$

32. If $\mathbf{a}\times\mathbf{b}=\mathbf{c}$ and $\mathbf{b}\times\mathbf{c}=\mathbf{a}$, then which one of the following is correct

(a) $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are orthogonal in pairs and $|\mathbf{a}|=|\mathbf{c}|$ and $|\mathbf{b}|=1$
(b) $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are non-orthogonal to each other
(c) $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are orthogonal in pairs but $|\mathbf{a}|\neq|\mathbf{c}|$
(d) $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are orthogonal in pairs but $|\mathbf{b}|\neq1$

(*a*) Given, $\mathbf{a}\times\mathbf{b}=\mathbf{b}$ …(i)

$\therefore$ vector $\mathbf{c}$ is perpendicular to both vectors $\mathbf{a}$ and $\mathbf{b}$.

and, $\mathbf{b}\times\mathbf{c}=\mathbf{a}$ …(ii)

$\therefore$ vector $\mathbf{a}$ is perpendicular to both vectors $\mathbf{b}$ and $\mathbf{c}$

$\therefore$ $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are orthogonal in pairs.

Now, $\because$ $\mathbf{a}\times\mathbf{b}=\mathbf{c}$

$\Rightarrow$ $|\mathbf{a}\times\mathbf{b}|=|\mathbf{c}|$

$\Rightarrow \quad |\mathbf{a}|\,|\mathbf{b}|\sin 90° = |\mathbf{c}|$
[$\because \mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other]
$\Rightarrow \quad |\mathbf{a}|\,|\mathbf{b}| = |\mathbf{c}|$ …(iii)
[$\because \sin 90° = 1$]
and $\mathbf{b} \times \mathbf{c} = \mathbf{a} \Rightarrow |\mathbf{b} \times \mathbf{c}| = |\mathbf{a}|$
$\Rightarrow \quad |\mathbf{b}|\,|\mathbf{c}|\sin 90° = |\mathbf{a}|$
[$\because \mathbf{b}$ and $\mathbf{c}$ are perpendicular to each other]
$\Rightarrow \quad |\mathbf{b}|\,|\mathbf{c}| = |\mathbf{a}|$ …(iv)
From Eqs. (iii) and (iv), we get
$|\mathbf{b}|\,|\mathbf{a}|\,|\mathbf{b}| = |\mathbf{a}|$
$\Rightarrow \quad |\mathbf{b}| = 1$ and $|\mathbf{a}| = |\mathbf{c}|$

33. If $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \lambda\hat{\mathbf{k}}$ are perpendicular, then what is the value of λ?
(a) 2 (b) 3 (c) 4 (d) 5

(*b*) Since, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other then
$2 \times 3 + 3 \times 2 + 4 \times (-\lambda) = 0$
$\Rightarrow \quad 6 + 6 - 4\lambda = 0$
$\Rightarrow \quad 12 - 4\lambda = 0 \Rightarrow \lambda = 3$

2016 (II)

Directions (Q. Nos. 34 and 35) *Let* $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, $\mathbf{b} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$ *and* $\mathbf{b} = \mathbf{c} + \mathbf{d}$, *where c is parallel to a and d is perpendicular to a.*

34. What is $\mathbf{c}$ equal to?
(a) $\frac{3(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{2}$ (b) $\frac{2(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{3}$
(c) $\frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{2}$ (d) $\frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{3}$

35. If $\mathbf{d} = x\,\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, then which of the following equations is/are correct?
I. $y - x = 4$ II. $2z - 3 = 0$
Select the correct answer using the code given below.
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Solutions (Q. Nos. 34 and 35)
Given, $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ … (i)
$\mathbf{b} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$ … (ii)
and $\mathbf{b} = \mathbf{c} + \mathbf{d}$ … (iii)
Since, $\mathbf{c}$ is parallel to $\mathbf{a}$ and $\mathbf{d}$ is perpendicular to $\mathbf{a}$, then
$\mathbf{c} = \lambda\mathbf{a}$ … (iv)
and $\mathbf{d} \cdot \mathbf{a} = 0$ … (v)
Now, $\mathbf{b} = \lambda\mathbf{a} + (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}})$
By using Eqs. (ii), (iii) and (iv), we get
$(3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}) = \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}})$
$\Rightarrow (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}) = (\lambda + x)\hat{\mathbf{i}} + (\lambda + y)\hat{\mathbf{j}} + z\hat{\mathbf{k}}$
Equating both sides, we get
$\lambda + x = 3$ … (vi)
$\lambda + y = 0$ … (vii)
and $z = 4$ …(viii)
From Eq. (vi) and (vii), we get
$x - y = 3$ …(ix)
Now, $\mathbf{d} \cdot \mathbf{a} = 0$
$\Rightarrow (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 0$
$\Rightarrow \quad x + y = 0$ …(x)
On solving Eqs. (ix) and (x), we get
$x = 3/2$ and $y = -3/2$
$\therefore \quad \lambda = 3 - x = 3 - \frac{3}{2} = \frac{3}{2}$

34. (*a*) $\because \mathbf{c} = \lambda\mathbf{a}$ [from Eq. (iv)]
$\Rightarrow \quad \mathbf{c} = \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
$\therefore \quad \mathbf{c} = \frac{3}{2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

35. (*d*) We have,
$y - x = -3$ [from Eq. (ix)]
and $2z - 3 = 5$ [from Eq. (x)]
Hence, both statements are incorrect.

Directions (Q. Nos. 36 and 37) *Let a, b and c be three vectors such that* $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ *and* $|\mathbf{a}| = 10, |\mathbf{b}| = 6$ *and* $|\mathbf{c}| = 14$.

36. What is $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$ equal to?
(a) –332 (b) –166 (c) 0 (d) 166

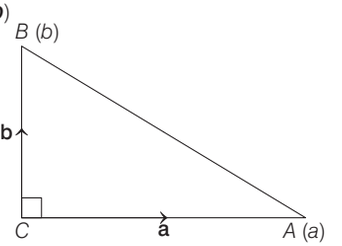
(*b*) Given, $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$
and $|\mathbf{a}| = 10, |\mathbf{b}| = 6$ and $|\mathbf{c}| = 14$
$(\mathbf{a} + \mathbf{b} + \mathbf{c})^2 = (|\mathbf{a}|)^2 + (|\mathbf{b}|)^2 + (|\mathbf{c}|)^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a})$
$\Rightarrow \quad 0 = (10)^2 + (6)^2 + (14)^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a})$
$\therefore \quad \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = -166$

37. What is the angle between $\mathbf{a}$ and $\mathbf{b}$?
(a) 30° (b) 45° (c) 60° (d) 75°

(*c*) $\because \mathbf{a} + \mathbf{b} + \mathbf{c} = 0$
$\Rightarrow \quad \mathbf{a} + \mathbf{b} = -\mathbf{c} \Rightarrow (\mathbf{a} + \mathbf{b})^2 = (-\mathbf{c})^2$
$\Rightarrow (|\mathbf{a}|)^2 + (|\mathbf{b}|)^2 + 2(\mathbf{a} \cdot \mathbf{b}) = (|\mathbf{c}|)^2$
$\Rightarrow \quad (10)^2 + (6)^2 + 2(\mathbf{a} \cdot \mathbf{b}) = (14)^2$
$\Rightarrow \quad \mathbf{a} \cdot \mathbf{b} = 30$
$\Rightarrow \quad |\mathbf{a}|\,|\mathbf{b}|\cos\theta = 30$
$\Rightarrow \quad (10 \times 6)\cos\theta = 30$
$\Rightarrow \quad \cos\theta = \frac{1}{2}$
$\therefore \quad \theta = 60°$

38. In a right-angled triangle ABC, if the hypotenuse $AB = p$, then what is $\mathbf{AB} \cdot \mathbf{AC} + \mathbf{BC} \cdot \mathbf{BA} + \mathbf{CA} \cdot \mathbf{CB}$ equal to?
(a) p (b) p^2
(c) $2p^2$ (d) $\frac{p^2}{2}$

(*b*)



Let position vector of A is $\mathbf{a}$ and position vector of B is $\mathbf{b}$ w.r.t. C as origin.
$\therefore \quad \mathbf{AB} = \mathbf{b} - \mathbf{a}$
and $\mathbf{a} \cdot \mathbf{b} = 0$
Similarly, $\mathbf{AC} = -\mathbf{a}$
$\mathbf{BC} = -\mathbf{b}$
$\mathbf{BA} = \mathbf{a} - \mathbf{b}$
$\mathbf{CB} = \mathbf{b}$
$\mathbf{CA} = \mathbf{a}$
$\therefore \mathbf{AB} \cdot \mathbf{AC} + \mathbf{BC} \cdot \mathbf{BA} + \mathbf{CB} \cdot \mathbf{CA}$
$= (\mathbf{b} - \mathbf{a}) \cdot (-\mathbf{a}) + (-\mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) + \mathbf{b} \cdot \mathbf{a}$
$= -\mathbf{a} \cdot \mathbf{b} + |\mathbf{a}|^2 - \mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 + \mathbf{b} \cdot \mathbf{a}$
$= |\mathbf{a}|^2 + |\mathbf{b}|^2$ [$\because (\mathbf{a} \cdot \mathbf{b}) = 0$]
$= CA^2 + BC^2 = AB^2 = p^2$

39. A force $\mathbf{F} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ is applied at the point $(1, -1, 2)$. What is the moment of the force about the point $(2, -1, 3)$?
(a) $\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ (b) $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
(c) $2\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ (d) $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$

(*c*) Given, $\mathbf{F} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$
$\because \mathbf{F}$ is applied at the point $(1, -1, 2)$, we have to find moment of the force about the point $(2, -1, 3)$.
$\therefore$ Position vector,
$\mathbf{r} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = -\hat{\mathbf{i}} - \hat{\mathbf{k}}$
$\therefore$ Moment, $M = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$
$= \hat{\mathbf{i}}(0 + 2) - \hat{\mathbf{j}}(4 + 3) + \hat{\mathbf{k}}(-2 - 0)$
$= 2\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

2016 (I)

40. What is a vector of unit length orthogonal to both the vectors $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$?
(a) $\frac{-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{26}}$ (b) $\frac{-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{26}}$
(c) $\frac{-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{14}}$ (d) $\frac{-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{14}}$

(*b*) Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Clearly, the vector which is orthogonal to both the vectors, is

$$\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 1 & 1\\ 2 & 3 & -1\end{vmatrix}$$

$$=\hat{\mathbf{i}}(-1-3)-\hat{\mathbf{j}}(-1-2)+\hat{\mathbf{k}}(3-2)$$
$$=-4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

Now, required vector = Unit vector along $\mathbf{a}\times\mathbf{b}$

$$=\frac{-4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{(-4)^2+(3)^2+1^2}}=\frac{-4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{26}}$$

41. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then which one of the following is correct?

(a) $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$
(b) $\mathbf{a}+\mathbf{b}+\mathbf{c}=$ unit vector
(c) $\mathbf{a}+\mathbf{b}=\mathbf{c}$
(d) $\mathbf{a}=\mathbf{b}+\mathbf{c}$

(*a*) Given, position vectors of vertices of equilateral triangle are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

$\therefore$ Orthocentre of an equilateral triangle $=\dfrac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$

Since, orthocentre of triangle is at the origin.

So, $\dfrac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}=0$

$\Rightarrow \quad \mathbf{a}+\mathbf{b}+\mathbf{c}=0$

[$\because$ in a equilateral triangle, centroid and orthocentre coincide]

42. What is the area of the parallelogram having diagonals $3\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$?

(a) $5\sqrt{5}$ sq units (b) $4\sqrt{5}$ sq units
(c) $5\sqrt{3}$ sq units (d) $15\sqrt{2}$ sq units

(*c*) Let diagonals of a parallelogram are $\mathbf{d}_1=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{d}_2=\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

Area of parallelogram $=\dfrac{1}{2}|\mathbf{d}_1\times\mathbf{d}_2|$

Then, $\mathbf{d}_1\times\mathbf{d}_2=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}$

$=\hat{\mathbf{i}}(4-6)-\hat{\mathbf{j}}(12+2)+\hat{\mathbf{k}}(-9-1)$

$\Rightarrow \mathbf{d}_1\times\mathbf{d}_2=-2\hat{\mathbf{i}}-14\hat{\mathbf{j}}-10\hat{\mathbf{k}}$

Now,
$|\mathbf{d}_1\times\mathbf{d}_2|=\sqrt{(-2)^2+(-14)^2+(-10)^2}$
$=\sqrt{4+196+100}=\sqrt{300}$
$=10\sqrt{3}$

Hence, area of parallelogram
$=\dfrac{1}{2}\times 10\sqrt{3}=5\sqrt{3}$ sq units

Directions (Q. Nos. 43 and 44) *Let* $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ *of two unit vectors and* θ *be the angle between them.*

43. What is $\cos\left(\dfrac{\theta}{2}\right)$ equal to?

(a) $\dfrac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{2}$ (b) $\dfrac{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}{2}$
(c) $\dfrac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{4}$ (d) $\dfrac{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}{4}$

(*b*) Given $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are unit vectors.
Now, $|\hat{\mathbf{a}}+\hat{\mathbf{b}}|^2=(\hat{\mathbf{a}}+\hat{\mathbf{b}})\cdot(\hat{\mathbf{a}}+\hat{\mathbf{b}})$
$=|\hat{\mathbf{a}}|^2+|\hat{\mathbf{b}}|^2+2|\hat{\mathbf{a}}||\hat{\mathbf{b}}|\cos\theta$
$=1+1+2\cos\theta$
$\Rightarrow|\hat{\mathbf{a}}+\hat{\mathbf{b}}|^2=2+2\cos\theta$
$=2\times 2\cos^2\dfrac{\theta}{2}$
$\Rightarrow \quad |\hat{\mathbf{a}}+\hat{\mathbf{b}}|^2=4\cos^2\dfrac{\theta}{2}$
$\Rightarrow \quad |\hat{\mathbf{a}}+\hat{\mathbf{b}}|=2\cos\dfrac{\theta}{2}$
$\therefore \quad \cos\dfrac{\theta}{2}=\dfrac{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}{2}$

44. What is $\sin\left(\dfrac{\theta}{2}\right)$ equal to?

(a) $\dfrac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{2}$ (b) $\dfrac{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}{2}$
(c) $\dfrac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{4}$ (d) $\dfrac{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}{4}$

(*a*) $|\hat{\mathbf{a}}-\hat{\mathbf{b}}|^2=(\hat{\mathbf{a}}-\hat{\mathbf{b}})\cdot(\hat{\mathbf{a}}-\hat{\mathbf{b}})$
$=|\hat{\mathbf{a}}|^2+|\hat{\mathbf{b}}|^2-2|\hat{\mathbf{a}}||\hat{\mathbf{b}}|\cos\theta$
$=1+1-2\cos\theta$
$=2-2\cos\theta=2(1-\cos\theta)$
$|\hat{\mathbf{a}}-\hat{\mathbf{b}}|^2=2\times 2\sin^2\dfrac{\theta}{2}$
$\Rightarrow \sin\dfrac{\theta}{2}=\dfrac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{2}$

2015 (II)

45. The area of the square, one of whose diagonals is $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$, is

(a) 12 sq units (b) 12.5 sq units
(c) 25 sq units (d) 156.25 sq units

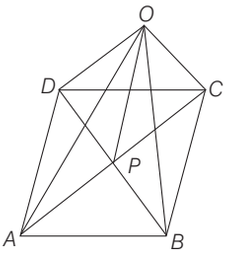
(*b*) The length of diagonal is $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$.
$\therefore \sqrt{9+16}=\sqrt{25}=5$
Let length of square be e.
$\therefore \quad e\sqrt{2}=5 \Rightarrow e=\dfrac{5}{\sqrt{2}}$

Now, area of the square $=e^2=\left(\dfrac{5}{\sqrt{2}}\right)^2$
$=\dfrac{25}{2}=12.5$ sq units

46. $ABCD$ is a parallelogram and P is the point of intersection of the diagonals. If O is the origin, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ is equal to

(a) 4OP (b) 2 OP
(c) OP (d) Null vector

(*a*) We know that P will be the mid-point of AC and BD.



$\therefore \quad \mathbf{OA}+\mathbf{OC}=2\mathbf{OP}$ ...(i)
$\mathbf{OB}+\mathbf{OD}=2\mathbf{OP}$ ...(ii)

On adding Eqs. (i) and (ii), we get
$\mathbf{OA}+\mathbf{OB}+\mathbf{OC}+\mathbf{OD}=4\cdot\mathbf{OP}$

47. If $\mathbf{b}$ and $\mathbf{c}$ are the position vectors of the points B and C respectively, then the position vector of the point D such that $\mathbf{BD}=4\,\mathbf{BC}$ is

(a) $4(\mathbf{c}-\mathbf{b})$ (b) $-4(\mathbf{c}-\mathbf{b})$
(c) $4\mathbf{c}-3\mathbf{b}$ (d) $4\mathbf{c}+3\mathbf{b}$

(*c*) Given, $\mathbf{BD}=4\,\mathbf{BC}$

It means D divides the join of BC externally in the ratio 4 : 3.
$\therefore$ Position vector of D
$=\dfrac{4\mathbf{c}-3\mathbf{b}}{4-3}=4\mathbf{c}-3\mathbf{b}$

48. If the position vector $\mathbf{a}$ of the point $(5, n)$ is such that $|\mathbf{a}|=13$, then the value(s) of n can be

(a) ± 8 (b) ± 12
(c) Only 8 (d) Only 12

(*b*) We have, $\mathbf{a}=5\hat{\mathbf{i}}+n\hat{\mathbf{j}}$

$|\mathbf{a}|=\sqrt{25+n^2}=13$
$\Rightarrow \quad 25+n^2=169$
$\Rightarrow \quad n^2=169-25=144$
$\Rightarrow \quad n=\pm 12$

49. If $|\mathbf{a}|=2$ and $|\mathbf{b}|=3$, then $|\mathbf{a}\times\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2$ is equal to

(a) 72 (b) 64 (c) 48 (d) 36

(*d*) We have, $|\mathbf{a}|=2, |\mathbf{b}|=3$

$|\mathbf{a}\times\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2$
$=|\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta+|\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta$
$=|\mathbf{a}|^2|\mathbf{b}|^2(\sin^2\theta+\cos^2\theta)$
$=|\mathbf{a}|^2|\mathbf{b}|^2=4\times 9=36$

50. Consider the following inequalities in respect of vectors $\mathbf{a}$ and $\mathbf{b}$

1. $|\mathbf{a}+\mathbf{b}| \le |\mathbf{a}|+|\mathbf{b}|$
2. $|\mathbf{a}-\mathbf{b}| \ge |\mathbf{a}|-|\mathbf{b}|$

Which of the above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) 1. Correct because modulus of sum of two vectors is less than or equal to the sum of modulus of these vectors separately.

2. Correct because modulus of difference of two vectors is greater than or equal to the difference of modulus of these vectors separately.

51. If the magnitude of difference of two unit vectors is $\sqrt{3}$, then the magnitude of sum of the two vectors is

(a) $\frac{1}{2}$ unit (b) 1 unit
(c) 2 units (d) 3 units

(**b**) $|\hat{\mathbf{a}}-\hat{\mathbf{b}}| = \sqrt{3}$

Squaring on both sides, we get

$|\hat{\mathbf{a}}|^2 + |\hat{\mathbf{b}}|^2 - 2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}} = 3$

[$\because \hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are unit vectors $\therefore |\hat{\mathbf{a}}| = |\hat{\mathbf{b}}| = 1$]

$2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}} = -1$

$|\hat{\mathbf{a}}+\hat{\mathbf{b}}|^2 = |\hat{\mathbf{a}}|^2 + |\hat{\mathbf{b}}|^2 + 2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}$

$= 1 + 1 - 1 = 1$

$\Rightarrow \hat{\mathbf{a}}+\hat{\mathbf{b}} = 1$

52. If the vectors $\alpha\hat{\mathbf{i}}+\alpha\hat{\mathbf{j}}+\gamma\hat{\mathbf{k}}, \hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\gamma\hat{\mathbf{i}}+\gamma\hat{\mathbf{j}}+\beta\hat{\mathbf{k}}$ lie on a plane, where α, β and γ are distinct non-negative numbers, then γ is

(a) arithmetic mean of α and β
(b) geometric mean of α and β
(c) harmonic mean of α and β
(d) None of the above

(**b**) Since, the vectors are coplanar.

$$\therefore \begin{vmatrix} \alpha & \alpha & \gamma \\ 0 & 1 & 1 \\ \gamma & \gamma & \beta \end{vmatrix} = 0$$

$\Rightarrow \alpha(\beta-\gamma) - \alpha(0-\gamma) + \gamma(0-\gamma) = 0$

$\Rightarrow \alpha\beta - \alpha\gamma + \alpha\gamma - \gamma^2 = 0$

$\Rightarrow \gamma^2 = \alpha\beta$

Hence, γ is GM of α and β.

53. The vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ are such that $\mathbf{a}\times\mathbf{b} = \mathbf{c}\times\mathbf{d}$ and $\mathbf{a}\times\mathbf{c} = \mathbf{b}\times\mathbf{d}$. Which of the following is/are correct?

1. $(\mathbf{a}-\mathbf{d})\times(\mathbf{b}-\mathbf{c}) = \mathbf{0}$
2. $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) = \mathbf{0}$

Select the correct answer using the code given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) We have, $\mathbf{a}\times\mathbf{b} = \mathbf{c}\times\mathbf{d}$ and $\mathbf{a}\times\mathbf{c} = \mathbf{b}\times\mathbf{d}$

1. $(\mathbf{a}-\mathbf{d})\times(\mathbf{b}-\mathbf{c})$

$= \mathbf{a}\times\mathbf{b} - \mathbf{a}\times\mathbf{c} - \mathbf{d}\times\mathbf{b} + \mathbf{d}\times\mathbf{c}$

$= \mathbf{c}\times\mathbf{d} - \mathbf{b}\times\mathbf{d} + \mathbf{b}\times\mathbf{d} - \mathbf{c}\times\mathbf{d} = 0$

[$\because \mathbf{a}\times\mathbf{b} = \mathbf{c}\times\mathbf{d}$, $\mathbf{a}\times\mathbf{c} = \mathbf{b}\times\mathbf{d}$]

It is correct.

2. $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})$

Taking $\mathbf{a}\times\mathbf{b} = \mathbf{l}$

$\therefore \mathbf{l}\times(\mathbf{c}\times\mathbf{d}) = (\mathbf{l}\cdot\mathbf{d})\mathbf{c} - (\mathbf{l}\cdot\mathbf{c})\mathbf{d}$

$= ((\mathbf{a}\times\mathbf{b})\cdot\mathbf{d})\mathbf{c} - ((\mathbf{a}\times\mathbf{b})\cdot\mathbf{c})\mathbf{d}$

$= ((\mathbf{c}\times\mathbf{d})\cdot\mathbf{d})\mathbf{c} - ((\mathbf{c}\times\mathbf{d})\cdot\mathbf{c})\mathbf{d}$

$= 0\cdot\mathbf{c} - 0\cdot\mathbf{d} = 0$

It is also correct.

2015 (I)

54. The adjacent sides AB and AC of a ΔABC are represented by the vectors $-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $-4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, respectively. The area of the ΔABC is

(a) 6 sq units (b) 5 sq units
(c) 4 sq units (d) 3 sq units

(**d**) Given, in ΔABC, $\mathbf{AB} = -2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{AC} = -4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

Area of $\Delta ABC = \frac{1}{2}|\mathbf{AB}\times\mathbf{AC}|$ ...(i)

Let us first find $\mathbf{AB}\times\mathbf{AC}$, which is given by

$$\mathbf{AB}\times\mathbf{AC} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -2 & 3 & 2 \\ -4 & 5 & 2 \end{vmatrix}$$

$= \hat{\mathbf{i}}(6-10) - \hat{\mathbf{j}}(-4+8) + \hat{\mathbf{k}}(-10+12)$

$= -4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\Rightarrow |\mathbf{AB}\times\mathbf{AC}| = \sqrt{(-4)^2 + (-4)^2 + (2)^2}$

$= \sqrt{32+4} = \sqrt{36} = 6$

From Eq. (i),

Area of $\Delta ABC = \frac{1}{2}\times 6 = 3$ sq units

55. A force $\mathbf{F} = 3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ is applied at the point P, whose position vector is $\mathbf{r} = 2\hat{\mathbf{i}}-2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$. What is the magnitude of the moment of the force about the origin?

(a) 23 units (b) 19 units
(c) 18 units (d) 21 units

(**a**) We have, $\mathbf{F} = 3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-3\hat{\mathbf{k}}$

and $\mathbf{OP} = \mathbf{r} = 2\hat{\mathbf{i}}-2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$

Clearly, the magnitude of moment of the force about origin

$= |\mathbf{r}\times\mathbf{F}|$ ...(i)

Let us first find $\mathbf{r}\times\mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -2 & -3 \\ 3 & 4 & -3 \end{vmatrix}$

$= \hat{\mathbf{i}}(6+12) - \hat{\mathbf{j}}(-6+9) + \hat{\mathbf{k}}(8+6)$

$= 18\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 14\hat{\mathbf{k}}$

From Eq. (i),

$|\mathbf{r}\times\mathbf{F}| = \sqrt{(18)^2 + (-3)^2 + (14)^2}$

$= \sqrt{324+9+196}$

$= \sqrt{529} = 23$ units

56. Given that the vector α and β are non-collinear. The values of x and y for which $\mathbf{u}-\mathbf{v} = \mathbf{w}$ holds true, if $\mathbf{u} = 2x\alpha + y\beta$, $\mathbf{v} = 2y\alpha + 3x\beta$ and $\mathbf{w} = 2\alpha - 5\beta$, are

(a) $x = 2, y = 1$ (b) $x = 1, y = 2$
(c) $x = -2, y = 1$ (d) $x = -2, y = -1$

(**a**) Given, α and β are non-collinear

i.e. $\alpha \neq \lambda\beta$ for any λ.

$\Rightarrow \alpha$ and β are linearly independent vectors.

Also, we have

$\mathbf{u} = 2x\alpha + y\beta$; $\mathbf{v} = 2y\alpha + 3x\beta$ and $\mathbf{w} = 2\alpha - 5\beta$ such that $\mathbf{u}-\mathbf{v} = \mathbf{w}$.

Consider, $\mathbf{u}-\mathbf{v} = \mathbf{w}$

$= (2x\alpha + y\beta) - (2y\alpha + 3x\beta) = 2\alpha - 5\beta$

$= (2x-2y)\alpha + (y-3x)\beta = 2\alpha - 5\beta$

$= (2x-2y-2)\alpha + (y-3x+5)\beta = 0$

Since, the vector α and β are linearly independent therefore, we have

$2x - 2y - 2 = 0$

$\Rightarrow x - y = 1$ ...(i)

and $y - 3x + 5 = 0$

$\Rightarrow y - 3x = -5$ ...(ii)

On solving Eqs. (i) and (ii), we get $x = 2$ and $y = 1$

57. If $|\mathbf{a}| = 7, |\mathbf{b}| = 11$ and $|\mathbf{a}+\mathbf{b}| = 10\sqrt{3}$, then $|\mathbf{a}-\mathbf{b}|$ is equal to

(a) 40 (b) 10 (c) $4\sqrt{10}$ (d) $2\sqrt{10}$

(**d**) We have, $|\mathbf{a}| = 7, |\mathbf{b}| = 11$ and $|\mathbf{a}+\mathbf{b}| = 10\sqrt{3}$

To find $|\mathbf{a}-\mathbf{b}|$

Consider, $|\mathbf{a}+\mathbf{b}|^2 = (\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})$

$\Rightarrow (10\sqrt{3})^2 = \mathbf{a}\cdot\mathbf{a} + \mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{a} + \mathbf{b}\cdot\mathbf{b}$

$\Rightarrow 300 = |\mathbf{a}|^2 + 2(\mathbf{a}\cdot\mathbf{b}) + |\mathbf{b}|^2$

[$\because$ dot product is commutative]

$\Rightarrow 300 = 49 + 2(\mathbf{a}\cdot\mathbf{b}) + 121$

$= 2(\mathbf{a}\cdot\mathbf{b}) = 300 - 170 = 130$

$\Rightarrow \mathbf{a}\cdot\mathbf{b} = 65$

Now, consider

$|\mathbf{a}-\mathbf{b}|^2 = (\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$

$= |\mathbf{a}|^2 - 2(\mathbf{a}\cdot\mathbf{b}) + |\mathbf{b}|^2$

$= 49 - 130 + 121$

$= 170 - 130 = 40$

$|\mathbf{a}-\mathbf{b}| = \sqrt{40} = 2\sqrt{10}$

58. Let α, β and γ be distinct real numbers. The points with position vectors $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$

(a) are collinear
(b) form an equilateral triangle
(c) form a scalene triangle
(d) form a right-angled triangle

(***b***) Let *A*, *B* and *C* be the given points whose position vectors are $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$; $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$, respectively.

Now,

$\mathbf{AB} = (\beta-\alpha)\hat{i}+(\gamma-\beta)\hat{j}+(\alpha-\gamma)\hat{k}$

$\mathbf{BC} = (\gamma-\beta)\hat{i}+(\alpha-\gamma)\hat{j}+(\beta-\alpha)\hat{k}$

$\mathbf{AC} = (\gamma-\alpha)\hat{i}+(\alpha-\beta)\hat{j}+(\beta-\gamma)\hat{k}$

$\Rightarrow |\mathbf{AB}| = \sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$

$= \sqrt{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2}$ …(i)

$|\mathbf{BC}| = \sqrt{(\gamma-\beta)^2+(\alpha-\gamma)^2+(\beta-\alpha)^2}$

$= \sqrt{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2}$…(ii)

and

$|\mathbf{AC}| = \sqrt{(\gamma-\alpha)^2+(\alpha-\beta)^2+(\beta-\gamma)^2}$

$= \sqrt{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2}$…(iii)

From Eqs. (i), (ii) and (iii), we get

$|\mathbf{AB}| = |\mathbf{BC}| = |\mathbf{AC}|$

Hence, *ABC* form an equilateral triangle.

59. If $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, then which of the following is/are correct?

I. $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar.

II. $\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{c} = \mathbf{c}\times\mathbf{a}$.

Select the correct answer using the code given below.

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(***c***) Given, $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$ …(i)

I. Consider $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$

$= -(\mathbf{b}+\mathbf{c})\cdot(\mathbf{b}\times\mathbf{c})$ [using Eq. (i)]

$= -(\mathbf{b}\cdot(\mathbf{b}\times\mathbf{c})+\mathbf{c}\cdot(\mathbf{b}\times\mathbf{c}))$

$= -([\mathbf{b}\,\mathbf{b}\,\mathbf{c}]+[\mathbf{c}\,\mathbf{b}\,\mathbf{c}])$

$= -(0+0) = 0$

Thus, the vectors are coplanar.

II. Consider, $\mathbf{a}\times\mathbf{b} = \mathbf{a}\times(-\mathbf{a}-\mathbf{c})$

$= -[\mathbf{a}\times(\mathbf{a}+\mathbf{c})]$ [using Eq. (i)]

$= -(\mathbf{a}\times\mathbf{a}+\mathbf{a}\times\mathbf{c})$

$= -(0+\mathbf{a}\times\mathbf{c}) = -(\mathbf{a}\times\mathbf{c})$

$= \mathbf{c}\times\mathbf{a}$ [using Eq. (i)]…(ii)

Similarly, $\mathbf{b}\times\mathbf{c} = \mathbf{b}\times(-\mathbf{a}-\mathbf{b})$

$= -(\mathbf{b}\times(\mathbf{a}+\mathbf{b}))$

$= -(\mathbf{b}\times\mathbf{a}+\mathbf{b}\times\mathbf{b})$

$= -(\mathbf{b}\times\mathbf{a}) = \mathbf{a}\times\mathbf{b}$ …(iii)

From Eqs. (ii) and (iii), we get

$\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{c} = \mathbf{c}\times\mathbf{a}$

60. If $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$, then which one of the following is correct?

(a) $\mathbf{a} = \lambda\mathbf{b}$ for some scalar λ
(b) $\mathbf{a}$ is parallel to $\mathbf{b}$
(c) $\mathbf{a}$ is perpendicular to $\mathbf{b}$
(d) $\mathbf{a} = \mathbf{b} = 0$

(***c***) We have, $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$

$\Rightarrow |\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}-\mathbf{b}|^2$

$\Rightarrow (\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b}) = (\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$

$\Rightarrow |\mathbf{a}|^2 + 2(\mathbf{a}\cdot\mathbf{b}) + |\mathbf{b}|^2 = |\mathbf{a}|^2 - 2(\mathbf{a}\cdot\mathbf{b}) + |\mathbf{b}|^2$

[$\because$ dot product is commutative]

$\Rightarrow 2(\mathbf{a}\cdot\mathbf{b}) = -2(\mathbf{a}\cdot\mathbf{b}) \Rightarrow 4(\mathbf{a}\cdot\mathbf{b}) = 0$

$\Rightarrow \mathbf{a}\cdot\mathbf{b} = 0$

$\Rightarrow \mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other.

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61. If $|\mathbf{a}|=2, |\mathbf{b}|=5$ and $|\mathbf{a}\times\mathbf{b}|=8$, then what is $\mathbf{a}\cdot\mathbf{b}$ equal to?

(a) 6 (b) 7 (c) 8 (d) 9

(***a***) Here, $|\mathbf{a}|=2, |\mathbf{b}|=5$ and $|\mathbf{a}\times\mathbf{b}|=8$

Also, $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\cdot|\mathbf{b}|\cdot|\sin\theta|$

$\Rightarrow |\sin\theta| = \frac{8}{2\cdot5} = \frac{4}{5}$

$\Rightarrow |\cos\theta| = \frac{3}{5} \Rightarrow \cos\theta = \pm\frac{3}{5}$

$\therefore \mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|\cdot|\mathbf{b}|\cos\theta = 2\times5\times\frac{3}{5} = 6$

62. If $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$, then which one of the following is correct?

(a) $|\mathbf{a}| = |\mathbf{b}|$ (b) $\mathbf{a}$ is parallel to $\mathbf{b}$
(c) $\mathbf{a}$ is perpendicular to $\mathbf{b}$
(d) $\mathbf{a}$ is a unit vector

(***c***) Since, $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$

$\Rightarrow [|\mathbf{a}+\mathbf{b}|]^2 = [|\mathbf{a}-\mathbf{b}|]^2$

$\Rightarrow \mathbf{a}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{a} = \mathbf{a}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{b}-\mathbf{a}\cdot\mathbf{b}-\mathbf{b}\cdot\mathbf{a}$

$\Rightarrow 4\mathbf{a}\cdot\mathbf{b} = 0$ [$\because \mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$]

$\Rightarrow \mathbf{a}\cdot\mathbf{b} = 0$

Hence, $\mathbf{a}$ is perpendicular to $\mathbf{b}$.

63. What is the area of ΔOAB, where O is the origin, $\mathbf{OA} = 3\hat{i}-\hat{j}+\hat{k}$ and $\mathbf{OB} = 2\hat{i}+\hat{j}-3\hat{k}$?

(a) $5\sqrt{6}$ sq units (b) $\frac{5\sqrt{6}}{2}$ sq units
(c) $\sqrt{6}$ sq units (d) $\sqrt{30}$ sq units

(***b***) Since, area of $\Delta OAB = \frac{1}{2}|\mathbf{OA}\times\mathbf{OB}|$

$\therefore \mathbf{OA}\times\mathbf{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix}$

$= \hat{i}[3-1] - \hat{j}[-9-2] + \hat{k}[3+2]$

$= 2\hat{i}+11\hat{j}+5\hat{k}$

and $|\mathbf{OA}\times\mathbf{OB}| = \sqrt{2^2+11^2+5^2}$

$= \sqrt{150} = 5\sqrt{6}$

$\therefore$ Required area $= \frac{1}{2}\times5\sqrt{6}$

$= \frac{5\sqrt{6}}{2}$ sq units

64. Which one of the following is the unit vector perpendicular to both $\mathbf{a} = -\hat{i}+\hat{j}+\hat{k}$ and $\mathbf{b} = \hat{i}-\hat{j}+\hat{k}$?

(a) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (b) $\hat{k}$ (c) $\frac{\hat{j}+\hat{k}}{\sqrt{2}}$ (d) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

(***a***) Since, unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b} = \pm\frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}$

$\therefore \mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}[1+1] - \hat{j}[-1-1] + \hat{k}[1-1]$

$= 2\hat{i}+2\hat{j}+0 = 2(\hat{i}+\hat{j})$

and $|\mathbf{a}\times\mathbf{b}| = \sqrt{4+4} = 2\sqrt{2}$

$\therefore$ Required unit vector $= \pm\frac{2(\hat{i}+\hat{j})}{2\sqrt{2}}$

$= \pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

65. What is the interior acute angle of the parallelogram whose sides are represented by the vectors $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}+\hat{k}$?

(a) 60° (b) 45° (c) 30° (d) 15°

(***a***) Let $\mathbf{a} = \frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\hat{k}$

and $\mathbf{b} = \frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}+\hat{k}$

$\therefore \cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$

$= \frac{\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\hat{k}\right)\cdot\left(\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}+\hat{k}\right)}{\sqrt{\frac{1}{2}+\frac{1}{2}+1}\sqrt{\frac{1}{2}+\frac{1}{2}+1}}$

$= \frac{1}{2}\left[\frac{1}{2}-\frac{1}{2}+1\right] = \frac{1}{2} = \cos 60°$

$\therefore \theta = 60°$

66. For what value of λ are the vectors $\lambda\hat{i}+(1+\lambda)\hat{j}+(1+2\lambda)\hat{k}$ and $(1-\lambda)\hat{i}+\lambda\hat{j}+2\hat{k}$ perpendicular?

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

(*a*) Let $\mathbf{a} = \lambda\hat{\mathbf{i}} + (1+\lambda)\hat{\mathbf{j}} + (1+2\lambda)\hat{\mathbf{k}}$ and $\mathbf{b} = (1-\lambda)\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

For $\mathbf{a}$ and $\mathbf{b}$ to be perpendicular we should have,

$$\mathbf{a}\cdot\mathbf{b} = 0$$

$$\Rightarrow [\lambda\mathbf{i}+(1+\lambda)\hat{\mathbf{j}} + (1+2\lambda)\hat{\mathbf{k}}] \cdot[(1-\lambda)\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 2\hat{\mathbf{k}}] = 0$$

$$\Rightarrow \lambda - \lambda^2 + \lambda + \lambda^2 + 2 + 4\lambda = 0$$

$$\Rightarrow 6\lambda = -2$$

$$\therefore \lambda = -\frac{2}{6} = -\frac{1}{3}$$

Directions (Q. Nos. 67-70) *Read the following information carefully and answer the questions given below.*

$\mathbf{a}+\mathbf{b}+\mathbf{c} = \mathbf{0}$, such that $|\mathbf{a}| = 3,\ |\mathbf{b}| = 5$ and $|\mathbf{c}| = 7$

67. What is the angle between $\mathbf{a}$ and $\mathbf{b}$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

68. What is $\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}$ equal to?

(a) –83 (b) $-\frac{83}{2}$ (c) 75 (d) $-\frac{75}{2}$

69. What is cosine of the angle between $\mathbf{b}$ and $\mathbf{c}$?

(a) $\frac{11}{12}$ (b) $\frac{13}{14}$ (c) $-\frac{11}{12}$ (d) $-\frac{13}{14}$

70. What is $|\mathbf{a}+\mathbf{b}|$ equal to?

(a) 7 (b) 8 (c) 10 (d) 11

Solutions (Q. Nos. 67-70)

We have, $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0$ ...(i)

On squaring both sides, we get

$$\mathbf{a}\cdot\mathbf{a} + \mathbf{b}\cdot\mathbf{b} + \mathbf{c}\cdot\mathbf{c} + 2\,\mathbf{a}\cdot\mathbf{b} + 2\mathbf{b}\cdot\mathbf{c} + 2\mathbf{c}\cdot\mathbf{a} = 0$$

$$[\because \mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}, \mathbf{b}\cdot\mathbf{c} = \mathbf{c}\cdot\mathbf{b} \text{ and } \mathbf{c}\cdot\mathbf{a} = \mathbf{a}\cdot\mathbf{c}]$$

$$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 = -2\,[\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}]$$

$$\Rightarrow (3)^2 + (5)^2 + (7)^2 = -2\,[\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}]$$

$$\Rightarrow \mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a} = \frac{9+25+49}{-2} = -\frac{83}{2}$$

Also, $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0$ [using Eq. (i)]

$$\Rightarrow \mathbf{a}+\mathbf{b} = -\mathbf{c}$$

$$\Rightarrow \mathbf{a}^2 + \mathbf{b}^2 + 2\mathbf{a}\cdot\mathbf{b} = c^2$$

$$\Rightarrow 2\mathbf{a}\cdot\mathbf{b} = 49 - 9 - 25 = 15$$

$$\Rightarrow \mathbf{a}\cdot\mathbf{b} = \frac{15}{2}$$

$$\Rightarrow |\mathbf{a}||\mathbf{b}|\cos\theta = \frac{15}{2}$$

$$\Rightarrow 3\cdot 5\cdot\cos\theta = \frac{15}{2}$$

$$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3}$$

From Eq. (i),

$$\mathbf{b}+\mathbf{c} = -\mathbf{a}$$

$$\Rightarrow \mathbf{b}^2 + \mathbf{c}^2 + 2\mathbf{b}\cdot\mathbf{c} = \mathbf{a}^2$$

$$\Rightarrow 2\mathbf{b}\cdot\mathbf{c} = \mathbf{a}^2 - \mathbf{b}^2 - \mathbf{c}^2 = 9 - 25 - 49 = -65$$

$$\Rightarrow \mathbf{b}\cdot\mathbf{c} = -\frac{65}{2}$$

$$\Rightarrow |\mathbf{b}||\mathbf{c}|\cos\theta = -\frac{65}{2}$$

$$\Rightarrow \cos\theta = -\frac{65}{2}\times\frac{1}{5}\times\frac{1}{7} = -\frac{13}{14}$$

Also, $|\mathbf{a}+\mathbf{b}| = |-\mathbf{c}| = |\mathbf{c}| = 7$

67. (*c*) **68.** (*b*)

69. (*d*) **70.** (*a*)

2014 (I)

Directions (Q. Nos. 71 and 72) *Consider the vectors* $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ *and* $\mathbf{b} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$.

71. What is the scalar projection of $\mathbf{a}$ on $\mathbf{b}$?

(a) 1 (b) 19/9 (c) 17/9 (d) 23/9

72. What is the vector perpendicular to both the vectors?

(a) $-10\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ (b) $-10\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

(c) $10\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ (d) None of these

Solutions (Q.Nos. 71 and 72)

Given vectors are

$\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

71. (*b*) Scalar projection of $\mathbf{a}$ on $\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|}$

$$= \frac{(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}})}{|4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}|}$$

$$= \frac{(4+8+7)}{\sqrt{(4)^2 + (-4)^2 + (7)^2}}$$

$$= \frac{19}{\sqrt{16+16+49}} = \frac{19}{\sqrt{81}} = \frac{19}{9}$$

Which is the required scalar projection of $\mathbf{a}$ on $\mathbf{b}$.

72. (*a*) The vector perpendicular to both the vectors $\mathbf{a}$ and $\mathbf{b} = \mathbf{a}\times\mathbf{b}$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -2 & 1 \\ 4 & -4 & 7 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(-14+4) - \hat{\mathbf{j}}(7-4) + \hat{\mathbf{k}}(-4+8)$$

$$= -10\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

Directions (Q. Nos. 73 and 74) *Let* $|\mathbf{a}| = 7, |\mathbf{b}| = 11$, *then* $|\mathbf{a}+\mathbf{b}| = 10\sqrt{3}$

73. What is $|\mathbf{a}-\mathbf{b}|$ equal to?

(a) $2\sqrt{2}$ (b) $2\sqrt{10}$

(c) 5 (d) 10

74. What is the angle between $(\mathbf{a}+\mathbf{b})$ and $(\mathbf{a}-\mathbf{b})$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) None of these

Solutions (Q.Nos. 73 and 74) Given that, $|\mathbf{a}| = 7, |\mathbf{b}| = 11$ and $|\mathbf{a}+\mathbf{b}| = 10\sqrt{3}$

73. (*b*) We have,

$$|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a}\cdot\mathbf{b} \quad ...(i)$$

$$\because |\mathbf{a}+\mathbf{b}| = 10\sqrt{3}$$

$$\Rightarrow |\mathbf{a}+\mathbf{b}|^2 = 100\times 3$$

$$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}\cdot\mathbf{b} = 300$$

$$\Rightarrow (7)^2 + (11)^2 + 2\mathbf{a}\cdot\mathbf{b} = 300$$

$$\Rightarrow 49 + 121 + 2\mathbf{a}\cdot\mathbf{b} = 300$$

$$\Rightarrow 2\mathbf{a}\cdot\mathbf{b} = 300 - 170$$

$$\Rightarrow 2\mathbf{a}\cdot\mathbf{b} = 130$$

Now, put the value of $|\mathbf{a}|, |\mathbf{b}|$ and $2\mathbf{a}\cdot\mathbf{b}$ in Eq. (i), we get

$$|\mathbf{a}-\mathbf{b}|^2 = (7)^2 + (11)^2 - 130 = 49 + 121 - 130 = 170 - 130 = 40$$

$$\therefore |\mathbf{a}-\mathbf{b}| = \sqrt{40} = 2\sqrt{10}$$

74. (*d*) We have, $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = |\mathbf{a}|^2 - |\mathbf{b}|^2$

Let θ be the angle between $(\mathbf{a}+\mathbf{b})$ and $\mathbf{a}-\mathbf{b}$.

Then,

$$\cos\theta = \frac{(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})}{|\mathbf{a}+\mathbf{b}||\mathbf{a}-\mathbf{b}|} = \frac{|\mathbf{a}|^2 - |\mathbf{b}|^2}{|\mathbf{a}+\mathbf{b}||\mathbf{a}-\mathbf{b}|}$$

$$= \frac{(7)^2 - (11)^2}{10\sqrt{3}\times 2\sqrt{10}} = \frac{(7+11)(7-11)}{20\sqrt{3}\times\sqrt{10}}$$

$$= \frac{18\times(-4)}{20\sqrt{30}} = \frac{-18}{5\sqrt{30}}$$

$$= \frac{-6\times 3}{5\sqrt{30}}\times\frac{\sqrt{30}}{\sqrt{30}} = \frac{-6\times 3\sqrt{30}}{5\times 30}$$

$$= -\frac{3\sqrt{30}}{25}$$

$$\therefore \theta = \cos^{-1}\left(\frac{-3}{5}\sqrt{\frac{6}{5}}\right)$$

Which is the required angle.

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STATISTICS

2019 (II)

1. If V is the variance and M is the mean of first 15 natural numbers, then what is $V + M^2$ equal to?

(a) $\frac{124}{3}$ (b) $\frac{148}{3}$

(c) $\frac{248}{3}$ (d) $\frac{124}{9}$

(c) (**c**) Mean of first 15 natural numbers, M

$$= \frac{1+2+3+4+5+6+7+8+9+10+11+12+13+14+15}{15}$$

$$= \frac{15(15+1)}{2\times 15}$$

$$\left[\because 1+2+3+\ldots+n=\frac{n(n+1)}{2}\right]$$

$= 8$

Variance of first 15 natural numbers, V

$$= \frac{1}{15}[(1-8)^2 + (2-8)^2 + (3-8)^2 + (4-8)^2 + (5-8)^2 + (6-8)^2 + (7-8)^2 + (8-8)^2 + (9-8)^2 + (10-8)^2 + (11-8)^2 + (12-8)^2 + (13-8)^2 + (14-8)^2 + (15-8)^2]$$

$$= \frac{1}{15}[(-7)^2 + (-6)^2 + (-5)^2 + (-4)^2 + (-3)^2 + (-2)^2 + (-1)^2 + 0 + (1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2 + (6)^2 + (7)^2]$$

$$= \frac{2}{15}[1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2]$$

$$= \frac{2}{15}\times\frac{7(7+1)(14+1)}{6}$$

$$\left[\because 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{2}{15}\times\frac{7\times 8\times 15}{6} = \frac{56}{3}$$

Now, $V + M^2 = \frac{56}{3} + 64$

$$= \frac{56+192}{3}$$

$$= \frac{248}{3}$$

2. If the range of a set of observations on a variable X is known to be 25 and if $Y = 40 + 3X$, then what is the range of the set of corresponding observations on Y?

(a) 25 (b) 40

(c) 75 (d) 115

(c) (**c**) Range of set of observations on a variable, $X = 25$

We know that

Range, $R_X = X_{max} = X_{min}$

$\Rightarrow$ $25 = X_{max} - 0$ $[\because X_{min} = 0]$

$\Rightarrow$ $X_{max} = 25$

$\because$ $Y = 40 + 3X$

$\therefore$ $Y_{min} = 40 + 3X_{min}$

$= 40 + 3(0)$ $[\because X_{min} = 0]$

$= 40$

and $Y_{max} = 40 + 3X_{max}$

$= 40 + 3(25)$ $[\because X_{max} = 25]$

$= 40 + 75$

$= 115$

Now, $R_Y = Y_{max} - Y_{min}$

$= 115 - 40$

$= 75$

3. Mean of 100 observations is 50 and standard deviation is 10. If 5 is added to each observation, then what will be the new mean and new standard deviation respectively?

(a) 50, 10 (b) 50, 15

(c) 55, 10 (d) 55, 15

(c) (**c**) Mean of 100 observations = 50

and standard deviation = 10

We know that, if k is added to each observation, then new mean will be more than k and standard deviation no change.

$\therefore$ After 5 added to each observation.

mean = 50 + 5 = 55

and standard deviation = 10

4. Consider the following discrete frequency distribution.

x	1	2	3	4	5	6	7	8
f	3	15	45	57	50	36	25	9

What is the value of median of the distribution?

(a) 4 (b) 5 (c) 6 (d) 7

(b) (**b**)

x	f	C
1	3	3
2	15	18
3	45	63
4	57	120
5	50	170
6	36	236
7	25	261
8	9	270
	$N = 270$	

Here, $N = 270$

$\therefore$ Median

$$= \frac{\text{Value of } \frac{N}{2}\text{th term} + \text{value of } \left(\frac{N}{2}+1\right)\text{th term}}{2}$$

$$= \frac{\text{Value of 135th term} + \text{Value of 136th term}}{2}$$

$$= \frac{5+5}{2} = 5$$

5. The mean of 5 observations is 4.4 and variance is 8.24. If three of the five observations are 1, 2 and 6, then what are the other two observations?

(a) 9, 16 (b) 9, 4
(c) 81, 16 (d) 81, 4

(***b***) Let x_1, x_2, x_3, x_4 and x_5 are five observations.

$\therefore \quad x_1 = 1, x_2 = 2$, and $x_3 = 6$

$$\therefore \quad \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$\Rightarrow 4.4 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 22$$

$$\Rightarrow \quad 1 + 2 + 6 + x_4 + x_5 = 22$$

$$\Rightarrow \quad x_4 + x_5 = 22 - 9$$

$$\Rightarrow \quad x_4 + x_5 = 13 \quad \ldots(i)$$

and variance,

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2 + (x_5 - \bar{x})^2}{5}$$

$$\Rightarrow 8.24 = \frac{(1 - 4.4)^2 + (2 - 4.4)^2 + (6 - 4.4)^2 + (x_4 - 4.4)^2 + (x_5 - 4.4)^2}{5}$$

$$\Rightarrow 8.24 \times 5 = 11.56 + 5.76 + 2.56 + (x_4 - 4.4)^2 + (13 - x_4 - 4.4)^2$$

[from Eq. (i)]

$$\Rightarrow 41.20 = 19.88 + (x_4 - 4.4)^2 + (8.6 - x_4)^2$$

$$\Rightarrow 41.20 - 19.88 = x_4^2 + 19.36 - 8.8\, x_4 + 73.96 + x_4^2 - 17.2x$$

$$\Rightarrow 21.32 = 2x_4^2 - 26x_4 + 93.32$$

$$\Rightarrow \quad 2x_4^2 - 26x_4 + 72 = 0$$

$$\Rightarrow \quad x_4^2 - 13x_4 + 36 = 0$$

$$\Rightarrow \quad x_4^2 - 9x_4 - 4x_4 + 36 = 0$$

$$\Rightarrow x_4 (x_4 - 9) - 4 (x_4 - 9) = 0$$

$$\Rightarrow \quad (x_4 - 9)(x_4 - 4) = 0$$

$$\therefore \quad x_4 = 4, 9$$

From Eq. (i), $x_5 = 9, 4$

Hence, other two observations are 9 and 4.

6. The class marks in a frequency table are given to be 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. The class limits of the first five classes are

(a) 3-7, 7-13, 13-17, 17-23, 23-27
(b) 2.5-7.5, 7.5-12.5, 12.5-17.5, 17.5-22.5, 22.5-27.5
(c) 1.5-8.5, 8.5-11.5, 11.5,-18.5, 18.5-21.5, 21.5-28.5
(d) 2-8, 8-12, 12-18, 18-22, 22-28

(***b***) Given, class marks in a frequency table are

5, 10, 15, 20, 25, 30, 35, 40, 45, 50.

Let L_1 and L_2 be the lower limit and upper limit of first interval.

$$\because \text{ Class mark } = \frac{L_1 + L_2}{2}$$

$$5 = \frac{L_1 + L_2}{2}$$

$$\Rightarrow \quad L_1 + L_2 = 10 \quad \ldots (i)$$

and $L_2 - L_1$ = Class interval

or $\quad L_2 - L_1 = 5 \quad \ldots (ii)$

Solving Eq. (i) and (ii),

$$L_2 = 7.5 \text{ and } L_1 = 2.5$$

$\therefore$ Class limit of first classes is 2.5 – 7.5

Similarly find class limit of other classes.

Hence, class limits of the first five classes are

2.5 – 7.5, 7.5 – 12.5, 12.5 – 17.5, 17.5 – 22.5, 22.5 –27.5.

7. Arithmetic mean of 10 observations is 60 and sum of squares of deviations from 50 is 5000. What is the standard deviation of the observations?

(a) 20 (b) 21
(c) 22.36 (d) 24.70

(***a***) Arithmetic mean of 10 observations = 60

$$\therefore \Sigma x_i = 60 \times 10 = 600 \qquad \left[\because \bar{x} = \frac{\Sigma x_i}{n}\right]$$

If, $A = 50$, then $\Sigma d_i^2 = 5000$

$\because \quad d_i = x_i - A$

$$\therefore \Sigma d_i = \Sigma(x_i - A) = \Sigma x_i - A\Sigma 1$$

$$= 600 - 50 \times n \qquad [\because \Sigma 1 = n]$$

$$= 600 - 50 \times 10 = 100$$

$$\text{Now, SD} = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$$

$$= \sqrt{\frac{5000}{10} - \left(\frac{100}{10}\right)^2}$$

$$= \sqrt{500 - 100}$$

$$= \sqrt{400}$$

$$= 20$$

8. The median of the observations 22, 24, 33, 37, $x + 1, x + 3$, 46, 47, 57, 58 in ascending order is 42. What are the values of 5th and 6th observations respectively?

(a) 42, 45 (b) 41, 43
(c) 43, 46 (d) 40, 40

(***b***) The observations in ascending order are

22, 24, 33, 37, $x + 1$, $x + 3$, 46, 47, 57, 58

Here, $n = 10$

$\therefore$ Median

$$= \frac{\text{Value of } \frac{N}{2}\text{th observations} + \text{Values of } \left(\frac{N}{2} + 1\right)\text{th observations}}{2}$$

$$= \frac{\text{Value of 5th observations} + \text{Value of 6th observation}}{2}$$

$$\Rightarrow 42 = \frac{x + 1 + x + 3}{2} \Rightarrow 2x = 84 - 4$$

$$\Rightarrow \quad x = \frac{80}{2} = 40$$

$\therefore$ 5th observation

$$= x + 1 = 40 + 1 = 41$$

and 6th observation

$$= x + 3 = 40 + 3 = 43$$

2019 (I)

9. If two variables X and Y are independent, then what is the correlation coefficient between them?

(a) 1 (b) –1
(c) 0 (d) None of these

(***c***) Correlation coefficient between two independent variables is zero.

10. Consider the following statements

1. The algebraic sum of deviations of a set of values from their arithmetic mean is always zero.
2. Arithmetic mean > Median > Mode for a symmetric distribution.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) We know that, the algebraic sum of deviations of a set of values from their arithmetic mean is always zero.

11. Let the correlation coefficient between X and Y be 0.6. Random variables Z and W are defined as $Z = X + 5$ and $W = \frac{Y}{3}$. What is the correlation coefficient between Z and W?

(a) 0.1 (b) 0.2 (c) 0.36 (d) 0.6

(***d***) Since, the correlation coefficient is independent of change of origin and scale. It is given that correlation coefficient between X and Y be 0.6. So, correlation coefficient between Z and W be 0.6.

12. If all the natural numbers between 1 and 20 are multiplied by 3, then what is the variance of the resulting series?

(a) 99.75 (b) 199.75
(c) 299.25 (d) 399.25

(**c**) Variance of first n natural number

$$= \frac{n^2 - 1}{12} = \frac{20^2 - 1}{12} = \frac{399}{12} = 33.25$$

If all the natural number between 1 and 20 multiplied by 3, then

$\therefore$ Required variance $= 9 \times 33.25 = 299.25$

13. The mean of 100 observations is 50 and the standard deviation is 10. If 5 is subtracted from each observation and then it is divided by 4, then what will be the new mean and the new standard deviation respectively ?

(a) 45, 5 (b) 11.25, 1.25
(c) 11.25, 2.5 (d) 12.5, 2.5

(**c**) Given, mean $(\bar{x}) = 50$

$$\text{The new mean} = \frac{50 - 5}{4} = \frac{45}{4} = 11.25$$

And standard deviation $(\sigma) = 10$

$\therefore$ The new standard deviation

$$= \frac{10}{4} = 2.5$$

Since, addition and subtraction does not effect standard deviation.

2018 (II)

14. The correlation coefficient computed from a set of 30 observations is 0.8. Then, the percentage of variation not explained by linear regression is

(a) 80% (b) 20%
(c) 64% (d) 36%

(**b**) Given that,

correlation coefficient = 0.8 = 80%, if the relation is 80% explained, then 20% of variation will not explained by near regression.

15. The average age of a combined group of men and women is 25 yr. If the average age of the group of men is 26 yr and that of the group of women is 21 yr, then the percentage of men and women in the group is respectively.

(a) 20, 80 (b) 40, 60
(c) 60, 40 (d) 80, 20

(**d**) In group of men, let number of men $= a$

$\bar{X}_1 = 26$ yr and $n_1 = a$ [let]

and in group of women, number of women $= b$ [let]

$\bar{X}_2 = 21$ yr

and combined mean $\bar{X} = 25$

Now, $$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$\Rightarrow \quad 25 = \frac{26a + 21b}{a + b}$$

$$\Rightarrow \quad 25a + 25b = 26a + 21b$$

$$\Rightarrow \quad 4b = a$$

$$\Rightarrow \quad \frac{a}{b} = \frac{4}{1}$$

By checking options (d) is correct.

16. Consider the following statements

1. If 10 is added to each entry on a list, then the average increases by 10.
2. If 10 is added to each entry on a list, then the standard deviation increases by 10.
3. If each entry on a list is doubled then the average doubles.

Which of the above statements are correct?

(a) 1, 2 and 3
(b) 1 and 2
(c) 1 and 3
(d) 2 and 3

(**c**) **Statement 1** The average is affected by the change of the origin.

So, if 10 is added to each entry on list then average increase by 10. Statement 1 is correct.

Statement 2 Standard deviation is independent on change in origin. Statement 2 is incorrect.

Statement 3 The average is affected by change in scale in same ratio as each entry is changes.

So, Statement 3 is correct.

17. The variance of 25 observations is 4. If 2 is added to each observation, then the new variance of the resulting observations is

(a) 2 (b) 4
(c) 6 (d) 8

(**b**) We know that,

Variance is independent on change in origin.

So, if 2 is added to each observation, then variance is remain same

Hence, variance = 4

18. If the regression coefficient of Y on X is -6 and the correlation coefficient between X and Y is $-\frac{1}{2}$, then the regression coefficient of X on Y would be

(a) $\frac{1}{24}$ (b) $-\frac{1}{24}$ (c) $-\frac{1}{6}$ (d) $\frac{1}{6}$

(**b**) Given that,

$$b_{yx} = -6 \text{ and } r_{xy} = -\frac{1}{2}$$

We know that,

$$r_{xy} = \sqrt{b_{yx} \times b_{xy}} \Rightarrow -\frac{1}{2} = \sqrt{-6 \times b_{xy}}$$

Squaring both sides,

$$\Rightarrow \frac{1}{4} = -6 \times b_{xy} \Rightarrow b_{xy} = -\frac{1}{24}$$

19. The set of bivariate observations $(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$ are such that all the values are distinct and all the observations fall on a straight line with non-zero slope. Then, the possible values of the correlation coefficient between x and y are

(a) 0 and 1 (b) 0 and –1
(c) 0, 1 and –1 (d) –1 and 1

(**d**) Given that,

All the observations fall on a straight line with non-zero, slope, then if slope is positive then $r = 1$

and if slope is negative then $r = -1$

So, values of the correlation coefficient between x and y are –1 and 1.

20. An analysis of monthly wages paid to the workers in two firms A and B belonging to the same industry gives the following result

	Firm A	Firm B
Number of workers	500	600
Average monthly wage	₹ 1860	₹ 1750
Variance of distribution of wages	81	100

The average of monthly wage and variance of distribution of wages of all the workers in the firms A and B taken together are

(a) ₹ 1860, 100 (b) ₹ 1750, 100
(c) ₹ 1800, 81
(d) None of the above

(**d**) For firm A

$n_1 = 500$, $\bar{X}_1 = 1860$

and variance $= \sigma_1^2 = 81$

So, S.D. $= \sigma_1 = 9$

For firm B

$n_2 = 600$, $\bar{X}_2 = 1750$

and variance $\sigma_2^2 = 100$

So, S.D. $= \sigma = 10$

Now, combined mean

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$= \frac{500 \times 1860 + 600 \times 1750}{500 + 600}$$

$\overline{X} = 1800$

Combined variance

$$= \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{500[81 + (-60)^2] + 600[100 + (50)^2]}{500 + 600}$$

$(\because\ d_1 = \overline{X} - X_1 = 1800 - 1860 = -60$ and $d_2 = 1800 - 1750 = 50)$

$$= \frac{500(81 + 3600) + 600(100 + 2500)}{1100}$$

$$= \frac{500(3681) + 600(2600)}{1100} = 3091.36$$

21. Which one of the following can be obtained from an ogive?

(a) Mean (b) Median
(c) Geometric Mean (d) Mode

(**b**) We know that, median is always calculated from less than or more than type ogive curve where both curve cuts each other that point median.

22. In any discrete series (when all values are not same), if x represents mean deviation about mean and y represents standard deviation, then which one of the following is correct?

(a) $y \geq x$ (b) $y \leq x$ (c) $x = y$ (d) $x < y$

(**d**) We know that,

$$MD = \frac{4}{5} S.D.$$

$\Rightarrow$ 5MD = 4.S.D.

$\Rightarrow$ $5x = 4y$ [$\because$ MD = x and SD = y]

$\therefore$ $x < y$

23. In which one of the following cases would you expect to get a negative correlation?

(a) The ages of husbands and wifes
(b) Shoe size and intelligence
(c) Insurance companies profits and the number of claims they have to pay
(d) Amount of rainfall and yield of crop

(**c**) In negative correlation, if x is increases then y is decreases by checking options Insurance companies profits and the number of claims they have to pay are negatively correlated.

2018 (I)

24. If the correlation coefficient between x and y is 0.6, covariance is 27 and variance of y is 25, then what is the variance of x?

(a) $\frac{9}{5}$ (b) $\frac{81}{25}$ (c) 9 (d) 81

(**d**) Given, $\sigma^2(y) = 25 \Rightarrow \sigma(y) = 5$

Correlation coefficient $= \frac{CoV(x, y)}{\sigma_x \cdot \sigma_y}$

$$0.6 = \frac{27}{\sigma_x \times 5} \Rightarrow \sigma_x = \frac{27}{0.6 \times 5} = \frac{27}{3} = 9$$

$\therefore$ Variance of $x = \sigma^2(x) = (9)^2 = 81$

25. Let $\bar{x}$ be the mean of $x_1, x_2, x_3, \ldots x_n$. If $x_i = a + cy_i$ for some constants a and c, then what will be the mean of $y_1, y_2, y_3, \ldots, y_n$?

(a) $a + c\bar{x}$ (b) $a - \frac{1}{c}\bar{x}$
(c) $\frac{1}{c}\bar{x} - a$ (d) $\frac{\bar{x} - a}{c}$

(**d**) Given that,
Mean of $x_1, x_2, x_3, \ldots x_n$ is $\bar{x}$
Now, we have $x_i = a + cy_i$

$$\Rightarrow y_i = \frac{1}{c}(x_i - a) \Rightarrow \bar{y} = \frac{1}{c}(\bar{x} - a)$$

26. Consider the following statements

I. If the correlation coefficient $r_{xy} = 0$, then the two lines of regression are parallel to each other.
II. If the correlation coefficient $r_{xy} = 1$, then the two lines of regression are perpendicular to each other.

Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(**d**) According to the correlation condition,
If correlation coefficient $r_{xy} = 0$, then lines of regression are perpendicular
And if $r_{xy} = 1$, then lines of regression are parallel.
So, both statements are wrong.

27. If $4x - 5y + 33 = 0$ and $20x - 9y = 107$ are two lines of regression, then what are the values of $\bar{x}$ and $\bar{y}$ respectively?

(a) 12 and 18 (b) 18 and 12
(c) 13 and 17 (d) 17 and 13

(**c**) Given lines of regression are

$4x - 5y + 33 = 0$ … (i)
and $20x - 9y - 107 = 0$ … (ii)

On multiplying Eq. (i) by 5 and subtract Eq. (ii) from it, we get

$20x - 25y + 165 = 0$
$20x - 9y - 107 = 0$
$- \quad + \quad +$
$-16y = -272$
$y = 17$

On putting the value of y in Eq. (i), we get

$4x - 85 + 33 = 0 \Rightarrow 4x = 52 \Rightarrow x = 13$

The mean of two regression lines are the solution set at given regression lines,

Here, $\overline{X} = 13$
and $\overline{Y} = 17$

28. Consider the following statements

I. Mean is independent of change in scale and change in origin.
II. Variance is independent of change in scale but not in origin.

Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(**d**) Since, mean changes with changes in origin. So, Statement I is wrong.
And variance is dependent to the change of scale. So, Statement II is also wrong.
Hence, both statements are wrong.

29. Consider the following statements

I. The sum of deviations from mean is always zero.
II. The sum of absolute deviations is minimum when taken around median.

Which of the above statements is/are correct.

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(**c**) By the property of deviation both statement are correct.

30. What is the median of the numbers 4.6, 0, 9.3, – 4.8, 7.6 2.3, 12.7, 3.5, 8.2, 6.1, 3.9, 5.2?

(a) 3.8 (b) 4.9
(c) 5.7 (d) 6.0

(**b**) On arranging the given number is ascending order, we have
– 4.8, 0, 2.3, 3.5, 3.9, 4.6, 5.2, 6.1, 7.6, 8.2, 9.3, 12.7
Here, $n = 12$
So, median

$$= \frac{\text{Value of } \left(\frac{12}{2}\right)\text{th number} + \text{Value of } \left(\frac{12}{2} + 1\right)\text{th number}}{2}$$

$$= \frac{\text{Value of 6th number} + \text{Value 7th number}}{2}$$

$$= \frac{4.6 + 5.2}{2} = 4.9$$

31. In a test in Mathematics, 20% of the students obtained "first class". If the data are represented by a pie chart, what is the central angle corresponding to "first class"?

(a) 20° (b) 36° (c) 72° (d) 144°

(**c**) Pie chart contains total angle equal to 360°.

So, central angle corresponding to "First class" = 20% of 360° = $\frac{20}{100} \times 360° = 72°$

32. The mean and standard deviation of a set of values are 5 and 2 respectively. If 5 is added to each value, then what is the coefficient of variation for the new set of values?

(a) 10 (b) 20 (c) 40 (d) 70

(**b**) Given, mean = 5

and standard deviation (σ) = 2

Since, 5 is added to each value.

So, new mean = 5 + 5 = 10

But standard deviation will remain same.

Hence, coefficient of variation

$$= \frac{\sigma}{\text{mean}} \times 100 = \frac{2}{10} \times 100 = 20$$

33. The standard deviation σ of the first N natural numbers can be obtained using which one of the following formulae?

(a) $\sigma = \frac{N^2 - 1}{12}$ (b) $\sigma = \sqrt{\frac{N^2 - 1}{12}}$

(c) $\sigma = \sqrt{\frac{N - 1}{12}}$ (d) $\sigma = \sqrt{\frac{N^2 - 1}{6N}}$

(**b**) $\because \sigma^2 = \frac{1}{N}\Sigma X_i^2 - (\bar{X})^2$

$$= \frac{1}{N}(1^2 + 2^2 + ... + N^2) - \left[\frac{1}{N}(1 + 2 + 3 + ... + N)\right]^2$$

$$= \frac{1}{N} \times \frac{N(N+1)(2N+1)}{6} - \left[\frac{(N+1)}{2}\right]^2$$

$$= \frac{N^2 - 1}{12} \Rightarrow \sigma = \sqrt{\frac{N^2 - 1}{12}}$$

2017 (II)

34. Consider the following statements :

1. Coefficient of variation depends on the unit of measurement of the variable.
2. Range is a measure of dispersion.
3. Mean deviation is least when measured about median.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(**a**) Both statement 1 and 2 are fundamental concept and are correct.

But, Mean deviation is least when measured about mean.

$\therefore$ Statement 3 is incorrect.

35. Given that the arithmetic mean and standard deviation of a sample of 15 observations are 24 and 0 respectively. Then which one of the following is the arithmetic mean of the smallest five observations in the data?

(a) 0 (b) 8 (c) 16 (d) 24

(**d**) Since standard deviation is 0.

$\because \quad \sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{N}} \Rightarrow 0 = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{N}}$

$\Rightarrow \quad \Sigma(x_i - \bar{x})^2 = 0 \Rightarrow x_i - \bar{x} = 0$

$[\because (x_i - \bar{x})^2$ is a positive value]

$\Rightarrow \quad x_i = \bar{x} \Rightarrow x_i = 24$

$\therefore$ Each observation will be 24.

$\therefore$ Mean of any five observation will be 24.

36. Which one of the following can be considered as appropriate pair of values of regression coefficient of y on x and regression coefficient of x on y?

(a) (1, 1) (b) (–1, 1)
(c) $\left(-\frac{1}{2}, 2\right)$ (d) $\left(\frac{1}{3}, \frac{10}{3}\right)$

(**a**) We know that regression coefficient of x on y and regression coefficient of y on x are always equal.

$\therefore$ Correct option is (a).

37. It is given that $\bar{X} = 10$, $\bar{Y} = 90$, $\sigma_X = 3$, $\sigma_Y = 12$ and $r_{XY} = 0.8$. The regression equation of X on Y is

(a) $Y = 3.2X + 58$
(b) $X = 3.2Y + 58$
(c) $X = -8 + 0.2Y$
(d) $Y = -8 + 0.2X$

(**c**) We have, $\bar{X} = 10$, $\bar{Y} = 90$, $\sigma_x = 3$

$\sigma_y = 12$, $r_{xy} = 0.8$

Now, we know that

Regression equation of X and Y is given by

$$X - \bar{X} = r_{xy} \cdot \frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$\Rightarrow \quad x - 10 = 0.8 \times \frac{3}{12}(y - 90)$$

$$\Rightarrow \quad X - 10 = 0.2(Y - 90)$$

$$\Rightarrow \quad X = 0.2Y - 18 + 10$$

$$\therefore \quad X = -8 + 0.2Y$$

38. If a variable takes values 0, 1, 2, 3, …, n with frequencies $1, C(n,1), C(n,2), C(n,3), \ldots, C(n,n)$ respectively, then the arithmetic mean is

(a) $2n$ (b) $n + 1$ (c) n (d) $\frac{n}{2}$

(**d**) Let $\bar{X}$ denote the required mean.

Then, $\bar{X} = \frac{\sum_{r=0}^{n} r \cdot {}^nC_r}{\sum_{r=0}^{n} {}^nC_r} = \frac{\sum_{r=0}^{n} r \cdot \frac{n}{r} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r}$

$$= \frac{\sum_{r=0}^{n} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r} = \frac{n \times 2^{n-1}}{2^n} = \frac{n}{2}$$

$[\because \sum_{r=0}^{n} {}^{n-1}C_{r-1} = 2^{n-1}]$

39. Consider the following statements

1. Variance is unaffected by change of origin and change of scale.
2. Coefficient of variance is independent of the unit of observations.

Which of the statements given above is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**b**) Variance is independent of change of origin but is not independent of change of scale. So, Statement 1 is in correct.

Coefficient of variance is independent of the unit of observations.

So, Statement 2 is correct.

40. The coefficient of correlation when coefficients of regression are 0.2 and 1.8 is

(a) 0.36 (b) 0.2 (c) 0.6 (d) 0.9

(**c**) Coefficient of correlation

$$= \sqrt{0.2 \times 1.8} = \sqrt{0.36} = 0.6$$

2017 (I)

41. The variance of 20 observations is 5. If each observation is multiplied by 3, then what is the new variance of the resulting observations?

(a) 5 (b) 10
(c) 15 (d) 45

(**d**) Let $x_1, x_2, \ldots x_{20}$ be the given observations, we have

$$\frac{1}{20}\sum_{i=1}^{20}(x_i - \bar{x})^2 = 5$$

Then we have to find variance of $3x_1$, $3x_2, \ldots, 3x_{20}$

Let $\bar{X}$ denotes the mean of new observation then,

$$\bar{X} = \frac{\sum_{i=1}^{20} 3x_i}{20} = \frac{3\sum_{i=1}^{20} x_i}{20} = 3\bar{x}$$

∴ variance of new observation

$$= \frac{1}{20}\sum_{i=1}^{20}(3x_i - 3\bar{x})^2$$

$$= \frac{1}{20}\sum_{i=1}^{20}(9)(x_i - \bar{x})^2$$

$$= \frac{9}{20}\sum_{i=1}^{20}(x_i - \bar{x})$$

$$= 9 \times 5 = 45$$

42. The mean of a group of 100 observations was found to be 20. Later, it was found that four observations were incorrect, which were recorded as 21, 21, 18 and 20. What is the mean if the incorrect observation are omitted?

(a) 18 (b) 20
(c) 21 (d) 22

(**b**) We have, $\bar{x} = 20$

$$\therefore \quad \bar{x} = \frac{\sum_{i=1}^{100} x_i}{100}$$

$$\Rightarrow \quad \sum_{i=1}^{100} x_i = 100\bar{x}$$

$$= 100 \times 20 = 2000$$

Four observation 21, 21, 18, 20 are incorrect and omitted

$$\therefore \quad \sum_{i=1}^{96} x_i = 2000 - (21 + 21 + 18 + 20)$$

$$= 2000 - 80$$

$$= 1920$$

Hence, new mean $= \frac{1920}{96} = 20$

43. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys in the class is 70 kg and that of girls is 55 kg. What is the number of boys in the class?

(a) 50 (b) 55 (c) 60 (d) 100

(**a**) We have

Total number of students = 150

Let the number of boys = x

So, the number of girls = $150 - x$

Mean weight of boys = 70

Mean weight of girls = 55

Mean weight of students = 60

∴ Mean weight of students

$$= \frac{(70 \times x + 55 \times (150 - x)}{150}$$

$$\Rightarrow \quad 60 = \frac{70x + 55 \times 150 - 55x}{150}$$

$$\Rightarrow \quad 60 \times 150 - 55 \times 150 = 15x$$

$$\Rightarrow \quad \frac{150 \times 5}{15} = x \Rightarrow x = 50$$

∴ Number of boys in the class = 50

44. If two regression lines between height (x) and weight (y) are $4y - 15x + 410 = 0$ and $30x - 2y - 825 = 0$, then what will be the correlation coefficient between height and weight?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(**b**) Given, regression lines

$$4y - 15x + 410 = 0 \quad \ldots (i)$$

and $\quad 30x - 2y - 825 = 0 \quad \ldots (ii)$

From (i)

$$4y = 15x - 410 = -410 + 15x$$

$$\Rightarrow \quad y = \frac{-410}{4} + \frac{15}{4}x$$

$$\therefore \quad b_{yx} = \frac{15}{4}$$

From (ii)

$$30x = 2y + 825$$

$$\Rightarrow \quad x = \frac{2y}{30} + \frac{825}{30} \Rightarrow x = \frac{825}{30} + \frac{1}{15}y$$

$$\therefore \quad b_{xy} = \frac{1}{15}$$

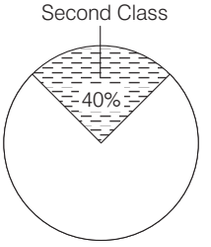
Now, correlation coefficient between height and weight

$$= \sqrt{b_{yx} \times b_{xy}} = \sqrt{\frac{15}{4} \times \frac{1}{15}} = \frac{1}{2}$$

45. In an examination, 40% of candidates got second class. When the data are represented by a pie chart, what is the angle corresponding to second class?

(a) 40° (b) 90° (c) 144° (d) 320°

(**c**)



∵ In the examination candidates who got second class = 40%

∴ Angle corresponding to second class

$$= \frac{40}{100} \times 360° = \frac{2}{5} \times 360°$$

$$= 2 \times 72° = 144°$$

46. Consider the following statements :

Statement 1 Range is not a good measure of dispersion.

Statement 2 Range is highly affected by the existence of extreme values.

Which one of the following is correct in respect of the above statements?

(a) Both Statement 1 and Statement 2 are correct and Statement 2 is the correct explanation of Statement 1
(b) Both Statement 1 and Statement 2 are correct but Statement 2 is not the correct explanation of Statement 1
(c) Statement 1 is correct but Statement 2 is not correct
(d) Statement 2 is correct but Statement 1 is not correct

(**a**) We know that, range is highly affected by the existence of extreme values. So, range is not a good measure of dispersion.

∴ Option (a) is correct.

47. If the data are moderately non-symmetrical, then which one of the following empirical relationships is correct?

(a) 2 × Standard deviation = 5 × Mean deviation
(b) 5 × Standard deviation = 2 × Mean deviation
(c) 4 × Standard deviation = 5 × Mean deviation
(d) 5 × Standard deviation = 4 × Mean deviation

(**c**) We know, If the data are moderately non-symmetrical, then the following empirical relationship holds.

Standard deviation = $\frac{5}{4}$ × Mean deviation.

⇒ 4 × Standard deviation = 5 × Mean deviation

48. Data can be represented in which of the following forms?

1. Textual form 2. Tabular form
3. Graphical form

Select the correct answer using the code given below.

(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) Only 1, 2 and 3

(**b**) We know, Data can be represent in Tabular form and Graphical form.

49. For given statistical data, the graphs for less than ogive and more than ogive are drawn. If the point at which the two curves intersect is P, then abscissa of point P gives the value of which one of the following measures of central tendency?

(a) Median (b) Mean
(c) Mode (d) Geometric mean

(**a**) Median of grouped data is the x-coordinate of the point of intersection of 'less than' and 'more than' ogive curves.

50. If the regression coefficient of x on y and y on x are $-\frac{1}{2}$ and $-\frac{1}{8}$, respectively then what is the correlation coefficient between x and y?

(a) $-\frac{1}{4}$ (b) $-\frac{1}{16}$
(c) $\frac{1}{16}$ (d) $\frac{1}{4}$

(**d**) Given,

Regression coefficient of x on $y = \frac{-1}{2}$

and

Regression coefficient of y on $x = \frac{-1}{8}$

We know, coefficient of correlation is the geometric mean between the regression coefficients.

$\therefore$ Correlation coefficient between x and y

$$= \left[\left(\frac{-1}{2}\right) \times \left(\frac{-1}{8}\right)\right]^{1/2} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

51. A sample of 5 observations has mean 32 and median 33. Later it is found that an observation was recorded incorrectly as 40 instead of 35. If we correct the data, then which one of the following is correct?

(a) The mean and median remains the same
(b) The median remains the same but the mean will decrease
(c) The mean and median both will decrease
(d) The mean remains the same but median will decrease

(**b**) From question for correcting the data we have to replace 40 by 35.

$\therefore$ Median of corrected data remains the same i.e. 33

and, as 35 is less than 40, so the mean of correct data will decrease.

2016 (II)

52. Consider the following statements.

I. The mean and median are equal in symmetric distribution.
II. The range is the difference between the maximum value and the minimum value in the data.
III. The sum of the areas of the rectangles in the histogram is equal to the total area bounded by the frequency polygon and the horizontal axis.

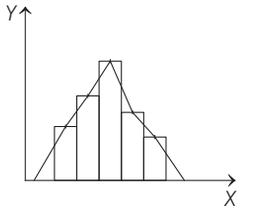
Which of the above statements are correct?

(a) I and II (b) II and III
(c) I and III (d) I, II and III

(**d**) **I.** In a symmetric distribution, values of variables occur at regular frequencies, hence mean and median are equal in symmetric distribution.

II. Range = Largest value – Smallest value.

III.



Clearly, sum of area of rectangles is equal to area under frequency polygon.

53. The scores of 15 students in an examination were recorded as 10, 5, 8, 16, 18, 20, 8, 10, 16, 20, 18, 11, 16, 14 and 12. After calculating the mean, median and mode, an error is found. One of the values is wrongly written as 16 instead of 18. Which of the following measures of central tendency will change?

(a) Mean and median
(b) Median and mode
(c) Mode only
(d) Mean and mode

(**d**) Given, scores 10, 5, 8, 16, 18, 20, 8, 10, 16, 20, 18, 11, 16, 14, 12

Mean $(\bar{x}) = \frac{\Sigma x_i}{n} = \frac{202}{15} = 13.46$

Scores in ascending order,

5, 8, 8, 10, 10, 11, 12, 14, 16, 16, 16, 18, 18, 20, 20

Here, $n = 15$ (odd)

$$\text{Median} = \left(\frac{n+1}{2}\right)\text{th} = \left(\frac{15+1}{2}\right)\text{th}$$

$$= \left(\frac{16}{2}\right)\text{th} = 8\text{th} = 14$$

and mode = 16

If score 16 change as 18 the mean

$$(\bar{x}) = \frac{208}{15} = 13.87$$

Median = 14

Mode = 18

So, mean will be changed and median will not be changed and mode will be changed.

54. For 10 observations on price (x) and supply (y), the following data was obtained.

$\Sigma x = 130, \Sigma y = 220, \Sigma x^2 = 2288,$

$\Sigma y^2 = 5506$ and $\Sigma xy = 3467$.

What is the line of regression of y on x?

(a) $y = 0.91x + 8.74$
(b) $y = 1.02x + 8.74$
(c) $y = 1.02x - 7.02$
(d) $y = 0.91x - 7.02$

(**b**) Given, $\Sigma x = 130$, $\Sigma y = 220$,

$\Sigma x^2 = 2288$, $\Sigma y^2 = 5506$ and $\Sigma xy = 3467$

We know that, regression line of y on x is $y = a + bx$.

To determine the values of a and b, we will solve the normal equations

$$\Sigma y = na + b\Sigma x \text{ and}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

Substituting the required value, the normal equations

$$220 = 10a + b(130)$$

$$\Rightarrow \quad a + 13b = 22 \qquad \ldots(i)$$

and $\quad 3467 = a(130) + b(2288)$

$$\Rightarrow \quad 130a + 2288b = 3467 \qquad \ldots(ii)$$

On multiplying Eq. (i) by 130 and subtracting from Eq. (ii), we get

$$130a + 2288b = 3467$$
$$\underline{130a + 1690b = 2860}$$
$$598b = 607$$

$$\Rightarrow \quad b = \frac{607}{598} = 1.02$$

From Eq. (i), we get

$$a + 13 \times 1.02 = 22$$

$$\Rightarrow \quad a + 13.26 = 22 \Rightarrow a = 8.74$$

So, equation of regression of line

$$y = 8.74 + 1.02x$$

55. In a study of two groups, the following results were obtained.

	Group A	Group B
Sample size	20	25
Sample mean	22	23
Sample standard deviation	10	12

Which of the following statements is correct?

(a) Group *A* is less variable than Group *B* because Group *A*'s standard deviation is smaller
(b) Group *A* is less variable than Group *B* because Group *A*'s sample size is smaller
(c) Group *A* is less variable than Group *B* because Group *A*'s sample mean is smaller
(d) Group *A* is less variable than group *B* because Group *A*'s coefficient of variation is smaller

(**d**) Coefficient of variation

$$= \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

$$\text{CV (Group } A) = \frac{10}{22} \times 100 = 45.45$$

$$\text{CV (Group } B) = \frac{12}{23} \times 100 = 52.17$$

So, group *A* is less variable than Group *B* because Group *A*'s coefficient of variation is smaller.

56. Consider the following statements in respect of class intervals of grouped frequency distribution.

I. Class-intervals need not be mutually exclusive.
II. Class-intervals should be exhaustive
III. Class-intervals need not be of equal width

Which of the above statements are correct?

(a) I and II (b) II and III
(c) I and III (d) I, II and III

(**a**) For grouped frequency distribution class-intervals should be exhausitive. Class-intervals are generally equal in width but this might not be the case always.

57. Two variates, x and y are uncorrelated and have standard deviations σ_x and σ_y respectively. What is the correlation coefficient between $x + y$ and $x - y$?

(a) $\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$ (b) $\frac{\sigma_x + \sigma_y}{2\sigma_x \sigma_y}$
(c) $\frac{\sigma_x^2 - \sigma_y^2}{\sigma_x^2 + \sigma_y^2}$ (d) $\frac{\sigma_y - \sigma_x}{\sigma_x \sigma_y}$

(**c**) $\because \text{var}(x - y) = \text{var}(x) + \text{var}(-y) = \text{var}(x) + \text{var}(y) = \sigma_x^2 + \sigma_y^2$
and $\text{var}(x + y) = \text{var}(x) + \text{var}(y) = \sigma_x^2 + \sigma_y^2$

$\text{cov}(x + y, x - y) = \text{cov}(x, x) - \text{cov}(x, y) + \text{cov}(y, x) - \text{cov}(y, y)$
$= \text{var}(x) - \text{var}(y) = \sigma_x^2 - \sigma_y^2$

$\therefore$ Correlation coefficient

$$= \frac{\text{cov}(x + y, x - y)}{\sqrt{\text{var}(x + y)}\sqrt{\text{var}(x - y)}} = \frac{\sigma_x^2 - \sigma_y^2}{\sigma_x^2 + \sigma_y^2}$$

58. If the covariance between x and y is 30, variance of x is 25 and variance of y is 144, then what is the correlation coefficient?

(a) 0.4 (b) 0.5 (c) 0.6 (d) 0.7

(**b**) Given, $\text{cov}(x, y) = 30$

$\sigma_x^2 = 25 \Rightarrow \sigma_x = 5$
and $\sigma_y^2 = 144 \Rightarrow \sigma_y = 12$

$\therefore$ Correlation coefficient

$$r(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \cdot \sigma_y} = \frac{30}{5 \times 12} = \frac{30}{60} = 0.5$$

59. A random sample of 20 people is classified in the following table according to their ages.

Age	Frequency
15-25	2
25-35	4
35-45	6
45-55	5
55-65	3

What is the mean age of this group of people?

(a) 41.0 (b) 41.5 (c) 42.0 (d) 42.5

(**b**)

Age	Frequency f_i	Mid- values x_i	$f_i x_i$
15-25	2	20	40
25-35	4	30	120
35-45	6	40	240
45-55	5	50	250
55-65	3	60	180
	$\Sigma f_i = 20$		$\Sigma f_i x_i = 830$

$\therefore \text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{830}{20} = 41.5$

2016 (I)

60. What is the mean deviation from the mean of the numbers 10, 9, 21, 16, 24?

(a) 5.2 (b) 5.0
(c) 4.5 (d) 4.0

(**a**) Given, $x_i = 10, 9, 21, 16, 24$

$\therefore \quad \Sigma x_i = 10 + 9 + 21 + 16 + 24 = 80$

Now, $\bar{X} = \frac{\Sigma x_i}{n} = \frac{80}{5} = 16$

$$\Rightarrow \text{MD} = \frac{\Sigma |x_i - \bar{X}|}{n} = \frac{|10-16| + |9-16| + |21-16| + |16-16| + |24-16|}{5} = \frac{6 + 7 + 5 + 0 + 8}{5} = \frac{26}{5} = 5.2$$

61. If the total number of observations is 20, $\Sigma x_i = 1000$ and $\Sigma x_i^2 = 84000$, then what is the variance of the distribution?

(a) 1500 (b) 1600
(c) 1700 (d) 1800

(**c**) Given, $N = 20$, $\Sigma x_i = 1000$ and $\Sigma x_i^2 = 84000$

$$\text{Now, variance} = \frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2 = \frac{84000}{20} - \left(\frac{1000}{20}\right)^2 = 4200 - (50)^2 = 4200 - 2500 = 1700$$

62. The mean of the series $x_1, x_2, \ldots, x_n$ is $\bar{X}$. If x_2 is replaced by λ, then what is the new mean?

(a) $\bar{X} - x_2 + \lambda$ (b) $\frac{\bar{X} - x_2 - \lambda}{n}$
(c) $\frac{\bar{X} - x_2 + \lambda}{n}$ (d) $\frac{n\bar{X} - x_2 + \lambda}{n}$

(**d**) We know, $\bar{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$

$\Rightarrow x_1 + x_2 + \ldots + x_n = n\bar{X}$

$\Rightarrow x_1 + x_3 + \ldots + x_n = n\bar{X} - x_2$

$\Rightarrow x_1 + x_3 + \ldots + x_n + \lambda = n\bar{X} - x_2 + \lambda$

$$\Rightarrow \text{Mean} = \frac{\text{Sum of all values}}{\text{Total number of values}} = \frac{x_1 + x_3 + \ldots + x_n + \lambda}{n} = \frac{n\bar{X} - x_2 + \lambda}{n}$$

63. For the data

3, 5, 1, 6, 5, 9, 5, 2, 8, 6

the mean, median and mode are x, y and z, respectively. Which one of the following is correct?

(a) $x = y \neq z$ (b) $x \neq y = z$
(c) $x \neq y \neq z$ (d) $x = y = z$

(*d*) Mean $= \frac{\Sigma x_i}{n}$

$= \frac{3+5+1+6+5+9+5+2+8+6}{10}$

$= \frac{50}{10} = 5$

Now, the data in 'Ascending' order is

1, 2, 3, 5, 5, 5, 6, 6, 8, 9

$$\text{Median} = \frac{\text{Valut of } \frac{n}{2} \text{ th term} + \text{Value of } \left(\frac{n}{2}+1\right)\text{th term}}{2}$$

$= \frac{5+5}{2} = 5$

and mode (most appeared value) is also 5.

$\therefore \quad x = y = z$

64. Consider the following statements in respect of a histogram.

1. The total area of the rectangles in a histogram is equal to the total area bounded by the corresponding frequency polygon and the X-axis.
2. When class intervals are unequal in a frequency distribution, the area of the rectangle is proportional to the frequency.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(*c*) Both 1 and 2 are correct.

Area of rectangles in a histogram = Total area bounded by frequency polygon and area of rectangle $\propto$ frequency in a frequency distribution

65. If m is the geometric mean of

$$\left(\frac{y}{z}\right)^{\log(yz)}, \left(\frac{z}{x}\right)^{\log(zx)} \text{ and } \left(\frac{x}{y}\right)^{\log(xy)},$$

then what is the value of m?

(a) 1 (b) 3
(c) 6 (d) 9

(a) Here,

$$m = \left[\left(\frac{y}{z}\right)^{\log(yz)} \times \left(\frac{z}{x}\right)^{\log(zx)} \times \left(\frac{x}{y}\right)^{\log(xy)}\right]^{1/3}$$

$\therefore m^3 = x^{\log(xy) - \log(zx)} \times y^{\log(yz) - \log(xy)} \times z^{\log(zx) - \log(yz)}$

$\Rightarrow m^3 = x^{\log\left(\frac{y}{z}\right)} \times y^{\log\left(\frac{z}{x}\right)} \times z^{\log\left(\frac{x}{y}\right)}$

Taking log on both sides, we get

$3\log m = \log\left(\frac{y}{z}\right)\log x + \log\left(\frac{z}{x}\right)\log y + \log\left(\frac{x}{y}\right)\log z$

$\Rightarrow 3\log m = \log y \log x - \log z \log x + \log z \log y - \log x \log y + \log x \log z - \log y \log z$

$\Rightarrow 3\log m = 0$

$\Rightarrow \log m = 0$

$\Rightarrow m = e^0 \Rightarrow m = 1$

2015 (II)

66. The regression coefficients of a bivariate distribution are -0.64 and -0.36. Then, the correlation coefficient of the distribution is

(a) 0.48 (b) – 0.48 (c) 0.50 (d) – 0.50

(*b*) We have, $b_{xy} = -0.64$, $b_{yx} = -0.36$

$\therefore$ Correlation coefficient

$(\sigma) = \sqrt{b_{xy} \times b_{yx}}$

$= \pm\sqrt{(-0.64)(-0.36)} = \pm 0.48$

$\Rightarrow \sigma = -0.48$

Because b_{xy} and b_{yx} both are negative.

67. The geometric mean of the observations $x_1, x_2, x_3, \ldots, x_n$ is G_1. The geometric mean of the observations $y_1, y_2, y_3, \ldots, y_n$ is G_2. The geometric mean of observations $\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, \ldots, \frac{x_n}{y_n}$ is

(a) G_1G_2 (b) $\ln(G_1G_2)$
(c) $\frac{G_1}{G_2}$ (d) $\ln\left(\frac{G_1}{G_2}\right)$

(*c*) Geometric mean of $x_1, x_2, x_3, \ldots, x_n$ is G_1.

$\Rightarrow \quad G_1 = (x_1 x_2 \ldots x_n)^{\frac{1}{n}}$

Geometric mean of $y_1, y_2, y_3, \ldots, y_n$ is G_2.

$\Rightarrow \quad G_2 = (y_1\ y_2\ y_3 \ldots y_n)^{\frac{1}{n}}$

$\therefore$ GM of $\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, \ldots, \frac{x_n}{y_n}$

$= \left(\frac{x_1}{y_1} \cdot \frac{x_2}{y_2} \cdot \frac{x_3}{y_3} \cdots \frac{x_n}{y_n}\right)^{\frac{1}{n}}$

$= \frac{(x_1x_2x_3 \ldots x_n)^{1/n}}{(y_1\ y_2\ y_3 \cdots y_n)^{1/n}} = \frac{G_1}{G_2}$

68. The arithmetic mean of 1, 8, 27, 64,... upto n terms is given by

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$
(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{n^2(n+1)^2}{4}$

(*c*) 1, 8, 27, 64, ... upto n terms $= 1^3, 2^3, 3^3, 4^3$ ... upto n terms

$$\therefore \text{AM} = \frac{1^3 + 2^3 + 3^3 + \ldots n^3}{n} = \frac{\left[\frac{n(n+1)}{2}\right]^2}{n}$$

$$= \frac{n^2(n+1)^2}{4n} = \frac{n(n+1)^2}{4}$$

$$\left[\because \Sigma n^3 = 1^2 + 2^2 + \ldots + n^2 = \left\{\frac{n}{2}(n+1)\right\}^2\right]$$

2015 (I)

69. The mean and the variance of 10 observations are given to be 4 and 2 respectively. If every observation is multiplied by 2, the mean and the variance of the new series will be respectively.

(a) 8 and 20
(b) 8 and 4
(c) 8 and 8
(d) 80 and 40

(*c*) Let the observations be x_i's, $i = 1, 2, \ldots, 10$ and the mean and variance of x_i's be $\bar{x} = 4$ and $\sigma^2 = 2$.

Now, let $y_i = 2x_i$'s and the mean and variance of y_i's and $\bar{y}$ is σ_1^2.

Then, $\bar{y} = \frac{\Sigma 2x_i}{10} = 2\frac{\Sigma x_i}{10} = 2\bar{x} = 8$

and $\sigma_1^2 = \text{Var}(y_i\text{'s}) = \text{Var}(2x_i\text{'s})$

$= 4\text{Var}(x_i\text{'s}) = 4 \times 2 = 8$

Thus, the mean and variance of new series are 8 and 8.

70. Which one of the following measures of central tendency is used in construction of index numbers?

(a) Harmonic mean (b) Geometric mean
(c) Median (d) Mode

(*a*) Harmonic mean is used in construction of index numbers.

Since, index numbers are specialized arranges a decision has to be made as to which particular average should be used for construction the index. Median, mode and mean are almost never used in the construction of index numbers.

71. The correlation coefficient between two variables X and Y is found to be 0.6. All the observations on X and Y are transformed using the transformations $U = 2 - 3X$ and $V = 4Y + 1$. The correlation coefficient between the transformed variables U and V will be

(a) – 0.5
(b) + 0.5
(c) – 0.6
(d) + 0.6

(**c**) We have, $r(x, y) = 0.6$

To find $r(U, V)$, where $U = 2 - 3x$ and $V = 4y + 1$

Clearly, $r(U, V) = \dfrac{\text{Cov}(U, V)}{\sqrt{\text{Var}(U) \cdot \text{Var}(V)}}$ ...(i)

Now, $\text{Var}(U) = \text{Var}(2 - 3x)$
$= (-3)^2 \text{Var}(x) = 9 \text{Var}(x)$
$\text{Var}(V) = \text{Var}(4y + 1)$
$= \text{Var}(4y) = 16 \text{Var}(y)$
$[\because V(x) = a^2 V(x)]$
$\text{Cov}(U, V) = E[(U - \bar{U})(V - \bar{V})]$
$= E[((2 - 3x) - (2 - 3\bar{x}))$
$((4y + 1) - (4\bar{y} + 1))]$
$= E[(-3x + 3\bar{x})(4y - 4\bar{y})]$
$= (-3)(4) E[(x - \bar{x})(y - \bar{y})]$
$= -12 \text{Cov}(X, Y)$

Thus, from Eq. (i), we have

$$r(U, V) = \frac{-12 \text{ Cov}(X, Y)}{\sqrt{9 \text{ Var}(x) \cdot 16 \text{ Var}(y)}}$$
$$= \frac{-12 \text{Cov}(x, y)}{3 \cdot 4\sqrt{\text{Var}(x) \cdot \text{Var}(y)}}$$
$$= -r(X, Y) = -0.6$$

72. Which of the following statements is/are correct in respect of regression coefficients?

I. It measures the degree of linear relationship between two variables.
II. It gives the value by which one variable changes for a unit change in the other variable.

Select the correct answer using the code given below.

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(**c**) Since, $y - \bar{y} = b_{yx}(X - \bar{X})$
and $(X - \bar{X}) = b_{xy}(Y - \bar{Y})$
Hence, both statements are true.

73. A set of annual numerical data, comparable over the years, is given for the last 12 yr.

Consider the following statements.

I. The data is best represented by a broken line graph, each corner (turning point) representing the data of one year.
II. Such a graph depicts the chronological change and also enables one to make a short-term forecast.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(**b**) For representation of these types of data usually used bar graph, the bar graph depicts comparison over the year of data whereas broken line graph does not make any sense in these types of data. So, only Statement II is correct.

74. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is

(a) 28 (b) 30 (c) 35 (d) 38

(**d**) Let the numbers be x_1, x_2, x_3, x_4 and x_5.
Then, we have,

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 30$$
$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 + x_5 = 150 \quad ...(i)$$

Now, suppose x_1 is excluded, then

$$\frac{x_2 + x_3 + x_4 + x_5}{4} = 28 \quad [\text{given}]$$
$$\Rightarrow \quad x_2 + x_3 + x_4 + x_5 = 112 \quad ...(ii)$$

From Eqs. (i) and (ii), we get
$x_1 = 150 - 112 = 38$

75. The 'less than' ogive curve and the 'more than' ogive curve intersect at

(a) median (b) mode
(c) arithmetic mean (d) None of these

(**a**) The median of grouped data is the x-coordinate of the point of intersection of 'less than' and 'more than' ogive curve.

2014 (II)

76. Consider the following statements in respect of histogram

1. The histogram is a suitable representation of a frequency distribution of a continuous variable.
2. The area included under the whole histogram is the total frequency.

Which of the above statement(s) is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**a**) **Statement 1** Because the help of a histogram, a suitable representation of a frequency distribution of a continuous variable can be made.
Hence, Statement 1 is correct.
Statement 2 We know that, the area of histogram is proportional to the frequency, so it is not true statement.

77. The regression lines will be perpendicular to each other, if the coefficient of correlation r is equal to

(a) Only 1 (b) 1 or –1 (c) Only –1 (d) 0

(**d**) Angle between the regression lines will be

$$\tan\theta = \left\{\left(\frac{1 - r^2}{r}\right)\left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)\right\}$$
$$\Rightarrow \quad \tan\frac{\pi}{2} = \left(\frac{1 - r^2}{r}\right)\left(\frac{\sigma_x \cdot \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$$
$$\Rightarrow \quad r \cdot (\sigma_x^2 + \sigma_y^2) = 0$$
$$\therefore \quad r = 0$$

78. If $\bar{x}$ and $\bar{y}$ are the means of two distributions such that $\bar{x} < \bar{y}$ and $\bar{z}$ is the mean of the combined distribution, then which one of the following statements is correct?

(a) $\bar{x} < \bar{y} < \bar{z}$ (b) $\bar{x} > \bar{y} > \bar{z}$
(c) $\bar{z} = \dfrac{\bar{x} + \bar{y}}{2}$ (d) $\bar{x} < \bar{z} < \bar{y}$

(**d**) It is obvious that, $\bar{x} < \bar{z} < \bar{y}$
Since, $\bar{z} = \dfrac{n_1\bar{x} + n_2\bar{y}}{n_1 + n_2}$

79. What is the mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13 and 17?

(a) 2.5 (b) 3 (c) 3.5 (d) 4

(**b**) Mean deviation about the mean

$$= \frac{\Sigma |x_i - \bar{x}|}{N}$$

Here,
$$\bar{x} = \frac{4 + 7 + 8 + 9 + 10 + 12 + 13 + 17}{8}$$
$= 10$
∴ Mean deviation about mean

$$= \frac{\left[\begin{array}{l} |4 - 10| + |7 - 10| + |8 - 10| \\ + |9 - 10| + |10 - 10| \\ + |12 - 10| + |13 - 10| \\ + |17 - 10| \end{array}\right]}{8}$$
$$= \frac{6 + 3 + 2 + 1 + 0 + 2 + 3 + 7}{8}$$
$$= \frac{24}{8} = 3$$

80. The variance of 20 observations is 5. If each observation is multiplied by 2, then what is the new variance of the resulting observations?

(a) 5 (b) 10 (c) 20 (d) 40

(**c**) Let $x_1, x_2, \ldots, x_{20}$ be the given observations.

We have, $\frac{1}{20}\sum_{i=1}^{20}(x_i - \bar{x})^2 = 5$

Then, according to the question, let $\bar{x}$ denotes the mean of new observation.

Clearly, $\bar{X} = \frac{\sum_{i=1}^{20} 2x_i}{20} = \frac{2\sum_{i=1}^{20} x_i}{20} = 2\bar{x}$

Now, variance of new observation

$$= \frac{1}{20}\sum_{i=1}^{20}(2x_i - \bar{X})^2$$

$$= \frac{1}{20}\sum_{i=1}^{20}(2x_i - 2\bar{x})^2$$

$$= \frac{1}{20}\sum_{i=1}^{20} 4(x_i - \bar{x})^2$$

$$= 4\left(\frac{1}{20}\sum_{i=1}^{20}(x_i - \bar{x})^2\right) = 4 \times 5 = 20$$

2014 (I)

81. For two variables x and y, the two regression, coefficients are $b_{yx} = -3/2$ and $b_{xy} = -1/6$. The correlation coefficient between x and y is

(a) –1/4 (b) 1/4
(c) –1/2 (d) 1/2

(**c**) Given that,

two regression coefficients are,

$b_{yx} = -3/2$ and $b_{xy} = -1/6$

Now, correlation coefficient between x and y i.e.

$$r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{(-1/6) \times (-3/2)}$$

$$= \sqrt{\frac{1}{2} \times \frac{1}{2}} = \pm\frac{1}{2}$$

Here, we have to take negative sign because b_{xy} and b_{yx} both have negative sign.

Hence, correlation coefficient $(r) = -\frac{1}{2}$.

82. The variance of numbers $x_1, x_2, x_3, \ldots, x_n$ is V. Consider the following statements

I. If every x_i is increased by 2, the variance of the new set of numbers is V.

II. If the numbers x_i is squared, the variance of the new set is V^2.

Which of the following statement(s) is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

(**c**) I. We know that, variance is not dependent on change of origin. i.e. independent on change of origin.

So, if every x_i is increased by 2, the variance of the new set of numbers is not change i.e. V.

II. We know that, variance is dependent on change of scale. So, if the number x_i is squared, the variance of the new set V^2.

i.e. if $V(x_i) = V$

Then, $V_{(x_i \times x_i)} = V_{(x_i)}\, V_{(x_i)}$

$= V \cdot V = V^2$

Hence, Statements I and II both are correct.

83. What is the mean of the squares of the first 20 natural numbers?

(a) 15 (b) 143.5 (c) 65 (d) 72

(**b**) Mean of first 20 natural numbers

$$= \frac{\text{Sum of the squares first of 20 natural numbers}}{\text{Number of observations}}$$

$$= \frac{(1^2 + 2^2 + 3^2 + \cdots + 20^2)}{20}$$

$$\left[\because \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{1}{20} \times \frac{20(20+1)(20\times 2+1)}{6}$$

$$= \frac{21 \times (40+1)}{6} = \frac{21 \times 41}{6}$$

$$= \frac{7 \times 41}{2} = \frac{287}{2} = 143.5$$

$\therefore$ Required mean = 143.5

84. p, q, r, s and t are five numbers such that the average of p, q and r is 5 and that of s and t is 10. What is the average of all the five numbers?

(a) 7.75 (b) 7.5 (c) 7 (d) 5

(**c**) Given that, p, q, r, s and t are five numbers,

$\therefore$ Average of p, q and $r = 5$

$$\Rightarrow \frac{p+q+r}{3} = 5$$

$$\Rightarrow p + q + r = 15 \quad \ldots(i)$$

and average of s and $t = 10$

$$\Rightarrow \frac{s+t}{2} = 10 \Rightarrow s + t = 20 \quad \ldots(ii)$$

Now, average of all five numbers

$$= \frac{p+q+r+s+t}{5}$$

$$= \frac{(p+q+r)+(s+t)}{5}$$

$$= \frac{15+20}{5} = \frac{35}{5} = 7$$

[from Eqs. (i) and (ii)]

Directions (Q. Nos. 85-87) *The number of telephone calls received in 245 successive one minute intervals at an exchange is given below in the following frequency distribution.*

Number	0	1	2	3	4	5	6	7
Frequency	14	21	25	43	51	40	39	12

85. What is the mean of the distribution?

(a) 3.76 (b) 3.84 (c) 3.96 (d) 4.05

86. What is the median of the distribution?

(a) 3.5 (b) 4 (c) 4.5 (d) 5

87. What is the mode of the distribution?

(a) 3 (b) 4 (c) 5 (d) 6

Solutions (Q.Nos. 85-87)

Given frequency distribution is

Number of calls (x_i)	0	1	2	3	4	5	6	7
Frequency (f_i)	14	21	25	43	51	40	39	12

85. (**a**) Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$$= \frac{[(0\times 14 + 1\times 21 + 2\times 25 + 3\times 43 + 4\times 51 + 5\times 40 + 6\times 39 + 7\times 12)]}{(14+21+25+43+51+40+39+12)}$$

$$= \frac{(0+21+50+129+204+200+234+84)}{245}$$

$$= \frac{922}{245} = 3.76$$

86. (**b**)

Number of calls (x_i)	Frequency (f_i)	Cumulative Frequency (f)
0	14	14
1	21	35
2	25	60
3	43	103
4	51	154
5	40	194
6	39	233
7	12	245
	$N = \Sigma f_i = 245$	

Here, $\frac{N}{2} = \frac{245}{2} = 122.5$

The cummulative frequency 154 which is equal or just greater than $\frac{N}{2}$.

∴ Required median = Value of the variable corresponding to the cummulative frequency 154 = 4

87. (***b***) We see that in the frequency distribution the higher frequency is 51.

∴ Required mode = Value of variable corresponding to the higher frequency = 4

Directions (Q. Nos. 88-90) *The mean and standard deviation of* 100 *items are* 50, 5 *and that of* 150 *items are* 40, 6 *respectively.*

88. What is the combined mean of all 250 items?

(a) 43 (b) 44
(c) 45 (d) 46

89. What is the combined standard deviation of all 250 items?

(a) 7.1 (b) 7.3
(c) 7.5 (d) 7.7

90. What is the variance of all the 250 items?

(a) 50.6
(b) 53.3
(c) 55.6
(d) 59.3

Solutions (Q. Nos. 88-90)

Given that, mean of 100 items i.e. $\bar{x}_{100} = 50$

Mean of 150 items i.e. $\bar{x}_{150} = 40$

and standard deviation of 100 items i.e. $\sigma_{100} = 5$

Standard deviation of 150 items i.e. $\sigma_{150} = 6$

88. (***b***) Here, $n_1 = 100$, $\bar{x}_{100} = 50$, $n_2 = 150$ and $\bar{x}_{150} = 40$

∴ Combined mean of all 250 items i.e.

$$\bar{x}_{250} = \frac{n_1 \cdot \bar{x}_{100} + n_2 \cdot \bar{x}_{150}}{n_1 + n_2} = \frac{100 \times 50 + 150 \times 40}{100 + 150} = \frac{5000 + 6000}{250} = \frac{11000}{250} = 44$$

89. (***c***) We know that, If n_1 and n_2 are the sizes, $\overline{X}_{100}$, $\overline{X}_{150}$ are the means and σ_{100}, σ_{150} are the standard deviation of the series, then the standard deviation of the combined series is

$$\sigma = \sqrt{\frac{n_1(\sigma_{100}^2 + d_1^2) + n_2(\sigma_{150}^2 + d_2^2)}{n_1 + n_2}} \quad \ldots(i)$$

where, $d_1 = \overline{X}_{100} - \overline{X}_{250}$ and $d_2 = \overline{X}_{150} - \overline{X}_{250}$

Here, $d_1 = 50 - 44 = 6$

$\Rightarrow \quad d_1^2 = 36$

and $d_2 = 40 - 44 = -4$

$\Rightarrow \quad d_2^2 = 16$

Combined standard deviation of all 250 items is

$$\sigma_{250} = \sqrt{\frac{100\{(5)^2 + 36\} + 150\{(6)^2 + 16\}}{100 + 150}}$$

[from Eq. (i)]

$$= \sqrt{\frac{100(25 + 36) + 150(36 + 16)}{250}} = \sqrt{\frac{100 \times 61 + 150 \times 52}{250}} = \frac{\sqrt{10 \times 61 + 15 \times 52}}{5} = \frac{\sqrt{610 + 780}}{5} = \frac{\sqrt{1390}}{5} = \frac{37.28}{5} = 7.456 \approx 7.5$$

90. (***c***) Variance of all the 250 items $= (\sigma_{250})^2 = (7.456)^2 = 55.59 \approx 55.6$

91. The cumulative frequency of the largest observed value must always be

(a) less than the total number of observations
(b) greater than the total number of observations
(c) equal to total number of observations
(d) equal to mid-point of the last class interval

(***a***) The cumulative frequency of the largest observed value must always be less than the total number of observations.

e.g.

Marks obtained	Number of students	Cumulative frequency
0-10	2	2
10-20	18	20
20-30	30	50
30-40	45	95
40-50	35	130
50-60	20	150
60-70	6	156
70-80	3	159
	N **= 159**	

(i) $95 < N = 159$

(ii) $95 \not> N = 159$

(iii) $95 \neq 159$

(iv) Mid-point of (70-80) $= 75 \neq 95$

92. Let X denotes the number of scores which exceed 4 in 18 tosses of a symmetrical die. Consider the following statements

I. The arithmetic mean of X is 6.
II. The standard deviation of X is 2.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

(***c***) Given that, n = Total number of tosses = 18 and X = Number of scores which exceed 4 in 18 tosses of a symmetrical die = {5,6}.

$\Rightarrow \quad n(X) = 2$

$\therefore \quad p = \frac{n(X)}{n(s)} = \frac{2}{6} = \frac{1}{3}$

and $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$ $[\because p + q = 1]$

I. Now, arithmetic mean of $X = np = 18 \times \frac{1}{3} = 6$

II. Standard deviation of $X = \sqrt{\text{Variance of } X} = \sqrt{npq} = \sqrt{18 \times \frac{1}{3} \times \frac{2}{3}} = \sqrt{4} = 2$

Hence, both statements are correct.

29

PROBABILITY

2019 (II)

1. A bag contains 20 books out of which 5 are defective. If 3 of the books are selected at random and removed from the bag in succession without replacement, then what is the probability that all three books are defective?

(a) 0.009 (b) 0.016 (c) 0.026 (d) 0.047

(***a***) Total books in bag = 20

Defective books = 5

∴ Undefective books = 20 − 5 = 15

∴ Probability to selected three books are defective without replacement

$$= \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{6}{684}$$

$$= 0.0087 = 0.009$$

2. A coin is biased so that heads comes up thrice as likely as tails. For three independent tosses of a coin, what is the probability of getting at most two tails?

(a) 0.16 (b) 0.48 (c) 0.58 (d) 0.98

(***d***) Let X be a random variable, it represents of the number tail comes of three tosses of a coin.

∴ Possible value of X are 0, 1, 2, 3.

According to the question, the coin is biased in which the probability to comes head is thrice as likely as tails.

$$\therefore P(\text{H}) = \frac{3}{4} \text{ and } P(\text{T}) = \frac{1}{4}$$

$$P(\text{X} = 0) = P(\{\text{HHH}\}) = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$P(X = 1) = \text{P}$ (2 heads and 1 tail)

$$= P(\{\text{HHT}\}) + P(\{\text{HTH}\}) + P(\{\text{THH}\})$$

$$= \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$$

$$= \frac{27}{64}$$

$P(X = 2) = P$ (1 head and 2 tails)

$$= P(\{\text{HTT}\}) + P(\{\text{THT}\}) + P(\{\text{TTH}\})$$

$$= \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}$$

$$= \frac{9}{64}$$

∴ Required probability

$$= P(\text{X} = 0) + P(\text{X} = 1) + P(\text{X} = 2)$$

$$= \frac{27}{64} + \frac{9}{64} + \frac{27}{64}$$

$$= \frac{63}{64} = 0.98$$

3. If a coin is tossed till the first head appears, then what will be the sample space?

(a) {H}
(b) {TH}
(c) {T, HT, HHT, HHHT, ...}
(d) {H, TH, TTH, TTTH, ...}

(***a***) A coin is tossed till the first head appears, then the sample space will be = {H}

4. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys is 70 kg and that of girls is 55 kg. What are the number of boys and girls respectively in the class?

(a) 75 and 75 (b) 50 and 100
(c) 70 and 80 (d) 100 and 50

(***b***) Let number of boys and girls be x and y respectively.

$$\therefore \quad x + y = 150 \qquad \ldots \text{(i)}$$

Mean weight of 150 students = 60 kg

∴ Total weight of 150 students

$$= 60 \times 150 = 9000 \text{ kg.}$$

Mean weight of boys = 70 kg

∴ Total weight of boys = $70x$ kg

and mean weight of girls = 55 kg

∴ Total weight of girls = $55\ y$ kg

∴ Total weight of 150 students = 9000 kg

$$\Rightarrow \quad 70x + 55y = 9000$$

$$\Rightarrow \quad 14x + 11y = 1800 \qquad \ldots \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$x = 50,\ y = 100$$

Hence, the number of boys and girls are 50 and 100 respectively.

5. If a fair die is rolled 4 times, then what is the probability that there are exactly 2 sixes?

(a) $\frac{5}{216}$ (b) $\frac{25}{216}$
(c) $\frac{125}{216}$ (d) $\frac{175}{216}$

(***b***) Let X be a random variable that represents to appearing 6 of rolled a die.

Probability of to get 6 to rolled a die,

$$p = \frac{1}{6}$$

∴ Probability of not get 6 to rolled a die,

$$q = 1 - \frac{1}{p} = 1 - \frac{1}{6} = \frac{5}{6}$$

Here, $n = 4$, $r = 2$

∴ Required probability = ${}^nC_r p^r q^{n-r}$

[By Bernoulli distribution]

$$= {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= \frac{4!}{2!2!} \times \frac{1}{36} \times \frac{25}{36}$$

$$= \frac{4\cdot 3}{2\cdot 1} \times \frac{1}{36} \times \frac{25}{36} = \frac{25}{216}$$

6. Consider the following statements:

1. If A and B are mutually exclusive events, then it is possible that $P(A) = P(B) = 0.6$.
2. If A and B are any two events such that $P(A/B) = 1$, then $P(\overline{B}/\overline{A}) = 1$.

Which of the above statement is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***b***) **Statement 1** : A and B are mutually exclusive events, then $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

$= 0.6 + 0.6$

$= 1.2$, it is not possible

So, Statement 1 is not correct.

Statement 2 : A and B are any two events such that

$$P\left(\frac{A}{B}\right) = 1$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = 1 \Rightarrow P(A \cap B) = P(B) \quad \ldots \text{(i)}$$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(B)$ [from Eq. (i)]

$$\Rightarrow P(A \cup B) = P(A) \quad \ldots \text{(ii)}$$

$$\text{Now, } P\left(\frac{\overline{B}}{\overline{A}}\right) = \frac{P(\overline{B} \cap \overline{A})}{P(\overline{A})} = \frac{P(\overline{A \cup B})}{P(\overline{A})}$$

$$= \frac{1 - P(A \cup B)}{1 - P(A)} = \frac{1 - P(A)}{1 - P(A)} = 1$$

So, Statement 2 is correct.

Hence, only the statement 2 is correct.

7. There are 3 coins in a box. One is a two-headed coin; another is a fair coin; and third is biased coin that comes up heads 75% of time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

(a) $\frac{2}{9}$ (b) $\frac{1}{3}$

(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

(***c***) Let E_1, E_2 and E_3 represent the events of two-headed coin, a fair coin and biased coin respectively.

$$\therefore P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P(E_3) = \frac{1}{4}$$

$$P\left(\frac{E}{E_1}\right) = \frac{1}{2}, P\left(\frac{E}{E_2}\right) = \frac{1}{2}, P\left(\frac{E}{E_3}\right) = \frac{1}{4}$$

Apply Baye's theorem,

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{4} + \frac{1}{16}}$$

$$= \frac{\frac{1}{4}}{\frac{4 + 4 + 1}{16}} = \frac{4}{9}$$

8. If 5 of a Company's 10 delivery trucks do not meet emission standards and 3 of them are chosen for inspection, then what is the probability that none of the trucks chosen will meet emission standards?

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{12}$ (d) $\frac{1}{4}$

(***c***) Total trucks of a company's = 10

Number of trucks that do not meet emission standards = 5

Number of trucks that are chosen for inspection = 3

$$\therefore \text{Required probability} = \frac{{}^5C_3}{{}^{10}C_3}$$

$$= \frac{\frac{5!}{3!2!}}{\frac{10!}{3!7!}} = \frac{5!7!}{10!2!} = \frac{5 \cdot 4 \cdot 3}{10 \cdot 9 \cdot 8} = \frac{1}{12}$$

9. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on them is a prime number?

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$

(c) $\frac{7}{12}$ (d) $\frac{2}{3}$

(***a***) Total number of sample space of two dice are thrown, $n(s) = 6 \times 6 = 36$

Total number of favourable outcomes the sum of numbers appearing on them is a prime number.

(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)

$\therefore \quad n(E) = 15$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)}$$

$$= \frac{15}{36} = \frac{5}{12}$$

2019 (I)

10. From a deck of cards, cards are taken out with replacement. What is the probability that the fourteenth card taken out is an ace?

(a) $\frac{1}{51}$ (b) $\frac{4}{51}$ (c) $\frac{1}{52}$ (d) $\frac{1}{13}$

(***d***) Total number of possible outcomes = 52

And number of favourable outcomes = 4

$$\therefore \text{Required probability} = \frac{4}{52} = \frac{1}{13}$$

11. If A and B are two events such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cap B) = 0.4$, then what is $P(\overline{A \cup B})$ equal to ?

(a) 0.9 (b) 0.7

(c) 0.5 (d) 0.3

(***d***) $P(\overline{A \cup B}) = 1 - P(A \cup B)$

We have,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.6 - 0.4$

$= 1.1 - 0.4$

$= 0.7$

$\therefore \quad P(\overline{A \cup B}) = 1 - 0.7 = 0.3$

12. A problem is given to three students A, B and C whose probabilities of solving the problem are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if they all solve the problem independently ?

(a) $\frac{29}{32}$ (b) $\frac{27}{32}$ (c) $\frac{25}{32}$ (d) $\frac{23}{32}$

(***a***) We have, $P(A) = \frac{1}{2}, P(\overline{A}) = \frac{1}{2}$

$$P(B) = \frac{3}{4}, P(\overline{B}) = \frac{1}{4}$$

and $\quad P(C) = \frac{1}{4}, P(\overline{C}) = \frac{3}{4}$

$\therefore$ Required probability

$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})$

$$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{29}{32}$$

13. A pair of fair dice is rolled. What is the probability that the second dice lands on a higher value than does the first?

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{5}{12}$ (d) $\frac{5}{18}$

(***c***) Total number of possible outcomes = 36

Favourable outcomes

= (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6) (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)

$\therefore$ Total number of favourable outcomes = 15

$$\therefore \text{Required probability} = \frac{15}{36} = \frac{5}{12}$$

14. A fair coin is tossed and an unbiased dice is rolled together. What is the probability of getting a 2 or 4 or 6 along with head?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

(c) (**c**) Total number of possible outcomes $= 2 \times 6 = 12$

And favourable outcomes
= (H, 2), (H, 4), (H, 6)

∴ Total number of possible outcomes = 3

∴ Required probability $= \frac{3}{12} = \frac{1}{4}$

15. If A, B and C are three events, then what is the probability that atleast two of these events occur together ?

(a) $P(A \cap B) + P(B \cap C) + P(C \cap A)$
(b) $P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$
(c) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
(d) $P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$

(c) (**c**) If A, B and C are three events, then atleast two events occur i.e.

$$(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C) \cup (A \cap B \cap C)$$

∴ Required probability

$$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$$

16. Two independent events A and B are such that $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. If $P(B) < P(A)$, then what is $P(B)$ equal to ?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

(b) (**b**) Given, $P(A \cup B) = \frac{2}{3}$

and $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{2}{3} = P(A) + P(B) - \frac{1}{6}$

$\Rightarrow P(A) + P(B) = \frac{2}{3} + \frac{1}{6}$

$\Rightarrow P(A) + P(B) = \frac{5}{6}$ ...(i)

And also, $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A)\,P(B) = \frac{1}{6}$ ...(ii)

From Eqs. (i) and (ii), we get

$P(A)$ or $P(B) = \frac{1}{2}$ or $\frac{1}{3}$

Also, given $P(B) < P(A)$

$\therefore P(B) = \frac{1}{3}$

17. If two fair dice are rolled, then what is the conditional probability that the first dice lands on 6, given that the sum of numbers on the dice is 8?

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{5}$ (d) $\frac{1}{6}$

(c) (**c**) Let E_1 = Event of first dice on 6

E_2 = Event of the sum of numbers on dices 8

∴ Total number of sample space of two dices are rolled, $n(s) = 36$

Possible outcomes of E_1 (6, 2)

Possible outcomes of E_2 (2, 6) (3, 5) (4, 4) (5, 3) (6, 2)

$\therefore P(E_1 \cap E_2) = \frac{1}{36}$

and $P(E_2) = \frac{5}{36}$

∴ Required probability $= P\left(\frac{E_1}{E_2}\right)$

$= \frac{P(E_1 \cap E_2)}{P(E_2)}$, when $P(E_2 \neq 0)$

$= \frac{\frac{1}{36}}{5/36} = \frac{1}{5}$

18. Two symmetric dice flipped with each dice having two sides painted red, two painted black, one painted yellow and the other painted white. What is the probability that both land on the same colour ?

(a) $\frac{3}{18}$ (b) $\frac{2}{9}$
(c) $\frac{5}{18}$ (d) $\frac{1}{3}$

(c) (**c**) P (two sides painted red) $= \frac{2}{6} \times \frac{2}{6}$

P (two sides painted black) $= \frac{2}{6} \times \frac{2}{6}$

P (one side painted yellow) $= \frac{1}{6} \times \frac{1}{6}$

and P (other side painted white) $= \frac{1}{6} \times \frac{1}{6}$

∴ Required probability that both land on the same colour

$$= \frac{2}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{4 + 4 + 1 + 1}{36}$$

$$= \frac{10}{36} = \frac{5}{18}$$

19. There are n socks in a drawer, of which 3 socks are red. If 2 of the socks are chosen randomly and the probability that both selected socks are red is $\frac{1}{2}$, then what is the value of n ?

(a) 3 (b) 4 (c) 5 (d) 6

(b) (**b**) Total number of socks = n

P (first socks is red) $= \frac{3}{n}$

P (second socks is red) $= \frac{2}{n-1}$

According to the question,

$$\frac{3}{n} \times \frac{2}{n-1} = \frac{1}{2}$$

$\Rightarrow n^2 - n = 12$

$\Rightarrow n^2 - n - 12 = 0$

$\Rightarrow n^2 - 4n + 3n - 12 = 0$

$\Rightarrow n(n-4) + 3(n-4) = 0$

$\Rightarrow (n-4)(n+3) = 0$

$\therefore n = 4, -3$

20. Two cards are chosen at random from a deck of 52 playing cards. What is the probability that both of them have the same value ?

(a) $\frac{1}{17}$ (b) $\frac{3}{17}$ (c) $\frac{5}{17}$ (d) $\frac{7}{17}$

(a) (**a**) ∴ Required probability $= \frac{{}^4C_2 \times 13}{{}^{52}C_2}$

$$= \frac{4 \times 3 \times 13}{52 \times 51} = \frac{1}{17}$$

21. In eight throws of a die, 5 or 6 is considered a success. The mean and standard deviation of total number of successes is respectively given by

(a) $\frac{8}{3}, \frac{16}{9}$ (b) $\frac{8}{3}, \frac{4}{3}$
(c) $\frac{4}{3}, \frac{4}{3}$ (d) $\frac{4}{3}, \frac{16}{9}$

(b) (**b**) We have, p (success) $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$\therefore q = 1 - p = \frac{2}{3}$

Given, $n = 8$

∴ Mean $= np = 8 \times \frac{1}{3} = \frac{8}{3}$

Standard deviation $= \sqrt{npq}$

$$= \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}}$$

$$= \sqrt{\frac{16}{9}} = \frac{4}{3}$$

22. A and B are two events such that $\overline{A}$ and $\overline{B}$ are mutually exclusive. If $P(A) = 0.5$ and $P(B) = 0.6$, then what is the value of $P(A/B)$?

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{2}{5}$ (d) $\frac{1}{3}$

(***b***) Given, $P(\overline{A} \cap \overline{B}) = 0$

$\Rightarrow \quad P(\overline{A \cup B}) = 0$

$\Rightarrow \quad 1 - P(A \cup B) = 0$

$\Rightarrow \quad P(A \cup B) = 1$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 1 = 0.5 + 0.6 - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.1$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.6} = \frac{1}{6}$

23. What is the probability that an interior point in a circle is closer to the centre than to the circumference?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$

(d) It cannot be determined

(***a***) Let radius of circle be r, then the points closer to centre if circumference will lie within radius of $\frac{r}{2}$.

So, the favourable outcome would be the points inside the area of circle with radius $\frac{r}{2}$ whereas the total possible outcomes could be all the points inside the area of circle with radius r.

$\therefore$ Required probability $= \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$

24. If A and B are two events, then what is the probability of occurrence of either event A or event B ?

(a) $P(A) + P(B)$ (b) $P(A \cup B)$
(c) $P(A \cap B)$ (d) $P(A)P(B)$

(***b***) If A and B are two events, then the probability of occurrence of either event A or event B is $P(A \cup B)$.

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25. If two dice are thrown and atleast one of the dice shows 5, then the probability that the sum is 10 or more is

(a) $\frac{1}{6}$ (b) $\frac{4}{11}$ (c) $\frac{3}{11}$ (d) $\frac{2}{11}$

(***c***) Let A be event of dice shows 5 and B be the event that the sum is 10 or more

Here, $n(S) = 36$

$n(A) = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)\}$

$n(B) = \{(5, 5), (6, 4), (4, 6), (6, 5), (5, 6), (6, 6)\}$

$n(A \cap B) = \{(5, 5), (6, 5), (5, 6)\}$

$$P\left(\frac{B}{A}\right) = \frac{\frac{3}{36}}{\frac{11}{36}} \left[\because P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}\right]$$

$$= \frac{3}{11}.$$

26. Let A, B and C be three mutually exclusive and exhaustive events associated with a random experiment. If $P(B) = 1.5\ P(A)$ and $P(C) = 0.5\ P(B)$, then $P(A)$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{4}{13}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(***b***) We have, $P(B) = 1.5\ P(A) = \frac{3}{2} P(A)$

and $P(C) = 0.5\ P(B) = \frac{1}{2} P(B)$

$$= \frac{1}{2} \times \frac{3}{2} P(A) = \frac{3}{4} P(A)$$

Now, A, B and C are mutually exclusive and exhaustive events

So, $P(A) + P(B) + P(C) = 1$

$\Rightarrow P(A) + \frac{3}{2} P(A) + \frac{3}{4} P(A) = 1$

$\Rightarrow \quad P(A)\left[1 + \frac{3}{2} + \frac{3}{4}\right] = 1$

$\Rightarrow \quad P(A)\left(\frac{13}{4}\right) = 1 \Rightarrow P(A) = \frac{4}{13}$

27. In a bolt factory, machines X, Y, Z manufacture bolts that are respectively 25%, 35% and 40% of the factory's total output. The machines X, Y, Z respectively produce 2%, 4% and 5% defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine X?

(a) $\frac{5}{39}$ (b) $\frac{14}{39}$ (c) $\frac{20}{39}$ (d) $\frac{34}{39}$

(***a***) Let

A : bolt manufactured from machine X

B : bolt manufactured from machine Y

C : bolt manufactured from machine Z

and E : bolt is defective

$\therefore \quad P(A) = 25\% = \frac{25}{100}$

$P(B) = 35\% = \frac{35}{100}$

$P(C) = 40\% = \frac{40}{100}$

and $P\left(\frac{E}{A}\right) = 2\% = \frac{2}{100}$

$P\left(\frac{E}{B}\right) = 4\% = \frac{4}{100}$

$P\left(\frac{E}{C}\right) = 5\% = \frac{5}{100}$

Probability of defective bulb that was manufactured by machine X,

$$P\left(\frac{A}{E}\right) = \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B) \cdot P\left(\frac{E}{B}\right) + P(C) \cdot P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{2}{100}}{\frac{25}{100} \times \frac{2}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{5}{100}}$$

$$= \frac{25 \times 2}{25 \times 2 + 35 \times 4 + 40 \times 5}$$

$$= \frac{50}{50 + 140 + 200} = \frac{50}{390} = \frac{5}{39}$$

28. 8 coins are tossed simultaneously. The probability of getting atleast 6 heads is

(a) $\frac{7}{64}$ (b) $\frac{57}{64}$ (c) $\frac{37}{256}$ (d) $\frac{229}{256}$

(***c***) We have,

8 coins are tossed simultaneously i.e

$n = 8$ probability of getting head $p = \frac{1}{2}$,

so $q = 1 - p = \frac{1}{2}$

Probability of getting atleast 6 heads.

$$= {}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 ({}^8C_6 + {}^8C_7 + {}^8C_8)$$

$$= \left(\frac{1}{2}\right)^8 (28 + 8 + 1) = \frac{37}{256}$$

29. Three groups of children contain 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consist of 1 girl and 2 boys is

(a) $\frac{13}{32}$ (b) $\frac{9}{32}$ (c) $\frac{3}{32}$ (d) $\frac{1}{32}$

(***a***) In first group, 3 girls and 1 boy

$P(G) = \frac{3}{4}$ and $P(B) = \frac{1}{4}$

In second group, 2 girls and 2 boys

$$P(G) = \frac{2}{4} = \frac{1}{2} \text{ and } P(B) = \frac{2}{4} = \frac{1}{2}$$

In third group 1 girl and 3 boys

$$P(G) = \frac{1}{4} \text{ and } P(B) = \frac{3}{4}$$

One child is selected at random from each group and consists of 1 girl and 2 boys are GBB or BGB or BBG.

So, required probability

$$= \frac{3}{4}\times\frac{1}{2}\times\frac{3}{4} + \frac{1}{4}\times\frac{1}{2}\times\frac{3}{4} + \frac{1}{4}\times\frac{1}{2}\times\frac{1}{4}$$
$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$$

30. If probability of simultaneous occurrence of two events A and B is p and the probability that exactly one of A, B occurs is q, then which of the following is/are correct?

1. $P(\overline{A}) + P(\overline{B}) = 2 - 2p - q$
2. $P(\overline{A} \cap \overline{B}) = 1 - p - q$

Select the correct answer using the code given below.

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**c**) Given that,

$$P(A \cap B) = P$$

and $P(A) + P(B) - 2P(A \cap B) = q$

Now, **Statement 1**

Given,

$$P(A) + P(B) - 2P(A \cap B) = q$$
$$\Rightarrow \quad 1 - P(\overline{A}) + 1 - P(\overline{B}) - 2p = q$$
$$\Rightarrow \quad P(\overline{A}) + P(\overline{B}) = 2 - 2p - q$$

Statement 1 is correct.

For **Statement 2**

$$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - [P(A) + P(B) - 2P(A \cap B) + P(A \cap B)]$$
$$= 1 - [q + p] = 1 - q - p$$

Statement 2 is also correct.

31. Two integers x and y are chosen with replacement from the set $[0, 1, 2, \ldots, 10]$. The probability that $|x - y| > 5$ is

(a) $\frac{6}{11}$ (b) $\frac{35}{121}$ (c) $\frac{30}{121}$ (d) $\frac{25}{121}$

(**c**) Given that,

$$S = \{0, 1, 2, \ldots, 10\}$$
$$n(S) = 11 \times 11 = 121$$

Now, $x - y > 5$

Now, E is the set of element such that $|x - y| > 5$

$E = \{(6, 0), (0, 6), (7, 1), (1, 7), (8, 2), (2, 8), (9, 3), (3, 9), (10, 4), (4, 10), (7, 0), (0, 7), (8,1), (1,8), (9, 2), (2, 9), (10, 3), (3, 10), (8, 0), (0, 8), (9, 1), (1, 9), (10, 2), (2, 10), (9, 0), (0, 9), (10, 1), (1, 10), (10, 0), (0, 10)\}$

$n(E) = 30$

So, required probability $= \frac{30}{121}$

32. Three dice having digits 1, 2, 3, 4, 5 and 6 on their faces are marked I, II, and III and rolled. Let x, y and z represent the number on die-I, die-II and die-III, respectively. What is the number of possible outcomes such that $x > y > z$?

(a) 14 (b) 16 (c) 18 (d) 20

(**d**) Three dice having digit 1, 2, 3, 4, 5 and 6 and given that $x > y > z$.

So, possiblities are

Case I If $x = 6$

$$x > y > z$$

Possible ways

$= \{(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 4, 1), (6, 4, 2), (6, 4, 3), (6, 3, 2), (6, 3, 1), (6, 2, 1)$

So, possible ways = 10

Case II If $x = 5$

Then, possible ways

$= \{(5, 4, 3), (5, 4, 2), (5, 4, 1), (5, 3, 2), (5, 3, 1), (5, 2, 1)\}$

So, possible ways = 6

Case III If $x = 4$

Then, possible ways

$= \{(4, 3, 2), (4, 3, 1), (4, 2, 1)$

So, possible ways = 3

Case IV If $x = 3$

Then, possible ways = 1 (3, 2, 1)

So, required possible outcomes

$= 10 + 6 + 3 + 1 = 20$

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33. In a Binomial distribution, the mean is three times its variance. What is the probability of exactly 3 successes out of 5 trials ?

(a) $\frac{80}{243}$ (b) $\frac{40}{243}$
(c) $\frac{20}{243}$ (d) $\frac{10}{243}$

(**a**) According to the question,

Mean = 3 (Variance)

$$\Rightarrow \quad np = 3npq$$

[where n = number of trials]

$$\Rightarrow \quad q = \frac{1}{3}$$

$$\because \quad p + q = 1$$
$$\Rightarrow \quad p + \frac{1}{3} = 1$$
$$\Rightarrow \quad p = \frac{2}{3}$$
$$\therefore \quad p(X = 3) = {}^5C_3 \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^2$$
$$= \frac{5!}{3!\,2!} \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^2 = \frac{80}{243}$$

34. Consider the following statements

I. $P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$
II. $P(A \cap \overline{B}) = P(B) - P(A \cap B)$
III. $P(A \cap B) = P(B)\,P(A/B)$

Which of the above statements are correct?

(a) I and II (b) I and III
(c) II and III (d) I, II and III

(**b**) Here,

Statement I

$P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$ is correct.

Statement II

$P(A \cap \overline{B}) = P(B) - P(A \cap B)$ is wrong as $P(A \cap \overline{B}) = P(A) - P(A \cap B)$.

Statement III

$P(A \cap B) = P(B) \times P\left(\frac{A}{B}\right)$ is correct. [by conditional theorem]

Hence, Statements I and III are correct.

35. The probabilities that a student will solve Question A and Question B are 0.4 and 0.5 respectively. What is the probability that he solves atleast one of the two questions?

(a) 0.6 (b) 0.7
(c) 0.8 (d) 0.9

(**b**) Given that, 7251024607

$$P(A) = 0.4 \text{ and } P(B) = 0.5$$
$$\because P(A \cup B) = 1 - P(A' \cap B')$$
$$= 1 - [(1 - 0.4) \times (1 - 0.5)]$$
$$= 1 - (0.6)(0.5)$$
$$= 1 - (0.3) = 0.7$$

36. Two fair dice are rolled. What is the probability of getting a sum of 7?

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

(***b***) Here, $n(S) = 36$

and E be the event of getting a sum of 7 on two fair dice.

$= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\therefore n(E) = 6$

So, required probability $= \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

37. If A and B are two events such that $2P(A) = 3P(B)$, where $0 < P(A) < P(B) < 1$, then which one of the following is correct?

(a) $P(A|B) < P(B|A) < P(A \cap B)$

(b) $P(A \cap B) < P(B|A) < P(A|B)$

(c) $P(B|A) < P(A|B) < P(A \cap B)$

(d) $P(A \cap B) < P(A|B) < P(B|A)$

(***b***) Given that, $2P(A) = 3P(B)$

$$\Rightarrow \quad 2\frac{P(A)}{P(A \cap B)} = \frac{3P(B)}{P(A \cap B)}$$

[dividing both sides by $P(A \cup B)$]

$$\Rightarrow \frac{1}{2} \times \frac{P(A \cap B)}{P(A)} = \frac{1}{3} \times \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{1}{2} \times P\left(\frac{B}{A}\right) = \frac{1}{3} P\left(\frac{A}{B}\right)$$

$$\Rightarrow \quad P\left(\frac{B}{A}\right) < P\left(\frac{A}{B}\right)$$

38. A box has ten chits numbered 0, 1, 2, 3, ..., 9. First, one chit is drawn at random and kept aside. From the remaining, a second chit is drawn at random. What is the probability that the second chit drawn is "9" ?

(a) $\frac{1}{10}$ (b) $\frac{1}{9}$

(c) $\frac{1}{90}$ (d) None of these

(***a***) Let E_1 be the event of drawing a chit which is not 9 and E_2 be the event of drawing second chit bearing number 9.

$$\therefore P(E_1) = \frac{{}^9C_1}{{}^{10}C_1} = \frac{9}{10} \text{ and } P(E_2) = \frac{{}^1C_1}{{}^9C_1} = \frac{1}{9}$$

$\therefore$ Required probability $= P(E_1) \cdot P(E_2)$

$$= \frac{9}{10} \times \frac{1}{9} = \frac{1}{10}$$

39. One bag contains 3 white and 2 black balls, another bag contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it, what is the chance that it is white?

(a) $\frac{3}{8}$ (b) $\frac{49}{80}$ (c) $\frac{8}{13}$ (d) $\frac{1}{2}$

(***b***) Let E_1 be the event of selecting the first bag and E_2 be the event of selecting the second bag. Let A be the event of drawing white ball.

So, by theorem of total probability.

$$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{{}^3C_1}{{}^5C_1} + \frac{1}{2} \times \frac{{}^5C_1}{{}^8C_1}$$

$$= \frac{1}{2}\left[\frac{3}{5} + \frac{5}{8}\right] = \frac{1}{2} \times \frac{24 + 25}{40}$$

$$= \frac{1}{2} \times \frac{49}{40} = \frac{49}{80}$$

40. Consider the following in respect to two events A and B

I. $P(A \text{ occurs but not } B) = P(A) - P(B)$ if $B \subset A$

II. $P(A \text{ alone or } B \text{ alone occurs}) = P(A) + P(B) - P(A \cap B)$

III. $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive

Which of the above is/are correct?

(a) Only I (b) I and III

(c) II and III (d) I and II

(***b***) If $B \subset A$, then $P(A - B)$

$$= P(A) - P(A \cap B) = P(A) - P(B)$$

$[\because B \subset A \Rightarrow A \cap B = B]$

So, Statement I is correct.

P (A alone or B alone)

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B)$$

So, Statement II is wrong.

If A and B are mutually exclusive,

then $P(A \cap B) = 0$

$\Rightarrow \quad P(A \cup B) = P(A) + P(B)$

So, Statement III is correct.

Hence, Statements I and III are correct.

41. A committee of three has to be chosen from a group of 4 men and 5 women. If the selection is made at random, what is the probability that exactly two members are men?

(a) $\frac{5}{14}$ (b) $\frac{1}{21}$

(c) $\frac{3}{14}$ (d) $\frac{8}{21}$

(***a***) Total number of selecting three members $= {}^9C_3$

Favourable numbers of selecting two members as men

$$= {}^4C_2 \times {}^5C_1$$

So, required probability $= \frac{{}^4C_2 \times {}^5C_1}{{}^9C_3}$

$$= \frac{\frac{4 \times 3}{2 \times 1} \times \frac{5}{1}}{\frac{9 \times 8 \times 7}{3 \times 2 \times 1}}$$

$$= \frac{2 \times 3 \times 5}{3 \times 4 \times 7} = \frac{5}{14}$$

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42. A committee of two persons is selected from two men and two women. The probability that the committee will have exactly one woman is

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(***b***) Let S be the event of selecting two persons from 2 men and 2 women.

$\therefore \quad n(s) = {}^4C_2$

Let E be the event that selected person has 1 woman.

$\therefore n(E) = {}^2C_1 \times {}^2C_1$

$$\therefore P(E) = \frac{n(E)}{n(s)} = \frac{{}^2C_1 \times {}^2C_1}{{}^4C_2} = \frac{2 \times 2}{\left(\frac{4 \times 3}{2}\right)} = \frac{2}{3}$$

43. Let a die be loaded in such a way that even faces are twice likely to occur as the odd faces. What is the probability that a prime number will show up when the die is tossed?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

(***c***) Let the probability of number to be even and odd be p and q respectively.

$\therefore \quad p + q = 1$

$\Rightarrow 2q + q = 1 \qquad [\because p(\text{even}) = 2\,p(\text{odd})]$

$$\Rightarrow \quad q = \frac{1}{3} \Rightarrow p = \frac{2}{3}$$

$\therefore$ Probability of each even number

$$= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

and probability of each odd number

$$= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$\therefore$ Required probability = Probability of prime number

= Probability of getting 2, 3, 5

$$= \frac{2}{9} + \frac{1}{9} + \frac{1}{9} = \frac{4}{9}$$

44. Let the sample space consist of non- negative integers upto 50, X denote the numbers which are multiples of 3 and Y denote the odd numbers. Which of the following is/are correct?

1. $P(X) = \frac{8}{25}$

2. $P(Y) = \frac{1}{2}$

Select the correct answer using the code given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(*d*) Let S = Set of all non-negative integers upto 50.

$\therefore \quad S = \{0,1,2,3,\ldots,49,50\}$

$\Rightarrow \quad n(s) = 51$

X = Number which are multiple of 3.

$= \{0,3,6,9\ldots,48\}$

$\Rightarrow \quad n(X) = 17$

Y = Numbers which are odd

$= \{1,3,5\ldots49\}$

$\Rightarrow \quad n(Y) = 25$

$\therefore \quad P(X) = \frac{n(X)}{n(s)} = \frac{17}{51} = \frac{1}{3}$

and $P(Y) = \frac{n(Y)}{n(s)} = \frac{25}{51}$

45. For two events A and B, let $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. What is $P(\overline{A} \cap B)$ equal to?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(*a*) We have,

$P(A) = \frac{1}{2}, P(A \cup B) = \frac{2}{3}, P(A \cap B) = \frac{1}{6}$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{2}{3} = \frac{1}{2} + P(B) - \frac{1}{6} \Rightarrow P(B) = \frac{1}{3}$

Again, $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

46. Let A and B be two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{6}$ and $P(A \cap B) = \frac{1}{12}$. What is $P(B|\overline{A})$ equal to?

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{10}$

(*c*) We have, $P(A) = \frac{1}{3}$

$P(B) = \frac{1}{6}, P(A \cap B) = \frac{1}{12}$

Now, $P(B/\overline{A}) = \frac{P(B \cap \overline{A})}{P(\overline{A})} = \frac{P(B) - P(A \cap B)}{1 - P(A)}$

$[\because P(B \cap \overline{A}) = P(B) - P(A \cap B)$ and $P(\overline{A}) = 1 - P(A)]$

$= \frac{\frac{1}{6} - \frac{1}{12}}{1 - \frac{1}{3}} = \frac{\frac{1}{12}}{\frac{2}{3}} = \frac{1}{8}$

47. In a binomial distribution, the mean is $\frac{2}{3}$ and the variance is $\frac{5}{9}$. What is the probability that $X = 2$?

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$ (c) $\frac{25}{216}$ (d) $\frac{25}{54}$

(*c*) We have, mean $= \frac{2}{3}$ and variance $= \frac{5}{9}$

$\therefore \quad np = \frac{2}{3}$ and $npq = \frac{5}{9}$

$\therefore \quad q = \frac{5/9}{2/3} = \frac{5}{6}$

$\Rightarrow \quad p = 1 - q = 1 - \frac{5}{6} = \frac{1}{6}$

$\therefore \quad n = \frac{2/3}{1/6} = 4$

Now, $p(x = 2) = {}^nC_2 p^{n-2} q^2$

$= {}^4C_2 p^{4-2} q^2 = {}^4C_2 p^2 q^2$

$= \frac{4 \times 3}{2 \times 1} \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{25}{216}$

48. The probability that a ship safely reaches a port is $\frac{1}{3}$. The probability that out of 5 ships, atleast 4 ships would arrive safely is

(a) $\frac{1}{243}$ (b) $\frac{10}{243}$ (c) $\frac{11}{243}$ (d) $\frac{13}{243}$

(*c*) According to the question,

$n = 5, P = \frac{1}{3}, q = 1 - P = \frac{2}{3}$

$\therefore$ Required probability $= P(x \geq 4)$

$= P(x = 4) + P(x = 5)$

$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 + {}^5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0$

$= \frac{5 \times 2}{3^5} + \frac{1 \times 1}{3^5} = \frac{10}{243} + \frac{1}{243} = \frac{11}{243}$

49. What is the probability that atleast two persons out of a group of three persons were born in the same month (disregard year)?

(a) $\frac{33}{144}$ (b) $\frac{17}{72}$
(c) $\frac{1}{144}$ (d) $\frac{2}{9}$

(*b*) P (None born in same month)

$= \frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} = \frac{110}{144}$

$\therefore P$ (atleast two person born in same month)

$= 1 - P$ (none born in same month)

$= 1 - \frac{110}{144} = \frac{144 - 110}{144} = \frac{34}{144} = \frac{17}{72}$

50. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \overline{C}) = \frac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$, then what is $P(B \cap C)$ equal to?

(a) $\frac{1}{12}$ (b) $\frac{3}{4}$
(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

(*a*) We know that,

$P(B \cap \overline{C}) = P(A \cap B \cap \overline{C}) + P(\overline{A} \cap B \cap \overline{C})$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ (given)

Also, $P(B) = P(B \cap C) + P(B \cap \overline{C})$

$\Rightarrow \quad \frac{3}{4} = P(B \cap C) + \frac{2}{3}$

$[\because P(B) = \frac{3}{4}$ and $P(B \cap \overline{C}) = \frac{2}{3}]$

$\Rightarrow P(B \cap C) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

51. In a multiple-choice test, an examinee either knows the correct answer with probability p, or guesses with probability $1 - p$. The probability of answering a question correctly is $\frac{1}{m}$, if he or she merely guesses. If the examinee answers a question correctly, the probability that he or she really knows the answer is

(a) $\frac{mp}{1 + mp}$ (b) $\frac{mp}{1 + (m-1)p}$
(c) $\frac{(m-1)p}{1 + (m-1)p}$ (d) $\frac{(m-1)p}{1 + mp}$

(*b*) Let E_1 be the event that examinee knows the answer.

E_2 be the event the examine guesses the answer.

A be the event that the answer is correct.

Now, according to the question

$P(E_1) = p$

$P(E_2) = 1 - p$

$P(A/E_1) = 1$

$P(A/E_2) = \frac{1}{m}$

Required probability $= P(E_1/A)$

$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$

$= \frac{p \times 1}{p \times 1 + (1-p) \times \frac{1}{m}} = \frac{mp}{pm + 1 - p}$

$= \frac{mp}{1 + (m-1)p}$

52. Five sticks of length 1, 3, 5, 7 and 9 feet are given. Three of these sticks are selected at random. What is the probability that the selected sticks can form a triangle?

(a) 0.5 (b) 0.4 (c) 0.3 (d) 0

(**c**) Let S be the event of selecting three sticks.

$\therefore \quad n(S) = {}^5C_3 = 10$

E be the event that selected sticks forms a friangle

$$n(E) = {}^5C_3 - 1 = 3$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{10} = 0.3$$

2017 (I)

53. A committee of two persons is constituted from two men and two women. What is the probability that the committee will have only women?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(**a**) Number of ways of forming a committee of two persons from two men and two women = ${}^4C_2$

and number of ways of forming a committee of two persons from two men and two women in which only women are there = ${}^2C_0 \times {}^2C_2$

$\therefore$ Required probability $= \frac{{}^2C_0 \times {}^2C_2}{{}^4C_2}$

$$= \frac{1 \times 1}{6} = \frac{1}{6}$$

54. A question is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the question will be solved?

(a) $\frac{1}{24}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{23}{24}$

(**c**) We have chances of solving the questions A, B and C are $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{4}$ respectively

$$\therefore \quad P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$$

$$P(A') = \frac{1}{2}, P(B') = \frac{2}{3} P(C') = \frac{3}{4}$$

Required probability

= 1 – probability of problem not solved

$= 1 - P(A') \times P(B') \times P(C')$

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

55. For two dependent events A and B, it is given that $P(A) = 0.2$ and $P(B) = 0.5$. If $A \subseteq B$, then the values of conditional probabilities $P(A/B)$ and $P(B/A)$ are respectively

(a) $\frac{2}{5}, \frac{3}{5}$

(b) $\frac{2}{5}, 1$

(c) $1, \frac{2}{5}$

(d) Information is insufficient

(**b**) We have,

$$P(A) = 0 \cdot 2, P(B) = 0.5$$

$$A \subseteq B$$

$$\therefore \quad A \cap B = A$$

$$\Rightarrow \quad P(A \cap B) = P(A)$$

$$P(A / B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5}$$

$$P(B / A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

56. A card is drawn from a well-shuffled ordinary deck of 52 cards. What is the probability that it is an ace?

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{1}{52}$

(**a**) $\because$ Number of ace in a deck of 52 cards = 4

and total number of cards = 52

$\therefore$ Required probability $= \frac{4}{52} = \frac{1}{13}$

57. Consider the following statements :

1. Two events are mutually exclusive if the occurrence of one event prevents the occurrence of the other.
2. The probability of the union of two mutually exclusive events is the sum of their individual probabilities.

Which of the above statement is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) We know,

two events are mutually exclusive if the occurance of one event prevents the occurance of the other and the probability of the union of two mutually exclusive events is the sum of their individual probabilities.

$\therefore$ Both Statements (1) and (2) are correct.

58. If two fair dice are thrown, then what is the probability that the sum is neither 8 nor 9?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{5}{6}$

(**c**) Total number of outcome when two fair dice are thrown = $6 \times 6 = 36$

Outcome in which sum is either 8 no 9 will be (2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (5, 3), (5, 4), (6, 2), (6, 3).

$\therefore$ Total number of outcome after throwing two dice in which sum is either 8 or 9 = 9

$\therefore$ Probability that the sum is either 8 no 9 = $\frac{9}{36}$

$\therefore$ Probability that the sum is neither 8 nor 9

$$= 1 - \frac{9}{36} = \frac{27}{36} = \frac{3}{4}$$

59. Let A and B are two mutually exclusive events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. What is the value of $P(\overline{A} \cap \overline{B})$?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{5}{12}$

(**d**) Given, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

$\because$ A and B are two mutually exclusive events

$\therefore \quad P(A \cap B) = 0$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{3} + \frac{1}{4} - 0 = \frac{7}{12}$$

Now

$\therefore \; P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

[By De-morgan's theorem]

$$= 1 - P(A \cup B) = 1 - \frac{7}{12} = \frac{5}{12}$$

60. The mean and standard deviation of a binomial distribution are 12 and 2 respectively. What is the number of trials?

(a) 2 (b) 12 (c) 18 (d) 24

(**c**) $\because$ Mean = 12 and standard deviation = 2

then, $np = 12$ and $npq = 4$

$$\Rightarrow \quad 12 \times q = 4 \Rightarrow q = \frac{1}{3}$$

$$\therefore \quad p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

and also, $np = 12$

$$\Rightarrow \quad n \times \frac{2}{3} = 12$$

$$\Rightarrow \quad n = 12 \times \frac{3}{2} = 18$$

2016 (II)

61. A special dice with numbers $1, -1, 2, -2, 0$ and 3 is thrown thrice. What is the probability that the sum of the numbers occurring on the upper face is zero?

(a) 1/ 72 (b) 1/8
(c) 7/72 (d) 25/216

(***d***) Let p, q and r be the numbers on the upper faces in first, second and third thrown respectively.

Now, number of favourable cases such that sum of upper faces is zero equals to number of integral solution of $p + q + r = 0$ subject to condition $-2 \le p, q, r \le 3$, i.e. equal to coefficient of t^0 in $(t^{-2} + t^{-1} + \cdots + t^3)^3$

$= $ coefficient of t^6 in $(1 + t + \cdots + t^5)^3$

$= $ coefficient of t^6 in $\left(\frac{1-t^6}{1-t}\right)^3$

$= $ coefficient of t^6 in $(1-t^6)^3(1-t)^{-3}$

$= $ coefficient of t^6 in $(1 - {}^3C_1 t^6 + \ldots)(1-t)^{-3}$

$= $ coefficient of t^6 in $(1-t)^{-3} - {}^3C_1 \times$ coefficient of t^0 in $(1-t)^{-3}$

$= {}^8C_2 - 3 = 25$

$\therefore$ Required probability $= \frac{25}{216}$

62. There is 25% chance that it rains on any particular day. What is the probability that there is atleast one rainy day within a period of 7 days?

(a) $1 - \left(\frac{1}{4}\right)^7$ (b) $\left(\frac{1}{4}\right)^7$
(c) $\left(\frac{3}{4}\right)^7$ (d) $1 - \left(\frac{3}{4}\right)^7$

(***d***) Given,
$p = P$(chance that it rains) = 25%

$\Rightarrow p = \frac{25}{100} = \frac{1}{4} \Rightarrow q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

P (atleast one rainy day within a period of 7 days) $= 1 - {}^7C_0 p^0 q^7 = 1 - \left(\frac{3}{4}\right)^7$

63. A salesman has a 70% chance to sell a product to any customer. The behaviour of successive customers is independent. If two customers A and B enter, what is the probability that the salesman will sell the product to customer A or B?

(a) 0.98 (b) 0.91
(c) 0.70 (d) 0.49

(***b***) Probability of salesman making a sale $= \frac{70}{100} = 0.70$

Let A and B be the events that salesman sell the product to A and B respectively.

$\therefore P(A) = 0.7$ and $P(B) = 0.7$

Since, A and B are independent.

$\therefore$
$P(A \cap B) = P(A) \cdot P(B) = 0.7 \times 0.7 = 0.49$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.7 + 0.7 - 0.49 = 0.91$

64. A student appears for tests I, II and III. The student is considered successful if he passes in tests I, II or I, III or all the three. The probabilities of the student passing in tests I, II and III are m, n and $1/2$ respectively. If the probability of the student to be successful is $1/2$, then which one of the following is correct?

(a) $m(1 + n) = 1$ (b) $n(1 + m) = 1$
(c) $m = 1$ (d) $mn = 1$

(***a***) Let the events be A = Passes in test I
B = Passes in test II
C = Passes in test III

Given, $P(A) = m, P(\bar{A}) = 1 - m$
$P(B) = n, P(\bar{B}) = 1 - n$
$P(C) = \frac{1}{2}, P(\bar{C}) = \frac{1}{2}$

Student is considered successful if he passes in test I, II or I, III or all the three.

$\therefore$ Required probability
$= P(AB\bar{C}) + P(A\bar{B}C) + P(ABC)$
$= P(A) \times P(B) \times P(\bar{C}) + P(A) \times P(\bar{B}) \times P(C) + P(A) \times P(B) \times P(C)$
$= \frac{mn}{2} + \frac{m(1-n)}{2} + \frac{mn}{2}$
$= \frac{mn + m - mn + mn}{2} = \frac{m(n+1)}{2}$

$\Rightarrow \frac{1}{2} = \frac{m(n+1)}{2}$

$\Rightarrow m(1 + n) = 1$

65. Three candidates solve a question. Odds in favour of the correct answer are 5 : 2, 4: 3 and 3 : 4 respectively for the three candidates. What is the probability that atleast two of them solve the question correctly?

(a) 209/ 343 (b) 134/343
(c) 149/343 (d) 60/343

(***a***) Let the students be E_1, E_2 and E_3 and $P(E_1)$, $P(E_2)$ and $P(E_3)$ denotes the probabilities of the students to solve question correctly.

$\because$ Odds in favour of the correct answer are 5 : 2, 4 : 3 and 3: 4, respectively.

$\therefore \quad P(E_1) = \frac{5}{5+2} = \frac{5}{7}$

$P(E_2) = \frac{4}{4+3} = \frac{4}{7}$

$P(E_3) = \frac{3}{3+4} = \frac{3}{7}$

$\therefore$ Probability that atleast two students solve question correctly
$= P(E_1E_2\bar{E}_3) + P(\bar{E}_1E_2E_3) + P(E_1\bar{E}_2E_3) + P(E_1E_2E_3)$
$= P(E_1) \cdot P(E_2) \cdot P(\bar{E}_3) + P(\bar{E}_1)\, P(E_2)\, P(E_3) + P(E_1)P(\bar{E}_2)P(E_3) + P(E_1)\, P(E_2)\, P(E_3)$
[$\because E_1, E_2$ and E_3 are independent to each other]

$= \frac{5}{7} \cdot \frac{4}{7}\left(1 - \frac{3}{7}\right) + \left(1 - \frac{5}{7}\right) \cdot \frac{4}{7} \cdot \frac{3}{7} + \frac{5}{7}\left(1 - \frac{4}{7}\right)\frac{3}{7} + \frac{5}{7} \cdot \frac{4}{7} \cdot \frac{3}{7}$

$= \frac{5}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} + \frac{2}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} + \frac{5}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} + \frac{5}{7} \cdot \frac{4}{7} \cdot \frac{3}{7}$

$= \frac{1}{7 \times 7 \times 7}(80 + 24 + 45 + 60)$

$= \frac{209}{343}$

66. A medicine is known to be 75% effective to cure a patient. If the medicine is given to 5 patients, what is the probability that atleast one patient is cured by this medicine?

(a) $\frac{1}{1024}$ (b) $\frac{243}{1024}$
(c) $\frac{1023}{1024}$ (d) $\frac{781}{1024}$

(***c***) Let p denotes the probability that medicine to be 75% effective to cure a patient, then

$p = 75\% = \frac{75}{100} = \frac{3}{4}$

$q = 1 - p = 1 - \frac{3}{4} = \frac{1}{4}$

Let X denotes the probability that at least one patient is cured by the medicine

$= 1 - {}^5C_0 (p)^0 (q)^5$

$= 1 - {}^5C_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^5$

$= 1 - \left(\frac{1}{4}\right)^5 = 1 - \frac{1}{1024}$

$= \frac{1023}{1024}$

67. For two events A and B, it is given that $P(A)=\frac{3}{5}, P(B)=\frac{3}{10}$ and $P(A/B)=\frac{2}{3}$. If $\overline{A}$ and $\overline{B}$ are the complementary events of A and B, then what is $P(\overline{A}/\overline{B})$ equal to?

(a) $\frac{3}{7}$ (b) $\frac{3}{4}$ (c) $\frac{1}{3}$ (d) $\frac{4}{7}$

(**a**) Given, $P(A)=\frac{3}{5}, P(B)=\frac{3}{10}$

and $P\left(\frac{A}{B}\right)=\frac{2}{3}$

$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{2}{3} \Rightarrow \frac{P(A\cap B)}{3/10}=\frac{2}{3}$

$\Rightarrow P(A\cap B)=\frac{2}{3}\times\frac{3}{10}=\frac{1}{5}$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{3}{5}+\frac{3}{10}-\frac{1}{5}$

$=\frac{6+3-2}{10}=\frac{7}{10}$

$\Rightarrow P(\overline{A\cup B})=1-P(A\cup B)=1-\frac{7}{10}=\frac{3}{10}$

and $P(\overline{B})=1-P(B)=1-\frac{3}{10}=\frac{7}{10}$

$\therefore P(\overline{A}/\overline{B})=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}=\frac{P(\overline{A\cup B})}{P(\overline{B})}$

$=\frac{3/10}{7/10}=\frac{3}{7}$

68. A machine had three parts, A, B and C, whose chances of being defective are 0.02, 0.10 and 0.05 respectively. The machine stops working if any one of the parts becomes defective . What is the probability that the machine will not stop working?

(a) 0.06 (b) 0.16 (c) 0.84 (d) 0.94

(**c**) Given, $P(A)=0.02=\frac{2}{100}=\frac{1}{50}$

$P(B)=0.10=\frac{10}{100}=\frac{1}{10}$

and $P(C)=0.05=\frac{5}{100}=\frac{1}{20}$

Probability of the machine will stop, if atleast one of its three components fails

$=P(A\cup B\cup C)=1-P(\overline{A\cup B\cup C})$

$=1-P(\overline{A}\cap\overline{B}\cap\overline{C})$

$=1-P(\overline{A})P(\overline{B})P(\overline{C})$

$=1-\left(1-\frac{1}{50}\right)\left(1-\frac{1}{10}\right)\left(1-\frac{1}{20}\right)$

$=1-\left(\frac{49}{50}\right)\left(\frac{9}{10}\right)\left(\frac{19}{20}\right)=1-\frac{8379}{10000}$

$=1-0.8379=0.1621$

Now, required probability of that the machine will not stop working

$=1-0.1621=0.8379=0.84$

69. Three independent events, A_1, A_2 and A_3 occur with probabilities $P(A_i)=\frac{1}{1+i}, i=1,2,3$. What is the probability that atleast one of the three events occurs?

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{24}$

(**c**) Given, A_1, A_2 and A_3 are independent events such that $P(A_i)=\frac{1}{1+i}$, where $i=1,2,3$

The probability that atleast one of three events occurs $=P(A_1\cup A_2\cup A_3)$

$=1-P(\overline{A}_1)P(\overline{A}_2)P(\overline{A}_3)$

$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$

$=1-\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)=1-\frac{1}{4}=\frac{3}{4}$

70. A coin is tossed three times. Consider the following events.

A : No head appears.
B : Exactly one head appears.
C : Atleast two heads appear.

Which one of the following is correct?

(a) $(A\cup B)\cap(A\cup C)=B\cup C$
(b) $(A\cap B')\cup(A\cap C')=B'\cup C'$
(c) $A\cap(B'\cup C')=A\cup B\cup C$
(d) $A\cap(B'\cup C')=B'\cap C'$

(**d**) If a coin is tossed three times, then

$S=\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

A = No head appears $=\{TTT\}$

B = Exactly one head appears $=\{TTH, THT, HTT\}$

C = At least two head appear $=\{HHH, HHT, HTH, THH\}$

Clearly, $B\cap C=\phi$

Now, $A\cap(B'\cup C')=A\cap(B\cap C)'$

$=A\cap(\phi)'$ [$\because B\cap C=\phi$]

$=A\cap S=A$

and $B'\cap C'=(B\cup C)'=A$

Hence, $A\cap(B'\cup C')=B'\cap C'$

71. In a series of 3 one-day cricket matches between teams A and B of a college, the probability of team A winning or drawing are 1 / 3 and 1 / 6 respectively. If a win, loss or draw gives 2, 0 and 1 point respectively, then what is the probability that team A will score 5 points in the series?

(a) $\frac{17}{18}$ (b) $\frac{11}{12}$ (c) $\frac{1}{12}$ (d) $\frac{1}{18}$

(**d**) We have, $P(\text{winning})=\frac{1}{3}$

$\Rightarrow P(\text{drawing})=\frac{1}{6}$

$P(\text{loss})=1-\left(\frac{1}{3}+\frac{1}{6}\right)=\frac{1}{2}$

Point of win, loss or draw are 2, 0, 1 respectively, 5 points are possible, in three cases (2, 2, 1) (2, 1, 2) (1, 2, 2)

$\therefore$ Required probability

$=P(WWD)+P(WDW)+P(DWW)$

$=3\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$

72. Let the random variable X follow $B(6, p)$. If $16P(X=4)=P(X=2)$, then what is the value of p?

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

(**c**) Given, binomial distribution $B(6, p)$.

Here, $n=6$

According to the question,

$16P(X=4)=P(X=2)$

$\Rightarrow 16\,{}^6C_4p^4q^2={}^6C_2p^2q^4$

$\Rightarrow 16\,{}^6C_2p^4q^2={}^6C_2p^2q^4$

$\Rightarrow 16p^2=q^2$

$\Rightarrow 16p^2=(1-p)^2$

$\Rightarrow 16p^2=1+p^2-2p$

$\Rightarrow 15p^2+2p-1=0$

$\Rightarrow 15p^2+5p-3p-1=0$

$\Rightarrow 5p(3p+1)-1(3p+1)=0$

$\Rightarrow (3p+1)(5p-1)=0$

$\Rightarrow 3p+1=0$ or $5p-1=0$

$\Rightarrow p=-\frac{1}{3}$ or $p=\frac{1}{5}$

$\therefore p=\frac{1}{5}$, since $p=-\frac{1}{3}$ is not possible.

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73. A fair coin is tossed 100 times. What is the probability of getting tails an odd number of times?

(a) 1 / 2 (b) 3 / 8
(c) 1 / 4 (d) 1 / 8

(**a**) Clearly, total number of outcomes $=2^{100}$ and number of favourable outcomes

$={}^{100}C_1+{}^{100}C_3+\ldots+{}^{100}C_{99}=2^{100-1}$ [$\because {}^nC_1+{}^nC_3+{}^nC_5+\ldots=2^{n-1}$]

$=2^{99}$

Hence, the required probability

$=\frac{2^{99}}{2^{100}}=\frac{1}{2}$

74. Three dice are thrown simultaneously. What is the probability that the sum on the three faces is atleast 5?

(a) $\frac{17}{18}$ (b) $\frac{53}{54}$ (c) $\frac{103}{108}$ (d) $\frac{215}{216}$

(**b**) Total outcomes = $6 \times 6 \times 6 = 216$

Now, required probability

$= 1 - P$ (sum = 3 and 4)

$= 1 - \left[\frac{1}{216} + \frac{3}{216}\right]$

[$\because$ sum 3 = {1, 1, 1} and sum 4 = {(1, 1, 2),(1, 2, 1), (2, 1, 1)}]

$= 1 - \frac{4}{216} = \frac{216 - 4}{216} = \frac{212}{216} = \frac{53}{54}$

75. Two independent events A and B have $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$. What is the probability that exactly one of the two events A or B occurs?

(a) 1 / 4 (b) 5 / 6
(c) 5 / 12 (d) 7 / 12

(**d**) Given, $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$

Now, P (exactly one)

$= P(A \cup B) - P(A \cap B)$

$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$

[$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$]

$= P(A) + P(B) - 2(P(A \cap B))$

$= P(A) + P(B) - 2P(A) \cdot P(B)$

[$\because$ event, A and B are independent]

$= \frac{1}{3} + \frac{3}{4} - 2 \cdot \frac{1}{3} \cdot \frac{3}{4}$

$= \frac{1}{3} + \frac{3}{4} - \frac{1}{2}$

$= \frac{4 + 9 - 6}{12} = \frac{7}{12}$

76. A coin is tossed three times. What is the probability of getting head and tail alternately?

(a) 1 / 8 (b) 1 / 4 (c) 1 / 2 (d) 3 / 4

(**b**) Given, a coin is tossed three times.

Now, required probability

$= P(HTH) + P(THT)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$\left[\because P(H) = P(T) = \frac{1}{2}\right]$

$= \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

77. A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is queen of spade?

(a) $\frac{1}{52}$ (b) $\frac{1}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

(**a**) A card can be chosen from a pack of 52 cards in ${}^{52}C_1$ ways = 52 ways.

Queen of spade can be chosen from pack in only 1 way.

So, probability of choosing queen of spade = $\frac{1}{52}$.

78. If two dice are thrown, then what is the probability that the sum on the two faces is greater than or equal to 4?

(a) $\frac{13}{18}$ (b) $\frac{5}{6}$ (c) $\frac{11}{12}$ (d) $\frac{35}{36}$

(**c**) For 2 dice, possible outcomes are,

(1, 1), (1, 2), ..., (6, 6)

A total of 36 possible outcomes out of these 36 outcomes, only 3 outcomes (1, 1), (2, 1) and (1, 2) are non-favourable and remaining 33 are favourable (sum $\geq$ 4).

So, probability of getting a sum $\geq$ 4

$= \frac{33}{36} = \frac{11}{12}$

79. A certain type of missile hits the target with probability $p = 0.3$. What is the least number of missiles should be fired, so that there is atleast an 80% probability that the target is hit?

(a) 5 (b) 6
(c) 7 (d) None of these

(**a**) Let n missiles be fired and r of them hits the target.

$\therefore \quad P(X = r) = {}^nC_r p^r q^{n-r}$

$= {}^nC_r (0.3)^r (0.7)^{n-r}$

Target is hit when atleast 1 missile strikes the target.

$\therefore \quad P(X \geq 1) = 1 - P(X = 0)$

This must be greater than 80%.

So, $1 - {}^nC_0 (0.3)^0 (0.7)^{n-0} \geq \frac{80}{100}$

$\Rightarrow \quad 1 - \left(\frac{7}{10}\right)^n \geq \frac{80}{100}$

$\Rightarrow \quad \left(\frac{7}{10}\right)^n \leq \frac{20}{100} \Rightarrow n \geq 5$

80. For two mutually exclusive events A and B, $P(A) = 0.2$ and $P(\overline{A} \cap B) = 0.3$. What is $P(A / (A \cup B))$ equal to?

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$
(c) $\frac{2}{7}$ (d) $\frac{2}{3}$

(**b**) As, $\overline{A} \cap B = B - A \cap B$

So, in given case, $P(\overline{A} \cap B) = P(B) = 0.3$

[$\because A$ and B are mutually exclusive, so $A \cap B = \phi$ $P(A \cap B) = 0$]

and $P(A / A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)}$

$= \frac{P(A)}{P(A) + P(B)} = \frac{0.2}{0.2 + 0.3} = \frac{2}{5}$

81. What is the probability of 5 Sunday in the month of December?

(a) 1 / 7 (b) 2 / 7
(c) 3 / 7 (d) None of these

(**c**) Number of days in December = 31

$\therefore$ Number of complete weeks = 4 (i.e. $7 \times 4 = 28$ days)

Remaining 3 days can be

(M, T, W), (T, W, Th), (W, Th, F), (Th, F, Sa), (F, Sa, S), (Sa, S, M), (S, M, T)

Out of these 7, 3 are favourable outcomes.

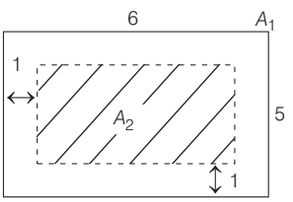
So, probability of having 5 Sunday in the month of December is $\frac{3}{7}$.

82. A point is chosen at random inside a rectangle measuring 6 inches by 5 inches. What is the probability that the randomly selected point is atleast one inch from the edge of the rectangle?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

(**d**) Let A_2 = Area in which ar andomly selected point lies

$= (6 - 2) \times (5 - 2) = 4 \times 3 = 12$ sq inch



A_1 = Total area = $6 \times 5 = 30$ sq inch

$\therefore$ Required probability = $\frac{A_2}{A_1} = \frac{12}{30} = \frac{2}{5}$

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83. Three digits are chosen at random from 1, 2, 3, 4, 5, 6, 7, 8 and 9 without repeating any digit. What is the probability that the product is odd?

(a) $\frac{2}{3}$ (b) $\frac{7}{48}$
(c) $\frac{5}{42}$ (d) $\frac{5}{108}$

(**c**) $n(S) = {}^9C_3$

Let favourable event $= E$

$\therefore \quad n(E) = {}^5C_3$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^5C_3}{{}^9C_3} = \frac{5}{42}$$

84. Two events A and B are such that P (not B) $= 0.8$, $P(A \cup B) = 0.5$ and $P(A / B) = 0.4$. Then, $P(A)$ is equal to

(a) 0.28 (b) 0.32
(c) 0.38 (d) None of these

(**c**) Given, $P(\bar{B}) = 0.8$

$\Rightarrow P(B) = 1 - P(\bar{B}) = 1 - 0.8 = 0.2$

$P(A \cup B) = 0.5, P(A / B) = 0.4$

$$\therefore \quad P(A / B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad 0.4 = \frac{P(A \cap B)}{0.2} \Rightarrow 0.08 = P(A \cap B)$$

Hence,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 0.5 = P(A) + 0.2 - 0.08$

$\Rightarrow \quad 0.5 = P(A) + 0.12$

$\Rightarrow \quad P(A) = 0.38$

85. If mean and variance of a Binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than 1, is

(a) $\frac{2}{3}$ (b) $\frac{4}{5}$ (c) $\frac{7}{8}$ (d) $\frac{11}{16}$

(**d**) Given, $np = 2$ and $npq = 1$

$$\Rightarrow \quad 2 \times q = 1 \Rightarrow q = \frac{1}{2}$$

$$\Rightarrow \quad p + q = 1 \Rightarrow p = \frac{1}{2}$$

$$\Rightarrow \quad n \times \frac{1}{2} \times \frac{1}{2} = 1 \Rightarrow n = 4$$

$$P(X > 1) = {}^4C_2 p^2 q^2 + {}^4C_3 p^3 q^1 + {}^4C_4 p^4 q^0$$

$$= 6\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + 1 \times \left(\frac{1}{2}\right)^4$$

$$= \left(\frac{1}{2}\right)^4 [6 + 4 + 1] = \frac{11}{16}$$

86. Seven unbiased coins are tossed 128 times. In how many throws would you find atleast three heads?

(a) 99 (b) 102 (c) 103 (d) 104

(**a**) Given, $p = q = \frac{1}{2}, n = 7, N = 128$

and $r = 3, 4, 5, 6, 7$

$$P(X \geq 3) = 128 [{}^7C_3 p^3 q^4 + {}^7C_4 p^4 q^3 + {}^7C_5 p^5 q^2 + {}^7C_6 p^6 q^1 + {}^7C_7 p^7 q^0]$$

$$= 128\left[35\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^4 + 35\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + 21\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^7\right]$$

$$= 128\left\{\left(\frac{1}{2}\right)^7 [35 + 35 + 21 + 7 + 1]\right\}$$

$$= \left[\frac{1}{128} \times 99\right] \times 128 = 99$$

87. A coin is tossed five times. What is the probability that heads are observed more than three times ?

(a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{1}{2}$ (d) $\frac{3}{32}$

(**b**) Given, $p = P$ (getting head) $= \frac{1}{2}$

$q = P$ (getting on head) $= \frac{1}{2}$

$n = 4, r = 4$ and $N = 5$

$$P(X > 3) = 5 [{}^4C_4 p^4 q^0] = 5\left[1 \times \left(\frac{1}{2}\right)^4\right]$$

$$= \frac{5}{16}$$

88. An unbiased coin is tossed until the first head appears or until four tosses are completed, whichever happens earlier. Which of the following statement(s) is/are correct ?

1. The probability that no head is observed is $\frac{1}{16}$.
2. The probability that the experiment ends with three tosses is $\frac{1}{8}$.

Select the correct answer using the code given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) Probability of no head is

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

Probability of the experiments end with three tosses if *TTH* comes

$\therefore$ Required probability is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Hence, both statements are correct.

89. If $x \in [0,5]$, then what is the probability that $x^2 - 3x + 2 \geq 0$?

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$
(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

(**d**) Let x be natural number.

Total number of solutions $= n(S) = 5$

Favourable event is 3, 4, 5.

$\therefore \quad n(E) = 3$

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{5}$

90. A bag contains 4 white and 2 black balls and another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, then the probability that one ball is white and one ball is black, is

(a) $\frac{5}{24}$ (b) $\frac{13}{24}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

(**b**) P (one ball is white and one ball is black).

$= P$(black ball from bag 1 and white ball from bag 2 or white ball from bag 1 and black ball from bag 2)

$$= \frac{2}{6} \times \frac{3}{8} + \frac{4}{6} \times \frac{5}{8} = \frac{26}{48} = \frac{13}{24}$$

91. A problem in Statistics is given to three students A, B and C whose chances of solving it independently are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$, respectively. The probability that the problem will be solved, is

(a) $\frac{1}{12}$ (b) $\frac{11}{12}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

(**d**) P(Problem will be solved) $= 1 - P$ (Problem will not solved by A, B and C)

$$= 1 - \left\{\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\right\}$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

92. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident involving a scooter driver, car driver and a truck driver are 0.01, 0.03 and 0.15, respectively. One of the insured persons meets with an accident. The probability that the person is a scooter driver, is

(a) $\frac{1}{52}$ (b) $\frac{3}{52}$ (c) $\frac{15}{52}$ (d) $\frac{19}{52}$

(**a**) Let $P(A) = P$(scooter) $= \frac{2000}{12000} = \frac{1}{6}$

$$P(B) = P(\text{car}) = \frac{4000}{12000} = \frac{1}{3}$$

$$P(C) = P(\text{Truck}) = \frac{6000}{12000} = \frac{1}{2}$$

Let E = Event that person meets with accident.

We have,

$$P\left(\frac{E}{A}\right)=\frac{1}{100}, P\left(\frac{E}{B}\right)=\frac{3}{100}, P\left(\frac{E}{C}\right)=\frac{15}{100}$$

Now,

$$P\left(\frac{A}{E}\right)=\frac{P(A)\cdot P\left(\frac{E}{A}\right)}{P(A)\cdot P\left(\frac{E}{A}\right)+P(B)\cdot P\left(\frac{E}{B}\right)+P(C)\cdot P\left(\frac{E}{C}\right)}$$

$$=\frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{100}+\frac{1}{2}\times\frac{15}{100}}$$

$$=\frac{\frac{1}{6}}{\frac{1}{6}+1+\frac{15}{2}}=\frac{\frac{1}{6}}{\frac{1+6+45}{6}}=\frac{1}{52}$$

93. A coin is tossed 5 times. The probability that tail appears an odd number of times, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

(**a**) We have,

p = Probability of tail appear = $\frac{1}{2}$

q = Probability of tail not appear

$=1-p=\frac{1}{2}$

Now, P (Tail appear odd number of times)

$=P(X=1)+P(X=3)+P(X=5)$

$[\because P(X=r)]$

$={}^{n}C_{r}p^{r}q^{n-r}$

$={}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{4}+{}^{5}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{2}+{}^{5}C_{5}\left(\frac{1}{2}\right)^{5}$

$=\frac{5}{32}+\frac{10}{32}+\frac{1}{32}=\frac{16}{32}=\frac{1}{2}$

94. What is the probability that the sum of any two different single digit natural numbers is a prime number ?

(a) $\frac{5}{27}$ (b) $\frac{7}{18}$

(c) $\frac{1}{3}$ (d) None of these

(**b**) We have single digit natural number $=\{1, 2, 3 \ldots 9\}$

$n(S)={}^{9}C_{2}=36$

The sum of two different numbers which are prime 3, 5, 7, 11, 13, 17, 19

$n(E)$ = (1, 2) (1, 4) (1, 6) (2, 3) (2, 5) (2, 9), (3, 4)(3, 8) (4, 7)(5, 6) (4, 9) (5, 8) (6, 7) (8, 9) = 14

Hence, $P(E)=\frac{14}{36}\Rightarrow P(E)=\frac{7}{18}$

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95. Two men hit at a target with probabilities $\frac{1}{2}$ and $\frac{1}{3}$, respectively. What is the probability that exactly one of them hits the target?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

(**a**) Let the men's be Mr. A and Mr. B.

Let A be the event that Mr. A hit the target and B be the event that Mr. B hit the target.

$\therefore P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$

Now, P (exactly one of them hits the target)

$=P(A\cap\bar{B}$ or $\bar{A}\cap B)$

$=P(A\cap\bar{B})+P(\bar{A}\cap B)$

$=P(A)\cdot P(\bar{B})+P(\bar{A})\cdot P(B)$

$=\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{3}=\frac{3}{6}=\frac{1}{2}$

96. Two similar boxes $B_i(i=1,2)$ contain $(i+1)$ red and $(5-i-1)$ black balls. One box is chosen at random and two balls are drawn randomly. What is the probability that both the balls are of different colours?

(a) $\frac{1}{2}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

(**d**) We have, box B_1 containing 2 red and 3 black balls and box B_2 containing 3 red and 2 black balls.

Let E_1 be the event that box B_1 is chosen, E_2 be the event that box B_2 is chosen and A be the event that balls are of different colours.

Clearly, $P(E_1)=P(E_2)=\frac{1}{2}$

$$P\left(\frac{A}{E_1}\right)=\frac{{}^{2}C_{1}\times{}^{3}C_{1}}{{}^{5}C_{2}}=\frac{6}{10}=\frac{3}{5}$$

and $P\left(\frac{A}{E_2}\right)=\frac{{}^{3}C_{1}\times{}^{2}C_{1}}{{}^{5}C_{2}}=\frac{6}{10}=\frac{3}{5}$

Now, by the theorem of total probability, we have

$$P(A)=P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)$$

$$=\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{3}{5}=\frac{1}{2}\left(\frac{2\times3}{5}\right)=\frac{3}{5}$$

97. In an examination, the probability of a candidate solving a question is $\frac{1}{2}$. Out of given 5 questions in the examination, what is the probability that the candidate was able to solve atleast 2 questions?

(a) $\frac{1}{64}$ (b) $\frac{3}{16}$ (c) $\frac{1}{2}$ (d) $\frac{13}{16}$

(**d**) Let solving a question is success and not solving a question is failure.

Let probability of success = $p=\frac{1}{2}$

and probability of failure = $q=\frac{1}{2}$

Let X be random variable that denotes the number of success in 5 trials.

Clearly, $X\sim B$ in $\left(5,\frac{1}{2}\right)$

$\therefore\quad P(X=x)={}^{5}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{5-x}$

$\Rightarrow\quad x=0,1,2,3,4,5={}^{5}C_{x}\left(\frac{1}{2}\right)^{5}$

Required probability = $P(X\geq2)4$

$=1-[P(x=0)+P(x=1)]$

$=1-\left({}^{5}C_{0}\left(\frac{1}{2}\right)^{5}+{}^{5}C_{1}\left(\frac{1}{2}\right)^{5}\right)$

$=1-\frac{6}{32}=\frac{26}{32}=\frac{13}{16}$

98. If $A\subseteq B$, then which one of the following is not correct?

(a) $P(A\cap\bar{B})=0$

(b) $P(A/B)=\frac{P(A)}{P(B)}$

(c) $P(B/A)=\frac{P(B)}{P(A)}$

(d) $P(A/(A\cup B))=\frac{P(A)}{P(B)}$

(**c**) As, $A\subseteq B$, then $A\cup B=B$ and $A\cap B=A$.

Clearly, $P(A\cap\bar{B})=P(\phi)=0$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}$

$\Rightarrow P\left(\frac{A}{(A\cup B)}\right)=P\left(\frac{A}{B}\right)=\frac{P(A)}{P(B)}$

but $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{P(A)}{P(A)}=1$

Hence, option (c) is not correct.

99. The mean and the variance in a binomial distribution are found to be 2 and 1 respectively. The probability $P(X=0)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(**d**) Let X be a random variable that follows binomial distribution with parameter n and p.

Then, Mean $= E(x) = np = 2$ ...(i)

and variance $= \text{var}(x) = npq = 1$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$q = \frac{1}{2} \Rightarrow p = \frac{1}{2} \quad [\because p + q = 1]$$

Now, on substituting the value of p in Eq. (i), we get $n = 4$

Thus, $X \sim B$ in $\left(\frac{1}{2}, 4\right)$ and

$$P(X = x) = {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x},$$

$x = 0, 1, 2, 3, 4$

$$= P(x = 0) = {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0}$$

$$= {}^4C_0 \left(\frac{1}{2}\right)^4$$

$$= 1 \times \frac{1}{16} = \frac{1}{16}$$

100. If A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$, then what is $P(B)$ equal to?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{8}$ (d) $\frac{2}{9}$

(**b**) We have, $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad \frac{3}{4} = (1 - P(\bar{A})) + P(B) - \frac{1}{4}$$

$$\Rightarrow \quad \frac{3}{4} + \frac{1}{4} = 1 - P(\bar{A}) + P(B)$$

$$\Rightarrow \quad 1 = 1 - P(\bar{A}) + P(B)$$

$$\Rightarrow \quad P(B) = P(\bar{A}) = \frac{2}{3}$$

101. In throwing of two dice, the number of exhaustive events that '5' will never appear on any one of the dice is

(a) 5 (b) 18 (c) 25 (d) 36

(**c**) Clearly, the possible outcomes in throwing of two dice are

$$\begin{Bmatrix} (1,1), & (1,2), & (1,3), & - & - & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & - & - & (2,5), & (2,6) \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ (5,1), & (5,2), & (5,3), & - & - & (5,5), & (5,6) \\ (6,1), & (6,2) & (6,3), & - & - & (6,5), & (6,6) \end{Bmatrix}$$

Since, 5 will never appear on any one of the dice, so number of exhaustive events are

$= 36 - 6 - 6 + 1 = 36 - 11 = 25$

102. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. The probability of drawing two aces is

(a) $\frac{1}{26}$ (b) $\frac{1}{221}$

(c) $\frac{4}{223}$ (d) $\frac{1}{13}$

(**b**) Total number of ways drawing 2 cards successively without replacement $= {}^{52}C_2$

Number of aces = 4

and number of ways 2 aces without replacement $= {}^4C_2$

$\therefore$ Required probability

$$= \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51}$$

$$= \frac{1}{13 \times 17} = \frac{1}{221}$$

2014 (II)

103. Suppose A and B are two events. Event B has occurred and it is known that $P(B) < 1$. What is $P(A / B')$ equal to?

(a) $\frac{P(A) - P(B)}{1 - P(B)}$

(b) $\frac{P(A) - P(AB)}{1 - P(B)}$

(c) $\frac{P(A) + P(B')}{1 - P(B)}$

(d) None of the above

(**a**) $P(A / B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(B)}{1 - P(B)}$

Directions (Q. Nos. 104-107) *Read the following information carefully and answer the questions given below.*

Consider events A, B, C, D and E of the sample space

$S = \{n : n$ is an integer such that $10 \le n \le 20\}$ (given)

A is the set of all even numbers.

B is the set of all prime numbers.

$C = 15$

D is the set of all integers ≤ 16.

E is the set of all double digit numbers expressible as a power of 2.

$S = \{n : n$ is an integer such that $10 \le n \le 20\}$

$= \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{10, 12, 14, 16, 18, 20\}$

$B = \{11, 13, 17, 19\}$

$C = \{15\}$

$D = \{10, 11, 12, 13, 14, 15, 16\}$

$E = \{16\}$

104. A, B and D are

(a) mutually exclusive events but not exhaustive events

(b) exhaustive events but not mutually exclusive events

(c) mutually exclusive and exhaustive events

(d) elementary events

(**b**) A, B and D are exhaustive events but not mutually exclusive events.

105. A, B and C are

(a) mutually exclusive events but not exhaustive events

(b) exhaustive events but not mutually exclusive events

(c) mutually exclusive and exhaustive events

(d) elementary events

(**b**) A, B and C are exhaustive events but not mutually exclusive events.

106. B and C are

(a) mutually exclusive events but not exhaustive events

(b) compound events

(c) mutually exclusive and exhaustive events

(d) elementary events

(**a**) B and C are mutually exclusive events but not exhaustive events.

107. C and E are

(a) mutually exclusive events but not elementary events

(b) exhaustive events but not mutually exclusive events

(c) mutually exclusive and exhaustive events

(d) elementary and mutually exclusive events

(**d**) C and E are mutually exclusive and elementary events.

108. For any two events A and B, which one of the following holds?

(a) $P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$
(b) $P(A \cup B) \le P(A) \le P(A \cap B) \le P(A) + P(B)$
(c) $P(A \cup B) \le P(B) \le P(A \cap B) \le P(A) + P(B)$
(d) $P(A \cap B) \le P(B) \le P(A) + P(B) \le P(A \cup B)$

(**a**) Clearly, $A \cap B \subseteq A$

$\Rightarrow \quad P(A \cap B) \le P(A)$ ...(i)

$A \subseteq A \cup B \Rightarrow P(A) \le P(A \cup B)$ ...(ii)

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cup B) \le P(A) + P(B)$ ...(iii)

From Eqs. (i), (ii) and (iii),

$P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$

109. The probability that in a random arrangement of the letters of the word 'UNIVERSITY', the two I's do not come together is

(a) 4/5 (b) 1/5 (c) 1/10 (d) 9/10

(**a**) $\because$ 1U, 1N, 2I, 1V, 1E, 1R, 1S, 1T, 1Y

$\therefore$ Total number of possible arrangements $= \frac{10!}{2!}$

and favourable arrangements $= \frac{10!}{2!} - 9!$

$\therefore$ Required probability $= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}}$

$= \frac{9!\,(5-1)}{9! \times 10} \times 2 = \frac{4}{5}$

110. There are 4 white and 3 black balls in a box. In another box, there are 3 white and 4 black balls. An unbiased die is rolled. If it shows a number less than or equal to 3, then a ball is drawn from the second box, otherwise from the first box. If the ball drawn is black, then the probability that the ball was drawn from the first box, is

(a) 1/2 (b) 6/7
(c) 4/7 (d) 3/7

(**d**) Box I $\rightarrow$ 4 W, 3 B

Box II $\rightarrow$ 3 W, 4 B

Probability for choosing first box $= \frac{1}{2}$

Probability for choosing second box $= \frac{1}{2}$

Probability of getting black ball in 1st box $= \frac{3}{7}$

Probability of getting black ball in 2nd box $= \frac{4}{7}$

$\therefore$ Required probability

$= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{4}{7}}$

$= \frac{\frac{3}{14}}{\frac{3}{14} + \frac{4}{14}} = \frac{3/14}{7/14} = \frac{3}{7}$

111. Two students X and Y appeared in an examination. The probability that X will qualify the examination is 0.05 and Y will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. What is the probability that only one of them will qualify the examination?

(a) 0.15 (b) 0.14
(c) 0.12 (d) 0.11

(**d**) Let A and B be the events that X and Y qualify the examination, respectively.

We have, $P(A) = 0.05$, $P(B) = 0.10$ and $P(A \cap B) = 0.02$

Clearly, P (only one of A and B will qualify the examination)

$= P(A \cap \bar{B}) + P(B \cap \bar{A})$
$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$= P(A) + P(B) - 2P(A \cap B)$
$= 0.05 + 0.1 - 2(0.02)$
$= 0.15 - 0.04 = 0.11$

112. A fair coin is tossed four times. What is the probability that at most three tails occur?

(a) 7/8 (b) 15/16
(c) 13/16 (d) 3/4

(**b**) Let S be the sample space of the experiment and E be the event that at most three tails occur.

Clearly, $n(S) = 2^4 = 16$

and $n(E) = {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$

$= 1 + 4 + \frac{4 \times 3}{2 \times 1} + 4$

$= 1 + 4 + 6 + 4 = 15$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{15}{16}$

2014 (I)

113. A box contains 3 white and 2 black balls. Two balls are drawn at random one after the other. If the balls are not replaced, what is the probability that both the balls are black?

(a) 2/5
(b) 1/5
(c) 1/10
(d) None of the above

(**c**) Total sample space, $n(S) = {}^5C_2$ because we select two balls out of 5 balls.

Now, favourable events, $n(E)$ = Two selected balls are black

$= {}^3C_0 \times {}^2C_2$

$\therefore$ Required probability

$= \frac{n(E)}{n(S)} = \frac{{}^3C_0 \times {}^2C_2}{{}^5C_2}$

$= \frac{1 \times 1}{\frac{(5 \times 4)}{2}} = \frac{1}{10}$

Directions (Q. Nos. 114-117) *Number X is randomly selected from the set of odd numbers and Y is randomly selected from the set of even numbers of the set* {1, 2, 3, 4, 5, 6, 7}. *Let* $Z = (X + Y)$.

114. What is $P(Z = 5)$ equal to?

(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/6

115. What is $P(Z = 10)$ equal to?

(a) 0 (b) 1/2
(c) 1/3 (d) 1/5

116. What is $P(Z > 11)$ equal to?

(a) 0 (b) 1/4
(c) 1/6 (d) 1/12

117. What is $P(Z$ is the product of two prime numbers) equal to?

(a) 0
(b) 1/2
(c) 1/4
(d) None of the above

(Solutions Q. Nos. 114-117)

Given that,

X = Set of odd numbers from the set A.

Y = Set of even numbers from the set A.

Let set $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $Z = X + Y$

114. (***d***) Now, $Z = 5$ is only possible when $X = 1, 3$ and $Y = 4, 2$

Sample space = {(1, 2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6), (7,2), (7,4), (7,6)}

$\therefore$ Total number of sample space, $n(S) = 12$

and favourable space = {(1,4), (3,2)}

$\therefore$ Total number of favourable cases, $n(E_1) = 2$

So, $P(Z = 5) = \dfrac{n(E_1)}{n(S)}$

$= \dfrac{2}{12} = \dfrac{1}{6}$

115. (***a***) We know that,

Sum of even and odd numbers never be an even number.

$\therefore$ Total number of favourable cases, $n(E_2) = 0$.

So, $P(Z = 10) = \dfrac{n(E_3)}{n(S)} = \dfrac{0}{12} = 0$

116. (***d***) Now, $Z > 11$ is only possible when $X = 7$ and $Y = 6$

$\therefore$ Favourable case = (7,6)

Total number of favourable cases, $n(E_3) = 1$

$\therefore \quad P(Z > 11) = \dfrac{n(E_3)}{n(S)} = \dfrac{1}{12}$

117. (***c***) $\therefore$ Z = Product of two prime numbers

$= \{2 \times 2, 3 \times 3, 5 \times 5, 7 \times 7, 2 \times 3, 2 \times 5, 2 \times 7, 3 \times 5, 3 \times 7, 5 \times 7\}$

$= (4, 9, 25, 49, 6, 10, 14, 15, 21, 35)$

$\because Z = X + Y = 7 + 6 = 13$

[*i.e.,* maximum of X is 7 and maximum of Y is 6].

So, maximum value of 13 *i.e., Z* cannot be exceed from 13.

$\therefore \quad Z \neq 25, 49, 14, 15, 21, 35.$

So, $Z = \{4, 9, 6, 10\}$, only the possible values of product of two prime numbers.

Also know that, sum of the odd and even number is always an odd number.

So, $Z \neq 4, 6, 10.$ [$\because Z = X + Y$]

$\therefore$ $Z = 9$ is only remaining case.

Favourable case = {(3, 6), (5, 4), (7, 2)}

$\therefore$ Total number of favourable cases, $n(E_3) = 3$

So, required probability

i.e., P (Z is the product of two prime numbers)

$= P(Z = 9) = \dfrac{n(E_3)}{n(S)} = \dfrac{3}{12} = \dfrac{1}{4}$

118. It has been found that, if A and B play a game 12 times, A wins 6 times, B wins 4 times and they draw twice. A and B take part in a series of 3 games. The probability that they win alternately, is

(a) 5/12 (b) 5/36 (c) 19/27 (d) 5/27

(***b***) $P(A \text{ wins}) = \dfrac{6}{12} = \dfrac{1}{2}$

and $P(B \text{ wins}) = \dfrac{4}{12} = \dfrac{1}{3}$

$\therefore$ Required probability

$= P(A) \cdot P(B) \cdot P(A) + P(B) \cdot P(A) \cdot P(B)$

$= \dfrac{1}{2} \cdot \dfrac{1}{3} \cdot \dfrac{1}{2} + \dfrac{1}{3} \cdot \dfrac{1}{2} \cdot \dfrac{1}{3}$

$= \dfrac{1}{12} + \dfrac{1}{18} = \dfrac{3+2}{36} = \dfrac{5}{36}$

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MISCELLANEOUS

2019 (II)

1. What is the value of
$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\infty}}}?$$
(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) 3 (d) 4

(***b***) Let, $x=2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\infty}}}$

$\Rightarrow \quad x=2+\frac{1}{x} \Rightarrow x^2=2x+1$

$\Rightarrow \quad x^2-2x-1=0$

$\Rightarrow \quad x=\frac{2\pm\sqrt{(-2)^2-4(1)(-1)}}{2}$

$=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm 2\sqrt{2}}{2}=1\pm\sqrt{2}$

$=\sqrt{2}+1 \qquad (\because x>2)$

2. Which one of the following is the second degree polynomial function $f(x)$ where, $f(0)=5$, $f(-1)=10$ and $f(1)=6$?
(a) $5x^2-2x+5$ (b) $3x^2-2x-5$
(c) $3x^2-2x+5$ (d) $3x^2-10x+5$

(***c***) From the option (c),

$f(x)=3x^2-2x+5$

$f(0)=3(0)^2-2(0)+5=5$

$f(-1)=3(-1)^2-2(-1)+5$

$=3+2+5=10$

and $f(1)=3(1)^2-2(1)+5=3-2+5=6$

Hence, the required polynomial

$f(x)=3x^2-2x+5.$

3. A car travels first 60 km at a speed of 3 v km/h and travels next 60 km at 2 v km/h. What is the average speed of the car?
(a) 2.5 v km/h (b) 2.4 v km/h
(c) 2.2 v km/h (d) 2.1 v km/h

(***b***) Time taken for first 60 km with speed 3 v km/h

$=\frac{60}{3v}=\frac{20}{v}$ h $\qquad \left[\because \text{Time}=\frac{\text{Distance}}{\text{Speed}}\right]$

Time taken for next 60 km with speed 2v km/h

$=\frac{60}{2v}=\frac{30}{v}$ h

$\therefore$ Average speed $=\frac{\text{Total distance}}{\text{Total time}}$

$=\frac{60+60}{\frac{20}{v}+\frac{30}{v}}=\frac{120\,v}{50}=2.4\,v$ km/h

4. For the variables x and y, the two regression lines are $6x+y=30$ and $3x+2y=25$. What are the values of $\bar{x}$, $\bar{y}$ and r respectively?
(a) $\frac{20}{3}, \frac{35}{9}, -0.5$ (b) $\frac{20}{3}, \frac{35}{9}, 0.5$
(c) $\frac{35}{9}, \frac{20}{3}, -0.5$ (d) $\frac{35}{9}, \frac{20}{3}, 0.5$

(***c***) Given lines, $6x+y=30$ …(i)

and $3x+2y=25$ …(ii)

where, x and y are two variables.

Solving these equations,

$x=\frac{35}{9}$, and $y=\frac{20}{3}$

These lines are regression,

Then, $\bar{x}=\frac{35}{9}$, $\bar{y}=\frac{20}{3}$

and $r=-\frac{3}{6}$ or $-\frac{1}{2}=-0.5$

2018 (II)

5. Consider the following expressions

1. $x+x^2-\frac{1}{x}$

2. $\sqrt{ax^2+bx+x-c+\frac{d}{x}-\frac{e}{x^2}}$

3. $3x^2-5x+ab$

4. $\frac{2}{x^2-ax+b^3}$ 5. $\frac{1}{x}-\frac{2}{x+5}$

Which of the above are rational expressions?
(a) 1, 4 and 5 (b) 1, 3, 4 and 5
(c) 2, 4 and 5 (d) 1 and 2

(***b***) We know that, rational expressions are those expression which can be write in the form of $\frac{p(x)}{q(x)}, q(x)\neq 0$

So, 1, 3, 4, 5 are rational expressions.

6. Suppose $f(x)$ is such a quadratic expression that, it is positive for all real x.
If $g(x)=f(x)+f'(x)+f''(x)$, then for any real x
(a) $g(x)<0$ (b) $g(x)>0$
(c) $g(x)=0$ (d) $g(x)\geq 0$

(***a***) Given that $f(x)$ is a quadratic expression.

Let $f(x)=ax^2+bx+c$, $a>0$

$\therefore \quad b^2-4ac<0 \qquad [\because f(x)>0]$

$\Rightarrow \quad b^2<4ac$

Now, $f'(x)=2ax+b$ and $f''(x)=2a$

We have, $g(x)=f(x)+f'(x)+f''(x)$

$=ax^2+bx+c+2ax+b+2a$

$=ax^2+(b+2a)x+2a+b+c$

Now, $(b+2a)^2-4a(2a+b+c)$

$=b^2+4ab+4a^2-8a^2-4ab-4ac$

$=b^2-4ac-4a^2<0\,[\because b^2-4ac<0]$

$\Rightarrow \quad g(x)<0$

7. What is the sum of all three digit numbers that can be formed using all the digits 3, 4 and 5, when repetition of digits is not allowed?
(a) 2664 (b) 3382 (c) 4044 (d) 4444

(**a**) Three digit number, that can be formed using 3, 4 and 5 when repetition not allowed are 543, 534, 453, 435, 354, 345

Sum

$= 543 + 534 + 453 + 435 + 354 + 345$

$= 2664$

8. A function $f : A \to R$ is defined by the equation $f(x) = x^2 - 4x + 5$, where $A = (1, 4)$. What is the range of the function?

(a) (2, 5) (b) (1, 5) (c) [1, 5) (d) [1, 5]

(**c**) We have,

A function $f : A \to R$ is defined by

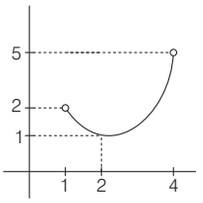
$f(x) = x^2 - 4x + 5$,

Where, $A = (1, 4)$

Let, $y = x^2 - 4x + 5$

$\frac{dy}{dx} = 2x - 4$

Now, $\frac{dy}{dx} = 0 \Rightarrow 2x - 4 = 0 \Rightarrow x = 2$



At $x = 2$, $y = 1$

At $x = 1$, $y = (1)^2 - 4(1) + 5 = 2$

At $x = 4$, $y = (4)^2 - 4(4) + 5 = 5$

So, $y \in [1, 5)$

$\therefore$ Range = [1, 5)

9. If $\sin\beta$ is the harmonic mean of $\sin\alpha$ and $\cos\alpha$ and $\sin\theta$ is the arithmetic mean of $\sin\alpha$ and $\cos\alpha$, then which of the following is/are correct?

1. $\sqrt{2}\sin\left(\alpha + \frac{\pi}{4}\right)\sin\beta = \sin 2\alpha$

2. $\sqrt{2}\sin\theta = \cos\left(\alpha - \frac{\pi}{4}\right)$

Select the correct answer using the code given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) Given that,

$\sin\beta$ is HM of $\sin\alpha$ and $\cos\alpha$

So, $\sin\beta = \frac{2\sin\alpha\cos\alpha}{\sin\alpha + \cos\alpha}$

$\Rightarrow \sin\beta(\sin\alpha + \cos\alpha) = \sin 2\alpha$

$\Rightarrow \sin\beta \times \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{\sqrt{2}}\cos\alpha\right)$

$= \sin 2\alpha$

$\Rightarrow \sin\beta \times \sqrt{2}\left(\cos\frac{\pi}{4}\sin\alpha + \sin\frac{\pi}{4}\cos\alpha\right)$

$= \sin 2\alpha$

$\Rightarrow \sqrt{2}\sin\beta\left[\sin\left(\alpha + \frac{\pi}{4}\right)\right] = \sin 2\alpha$

$\Rightarrow \sqrt{2}\sin\left(\alpha + \frac{\pi}{4}\right)\sin\beta = \sin 2\alpha$

Statement 1 is true.

Now, $\sin\theta$ is AM of $\sin\alpha$ and $\cos\alpha$

So, $\sin\theta = \frac{\sin\alpha + \cos\alpha}{2}$

$\Rightarrow 2\sin\theta = \sin\alpha + \cos\alpha$

$\Rightarrow 2\sin\theta = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{\sqrt{2}}\cos\alpha\right)$

$\Rightarrow 2\sin\theta = \sqrt{2}\left(\sin\frac{\pi}{4}\sin\alpha + \cos\frac{\pi}{4}\cos\alpha\right)$

$\Rightarrow \sqrt{2}\sin\theta = \cos\left(\alpha - \frac{\pi}{4}\right)$

Hence, Statement 2 is also correct.

2018 (I)

10. If $n \in N$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by which one of the following?

(a) 1904 (b) 2000 (c) 2002 (d) 2006

(**b**) We have,

$121^n - 25^n + 1900^n - (-4)^n$

On putting $n = 1$, we get

$(121)^1 - (25)^1 + (1900)^1 - (-4)^1$

$= 121 - 25 + 1900 + 4 = 2000$

Which is divisible by 2000.

11. What is the greatest integer among the following, by which the number $5^5 + 7^5$ is divisible?

(a) 6 (b) 8 (c) 11 (d) 12

(**d**) We know that, when m is odd, then $(x^m + y^m)$ is divisible by $(x + y)$.

$\therefore 5^5 + 7^5$ is divisible by $5 + 7 = 12$ as $m = 5$ is odd.

12. A train covers the first 5 km of its journey at a speed of 30 km/h and the next 15 km at a speed of 45 km/h. What is the average speed of the train?

(a) 35 km/h (b) 37.5 km/h
(c) 39.5 km/h (d) 40 km/h

(**d**) Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{5 + 15}{\frac{5}{30} + \frac{15}{45}} = \frac{20}{\frac{1}{6} + \frac{1}{3}} = \frac{20}{\frac{1 + 2}{6}}$

$= \frac{20 \times 6}{3} = 40$ km/h

2017 (II)

13. If $|a|$ denotes the absolute value of an integer, then which of the following are correct?

1. $|ab| = |a||b|$ 2. $|a + b| \le |a| + |b|$
3. $|a - b| \ge |a| - |b|$

Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(**d**) For absolute value

1. $|ab| = |a||b|$ is true
2. $|a + b| \le |a| + |b|$ is true
3. $|a - b| \ge ||a| - |b||$ is true

$\therefore$ For absolute value all the given conditions are true.

14. Consider the following statements :

1. $\frac{dy}{dx}$ at a point on the curve gives slope of the tangent at that point.
2. If $a(t)$ denotes acceleration of a particle, then $\int a(t)dt + c$ gives velocity of the particle.
3. If $s(t)$ gives displacement of a particle at time t, then $\frac{ds}{dt}$ gives its acceleration at that instant.

Which of the above statements is/are correct?

(a) Only 1 and 2 (b) Only 2
(c) Only 2 (d) 1, 2 and 3

(**a**) If $y = f(x)$, then $\frac{dy}{dx}$ gives the slope of the tangent at that point.

So, Statement 1 is correct.

We know that, $a(t) = \frac{dv}{dt}$

$\Rightarrow v(t) = \int a(t)dt + c$

where, $v(t)$ and $a(t)$ are velocity and acceleration respectively.

So, Statement 2 is correct.

We know that, $v(t) = \frac{ds}{dt}$

where, $s(t)$ is displacement and $v(t)$ is velocity.

So, Statement 3 is incorrect.

15. If x_1 and x_2 are positive quantities, then the condition for the difference between the arithmetic mean and the geometric mean to be greater than 1 is

(a) $x_1 + x_2 > 2\sqrt{x_1 x_2}$

(b) $\sqrt{x_1} + \sqrt{x_2} > \sqrt{2}$

(c) $|\sqrt{x_1} - \sqrt{x_2}| > \sqrt{2}$

(d) $x_1 + x_2 < 2(\sqrt{x_1 x_2} + 1)$

(c) We have, AM – GM > 1

$\Rightarrow \quad x_1 + x_2 - 2\sqrt{x_1 x_2} > 2$

$\Rightarrow \quad (\sqrt{x_1})^2 + (\sqrt{x_2})^2 - 2\sqrt{x_1}\sqrt{x_2} > 2$

$\Rightarrow \quad (\sqrt{x_1} - \sqrt{x_2})^2 > 2 \Rightarrow |\sqrt{x_1} - \sqrt{x_2}| > \sqrt{2}$

16. If $I_1 = \dfrac{d}{dx}(e^{\sin x})$

$$I_2 = \lim_{x \to 0} \frac{e^{\sin(x+h)} - e^{\sin x}}{h}$$

$$I_3 = \int e^{\sin x} \cos x \, dx$$

then which one of the following is correct?

(a) $I_1 \neq I_2$ (b) $\dfrac{d}{dx}(I_3) = I_2$

(c) $\int I_3 dx = I_2$ (d) $I_2 = I_3$

(c) We have,

$$I_1 = \frac{d}{dx}(e^{\sin x}) = e^{\sin x} \cdot \frac{d(\sin x)}{dx}$$

$$= e^{\sin x} \cos x$$

$$I_2 = \lim_{h \to 0} \frac{e^{\sin(x+h)} - e^{\sin x}}{h}$$

$$= \frac{d}{dx} e^{(\sin x)}$$

$$= e^{\sin x} \frac{d}{dx}(\sin x) = e^{\sin x} \cos x$$

$$I_3 = \int e^{\sin x} \cos x \, dx = \int e^t dt$$

$[\because \sin x = t \Rightarrow \cos x \, dx = dt]$

$$= e^t + C = e^{\sin x} + C$$

$$\therefore \frac{d}{dx}(I_3) = \frac{d}{dx}(e^{\sin x} + C)$$

$$= e^{\sin x} \cdot \frac{d}{dx} \sin x = e^{\sin x} \cos x$$

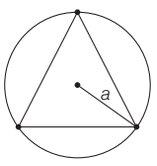
2017 (I)

17. What is the maximum area of a triangle that can be inscribed in a circle of radius a?

(a) $\dfrac{3a^2}{4}$ (b) $\dfrac{a^2}{2}$

(c) $\dfrac{3\sqrt{3}a^2}{4}$ (d) $\dfrac{\sqrt{3}a^2}{4}$

(c)



For area to be maximum triangle inscribed in the circle must be equilateral triangle.

Given, radius of the circle = a

$\therefore$ Side of the equilateral triangle = $\sqrt{3}a$

$\therefore$ Area of the equilateral triangle

$$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(\sqrt{3}a)^2 = \frac{3\sqrt{3}a^2}{4}$$

2015 (II)

Directions (Q. Nos. 18 and 19)

Consider $f'(x) = \dfrac{x^2}{2} - kx + 1$ such that $f(0) = 0$ and $f(3) = 15$.

18. The value of k is

(a) $\dfrac{5}{3}$ (b) $\dfrac{3}{5}$ (c) $-\dfrac{5}{3}$ (d) $-\dfrac{3}{5}$

(c) Given, $f'(x) = \dfrac{x^2}{2} - kx + 1$

On integrating, we get

$$\int f'(x)\,dx = \int \left(\frac{x^2}{2} - kx + 1\right) dx$$

$$f(x) = \frac{x^3}{6} - \frac{kx^2}{2} + x + C \quad \ldots(i)$$

Given, $f(0) = 0$

$\therefore \quad 0 = 0 - 0 + 0 + C \Rightarrow C = 0$

$\therefore \quad f(x) = \dfrac{x^3}{6} - \dfrac{kx^2}{2} + x$

Given, $f(3) = 15$

$\therefore \quad 15 = \dfrac{(3)^3}{6} - \dfrac{k(3)^2}{2} + 3$

$\Rightarrow \quad 15 = \dfrac{27}{6} - \dfrac{9}{2}k + 3 \Rightarrow k = -\dfrac{5}{3}$

19. $f''\left(-\dfrac{2}{3}\right)$ is equal to

(a) – 1 (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) 1

(d) $\because \quad f'(x) = \dfrac{x^2}{2} - kx + 1$

$$\Rightarrow f'(x) = \frac{x^2}{2} - \left(\frac{-5}{3}\right)x + 1 \quad \left[\because k = -\frac{5}{3}\right]$$

$$= \frac{x^2}{2} + \frac{5}{3}x + 1$$

Now, differentiating both sides w.r.t. x, we get

$$f''(x) = \frac{2x}{2} + \frac{5}{3} = x + \frac{5}{3}$$

$$\therefore \quad f''\left(-\frac{2}{3}\right) = -\frac{2}{3} + \frac{5}{3} = 1$$

2015 (I)

20. Consider the following statements in respect of the given equation

$(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$

I. All the roots of the equation are complex.

II. The sum of all the roots of the equation is 6.

Which of the above statement(s) is/are correct?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(c) We have,

$(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$ ...(i)

$\Rightarrow \quad x^4 + 4 + 4x^2 + 8x^2 = 6x^3 + 12x$

$\Rightarrow \quad x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$

Let $P(x) = x^4 - 6x^3 + 12x^2 - 12x + 4$

Again, let $(x^2 + 2) = y$

From Eq. (i)

$\therefore \quad y^2 + 8x^2 = 6xy$

$\Rightarrow \quad y^2 - 6xy + 8x^2 = 0$

$\Rightarrow \quad (y - 4x)(y - 2x) = 0$

$\Rightarrow \quad (x^2 - 4x + 2)(x^2 - 2x + 2) = 0$

So, it is clear that no real roots, then all roots are complex.

Also, it is also clear that the sum of all the roots of the equation is 6. Hence, both the statements are true.

21. If $2p + 3q = 18$ and $4p^2 + 4pq - 3q^2 - 36 = 0$, then what is $(2p + q)$ equal to ?

(a) 6 (b) 7 (c) 10 (d) 20

(c) We have, $2p + 3q = 18$

and $4p^2 + 4pq - 3q^2 - 36 = 0$

$\Rightarrow \quad (2p + 3q)^2 - 8pq - 12q^2 = 36$

$\Rightarrow \quad 18^2 - 4q(2p + 3q) = 36$

$\Rightarrow \quad 324 - 36 = 4q(18)$

$\Rightarrow \quad \dfrac{288}{18} = 4q \Rightarrow q = 4$

$\therefore 2p + 3 \times 4 = 18 \Rightarrow p = \dfrac{18 - 12}{2} = 3$

$\therefore \quad 2p + q = 2 \times 3 + 4 = 10$

22. Let θ be a positive angle. If the number of degrees in θ is divided by the number of radians in θ, then an irrational number $\dfrac{180}{\pi}$ results. If the number of degrees in θ is multiplied by the number of radians in θ, then an irrational number $\dfrac{125\pi}{9}$ results.

The angle θ must be equal to

(a) 30° (b) 45° (c) 50° (d) 60°

(c) Given, $\dfrac{\theta}{\theta^c} = \dfrac{180°}{\pi} \Rightarrow \theta^c = \dfrac{\pi \times \theta}{180°}$

and $\theta \times \theta^c = \dfrac{125\pi}{9}$

$$\therefore \quad \theta \times \frac{\pi \times \theta}{180°} = \frac{125\,\pi}{9} = 125 \times 20°$$

$$= 25 \times 100° = (5 \times 10°)^2$$

$\Rightarrow \quad \theta = 50°$

ENGLISH

31

SPOTTING ERRORS

Directions (Q. Nos. 1-105) *Each item in this section has a sentence with three underlined parts labelled (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part. If you find no error, your response should be indicated as (d).*

2019 (II)

1. He had arrived at Cairo *(a)* a few months before *(b)* protests shook the Arab world. *(c)* No error *(d)*

(a) Here part (a) has an error. As sentence is in past tense, so we should use Simple Past tense 'arrived' without the use of had.

2. Most of us who are older competitive runners *(a)* are not able to race anywhere at the same speed *(b)* as we do when we were 30. *(c)* No error *(d)*

(b) Here part (b) has an error. 'At the same speed' is incorrect. It should be 'with the same speed', to make the given sentence grammatically correct. Because 'at' is used to show only a definite speed; e.g. at 80mph etc.

3. Work hard *(a)* lest you do not *(b)* fail. *(c)* No error *(d)*

(b) Here part (b) has an error. With 'lest' we should always use 'should'. Replace 'do not' with 'should' to make the given sentence grammatically correct.

4. The Eastern Ghats are home of 2600 plant species *(a)* and this habitat fragmentation *(b)* can pose a serious threat to endemic plants. *(c)* No error *(d)*

(a) Here part (a) has an error. 'Home of' is incorrect expression. It should be replaced with 'home to' as it refers to the place where you live or from where you belonged.

5. Turbidity current is a fast-moving current *(a)* that sweeps down submarine canyons, *(b)* carrying sand and mud into the deep sea. *(c)* No error *(d)*

(c) Here part (c) has an error. Replace 'carrying sand' with 'that carries sand' as the reference is to a particular thing (Turbidity current).

6. Every one *(a)* of the boys *(b)* love to ride. *(c)* No error *(d)*

(c) Here part (c) has an error. With 'everyone' singular verb should be used. So, use 'loves' in place of 'love' to make the given sentence grammatically correct.

7. Neither praise nor blame *(a)* seem *(b)* to affect him. *(c)* No error *(d)*

(b) Here part (b) has an error of use of verb. 'Seem' should be replaced with 'seems' as the sentence is in Simple Present tense.

8. Many a man *(a)* has succumbed *(b)* to this temptation. *(c)* No error *(d)*

(d) The given sentence has no error.

9. A time slot of fifteen minutes *(a)* are allowed *(b)* to each speaker. *(c)* No error *(d)*

(b) Here part (b) has an error. As 'time slot' given in part (a) is singular noun, so the verb used with it, should also be singular. Hence, 'is allowed' should be used in place of 'are allowed' to make the given sentence grammatically correct.

10. He asked (a) whether either of the brothers (b) were at home. (c) No error (d)

(c) Here part (c) has an error. 'Were at home' is incorrect. It should be replaced with 'was at home', as either is always followed by the singular verb.

2019 (I)

11. Opening his letters, (a)/ reading them carefully and sending for his clerk, (b)/ he dictated answers with them. (c)/ No error (d)

(**c**) Here, the use of preposition 'with' is incorrect here. Replace it with 'to', to make the given sentence grammatically correct. 'Dictated to' refers 'to say something aloud so that somebody else can write or type it.'

12. He was my school-friend, (a)/ but becoming a great man, (b)/ he has grown proud enough to forget his old friends. (c)/ No error (d)

(**b**) Here 'but becoming' is incorrect in part (b). It should be replaced with 'but after becoming' as preposition 'after' indicates 'at a later time' which makes the given sentence grammatically correct.

13. Rabindranath Tagore, (a)/ a Nobel laureate and the author of the National Anthem, (b)/ found Shantiniketan. (c)/ No error (d)

(**c**) As the given sentence is in Simple Past Tense, verb 'found' should be replaced with its past form 'founded' to make the given sentence grammatically correct.

14. The art of printing was introduced into England (a)/ during the reign of Edward IV (b)/ by William Caxton, a native of Kent. (c)/ No error (d)

(**a**) Here part (a) has an error. Use of preposition 'into' is incorrect. Replace it with preposition 'in' as it is used to indicate a location or place while preposition 'into' is used to indicate movement toward the inside of a place.

15. From thirty years (a)/ he devoted himself to public affairs (b)/without taking a holiday. (c)/ No error (d)

(**a**) Here, use 'for' in place of 'from' to start the sentence. Use of preposition 'for' shows time period, which is grammatically correct.

16. If Ramesh will be promoted (a)/ he will get (b)/ a higher salary. (c) No error (d)

(**a**) Here use of 'is promoted' in place of 'will be promoted' make the sentence grammatically correct. In conditional sentences, condition is expressed by 'If' clause and Simple Present tense is used to show a future result of that activity.

17. My brother goes (a)/ to the office (b)/ five day week. (c) No error (d)

(**c**) Here, the use of 'Five days a week' is more appropriate in place of 'five day week' to make the given sentence grammatically correct. Five is plural, so days should be used and 'a' is used for denoting one week.

18. If you lend Mohan a pen (a)/ he will lend it to someone else (b)/ and never you will get it back. (c)/ No error (d)

(**c**) Here, part (c) of the given sentence has an error. 'And never you will get it back' is incorrect because adverb should be followed by helping verb. So, the correct phrase will be - 'and you will never get it back'.

19. One of most widely spread (a)/ bad habits (b)/ is the use of tobacco. (c) No error (d)

(**a**) Here part (a) has an error. Article 'the' should be used before superlative degree (most). 'One of the most' is the correct phrase to make the given sentence grammatically correct.

20. A great part (a)/ of Arabia (b)/ is desert. (c) No error (d)

(**d**) There is no error. The sentence is grammatically correct.

2018 (II)

21. He appears (a) to be (b) a honest man. (c) No error (d)

(**c**) Here before the word 'honest', article 'an' should be used as 'h' is mute here and the word gives us vowel sound. The words start with vowel sound are followed by article 'an'.

22. One of the members (a) expressed doubt if (b) the Minister was an atheist. (c) No error (d)

(**b**) In place of 'if' we should use 'that' as 'that the minister was an atheist' is a noun clause and noun clauses begin with some words such as how, that, what, when, where etc, but not with 'if'.

23. This view has been taken (a) by one of the ablest persons (b) who has written on this subject. (c) No error (d)

(**c**) Here use of verb 'has' should be replaced by plural verb 'have' as to agree with the plural subject (persons).

24. Slow and steady (a) owns the race, (b) as the wise would say. (c) No error (d)

(**b**) Here part (b) has an error because 'owns' will be replaced by 'wins' to make the given proverb correct and meaningful.

25. We often hear people say (a) that most human beings have not evolved (b) or grow up enough to stick to truth or non-violence. (c) No error (d)

(***c***) Here part (c) has an error of use of verb. As noun clause is in Present Perfect tense, use of 'grow up' is incorrect. Past participle of 'grow' i.e. 'grown up' should be used here, to make the given sentence grammatically correct.

2018 (I)

26. The politician lost face in his constituency *(a)*/when he broke the pre-election promises *(b)*/he made to his people. *(c)* No error *(d)*

(***c***) Here part (c) has an error. To describe one event or happening following another in the past then, Past Perfect tense is used to describe the earlier event and Past Indefinite is used for the event that took place later on. So, 'he had made' should be used in place of 'he made' in part (c), to make the given sentence grammatically correct.

27. At the request of the Defence Attorney, *(a)*/the jury were called *(b)*/and their individual verdicts were recorded. (c) No error *(d)*

(***b***) Here part (b) has an error of correct use of verb. With the word 'jury' which is taken as singular noun, verb 'was' will be used here to make the given sentence grammatically and contextually correct.

28. Frank Lloyd Wright has been acclaimed *(a)*/by colleagues *(b)*/as the greater of all modern architects. *(c)* No error *(d)*

(***c***) Here part (c) has an error of use of correct degree of adjective. Here, we will use superlative degree (greatest) in place of 'greater' as we are comparing one person with many persons.

29. In my younger days *(a)*/I could run four miles *(b)*/at a stretch. *(c)* No error *(d)*

(***a***) Here part (a) has an error of use of syntax. 'When I was young' is better construction in place of 'In my younger day' to make the sentence more appropriate.

30. The owner *(a)*/as well as his servants *(b)*/is honest. *(c)* No error *(d)*

(***d***) There is no error. The given sentence is grammatically correct.

2017 (II)

31. An electrical circuit (a) is the complete path (b) traversed by electric current. (c) No error (d)

(***c***) The words 'Electric current' will be followed by the article 'an'. So, 'by an electric current' should be used here to make the sentence correct and meaningful.

32. He waved us a by-by (a) as he boarded the train (b) which disappeared into the tunnel. (c) No error (d)

(***a***) Here in part (a) 'Bye-bye' will be the correct expression to use in place of by-by as 'bye-bye' is used to express 'farewell'.

33. There was great deal that had to be scrapped, that (a) must be scrapped; but surely India could not have (b) been what she undoubtedly was, and could not have continued a cultured existence for thousands of years. (c) No error (d)

(***d***) There is no error. The given sentence is grammatically correct.

34. With regard to interior decoration, it is the (a) attention given to the less overt aspects of using space that give it life, an identity, a quality (b) that makes it exciting and unusual. (c) No error (d)

(***b***) Here, in part (b) of the given sentence, 'gives' will be used in place of 'give' before 'it life' as it represents an action which is regular or normal.

35. A small parcel of novels is (a) better than none. (b) No error (c) (d)

(***d***) The sentence has no error and the sentence is grammatically correct.

2017 (I)

36. I can fly *(a)*/ if I will be *(b)*/ a bird. *(c)* No error *(d)*

(***b***) The given sentence indicates an imaginary condition so 'will be' is to be replaced by 'were' to make it grammatically correct. The syntax is as follows–

If + subject + were + object.

37. As soon as *(a)*/ the train arrived *(b)*/ the passengers entered the compartments. *(c)* No error *(d)*

(***d***) There is no error. The sentence is grammatically correct.

38. Never *(a)*/ I have *(b)*/ seen such a town. *(c)* No error *(d)*

(***b***) Here, the sentence requires the form of inversion. Hence, 'I have' is to be written as 'have I', to make the given sentence more appropriate.

39. He goes to his office/ *(a)* by bicycle *(b)*/ every day. *(c)* No error *(d)*

(***b***) Here, in part (b) of the given sentence 'by' should be replaced with preposition 'on'. With two wheelers, preposition 'on' should be used.

40. Far from being appreciated, *(a)*/ his conduct was discussed *(b)*/ and sharply commented. *(c)* No error *(d)*

(***c***) Here, use preposition 'upon' after 'commented' as it is a more formal term for on, especially in abstract senses.

2016 (II)

41. My friend was *(a)*/habitual to drive a car *(b)* /at very high speed. *(c)*/No error *(d)*

(***b***) Here in part (b) correct expression is 'habitual of driving' to make it a grammatically correct sentence. Preposition 'of' is used to indicate reference.

42. Being a day of national mourning*(a)*/the offices were closed *(b)*/and the national flag flew half mast. *(c)*/No error *(d)*

(***a***) Here, we should not start a sentence from 'being'. Instead, we should use the introductory word 'it'. So, the expression is to be written as 'It being' to make the given sentence grammatically correct.

43. If you are wanting to watch *(a)*/the birds nesting *(b)*/you must not let them see you. *(c)*/No error *(d)*

(***a***) Stative verbs are not written in the progressive forms. So, here 'are wanting' should be written as 'want' to make the given sentence grammatically and contextually correct.

44. He does not listen to *(a)*/what I say, *(b)*/although I am advising him for a long time. *(c)*/No error *(d)*

(***c***) If the action is continued for a long period of time and there is a time reference to it, we should use Present Perfect Continuous Tense. So, 'I have been' should be used in place of 'I am' to make the given sentence grammatically correct and meaningful.

45. It is most essential for us *(a)*/to reach the station before noon *(b)*/lest we should miss the train. *(c)*/No error *(d)*

(***a***) Remove 'most' from the expression given in part (a) as it is superfluous here. 'Essential' itself is proper word to convey the complete and meaningful sense.

46. I asked him *(a)*/what had brought *(b)*/him there. *(c)*/No error *(d)*

(***d***) There is no error. The given sentence is grammatically correct.

47. The guards *(a)*/dare not *(b)*/to harm him. *(c)*/No error *(d)*

(***c***) After 'dare', we should not use infinitive 'to'. Hence, the correct expression should be 'dare not harm him'.

48. The Principal looks very worried *(a)*/because students are on strike for the last fifteen days as a protest against *(b)*/his disciplinary action against some miscreants. *(c)*/No error *(d)*

(***d***) There is no error. The given sentence is grammatically correct.

49. The king was embarrassed *(a)*/to find evidences *(b)*/against his own queen. *(c)*/No error *(d)*

(***c***) Here 'his own' is the superfluous usage. It is very obvious that only the king has a queen, so, remove 'his own', to make the sentence more appropriate.

50. As I prefer coffee than tea *(a)*/my friends always take the trouble *(b)*/to get me a cup of coffee whenever I visit them. *(c)*/No error *(d)*

(***a***) The word 'prefer' is followed by 'to', if the comparatives are 'nouns'. So, 'prefer coffee to tea' is correct phrase, to make the given sentence grammatically correct.

2016 (I)

51. She said that she was understanding (a)/ his point of view (b)/ very well. (c)/ No error (d)

(***a***) Here part (a) has an error of tense. As the sentence is in Past Perfect tense, so the verb should also be used in Past Perfect tense. Hence 'She had understood' should be used.

52. All the houses having been washed away (a) by the floods, (b) the villagers sought shelter (c) in the panchayat office. No error (d)

(***b***) Here part (b) has an error in use of preposition. Instead of 'by' we should use 'due' to floods, which makes the sentence more appropriate and grammatically correct.

53. It is necessary for us to familiarise with (a)/ and get used to (b)/ the ways of the people among whom we live. (c) No error (d)

(***b***) Here, in part (b) of the sentence, before 'get' we must use infinitive 'to', to make the sentence grammatically and contextually correct.

54. The mother asked the child (a)/ why did she cry out (b)/ in her sleep. (c) No error (d)

(***b***) Here part (b) has an error in usage of tense. With asked 'why did she cry' should not be used. It should be 'why she cried' as the sentence is in Past tense.

55. I asked him that (a) why he was angry (b) but he did not answer. (c) No error (d)

(***a***) Here part (a) has an error. With question word (why) there is no need to use 'that' before it as conjunction words are not used before question word.

56. If not taken ill, (a)/ I would have sent you (b)/ my research paper much earlier. (c) No error (d)

(***a***) Here part (a) has an error. The sentence should begin with 'I had not taken ill' as the sentence is conditional or shows condition. The syntax of conditional sentence is as follows–

If + subject + had + verb (3rd form) + object + (,) + subject + would have + verb (3rd form) +remaining part.

57. The barn owl helps the farmer by destroying (a)/ rats which could, if left unchecked (b)/ do a lot of damage to the crops. (c)/ No error (d)

(***a***) Here part (a) has an error of tense. Instead of 'helps' (Simple Present tense) it should be 'helped' (Past tense) as the rest of the sentence indicates the Past tense.

58. He had gone (a)/ home three weeks ago and was expected (b)/ back yesterday, but he has not come yet. (c) No error (d)

(***a***) Here part (a) has an error. Simple Past tense is to be used here as it is used to show a complete action that took place at a specific time in the past and it should be 'He went' in place of 'had gone' to make the sentence grammatically correct.

59. We must sympathise (a)/ for others (b)/ in their troubles. (c)/ No error (d)

(***b***) Here, part (b) has an error in usage of preposition. The verb 'Sympathise' is always followed by preposition 'with'. So, 'with' should be used in place of 'for' to make the sentence correct.

60. The scientists in America (a)/ are trying for long (b)/ to discover the genes responsible for ageing. (c)/ No error (d)

(***b***) Here, in part (b) use of Present Perfect tense is required here as it is used to express a past event that has present consequences. It should be 'The scientist in America have been trying for long' instead of 'are trying for long'.

2015 (II)

61. I am told (a)/John is ill (b)/ since Monday. (c)/No error (d)

(***b***) Here, in part (b) use 'John has been ill' in place of 'John is ill' to make the syntax correct as there is a time reference.

62. I shall leave school (a)/as soon as (b)/ I shall find a job. (c)/ No error (d)

(***c***) Here, in part (c) remove 'shall' from the part of the sentence to make it correct. See the syntax: sub+shall/will+…+sub+v_1/v_5… .

63. He is a rich man (a)/ he is owning (b)/ a number of buildings in Bombay. (c)/No error (d)

(***b***) Here, in part (b) 'He is owning' is to be replaced by 'He owns' because 'own' is not used in progressive form.

64. We were able to make it (a)/ in spite of (b)/ a bad weather. (c)/ No error (d)

(***d***) There is no error. The sentence is grammatically correct.

65. Neither of the two letters of credit (a)/ were useful (b)/ when he needed it. (c)/ No error (d)

(***b***) 'Neither' is followed by singular verb. So, here use 'was' in place of 'were' to make the given sentence grammatically correct.

66. Little he realised (a)/ that he wouldn't live (b)/ to see his grandson married. (c)/ No error (d)

(***c***) Here, in part (c) use 'get married' to make the syntax correct. 'Marry' is a verb that is used in collocation.

67. A difficult syllabus (a)/ is supposed to hurt (b)/ juvenile mind. (c)/No error (d)

(***b***) In part (b), replace 'is supposed to hurt' by 'is difficult to'. According to the given sentence, 'Hurt' is not appropriate to use here.

68. The voice of sanity and patriotism (a)/ are important to dispel (b)/ the prevailing confusion. (c)/ No error (d)

(***b***) Here, in part (b) use 'is' in place of 'are' to make the sentence grammatically correct. When two things are meant for same thing then singular verb is used.

69. You were going to explain (a)/ your late arrival, (b)/ isn't it? (c)/ No error (d)

(***c***) As per the rules of question tag, we should use 'weren't you' in place of 'isn't it'. A question tag takes the verb of the main sentence (subject). Here, 'were' is the verb of the main sentence.

2015 (I)

70. Unless you stop to make noise at once, (a)/ I will have no option but to (b)/ bring the matter to the attention of the police. (c)/ No error (d)

(***a***) 'Unless you stop making noise at once' is better expression. We have to remove 'to make' in part (a) as it is superfluous here.

71. He couldn't but help (a)/ shed tears at the plight of the villagers (b)/ rendered homeless by a devastating cyclone. (c)/ No error (d)

(***a***) Here, in part (a) of the given sentence 'But' should come after 'help' because 'but' is the conjunction which add two words, sentences etc. Here, 'but' will come between 'help' and shed', which is grammatically not correct.

72. Since, it was his first election campaign, the candidate was confused; (a)/ none could clearly understand (b)/ either the principles he stood for or the benefits he promised. (c)/ No error (d)

(***c***) Here, in part (c) of the given sentence 'had' should come in between 'he' and 'promised' as the given sentence is in Past Perfect tense. i.e. the benefits he had promised, to make the sentence more appropriate and grammatically correct.

73. It is an established fact that the transcendental American poets and philosophers, (a)/ who lived in the latter half of the 19th century, (b)/ were more influenced by Indian philosophy, in particular by Upanishadic Philosophy. (c) / No error (d)

(c) *(**c**)* In the given sentence, 'much' will be used in place of 'more' as 'much' is used with uncountable noun while 'more' is used to denote additional or a greater quantity than before.

74. No sooner did I open the door (a)/ when the rain, heavy and stormy, rushed in (b)/ making us shiver from head to foot. (c)/ No error (d)

(b) *(**b**)* Here, in part (b) only 'when' should be replaced with 'than'. Because 'No sooner' is always followed by 'than' not 'when'.

75. After opening the door (a)/ we entered into the room (b)/ next to the kitchen. (c)/ No error (d)

(b) *(**b**)* Here, in part (b) use of preposition (into) is not required with 'enter'. The use of 'into' here, is superfluous.

76. When the dentist came in (a)/ my tooth was stopped aching (b)/ out of fear that I might lose my tooth. (c)/ No error (d)

(b) *(**b**)* Here, in part (b) of the sentence, 'my toothache stopped' should be used in place of 'my tooth was stopped', to make the given sentence grammatically correct. As toothache is used for 'aching of tooth'.

77. Emphasis on equality of life ensures (a)/ for the health and happiness (b)/ of every individual. (c)/ No error (d)

(b) *(**b**)* The verb 'ensures' does not take 'for' after it. So 'for' should be removed in part (b) of the sentence.

78. The students were (a)/ awaiting for (b)/ the arrival of the chief guest. (c)/ No error (d)

(b) *(**b**)* Here, in part (b) 'Awaiting' should be replaced by 'waiting', to make the sentence grammatically correct. 'Awaiting' means 'to wait for somebody' while 'waiting' means 'to stay in a particular place until somebody arrives'.

79. You will come (a)/ to my party tomorrow, (b)/ isn't it? (c)/ No error (d)

(c) *(**c**)* Here in part (c) 'Isn't it ?' is wrong question tag as it should be 'won't you ?' because the subject of the sentence is 'will'.

80. Having read a number of stories (a)/ about space travel (b)/ his dream now is about to visit the Moon. (c)/ No error (d)

(c) *(**c**)* Here, in part (c) 'About' should not be used. The sentence should be 'his dream now is to visit the Moon'. As, 'about' is used to indicate movement, within a particular area.

81. The meeting adjourned abruptly (a)/ by the Chairman after (b)/ about three hours of deliberation. (c)/ No error (d)

(a) *(**a**)* Here, in part (a) of the sentence, 'was' should be used before the word 'Adjourned' as the given sentence is in Passive voice.

82. Not one of the hundreds (a)/ of striking workers (b)/ were allowed to go near the factory. (c)/ No error (d)

(c) *(**c**)* Here, subject (one) is singular so in part (c) the verb should also be singular. Therefore, 'was' should be used in place of 'were' to make the sentence grammatically correct.

83. If I had known (a)/ this earlier (b)/ I will have helped him. (c) / No error (d)

(c) *(**c**)* Here the sentence is an imaginative sentence, so in part (c) 'will' has to be replaced by 'would', to make the sentence grammatically correct.

84. Mr Smith was accused for murder (a)/ but the court found him not guilty (b)/ and acquitted him. (c)/ No error (d)

(a) *(**a**)* Here 'Accused of' is the correct use of preposition as 'accused' is always followed by preposition 'of'.

2014 (II)

85. It was nearly thirty years ago (a)/ since this magazine (b)/ was first published. (c)/ No error (d)

(a) *(**a**)* Here, in part (a) use 'about' in place of 'nearly'. 'Nearly' is used when the sense is vague, incomplete and uncertain but here the duration has been definite as to be thirty years. So, use of 'about' is more appropriate which gives the sense of 'definite nature'.

86. Ten years ago he was having an income (a)/ of over ten thousand rupees a month; (b)/ he must indeed be a wealthy man by now. (c)/ No error (d)

(c) *(**c**)* Use of 'indeed' is not required in part (c) of the given sentence as it is superfluous. Using 'must' alone gives the proper sense here. So remove the word 'indeed' to make the sentence grammatically correct.

87. In spite of the fact (a)/ that the meeting was about to end, (b)/ he insisted to ask several questions. (c)/ No error (d)

(c) *(**c**)* Here, in part (c) use 'intended' in place of 'insisted' as 'intended' refers 'to plan or mean to do something' while 'insisted' refers 'to say strongly that you must have or do something.' Use of 'insisted' is wrong as it gives no proper sense here and make the sentence inappropriate.

88. Hardly had he entered into the room (a)/ and taken his seat, (b)/ when the girls began to giggle. (c)/ No error (d)

(a) *(**a**)* Use of 'into' is not required in part (a) of the given sentence as we know that 'enter' agrees with no preposition. So, remove 'into', to make the sentence grammatically correct.

89. I was disappointed (a)/ when I came for seeing you last evening (b)/ but could not find you at home. (c)/ No error (d)

(b) *(**b**)* Here, part (b) has an error. It is wrong to use a preposition or an adverb with the verb 'see'. As per the rule, infinitive 'to' is used with the verb 'see'.

90. The number of jobs (a)/that would be satisfactory in all respects (b)/ are small. (c)/ No error (d)

(c) *(**c**)* Here, the subject (The number of jobs) is singular, so helping verb 'is' should be used in place of 'are', to make the given sentence grammatically correct.

91. I have found the man, (a)/ whom I know (b)/ is the thief. (c)/ No error (d)

(***b***) Here in part (b) use 'who' in place of 'whom' as the sentence is in need of adjective clause to make it grammatically correct.

92. There is no reason (a)/ why he should not come (b)/ since he is in town and is free. (c)/ No error (d)

(***b***) Here in part (b) use 'why should he' in place of 'why he should' for proper framing of sentence and grammatical accuracy.

93. It is high time (a)/ we cried a halt to the existing corrupt practices (b)/ followed in the educational sphere. (c)/ No error (d)

(***c***) Here, in part (c) use 'being' before 'followed' as certain verbs like 'practice' is followed by gerund (ing) form.

94. The situation is perilous (a)/ but if we are prepared promptly to act, (b)/ there is still one chance of escape. (c)/ No error (d)

(***b***) Here, in part (b) use 'prepared to act promptly' in place of 'prepared promptly to act' to make the given sentence grammatically correct.

2014 (I)

95. The pile of books(a)/ are(b)/ missing. (c) / No error (d)

(***b***) Here, in part (b) 'is' will be used in place of 'are'. Because the subject (The pile of books) of the sentence is singular. So, verb also used in singular form.

96. Either he(a)/ or I(b)/ are wrong. (c)/ No error (d)

(***c***) Here, in part (c) 'am wrong' is to be used instead of 'are wrong'. Helping verb is used according to the nearest subject which is 'I'.

97. Please(a)/ tell to him(b)/ to do his work. (c)/ No error (d)

(***b***) Here in part (b) remove 'to' after 'tell'. It is wrong use of preposition with the objective case of a pronoun. 'to' is not followed by objective case of pronoun.

98. Though he is good (a)/ he is (b)/ mischievous. (c)/ No error (d)

(***b***) Here, in part (b) Add 'yet' before 'he'. 'Yet' is used in the sentence, which starts with 'though/although'.

99. The thief (a)/ hit me (b)/ suddenly and hardly. (c)/ No error (d)

(***b***) Here in part (b) replace 'hardly' with 'hard'. Hardly means scarcely or barely.

Hard means harsh, severe (a hard blow), which is correct in context of the sentence.

100. The number of books (a)/ in our library (b)/ is less. (c)/ No error (d)

(***d***) The sentence has no error and it is grammatically correct.

101. The general said (a)/ 'soldiers do not fire (b)/ till I will give the order'. (c)/ No error (d)

(***c***) Here, in part (c) remove 'will' after 'I'. 'I will give the order' shows future tense which is incorrect. 'Till I give order' is the correct usage because it would make soldiers' fire or not.

102. Neither of the candidates (a)/ are (b)/ good. (c)/ No error (d)

(***b***) Here in part (b) replace 'are' with 'is' because singular form of verb is used after 'neither'.

103. The machine (a)/ is more efficient (b)/ than any other equipments in the workshop. (c)/ No error (d)

(***c***) In part (c) of the given sentence, write 'equipment' in place of 'equipments'. After 'any other' singular noun (equipment) should be used.

104. The tin deposits in that area (a)/ would probably be exhausted (b)/ into near future. (c)/ No error (d)

(***c***) Here, replace 'into' with 'in the'. 'Into' shows the motion of subject and here 'in the' will make the sentence senseful and also grammatically correct.

105. The operation of this machine (a)/ is different (b)/ to that of other. (c)/ No error (d)

(***c***) Here, in part (c) of the given sentence has an error. The word 'different' is followed by preposition 'from', so 'from' will be used in place of 'to' to make the sentence correct.

32

IMPROVEMENT OF SENTENCE

Directions (Q. Nos. 1-80) *In this section, look at the underlined part of each sentence. Below each sentence are given three possible substitutions for the underlined part. If one of them (a), (b) or (c) is better than the underlined part, indicate your response against the corresponding letter (a), (b) or (c). If none of the substitutions improves the sentence, indicate (d) as the answer. Thus, a 'No improvement' response will be signified by the option (d).*

2017 (I)

1. I disliked him not so much for his meanness <u>but for</u> his dishonesty.

(a) as for (b) but because
(c) but due to (d) No improvement

› *(**d**)* The given sentence is correct.

2. He preferred death <u>rather than</u> imprisonment.

(a) for (b) to
(c) than (d) No improvement

› *(**b**)* 'Prefer' is followed by preposition 'to' not 'than'. So, option (b) is appropriate.

3. They kept the idea <u>secretly</u>.

(a) secretive (b) secret
(c) secretively (d) No improvement

› *(**b**)* Here, we don't require an adverb, according to the given context. So, 'secretly' is not proper. We should use 'secret' here.

4. He ought not to <u>tell</u> me your secret, but he did.

(a) telling (b) have told
(c) having told (d) No improvement

› *(**b**)* The sentence requires Present Perfect tense as it is used to express a past event that has present consequences. So, 'have told' is the suitable choice.

5. If I don't know the meaning of a word, I <u>look it after</u> in the dictionary.

(a) look it out (b) look it for
(c) look it up (d) No improvement

› *(**c**)* 'Look it up' is a phrasal verb which means 'to search'. So, option (c) is the correct improvement in the given context.

6. Mr. and Mrs. Rao stay <u>in their home</u> every evening.

(a) at home
(b) inside home
(c) within their home
(d) No improvement

› *(**a**)* The verb 'Stay' is followed by preposition 'at'. Hence, option (a) 'at home' is the correct improvement of the underlined part.

7. When we saw him last, he <u>ran</u> to catch a bus.

(a) has run (b) had run
(c) was running (d) No improvement

› *(**c**)* Sentences that are joined by 'when' take Simple Past and Past Continuous Tenses. Here, 'was running' is suitable option in place of 'ran'.

8. She <u>cut a sad figure</u> in her first performance on the stage.

(a) made a sorry figure
(b) cut a sorry face
(c) cut a sorry figure
(d) No improvement

› *(**c**)* Here, 'cut a sad figure' is an incorrect idiom. 'Cut a sorry figure' is correct one and it means 'to be ashamed' which is correct according to the given context. Hence, option (c) is the correct answer.

9. Last evening I went to the optician and bought <u>spectacles</u>.

(a) a spectacle (b) two spectacles
(c) a pair of spectacles
(d) No improvement

› *(**c**)* It is correct to write 'a pair of spectacles' in place of 'spectacles.' Hence, option (c) is suitable here.

10. I would like <u>to avail</u> a fifteen days' holiday this summer.

(a) to avail of (b) to avail myself of
(c) to avail myself (d) No improvement

› *(**b**)* The word 'avail' is followed by preposition 'of'. Also, it takes a reflexive pronoun. So, the correct replacement will be 'to avail myself of'.

2016 (II)

11. My old teacher gave many poor students money <u>in spite</u> of he was poor.

(a) even though (b) as if
(c) however (d) No improvement

› *(**a**)* The sentence shows a contrast. Hence, a contrasting expression should be used here. So, 'even though' is suitable here.

12. You <u>should to</u> read the newspapers regularly, if you want to be well-informed.

(a) ought (b) have to
(c) should be (d) No improvement

› *(**b**)* Necessity for an action is shown by the use of 'have to'. Therefore, option (b) is correct replacement here.

13. If he had <u>listening to me</u> he would not have got lost.

(a) listened me (b) listened to me
(c) listen me (d) No improvement

(***b***)Auxiliary 'has/have/had' is followed by the Past Participle form of the verb. So, 'listened to me' is the correct replacement here.

14. He asked for a bank loan <u>also</u> he didn't have sufficient resources.
(a) because (b) even if
(c) requiring (d) No improvement

(***a***) To state a reason, we should use 'because'. Therefore, option (a) is the correct replacement here.

15. I went to the shop to buy some <u>stationeries</u>.
(a) stationery (b) stationary
(c) stationaries (d) No improvement

(***a***)'Stationery' is the correct usage here even if we use it in plural form. Hence, option (a) is suitable here.

16. I have been waiting here <u>for</u> the last two hours.
(a) from (b) since
(c) by (d) No improvement

(***d***) The sentence does not require any improvement.

17. The table needs to be <u>painted</u>.
(a) paint (b) painting
(c) having paint (d) No improvement

(***d***) The sentence does not require any improvement.

18. It began to rain heavily just when I <u>prepare</u> to go out.
(a) prepared
(b) was preparing
(c) have prepared
(d) No improvement

(***b***) The sentence requires Past Continuous tense as it is often used to describe conditions that existed in the past. So, use 'was preparing' instead of 'prepare' to make the syntax correct.

19. The couple seemed to love <u>one another</u> very much.
(a) every other
(b) each
(c) each other
(d) No improvement

(***c***) When two persons are involved, then pronoun should be used 'each other'not 'one another'. Hence, option (c) is the correct replacement here.

20. It is high time that we <u>did</u> something.
(a) had done (b) would do
(c) have done
(d) No improvement

(***d***) The sentence does not require any improvement.

21. If I were a millionaire, I <u>would have helped</u> the poor.
(a) am capable of helping
(b) could have helped
(c) would help
(d) No improvement

(***c***) For imaginary conditions, 'would $+V_1$' is suitable usage. Therefore, option (c) is the correct replacement here.

22. The training programme was extended <u>for</u> a month.
(a) from (b) until
(c) since (d) No improvement

(***d***) The sentence does not require any improvement.

23. The painting <u>which</u> they were looking at was done by my sister.
(a) what (b) that
(c) whom (d) No improvement

(***b***) Use 'that' in place of 'which' because 'that' is used to refer to a thing, especially when it is not near the person speaking.

2016 (I)

24. When he heard the rhetorical speech of the leader, he was <u>carried along</u> by his enthusiasm.
(a) carried aloft (b) carried down
(c) carried away (d) No improvement

(***c***) From the given options, the idiom 'carried away' is appropriate here. 'Carried away' means to become overly excited or involved in something.

25. After the heavy rains last week, the water in the lake <u>raised another two feet</u>.
(a) rose another two feet
(b) arisen another two feet
(c) would raise another two feet
(d) No improvement

(***a***) The given sentence is in the Past Tense. The Past form of the verb 'rise' is 'rose'. Therefore, option (a) is the correct replacement here.

26. One can live and work in a town without being aware of the daily march of the Sun across the sky without <u>never</u> seeing the Moon and stars.
(a) seldom (b) hardly
(c) ever
(d) No improvement

(***c***) The word 'ever' is an appropriate substitution of 'never'. 'Ever' is used for emphasis while comparing two things. Therefore, option (c) is the correct replacement here.

27. Applications of those <u>who are graduates</u> will be considered.
(a) whom are graduates
(b) whose are graduates
(c) whom are only graduates
(d) No improvement

(***d***) The sentence does not require any improvement.

28. It <u>is raining</u> heavily all through this week.
(a) has rained (b) rains
(c) rained (d) No improvement

(***c***) The sentence is in Past Tense. So, option (c) 'rained' should be the correct replacement here.

29. The sparrow took no <u>notice about</u> the bread.
(a) notice of (b) notice from
(c) notice to (d) No improvement

(***a***) The preposition 'of' is used with the word 'notice' to form a meaningful expression. Hence, option (a) is the correct replacement here.

30. As he spoke about his achievements, his <u>high</u> claims amused us.
(a) big (b) tall
(c) long (d) No improvement

(***b***) 'Tall claims' is the correct expression meaning 'over statement'. So, 'tall' will be used in place of 'high' to make the sentence correct.

31. We ought to <u>stand</u> for what is right.
(a) stand at (b) stand on
(c) stand up (d) No improvement

(***c***) The phrase 'stand up' means 'to support'. Therefore, option (c) conveys the correct meaning of the given sentence.

32. He enjoys <u>to tell stories</u> to children.
(a) how to tell stories
(b) telling stories
(c) to narrate stories
(d) No improvement

(***b***) After some verbs such as enjoy, admit, deny, avoid, etc gerund (-ing form) is used. So, option (b) 'telling stories' is the correct replacement here.

33. In a few minutes time, when the clock strikes six, I <u>would be waiting</u> here for an hour.
(a) shall be waiting on
(b) shall have been waiting
(c) shall wait (d) No improvement

(***a***) The given sentence is in Future Continuous Tense. Therefore, option (a) 'shall be waiting on' is the appropriate replacement here.

2015 (II)

34. Had she been hungry, she would have devoured the whole lot of it.
(a) Unless she had been hungry
(b) However she had been hungry
(c) As if she had been hungry
(d) No improvement

(*d*) The given sentence does not require any improvement.

35. Until you don't finish the work, you won't be given leave.
(a) Until you must finish
(b) Until finishing
(c) Until you finish
(d) No improvement

(*c*) 'Until' is itself used in negative form, so negative words (like 'don't') are not used with it. Hence, option(c) is the correct improvement for the given sentence.

36. The names of the defaulters have been cut off the register.
(a) cut out (b) struck out
(c) struck off (d) No improvement

(*c*) The correct verb to be used here is 'struck off'.
'Cut off' means 'to cut short' or 'to terminate supply' which is not suitable. So, 'struck off' is the appropriate usage here as 'struck off' means removed from a list or register (the name of a person or thing).

37. This is a matter I'd rather not talk about.
(a) of which I'd rather not talk about
(b) I rather not talk about
(c) than I'd rather not talk about
(d) No improvement

(*a*) Here we need a sentence connector. So, 'of which' is the suitable usage here as a proper connector. Hence, option (a) is the correct improvement here.

38. Only when you left I did sleep.
(a) I slept (b) did I sleep
(c) had I slept (d) No improvement

(*b*) 'Only when ...' is an adverbial phrase here and for this, we have to use inversion form of the verb. So, 'did I sleep' is the correct improvement here.

39. I will be happy if you will buy me apples.
(a) buy (b) bought
(c) will be buying (d) No improvement

(*a*) Here use of 'buy', as the conditional clause needs to be in Simple Present tense. Hence, option (a) is the correct improvement here.

40. After I saved some money, I shall go abroad.
(a) shall save (b) should have saved
(c) have saved (d) No improvement

(*c*) 'Saved' should be replaced with 'have saved' to show the condition of the action in the given sentence. So, option (c) is the correct choice.

41. Can you arrange the car to be ready this evening?
(a) arrange with the car
(b) arrange for the car
(c) arrange that the car
(d) No improvement

(*b*) 'Arrange for the car' is the suitable replacement for this sentence. 'Arrange for' is an idiomatic expression which means 'to organise or plan something for a specific time.'

42. Hardly had he reached home when the telephone rang.
(a) he had reached (b) did he reach
(c) he reached (d) No improvement

(*d*) The sentence does not require any improvement.

43. He is likely to win the elections by the sweeping majority.
(a) with the sweeping majority
(b) in sweeping majority
(c) by a sweeping majority
(d) No improvement

(*c*) Use 'by a sweeping majority' which is an idiomatic expression to make the syntax correct. 'Sweeping majority' means 'affecting many things or people'.

2015 (I)

44. Although, India is still by far a poor country, it can become rich if its natural and human resources are fully utilised.
(a) few and far between
(b) by and large
(c) by and by
(d) No improvement

(*b*) 'By and large' should be used here in place of 'by far' to make the given sentence grammatically correct. 'By and large' means 'mostly or in general' while 'by far' means 'by a large amount'.

45. The more they earn, more they spend.
(a) More they earn, more they spend
(b) More they earn, the more they spend
(c) The more they earn, the more they spend
(d) No improvement

(*c*) The correct improvement of the given sentence is 'the more they spend' in place of 'more they spend'. See the syntax:
The more + subject +verb ... + the more + sub + verb

46. But in all these cases, conversion from one scale to another is easy because scales have well-formulated.
(a) can be well-formulated
(b) are well-formulated
(c) well-formulated
(d) No improvement

(*b*) The given sentence is in Passive voice and the usage of 'is' indicates that the sentence is in Present Continuous tense. So, option (b) 'are well-formulated' is the correct improvement.

47. Five years ago on this date, I am sitting in a small Japanese car, driving across Poland towards Berlin.
(a) was sitting
(b) sat
(c) have been sitting
(d) No improvement

(*a*) The sentence is about Past, so verb should also be used in Past tense. Hence, 'was sitting' is the correct improvement for the given sentence.

48. The old man felled some trees in the garden with hardly no effort at all.
(a) hard effort
(b) hardly any effort
(c) a hardly any effort
(d) No improvement

(*b*) 'Hardly no effort' is wrong because adverb 'hardly' is not followed by any negative word. It should be replaced with 'hardly any effort' to make the given sentence grammatically correct.

49. She says she's already paid me back, but I can't remember, so I'll have to take her word.
(a) to take her word true
(b) to take her at her word
(c) to take her word for it
(d) No improvement

(*b*) 'To take her at her word' should be used in place of ' to take her word' as it means 'to trust her', which is correct according to the given sentence.

50. The workers are hell bent at getting what is due to them.
(a) hell bent on getting
(b) hell bent for getting
(c) hell bent to getting
(d) No improvement

› *(**a**)* 'Hell bent on getting' is the correct replacement for the given underlined part.
'Hell bent on getting' means 'to be extremely determined to do something, without considering the risks or possible dangerous results'.

51. You are warned <u>against committing</u> the same mistake again.
(a) to commit
(b) for committing
(c) against to commit
(d) No improvement

› *(**d**)* The given sentence does not require any improvement.

52. While we would like <u>that all Indian children</u> to go to school, we need to ponder why they do not.
(a) that all the Indian children
(b) if all the children of India
(c) all Indian children
(d) No improvement

› *(**c**)* Remove 'that' before 'all Indian children' to make it correct because 'that' is used to indicate the person or thing already mentioned but 'all Indian Children' are not mentioned in the given sentence. Hence, option (c) is the correct improvement of the given sentence.

53. <u>Due to these reasons</u>, we are all in favour of universal compulsory education.
(a) Out of these reasons
(b) For these reasons
(c) By these reasons
(d) No improvement

› *(**b**)* 'For these reasons' is the most appropriate expression as it suiting the purposes or needs. So, it should replace 'due to these reasons' in the given sentence.

54. When it was feared that the serfs might go too far and gain their freedom from serfdom, the protestant leaders joined the princes <u>at crushing</u> them.
(a) into crushing
(b) in crushing
(c) without crushing
(d) No improvement

› *(**b**)* Use of preposition 'at' is incorrect here 'at' is used to indicate an activity while preposition 'in' is used to indicate a belief, opinion, interest or feeling. So, preposition 'in' will be used in place of 'at' to make the sentence grammatically correct.

55. In India, today many of our intellectuals still talk in terms of the French revolution and the Rights of Man, not appreciating <u>that much has happened</u> since then.
(a) much has been happening
(b) much had happened
(c) much might happen
(d) No improvement

› *(**b**)* 'Much had happened' should be used here as the Past Perfect refers to a time earlier than before now.

56. Taxpayers <u>are to</u> be conscious of their privileges.
(a) have to (b) need
(c) ought to (d) No improvement

› *(**a**)* 'Have to' should be used in place of 'are to', to improve the given sentence as 'have to' is used to express certainty, necessity and obligation.

57. I would have waited for you at the station if I <u>knew</u> that you would come.
(a) had known
(b) was knowing
(c) have known
(d) No improvement

› *(**a**)* 'Had known' (past imaginative case) is the correct expression in place of 'knew' to make the given sentence grammatically correct.

58. No one could explain how a calm and balanced person like him could <u>penetrate</u> such a mindless act on his friends.
(a) perpetuate (b) perpetrate
(c) precipitate (d) No improvement

› *(**b**)* Use of 'penetrate' is wrong here which means 'to go inside'. It should be 'perpetrate' which means 'to commit' or 'to be responsible for'. And it is the correct improvement in the context of the given sentence.

2014 (II)

59. As the parties failed to reach any agreement, the meeting <u>broke through.</u>
(a) broke out
(b) broke away
(c) broke up
(d) No improvement

› *(**b**)* Use of 'broke away' is suitable as it is a phrase which means 'to stop something'. 'Broke out', 'Broke up' have no sense here, so use of these phrases is unsuitable in this context.

60. If I <u>have reached</u> there by this time tomorrow, I will write to you immediately.
(a) reached (b) reach
(c) am reaching (d) No improvement

› *(**b**)* According to the proper tense structure, if any sentence contains condition, then we should use Present and Future tenses. So, use of 'reach' is proper here, rest all are not in Simple Present, so we should omit them.
Here is the correct conditional structure for an action that is probable in future:
If + Subject + V_1/V_5 + ... + Subject + Auxiliary verb + (shall/will) V_1 + Object.

61. My friend met me, unexpectedly, when I <u>looked</u> for a taxi, desperately.
(a) had looked
(b) was looking
(c) had been looking
(d) No improvement

› *(**b**)* Use of 'was looking' is appropriate here, according to the correct tense structure. When two simultaneous actions are in the past, then the syntax used is as follows :
When + Subject + Was/Were + [V_1 + ing] + Object + Subject + V_2 + Object.

62. Her learning makes up <u>towards</u> her want of beauty.
(a) for (b) to
(c) against (d) No improvement

› *(**a**)* Use 'makes up for' in place of 'makes up towards' as 'make up for' is used to compensate for something bad with something good.

63. If he <u>got</u> his enemy in his hand, he would have crushed him to death.
(a) has got (b) was getting
(c) had got (d) No improvement

› *(**c**)* Use of 'had got' is suitable here because here Past Perfect tense is required. Rest options are not useful as they are not in Past Perfect tense.
For a condition that didn't fulfil in the Past, we should use the following structure:
If + Subject + had + V_3 + ... + Subject + could/would/might + have + V_3.

64. Please ask your son to turn <u>off</u> the radio, so that it is not quite so loud.
(a) in (b) back
(c) down (d) No improvement

› *(**c**)* 'Turn off' means to switch the radio off. Here, there is a sense of low sound with radio still playing. Hence, 'turn down' is suitable because it means to lower the volume.

65. He said that when I <u>have worked</u> for him for six months I would get a pay rise.
(a) had worked (b) have had worked
(c) did work (d) No improvement

› *(**a**)* If two actions, out of which, one is of near past we should use Simple Past with it and with another which is of remote past we should use Past Perfect. Hence, 'had worked' should be used to improve the given sentence.

66. I am looking forward to <u>joining</u> your organisation.
(a) join (b) join in
(c) having joined (d) No improvement

› *(**d**)* The given sentence does not require any improvement.

67. I am not sure why <u>she is wanting</u> to see him.
(a) she wants (b) is she wanting
(c) she want (d) No improvement

› *(**a**)* Here, we should use 'she wants' because sense related verbs are always used in Present tense, not in progressive. So, option (a) is the correct choice.

68. Everybody who <u>finished</u> writing can go home.
(a) had finished (b) have finished
(c) has finished (d) No improvement

› *(**c**)* Use Present Perfect tense here because it is the requirement of the sentence. The action is completed recently, so except Present Perfect any other tense cannot be used. Subject denotes singular, so use of 'has finished' is appropriate.

2014 (I)

69. I don't remember exactly <u>when did I go</u> to Shimla last year.
(a) when I did go
(b) when I was going
(c) when I went
(d) No improvement

› *(**c**)* 'When I went' will be used in place of 'when did I go'. The use of Simple Past is correct as it is used to indicate an action completed in the past. It often occurs with adverbs or adverb phrases of past time.

70. <u>Even</u> he worked hard, he failed in the examination.
(a) Since (b) Although
(c) For (d) No improvement

› *(**b**)* Here 'Although' is the correct usage because in such sentences 'although' is used as a conjunction and it means 'in spite of the fact that'.

71. He was asked <u>to arbitrate with</u> two merchants in a dispute.
(a) to arbitrate between
(b) to arbitrate at
(c) to arbitrate
(d) No improvement

› *(**a**)* Here 'to arbitrate between' is the correct improvement. 'Arbitrate' means 'to settle dispute'. So, one arbitrates 'between two persons' not 'with two persons'.

72. He is definitely the cleverer <u>among</u> the two.
(a) in (b) of
(c) than (d) No improvement

› *(**b**)* 'Of' will be used instead of 'among'. The sentence compares only the two persons and so comparative degree should be used. Hence, 'of' is the correct improvement of the given sentence.

73. He <u>is resembling</u> his father.
(a) has resembled (b) was resembling
(c) resembles (d) No improvement

› *(**c**)* Here 'resembles' will be used. The use of Simple Present Tense is appropriate in the given sentence as the Simple Present Tense is used to describe people, things, etc.

74. Would you mind <u>closing</u> the window?
(a) to close (b) in closing
(c) for closing (d) No improvement

› *(**d**)* The sentence is correct and does not require any improvement.

75. I <u>can be</u> grateful if you could write to me about it.
(a) would be (b) should be
(c) must be (d) No improvement

› *(**a**)* Here 'would be' is the correct usage. In such sentences, 'would be' is used as the polite form of 'will be'.

76. A majority of the passengers <u>could have been saved</u> if the driver had applied the brakes in time.
(a) had been saved
(b) may have been saved
(c) could have saved
(d) No improvement

› *(**d**)* The given sentence is correct and does not require any improvement.

77. He <u>is living</u> in this house for the <u>past ten</u> years.
(a) was living
(b) had been living
(c) has been living
(d) No improvement

› *(**c**)* Here 'has been living' will be used because Present Perfect Continuous Tense is used for something which began at some time in the past and is still continuing.

78. He <u>is knowing</u> the secret.
(a) knows
(b) has been knowing
(c) was knowing
(d) No improvement

› *(**a**)* Here 'knows' is the most appropriate choice. Simple Present Tense suits best to the given sentence as the given sentence describes the situation.

79. The meeting has been <u>put off</u> to Friday next.
(a) put on
(b) put out
(c) put for
(d) No improvement

› *(**d**)* The given sentence is correct and does not require any improvement.

80. If you <u>will come with me</u> I shall be happy.
(a) come with me
(b) had come with me
(c) came with me
(d) No improvement

› *(**a**)* Here 'come with me' will be used because in the sentence the indicative mood is used in expressing a supposition which is assumed as a fact.

33

SENTENCE COMPLETION

Directions (Q. Nos. 1-103) *Each of the following sentences in this section has a blank space for word or group of words given after the sentence. Select the word or group of words you consider must appropriate for the blank space.*

2019 (II)

1. The Election Commission on Saturday …… that the Assembly elections in the five States will be held from November 12th to December 7th.

(a) pronounced (b) announced
(c) promulgated (d) issued

(*b*) 'Announced' is the appropriate word to fill the given blank.
Announce means 'make a formal public statement about a fact, occurrence or intention'.

2. The victims were fruit vendors and they were going in an auto when they … an accident on the way.

(a) met with (b) ran into
(c) experienced (d) heard

(*a*) According to the given sentence, 'Met with' is the correct alternative to fill the blank as it means 'to experience something, such as an accident'.

3. Scores of villagers are …… a sit-in protest against the consruction of a new underpass.

(a) performing (b) sitting
(c) staging (d) standing

(*c*) 'Staging' is the appropriate word for the given blank.
Staging means 'the method of presenting a play or other dramatic performance.'

4. It is common for patients to stop …… medicine as soon as they start feeling better.

(a) earing (b) gulping
(c) swallowing (d) taking

(*d*) According to the given sentence, option (d) 'taking' is the correct alternative to fill the blank as 'taking medicine' refers to swallow medicine. Other words are irrelevant.

5. A four-year-old girl got a new lease of life after doctors at a hospital …… a cancerous tumour from one of her kidneys.

(a) rejuvenated (b) removed
(c) displaced (d) replaced

(*b*) Here, 'removed' is the correct alternative to fill the given blank as 'removed' means 'take something off or away'. Other words do not fit in the blank.

2019 (I)

6. ..... forest prevent erosion.

(a) Lean (b) Dense (c) Sparse (d) Tidy

(*b*) 'Dense' is the appropriate word for the blank as it means 'containing a lot of things (trees) close together.' Other words do not fit in the blank.

7. Three people were arrested and an illegal arms unit was ..... by the police in a raid.

(a) revealed (b) searched
(c) discovered (d) busted

(*d*) 'Busted' is the appropriate word for the blank. It means 'raid or search by the police where illegal activity is suspected.'
Other words do not fit in the blank to make the sentence meaningful.

8. A woman got into the car and ... .

(a) drove off (b) broke down
(c) rode in (d) drove in

(*a*) 'Drove off' is the correct phrase to make the sentence meaningful. It means 'force someone to go away and to stop attacking or threatening you.'

9. The lecture was not very interesting. Infact I ..... in the middle of it.

(a) showed off (b) put off
(c) dozed off (d) plugged off

(*c*) 'Dozed off' is correct phrasal verb to be filled in the blank. It means 'to fall into a light sleep unintentionally.'

10. The cops ..... murder by kin.

(a) suspect (b) afford
(c) manage (d) administer

(*a*) 'Suspect' is the appropriate word for the blank to make the sentence meaningful. Suspect means 'to have doubts or mistrust.'
Other words are not relevant here.

2018 (II)

11. We fail to understand your reasons for …… the college without completing the degree.

(a) attending (b) joining
(c) leaving (d) refusing

(*c*) 'Leaving' is the correct word for the given blank. The word means 'to go away permanently.'

12. She …… her energy and started shouting only when she heard the noise of bulldozers and cranes.

(a) checked (b) conserved
(c) maintained (d) controlled

(b) 'Conserved' is the appropriate word for the given blank. The word 'conserved' means 'protect from harm and destruction.'

13. The British …… all over Africa and Asia collapsed in the first half of the twentieth century.

(a) hegemony (b) domicile
(c) residence (d) inheritance

(a) 'Hegemony' is the appropriate word to fill the given blank. Hegemony means 'leadership or dominance especially by one state or social group over others.'

14. The football match has to be …… because of the weather.

(a) called off (b) continued
(c) put off (d) turned off

(a) The phrase 'called off' means 'to cancel an event or agreement,' which is suitable according to the given sentence.

15. If I were rich, …… a lot.

(a) I'll travel (b) I can travel
(c) I would travel (d) I travelled

(c) 'I would travel' is the appropriate phrase for the given blank. The given sentence is conditional and the verb in the 'if' clause is in past tense. Hence, option (c) is its correct answer.

16. They apologised …… me for what happened.

(a) to (b) at (c) for (d) with

(a) With the word 'apologised', preposition 'to' should be used.

17. If you are tired of swimming, just …… for a while.

(a) struggle (b) stroke
(c) float (d) streak

(c) 'Float' is the appropriate word here as floating in the water will not tire anyone.

18. He had …… spoken for two minutes when there was a commotion in the crowd.

(a) even (b) hardly
(c) often (d) little

(b) 'Hardly' is the correct word here. As it means 'not much or scarcely'.

19. I would not commit myself …… that course of action.

(a) of (b) to
(c) by (d) with

(b) Here, preposition 'to' is appropriate to fill the given blank as it is used for expressing direction towards something. Other words are not relevant here.

20. The soldiers waiting to go into battle for the first time were full of …… .

(a) apprehension (b) consideration
(c) anticipation (d) frivolity

(a) 'Apprehension' is the suitable word for the given blank. The word means 'anxiety or fear that something bad may happen'.

2018 (I)

21. The tired traveller …… on in the hope of finding some resting place.

(a) strolled (b) scurried
(c) paraded (d) plodded

(d) The word 'plodded' means to walk with slow heavy steps. So, from the given options it is most suitable to fill the given blank.

22. The car was damaged beyond repair in the …… accident.

(a) outrageous (b) ghastly
(c) nasty (d) heinous

(b) 'Ghastly' means 'unpleasant and shocking.' So, it should be appropriate to fill the given blank.

23. They gave a …… dinner to celebrate the occasion, which impressed every guest.

(a) austere (b) public
(c) sumptuous (d) summary

(c) 'Sumptuous' means 'of high quality and often expensive'. So, from the given options it is the most suitable word to fill the given blank.

24. Once the …… manuscript is received by the publishers, it is typed in double space.

(a) total (b) full
(c) complete (d) filled

(c) 'Complete' is the most suitable word to fill the given blank. The word means 'having or including all parts.'

25. I am used to …… in queues.

(a) stand (b) standing
(c) stand up (d) standing still

(c) 'Stand up' means 'to remain sound and intact under stress, attack or close scrutiny'.
So, from the given options, it is the most suitable word to fill the given blank.

2017 (II)

26. He looks as if he ..... weary.

(a) is (b) was
(c) would be (d) were

(a) 'Is' will be used in the blank because it shows that the sentence is in Present tense.

27. My house is insured ..... theft and fire.

(a) for (b) against
(c) in (d) towards

(b) Among the given options, only the word 'against' is suitable with the word 'insured'. As insure against something means 'to guard or protect against something'.

28. The result of the prolonged discussion was..... .

(a) disappointment (b) disappointing
(c) disappointed (d) to disappoint

(b) 'Disappointing' is the correct word to be filled in the blank as it means 'not satisfactory or failing to fulfil one's hopes or expectations'.

29. You are lucky ..... in the 20th century.

(a) by being born (b) to have been born
(c) for being born (d) to have born

(b) 'In the given sentence, time reference is given, so Present Perfect Continuous tense will be used. Hence, option (b) 'to have been born' is the correct choice.

30. Sita is true to..... .

(a) word (b) her words
(c) the words (d) words

(b) 'her words' is the correct expression to be used in the sentence, as true to one's word is a phrase which means 'keeping one's promise'.

31. Years ..... since I saw her last.

(a) have passed
(b) had passed
(c) had been passing
(d) have been passing

(a) 'have passed' is the correct form of verb to be used in the sentence to make it meaningful. The Present Perfect tense is used to express a past event.

32. When he heard the terrible noise he asked me what was ..... on.

(a) happening (b) being
(c) getting (d) going

(d) 'going on' means 'happenings'. So going on will be used in the blank to make it meaningful and grammatically correct.

33. Could you lend me some money? I am very ..... of cash at the moment.

(a) down (b) low (c) short (d) scarce

(c) 'short of something' means 'not having enough of something'. So 'short' will be used in the blank to make the sentence meaningful.

34. I saw her when she was standing .....the side of the old statue.
(a) by (b) at
(c) in (d) beyond

(a) 'by' is the correct preposition to fill the given blank. 'By the side of' is a phrase which means 'close to'.

35. True friends never ..... their loved ones in adversity.
(a) abuse (b) criticise
(c) befool (d) desert

(d) 'desert' means 'leave', which fits in the context of the sentence. So, option (d) is the correct choice to fill the given blank.

2017 (I)

36. I ..... you to be at the party this evening.
(a) look forward to
(b) hope
(c) expect
(d) think

(c) When there is an informal anticipation, we should use 'expect'. Hence, option (c) is the correct answer.

37. When I met John yesterday, it was the first time I ..... him since Christmas.
(a) saw
(b) have seen
(c) had seen
(d) have been seeing

(c) Here, Past Perfect tense 'had seen' should be used in the given blank.
To describe one event or happening following another in the past, then Past Perfect tense is used to describe the earlier event and Past Indefinite tense is used for the event, that took place later on.

38. He ..... to listen my arguments and walked away.
(a) denied (b) disliked
(c) prevented (d) refused

(d) 'denied' and 'refused' are probable usages. But, here 'refused' is more appropriate usage. As 'denied' means refuse to give something requested or desired to someone. And 'Refused' means you will not do or accept something.

39. The flow of blood was so ..... that the patient died.
(a) intense (b) adequate
(c) profuse (d) extensive

(a) To specify the extent of something, 'intense' is the appropriate word.
Hence, option (a) is the correct answer.

40. You have never ..... me about your experiences in America.
(a) said (b) told
(c) explained (d) spoken

(b) There is no preposition after the blank. So, 'told' is the only suitable option.

41. I always felt hungry ..... I heard the dinner bell.
(a) as much as (b) as well as
(c) as soon as (d) as close as

(c) Here, the two actions are simultaneous. So, we should use 'as soon as' to fill the given blank.

42. Although they took every precaution, they could not ..... the accident.
(a) defer (b) allow
(c) avoid (d) block

(c) 'avoid' is the most appropriate usage in the blank as it gives the sense of making all the possible effort to null the change.

43. The ambitious nobleman ..... to marry the king's daughter.
(a) transpired (b) perspired
(c) aspired (d) expired

(c) 'Ambition' and 'Aspiration' have the same collocation.
Aspired means 'one's hopes or ambitions towards achieving something'. Hence, 'aspire' is more appropriate to fill the given blank.

44. The dictator of that country was a monster of wickedness, insatiable in his ..... for blood and plunder.
(a) idea (b) vision
(c) lust (d) intention

(c) When vices and negativities become the extreme, then 'lust' is the proper usage to fill the given blank.
Lust is a psychological force producing intense wanting or longing for an object or circumstance fulfilling the emotion.

45. Please don't give me any more, I have had ..... .
(a) few (b) too little
(c) little (d) enough

(d) When things are more than the required, 'enough' is the suitable word to be used. So, option (d) is the correct answer.

2016 (II)

46. She has been lying in bed for the last fortnight. I hope she will ..... .
(a) come out (b) pull out
(c) pull through (d) go out

(c) Phrasal verb 'pull through' means 'to recover from illness', which gives the correct sense here.

47. The path of progress is beset ..... difficulties.
(a) with (b) by
(c) through (d) along

(a) 'Beset' is always followed by preposition 'with'. So, 'with' is the correct word to fill the given blank.

48. I feel ..... for those, who are cruel to their children.
(a) sympathy (b) contempt
(c) admiration (d) craving

(b) The blank space needs a word with negative sense. Only 'contempt' is the negative word, rest are used in the positive sense.
Contempt means 'the feeling that a person or a thing is worthless or beneath consideration'.

49. Both the parties were keen to have an ..... settlement of the dispute.
(a) enviable (b) inimical
(c) worthy (d) amicable

(d) Settlement of dispute needs to be in an 'amicable' manner.
The word 'Amicable' means friendliness or goodwill between people or groups and absence of discord. Hence, option (d) is suitable usage.

50. I have come to know that the two brothers have ..... .
(a) fallen through (b) fallen out
(c) fallen for (d) fallen short

(d) Phrasal verb 'fallen short' means 'being unsuccessful', fits in the blank properly. So, option (d) is the suitable usage.

51. Everybody finds his own work ..... whereas he feels that others have delightful jobs.
(a) tedious (b) fabulous
(c) unprofitable (d) indecent

(a) The blank should be filled with a contrasting word because the next part has a contrasting word (delightful). So, option (a) 'tedious' is suitable word to fill the given blank.
'Tedious' means boring or tiresome.

52. The accident took place because of the criminal ..... of the driver.
(a) performance (b) disregard
(c) negligence (d) slackness

*(**c**)* Accident is an outcome of 'negligence' of the driver So, option (c) is the proper usage for the given blank.

53. The belief in the ..... of vaccination is gaining ground.
(a) immunity (b) prevalence
(c) efficacy (d) workability

*(**c**)* Word 'efficacy' means 'fruitfulness or potency', which fits in the sense of the given sentence.

54. Our new leader is a ..... young man and will take us forward.
(a) haughty (b) intoxicated
(c) fanciful (d) dynamic

*(**d**)* Here, 'dynamic' is the appropriate word for the given blank and gives the sentence its true meaning.
Dynamic means 'positive in attitude and full of energy and new ideas'.

55. They found a world of ..... between what he said and what he did.
(a) chaos (b) bitterness
(c) difference (d) hope

*(**c**)* 'Difference' is the appropriate word for the given blank, according to the context of the sentence. 'World of difference' refers to two different things being compared which have a vast difference between them.

2016 (I)

56. When the bus was at full speed, its brakes failed and an accident was .....
(a) inevitable (b) undeniable
(c) fatal (d) miserable

*(**a**)* 'Inevitable' is the appropriate word for the given blank and it means 'difficult to avoid or prevent'.

57. To explain his design to his visitors, the architect ..... a simple plan on the blackboard.
(a) built
(b) finalised
(c) sketched
(d) arranged

*(**c**)* 'Sketch' means to draw a simple outline giving less details. Therefore, option (c) suits the context of the sentence.

58. Though Bonsai, a well-known art form, originated in China, it was ..... by the Japanese.
(a) cultivated (b) finished
(c) perfected (d) enlarged

*(**c**)* 'Perfected' means 'to complete without fault'. Therefore, option (c) completes the sentence meaningfully.

59. He is greatly admired for his ..... behaviour.
(a) decrepit (b) decadent
(c) decorative (d) decorous

*(**d**)* To fill the given blank, 'decorous' is the appropriate word.
Decorous means 'polite or restraint', which suits the context of the given sentence.

60. Would you mind if you ..... to the principal how the trouble started?
(a) remarking (b) saying
(c) explaining (d) talking

*(**c**)* 'Explain' means 'to express in a detailed and easy manner.' Therefore, option (c) completes the meaning of the given sentence.

61. Vaccination will make people immune ..... certain diseases for a given period.
(a) against (b) to (c) with (d) for

*(**b**)* Preposition 'to' is appropriate here to fill the given blank. The preposition 'to' is used as an infinitive of purpose to mean 'in order to'.

62. The two boys looked so alike that it was impossible to ..... between them.
(a) discriminate (b) discern
(c) distinguish (d) identify

*(**c**)* 'Distinguish' means 'difficult to understand the difference.' Therefore, option (c) is the correct choice here.

63. The campers ..... their tents at the base of the mountain.
(a) installed (b) dug
(c) pitched (d) established

*(**c**)* The word 'pitch' means 'to set-up and camp for a short time.' Therefore, option (c) suits the context of the given sentence.

64. The enemy had captured him and his life was at stake, still he refused to ..... the state secrets.
(a) divulge
(b) divert
(c) indulge
(d) invert

*(**a**)* 'Divulge' means 'to reveal or make known private or sensitive information'. Therefore, option (a) suits the context of the given sentence.

2015 (II)

65. Man has won his dominant position on this planet by his ..... of technology.
(a) command (b) emphasis
(c) belief (d) stress

*(**a**)* Domination is shown by the use of 'command'. Hence, option (a) is the appropriate word to fill the given blank.

66. The day was extremely hot and, in no time, my back was drenched with ..... .
(a) prickly heat (b) perspiration
(c) sores (d) fatigue

*(**b**)* 'Prickly heat', 'sores' and 'fatigue', all are the outcomes of strong heat. But with the word 'drenched', we can't use any of them. 'Perspiration' means 'sweat' that can drench someone, hence, it is suitable for the given blank.

67. The government is encouraging village upliftment ..... in the country.
(a) programmes (b) designs
(c) talks (d) propaganda

*(**a**)* In the given sentence, with respect to the word 'encouraging', 'programmes' is the suitable word to fill the blank. Encouraging is used for giving hope or promise.

68. Satish was endowed ..... a natural talent for music.
(a) in (b) by (c) for (d) with

*(**d**)* 'Endowed' is always followed by the preposition 'with'. Hence, option (d) is its correct answer.

69. Mr Ghosh is very happy ..... his son's excellent result.
(a) for seeing (b) to see
(c) by seeing (d) see

*(**b**)* Use infinitive 'to see' to state the reason/purpose involved in it. Hence, 'to see' is the correct word to fill the given blank.

70. Sunlight filtering ..... the stained glass window created a mosaic of colours on the floor.
(a) in (b) through (c) at (d) into

*(**b**)* 'Filter' is followed by preposition 'through' and it makes the sentence meaningful.

71. Are you feeling doubtful …… your decision?
(a) about (b) upon
(c) at (d) for

(a) 'Doubtful' is followed by the preposition 'about'. Hence, option (a) is the correct answer.

72. I want to study Geology now for I …… Zoology for the last three years.
(a) am studying
(b) have been studying
(c) had studied
(d) had been studying

(b) When time reference has been given, then Present Perfect Continuous tense is used. So, 'have been studying' is appropriate one.

73. Increase in storage facilities has made it possible to store goods at places …… to people all over the country.
(a) safe (b) easy
(c) proper (d) convenient

(d) 'Safe', 'easy' and 'proper', all are nearly appropriate to be used. But, 'convenient' alone is enough to convey the essence of all the words given in the option. Hence, option (d) is the correct choice.

74. To say that we were surprised at the cleverness of the child is an understatement; we were really …… .
(a) annoyed (b) astounded
(c) flattered (d) confused

(b) Sentence gives the idea of cunningness of a child that is really a surprise. Hence, we should provide such a word in the blank that gives the idea of the surprise. The only suitable word for the surprise is 'astounded'. 'Astounded' means 'shock or greatly surprise'.

75. Inspite of the old woman's repeated entreaties, he remained …… .
(a) ashamed (b) docile
(c) indifferent (d) lethargic

(c) 'Indifferent' means 'not being interested' and this meets the sense of the sentence which involves repeated requests. Rest options are out of context.

76. There was …… competition for electoral seats.
(a) diligent (b) rapid
(c) cut-throat (d) sparse

(c) The word 'cut-throat' is suitable here as it shows the level of the competition. Cut-throat means ''competing in a strong and unfair way and makes the sentence meaningful''.

77. The cricket team …… mainly of State players.
(a) composed
(b) consist
(c) made with
(d) comprises

(d) 'Comprises' gives the proper sense to the sentence. Comprises means 'consist of or to be the part of.' Rest of the options need preposition with them.

78. His love for money is the only …… that drives him to work so hard.
(a) programme (b) plan
(c) reason (d) greed

(c) The sentence demands an explanation for the cause. It can be fulfilled by supplying the word 'reason', which indicates the explanation for his working hard.

79. According to …… sources, the Chief Minister did not consult his Cabinet Members on this issue.
(a) intelligent (b) simple
(c) reliable (d) fundamental

(c) When something is confidential and of utmost importance, then sources is said to be 'reliable'. Hence, option (c) is the appropriate word to fill the given blank.

2014 (II)

80. You haven't had your lunch yet, ... you?
(a) are (b) aren't
(c) have (d) haven't

(c) Here, the subject of the sentence (You haven't) is negative. And negative statement is followed by a positive question tag. Hence, 'have' is the correct one to fill the blank.

81. Life is to death as pleasure is to .....
(a) poverty (b) suffering
(c) anguish (d) pain

(d) Here, use of 'pain' is suitable as we know that 'pain' is opposite to the word 'pleasure' as per antonym analogy. Rest words are not the antonyms of the word 'pleasure'.

82. The French ..... reputed to have a very good sense of humour.
(a) is (b) was
(c) are (d) will be

(c) Here, a complete race is addressed which is obviously a plural subject, so we will use plural helping verb according to the subject for proper sentence formation, i.e. 'are'.

83. 'Please' and 'Thank you' are the little courtesies by which we keep the ..... of life oiled and running smoothly.
(a) path (b) machine
(c) garden (d) river

(b) Use of 'machine' is proper here because it needs oiling for smooth run. So, use of 'machine' is suitable here and it is the correct answer.

84. Many of the advances of civilisation have been conceived by young people just on the ..... of adulthood.
(a) boundary
(b) threshold
(c) peak
(d) horizon

(b) We get the sense of beginning phase of the stated stage of life, so we should use word 'threshold'. Here, threshold means 'the entrance or starting of something.'

85. The more your action and thought are allied and ....., the happier you grow.
(a) divergent
(b) unravelled
(c) integrated
(d) invincible

(c) According to the sense of sentence, here we should use a collective term which can show the coupled relation of both words. So, use of 'integrated' is proper here, as it means 'with various parts or aspects linked or coordinated.'

86. He ..... in wearing the old-fashioned coat in spite of his wife's disapproval.
(a) insists (b) persists
(c) desists (d) resists

(b) Use of 'persists' is proper here as it gives required sense to go on continually or stubbornly inspite of opposition, importunity or warning.

87. It is not what you say that ....., but what you do.
(a) matches (b) implies
(c) matters (d) moves

(c) *(c)* Use of 'matters' gives a proper sense here. Use of 'implies' is not suitable here because it is not according to context of sentence. 'Matches' and 'moves' also give non-contextual meaning here, so we should use 'matters' for complete sense.

88. Physically we are now all neighbours, but psychologically we are ..... to each other.

(a) primitives (b) complementary
(c) strangers (d) cowards

(c) *(c)* According to sense of the sentence, we need a word opposite to 'neighbours', which is 'strangers'. So, 'strangers' is the correct word for the given blank.

89. The old 'Nature versus .....' debate regarding crime continues even today.

(a) man (b) universe
(c) culture (d) nurture

(c) *(d)* Best suitable phrase would be 'Nature versus nurture' when it is related to crime. Nature is something that reflects the internal motto while 'nurture' reflects the idea of the upbringing. Crimes are often an outcome of a poor and faulty upbringing. Hence 'Nature versus nurture' is suitable.

2014 (I)

90. When the police arrived, the thief.....away.

(a) strolled (b) moved
(c) galloped (d) ran

(c) *(d)* At the sudden arrival of the police, a thief will ran away, he will not stroll or just move. Hence, 'ran' is the suitable word for the given blank.

91. His.....as an officer was not quite satisfactory.

(a) acting (b) doing
(c) deed (d) record

(c) *(d)* 'Record' is the correct usage here to fill the given blank according to the context of the sentence. Here 'record' refers to an official document that records the acts of a public body or officer.

92. The two children stood at the.....of the table.

(a) conclusion (b) terminal
(c) end (d) termination

(c) *(c)* Here with respect to 'table', 'end' is the most suitable usage in the given sentence. Terminal describes a bus or train station at the end of the line.

93. The old man was.....weak to open the door.

(a) very (b) far
(c) much (d) too

(c) *(d)* 'Too' will be the most appropriate use here as the sentence has 'to' after the word 'weak'. 'Too' is followed by an adjective+to infinitive.

94. Universities in Germany and Denmark will have an input ..... the project.

(a) for (b) into
(c) about (d) at

(c) *(c)* Here, 'input' means in general. So, 'about' will be the most appropriate use here.

95. He gave me a leg ..... when I was completely new to the business.

(a) up (b) down
(c) off (d) for

(c) *(a)* 'Give somebody a leg up' means 'to help someone to be more successful'. So, 'up' is the most suitable word to fill the given blank.

96. You should have nothing to do with those criminals, they all have violent attitudes and long records. They are men ..... the same kidney.

(a) with (b) in
(c) of (d) on

(c) *(c)* 'Of the same kidney' means 'alike, identical or equivalent'. So, it is the appropriate word for the given blank.

97. He didn't have the guts to stand ..... his friend when he was in trouble.

(a) with (b) for
(c) by (d) beside

(c) *(c)* Here 'to stand by' means 'to support'. So, 'by' is the correct word to fill the given blank.

98. He broke out of the prison ..... dressing as a woman.

(a) in (b) as
(c) by (d) with

(c) *(c)* 'By' here implies through action or means of. So, it is the appropriate word for the given blank.

99. When electricity failed, emergency generators .....

(a) cut out (b) cut off
(c) cut on (d) cut in

(c) *(b)* Here 'cut off' is the appropriate one to fill the blank and it means 'an electricity cut off'. Hence, option (b) is the correct usage.

100. We need two more hands to ..... the heavy rush of work.

(a) cope up (b) cope with
(c) cope to (d) cope in

(c) *(b)* 'Cope with' means 'deal effectively or manage'. Hence, option (b) is the correct usage.

101. Twenty-five candidates ..... each other for the first prize.

(a) compete for
(b) compete with
(c) compete to
(d) compete on

(c) *(b)* With respect to the given context (Twenty-five candidates) 'compete with' is the suitable word for the given blank. Compete with someone or something seems to imply a loss targeted, more individualist and less antagonist form of competition.

102. We must eliminate the ..... rich and poor.

(a) disparity between (b) disparity for
(c) disparity in (d) disparity from

(c) *(a)* According to the given context of the sentence 'Disparity between' rich and poor means difference between rich and poor.

103. As an innovator, he ..... the beaten track and explored religions.

(a) deviated from (b) deviated to
(c) deviated in (d) deviated for

(c) *(a)* 'One deviate from a track' is the correct usage. Thus, after the word 'deviate', preposition 'from' is used.

34

RECONSTRUCTION OF SENTENCES

Directions (Q. Nos. 1-40) *Each of the following items in this section consists of a sentence, the parts of which have been jumbled. These parts have been labelled P, Q, R and S. Given below each sentence have four sequences namely (a), (b), (c) and (d). You are required to re-arrange the jumbled parts of the sentence and mark your response accordingly.*

2019 (II)

1. closely monitor the situation (P) by the disturbing (Q) allegations we are deeply concerned (R) and will continue to (S)

(a) R Q S P (b) S P Q R (c) S P R Q (d) P R S Q

➢ ***(a)*** RQSP is the correct sequence.

2. relieve nausea, pain and stress (P) aromatherapy may (Q) also help (Q) using lavender oil on their skin (R) but patients are cautioned against (S)

(a) R Q P S (b) S Q P R (c) Q P S R (d) R P S Q

➢ ***(c)*** QPSR is the correct sequence.

3. without any mandate of law (P) the plea filed through the advocate (Q) online is illegal and submitted (R) that the sale of drugs and prescription medicines (S)

(a) R Q S P (b) Q S R P
(c) Q R S P (d) P R S Q

➢ ***(b)*** Q S R P is the correct sequence.

4. humans and machines have co-existed (P) dating back to the invention (Q) of the potter's wheel in ancient (R) Mesopotamia at the physical level for millennia. (S)

(a) R S P Q (b) Q S P R (c) Q R P S (d) P S Q R

➢ ***(d)*** P S Q R is the correct sequence.

5. a sell-off as US Treasury yields (P) surged to multi-year highs (Q) on robust economic data (R) global markets also witnessed (S)

(a) R S Q P (b) P Q S R
(c) Q R P S (d) S P Q R

➢ ***(d)*** SPQR is the correct sequence.

2019 (I)

6. the urban local body elections (P)/unidentified gunmen (Q)/and injured another during (R)/ shot dead two workers (S)

(a) Q S R P (b) P Q S R (c) S P R Q (d) R P S Q

➢ ***(a)*** QSRP is the correct sequence.

7. both intense political and (P)/ this state has a history of (Q)/of syncretic accomplishments (R)/ religious contestation and (S)

(a) S Q P R (b) P Q S R (c) S Q R P (d) Q P S R

➢ ***(d)*** QPSR is the correct sequence.

8. the father also (P)/in his quest for justice (Q)/ by the system (R)/ feels let down (S)

(a) S Q P R (b) P S R Q (c) S Q R P (d) P Q R S

➢ ***(b)*** PSRQ is the correct sequence.

2018 (II)

9. Domestic fires in Indian villages
P. as the heat is dispersed
Q. in unhealthy smoke accumulates in unventilated houses
R. are wasteful
S. lit in Chulhas
The correct sequence should be
(a) PQSR (b) PQRS (c) SPQR (d) SRPQ

➢ *(**d**)* SRPQ is the correct sequence.

10. Bold rocks near at hand
P. makes a fine forest for the imagination
Q. than distant Alps
R. are more inspiring
S. and the thick fern upon a heath
The correct sequence should be
(a) QRSP (b) RSQP (c) RQSP (d) SRQP

➢ *(**c**)* RQSP is the correct sequence.

11. Newton discovered that
P. due to the gravitational pull
Q. of the earth
R. the apple falls
S. on the ground
The correct sequence should be
(a) QPRS (b) RSPQ (c) QSPR (d) SQRP

➢ *(**b**)* RSPQ is the correct sequence.

12. The knocking
P. were still in the house Q. for some time
R. although the echoes of it S. ceased
The correct sequence should be
(a) SRPQ (b) SQRP (c) PQSR (d) RSPQ

➢ *(**b**)* SQRP is the correct sequence.

13. The history of mankind is
P. and steady progress
Q. continuous change
R. the history of
S. from barbarism to refinement
The correct sequence should be
(a) PQRS (b) RQPS (c) QSPR (d) SQRP

➢ *(**b**)* RQPS is the correct sequence.

2017 (II)

14. The spirit of man has slowly and painfully
P
surmounted and his growing intelligence
Q
all the obstacles that have come in his way
R
has faced all kinds of danger
S
(a) Q P S R (b) S Q P R (c) R P Q S (d) P R Q S

➢ *(**b**)* SQPR is the correct sequence.

15. After our school boys had won a well-contested hockey match
so that they might communicate the
P
news of their victory to the
headmaster who is a keen sportsman
Q
they came to school in high spirits
R
and takes a very lively interest in school games
S
(a) Q P S R (b) S Q P R (c) R P Q S (d) P R Q S

➢ *(**c**)* R P Q S is the correct sequence.

16. Even a leisurely game like cricket demanding grace rather than strength
P
and over the rough tactics of the Australian team
Q
that visited England in 1921 as we saw in the
R
controversy over body-line bowling
can cause much ill-will
S
(a) P S R Q (b) R S P Q
(c) S R Q P (d) Q P R S

➢ ***(a)*** PSRQ is the correct sequence.

17. Scientists point out of sunspot activity
P
that it is an aftermath that
Q
has now reached its peak
R
of the eleven-year cycle
S
(a) R S P Q (b) P Q S R
(c) Q R P S (d) Q S P R

➢ *(**c**)* QRPS is the correct sequence.

18. As the ship streams from San Diego as walls of gray water from a distant storm in the North Pacific
P
making the greener among us
Q
miserable with sea sickness rock and toss the ship
R
those of us aboard have a personal demonstration
S
of powerful ocean movement

(a) P Q R S (b) S R P Q
(c) S P R Q (d) Q S R P

(c) (***c***) SPRQ is the correct sequence.

2017 (I)

19. Farm workers (P)/ spend (Q)/ outdoors (R)/ most of their time (S)

The proper sequence should be

(a) P Q R S (b) P R Q S
(c) P Q S R (d) R P Q S

(c) (***c***) PQSR is the correct sequence.

20. He (P)/ shuffled the papers (Q)/ in a drawer (R)/ together (S)

The proper sequence should be

(a) P Q S R (b) P Q R S (c) P S Q R (d) R S P Q

(c) (***a***) PQSR is the correct sequence.

21. Do you think (P)/ will (Q)/ this soap (R)/ shrink woollen clothes (S) ?

The proper sequence should be

(a) P Q R S (b) P R Q S (c) Q R P S (d) Q P R S

(c) (***b***) PRQS is the correct sequence.

22. We advised (P)/ the hijackers to surrender (Q)/ to the police (R)/ themselves (S)

The proper sequence should be

(a) P Q R S (b) P R Q S (c) P Q S R (d) R P Q S

(c) (***c***) PQSR is the correct sequence.

23. Sports cars (P)/ appeal (Q)/ to some motorists only (R)/ with noisy exhausts (S)

The proper sequence should be

(a) R Q S P (b) P S Q R (c) R S P Q (d) P Q S R

(c) (***b***) PSQR is the correct sequence.

24. He (P)/ almost (Q)/ planned the entire strategy of operation (R)/ single-handed (S)

The proper sequence should be

(a) R S P Q (b) P R Q S (c) S Q R P (d) Q P S R

(c) (***b***) PRQS is the correct sequence.

25. She has (P)/ more (Q)/ intelligence (R)/ than we suspected her to possess (S)

The proper sequence should be

(a) P Q S R (b) P R Q S
(c) P S Q R (d) P Q R S

(c) (***d***) PQRS is the correct sequence.

26. They (P)/ should implant (Q)/ in the minds of young children (R)/ sound principles (S)

The proper sequence should be

(a) P Q R S (b) P Q S R
(c) R P Q S (d) P R Q S

(c) (***b***) PQSR is the correct sequence.

27. When I was a student (P)/ I learnt swimming at the age of 15 (Q)/ of class X (R)/ in a government school (S)

The proper sequence should be

(a) P Q R S (b) P R Q S
(c) Q P R S (d) Q S P R

(c) (***c***) QPRS is the correct sequence.

2016 (II)

28. The statement therefore you must listen carefully (P)/what the speaker has said (Q)/in order to understand (R)/will be made just once. (S)

The proper sequence should be

(a) SPRQ (b) SRQP (c) RSPQ (d) SPQR

(c) (***a***) SPRQ is the correct sequence.

29. The small boy who broke his leg (P)/fell down (Q)/and (R)/climbed the wall. (S)

The proper sequence should be

(a) PSRQ (b) SQRP (c) QSRP (d) SPRQ

(c) (***b***) SQRP is the correct sequence.

30. According to an engineer might hit the market next year (P)/a newly developed air-cooler system (Q)/that employed in conventional room coolers (R)/, which is based on a principle radically different from. (S)

The proper sequence should be

(a) SQRP (b) RSQP
(c) QSRP (d) PQRS

(c) (***c***) QSRP is the correct sequence.

31. The clerk on the desk (P)/left the money (Q)/in the safe (R)/which he should have locked up. (S)

The proper sequence should be

(a) SRQP (b) QRSP
(c) PQRS (d) QPSR

(c) (***d***) QPSR is the correct sequence

32. Hardly had my brother descended from the plane when the people (P)/waved and cheered (Q)/who had come to receive him (R)/from the lounge. (S)

The proper sequence should be

(a) PQRS (b) RSPQ
(c) RPQS (d) PRQS

› *(**d**)* PRQS is the correct sequence.

33. The essay like that of the human face (P)/because its variety is infinite (Q)/is a literary form (R)/which is not easy to define. (S)

The proper sequence should be

(a) PQRS (b) PQSR
(c) RSQP (d) RSPQ

› *(**c**)* RSQP is the correct sequence.

34. The exhibition committee attractive and useful (P)/to make exhibition (Q)/making efforts (R)/has been. (S)

The proper sequence should be

(a) QSRP (b) SRQP
(c) QPSR (d) SPQR

› *(**b**)* SRQP is the correct sequence.

2015 (II)

35. His uncle for success in life, (P) always advised his son, (Q) who was a self-made man (R) to depend on his own efforts (S)

The proper sequence should be

(a) SQPR (b) RQSP
(c) PRSQ (d) QPSR

› *(**b**)* RQSP is the proper sequence.

36. The doctor did not like the behaviour of the patients, (P) who was very competent in his profession (Q) when they talked at length, (R) about their problems (S)

The proper sequence should be

(a) RPSQ (b) SRPQ
(c) QPRS (d) PRQS

› *(**c**)* QPRS is the proper sequence.

37. from leadership in culture in military situations (P)
and in face-to-face small groups (Q)
leadership has wide range of expressions (R)
to leadership in politics (S)

The proper sequence should be

(a) RSQP (b) PQRS (c) RPSQ (d) SQRP

› *(**c**)* RPSQ is the proper sequence.

38. He sat glancing occasionally (P)
peering through the window, (Q)
at the figure of the old woman, (R)
until he was chilled with the cold (S)

The proper sequence should be

(a) PSRQ (b) QRPS (c) SPRQ (d) PRSQ

› *(**b**)* QRPS is the proper sequence.

39. After the earthquake tremors, the TV showed a haggard man
shaking his fist at the sky (P) clambering over the ruins (Q)
and collapsing with a howl of revolt (R)
of his house and factory (S)

The proper sequence should be

(a) SRQP (b) QSPR (c) PQRS (d) RPSQ

› *(**b**)* QSPR is the proper sequence.

40. Everyone acknowledges (P) when he considers the case calmly (Q)
who knows you (R) that you have been wronged (S)

The proper sequence should be

(a) RSQP (b) RPSQ (c) PQRS (d) QRPS

› *(**b**)* RPSQ is the proper sequence.

35

REARRANGING SENTENCES IN A PARAGRAPH

Directions (Q.Nos. 1-13) *In this section, each item consists of six sentences of a passage. The first and sixth sentences are given in the beginning as S1 and S6. The last four sentences in each have been jumbled up and labelled as P, Q, R and S. You are required to find the proper sequence of the four sentences.*

2019 (I)

1. S1 : We do not know what to do with our knowledge.
S6: In the course of time they may rule over us altogether.
P : For example, we are unable to manage our machines.
Q : We already find it difficult to do without machines.
R : Machines should be fed properly and waited upon attentively; otherwise they refuse to work or cause destruction.
S : Science has given us superhuman powers, which we do not use properly.

The proper sequence should be
(a) S P R Q (b) P S Q R
(c) Q R P S (d) S R P Q

› ***(a)*** SPRQ is the correct sequence of the given sentences to form a meaningful passage.

2. S1 : The British rule in India has brought about moral, material, cultural and spiritual ruination of this great country.
S6 : We are not to kill anybody but it is our *dharma* to see that the curse of this government is blotted out.
P : I regard this rule as a curse.
Q : Sedition has become my religion.
R : Ours is a non-violent battle.
S : I am out to destroy this system of government.

The proper sequence should be
(a) S P R Q (b) P S Q R
(c) Q R P S (d) S R P Q

› ***(b)*** PSQR is the correct sequence of the given sentences to form a meaningful passage.

2018 (I)

3. S1: The Subsidiary Alliance system was extremely advantageous to the British.
S6: They controlled the defence and the foreign relations of the protected ally.
P : They could now maintain a large army at the cost of Indian states.
Q : if many war occurred in the territories
R : either of the British ally or of the Britishers
S : This enabled them to fight wars far away from their own territories

The proper sequence should be
(a) P Q R S (b) P S Q R
(c) Q R P S (d) S R P Q

› ***(b)*** PSQR is the correct sequence of the given sentences to form a meaningful passage.

4. S1: In reality, by signing a Subsidiary Alliance, an Indian State virtually signed away its independence.
S6: Infact, the Indian ruler lost all vestiges of sovereignty in external matters.
P : of maintaining diplomatic relations
Q : It lost the right of self-defence,
R : with its neighbours
S : and of settling its disputes

The proper sequence should be
(a) P Q R S (b) R S P Q
(c) Q P S R (d) Q S R P

› ***(c)*** QPSR is the correct sequence of the given sentences to form a meaningful passage.

5. S1: A mighty popular Revolt broke out in Northern and Central India in 1857
S6: Millions of peasants, artisans and soldiers fought heroically and wrote a glorious chapter.
P : Sepoys or the Indian soldiers of the Company's army
Q : but soon engulfed wide regions and involved the masses
R : and nearly swept away the British rule
S : It began with a mutiny of the

The proper sequence should be

(a) R S P Q (b) P Q R S
(c) S R P Q (d) Q R P S

› ***(a)*** RSPQ is the correct sequence of the given sentences to form a meaningful passage.

6. S1: The Indian Civil Service gradually developed into one of the most efficient and powerful civil services in the world.
S6: though these qualities obviously served. British and not Indian interests.
P : and often participated in the making of policy
Q : independence, integrity and hard work
R : They developed certain traditions of
S : Its members exercised vast power

The proper sequence should be

(a) P Q R S (b) Q R S P
(c) R S Q P (d) S P R Q

› ***(d)*** SPRQ is the correct sequence of the sentences to form a meaningful passage.

7. S1: The ruin of Indian handicrafts was reflected in the ruin of the towns and cities which were famous for their manufactures.
S6 : Centres were developed and laid waste.
P : Dacca, Surat, Murshidabad and many other rising industrial
Q : ravages of war and plunder, failed to
R : survive British conquest
S : Cities which had withstood the

The proper sequence should be

(a) P Q R S (b) S Q R P
(c) S R P Q (d) Q R S P

› ***(b)*** SQRP is the correct sequence of the sentences to form a meaningful passage.

2015 (I)

8. S1 : At the roadside, the driver will be asked to blow through a small glass tube into a plastic bag.
S6 : The driver will be asked to go to the police station.
P : and if the colour change does not reach the line, the driver cannot be punished under the new law
Q : Inside the tube are chemically treated crystals which change colour if the driver has alcohol on his breath
R : but if the colour change does reach the line then the test has proved positive
S : if the colour change goes beyond a certain line marked on the tube, this indicates that the driver is probably over the specified limit

The proper sequence should be

(a) PSQR (b) SQRP
(c) RPSQ (d) QSPR

› ***(d)*** QSPR is the correct sequence of the given sentences to form a meaningful passage.

9. S1: Hope springs eternal in the heart of man.
S6: This is the central idea of the poem.
P : but hope is everlasting
Q : love, friendship and youth perish
R : it is nursed by the glorious elements of nature
S : man derives hope from nature in his gallant struggle after some noble ideal

The proper sequence should be

(a) QPRS (b) SRQP
(c) RSQP (d) QPSR

› ***(c)*** RSQP is the correct sequence of the sentences to form a meaningful passage.

10. S1: Mr. Sherlock Holmes and Dr. Watson were spending a weekend in a University town.
S6: It was clear that something very unusual happened.
P : one evening, they received a visit from an acquaintance, Mr. Hilton Soames
Q : on that occasion, he was in a state of great agitation
R : they were staying in furnished rooms, close to the library
S : Mr Soames was a tall, thin man of a nervous and excitable nature

The proper sequence should be

(a) PRSQ (b) RPSQ
(c) PQRS (d) RPQS

› ***(b)*** RPSQ is the correct sequence of the sentences to form a meaningful passage.

11. S1: The machines that drive modern civilisation derive their power from coal and oil.
S6: Nuclear energy may also be effectively used in this respect.
P : but they are not inexhaustible
Q : these sources may not be exhausted very soon
R : a time may come when some other sources have to be tapped and utilised
S : power may, of course, be obtained in future from forests, water, wind and withered vegetables

The proper sequence should be

(a) PQRS (b) QPRS
(c) SRQP (d) SPQR

› ***(b)*** QPRS is the correct sequence of the sentences to form a meaningful passage.

12. S1 : The body can never stop.
S6 : It comes from food.
P : to support this endless activity, the body needs all the fuel for action
Q : sometimes, it is more active than at other times, but it is always moving
R : even in the deepest sleep, we must breathe
S : the fuel must come from somewhere

The proper sequence should be

(a) PQRS (b) PRQS
(c) QRPS (d) SRQP

› ***(c)*** QRPS is the obvious choice of the sentences to form a meaningful passage.

36

SYNONYMS

Directions (Q. Nos. 1-65) *Each item in this section consists of a sentence with an underlined word followed by four options. Select the option that is nearest in meaning to the underlined word.*

2019 (II)

1. He is always <u>anxious</u>.

(a) worried (b) dispassionate
(c) sluggish (d) torpid

(a) *(**a**)* 'Worried' is the correct synonym of 'anxious'. Both words mean 'feeling uneasy about something with an uncertain outcome.'
Some other synonyms are - uneasy, agitated, etc.

2. The poems of Kabir are <u>ecstatic</u> in nature.

(a) efficacious (b) eerie
(c) rapturous (d) reverential

(c) (**c**) 'Rapturous' is the correct synonym of 'ecstatic'. Both words mean 'feeling or expression overwhelming happiness'.
Some other synonyms are - joyful, overjoyed, etc.

3. Ravi loves <u>seclusion</u>. Therefore, he lives in the mountain.

(a) nature (b) scripture
(c) seafaring (d) solitariness

(d) *(**d**)* 'Solitariness' is the correct synonym of 'seclusion'. Both words mean 'being separate and apart from others'.
Some other synonyms are - solitude, isolation, etc.

4. Hitler was a <u>despot</u>.

(a) conservative (b) dictator
(c) passionate (d) monstrous

(b) (**b**) 'Dictator' is the correct synonym of 'despot'. Both words refer to 'a ruler or other person who holds absolute power.'
Some other synonyms are - autocrat, authoritarian, etc.

5. The imagery used in the poem is <u>vivid</u>.

(a) lively (b) inert
(c) ebullient (d) caustic

(a) (**a**) 'Lively' is the correct synonym of 'vivid'. Both words refer to 'something which is very clear and true to life.'
Some other synonyms are - realistic, lifelike, etc.

6. A human being is always <u>vulnerable</u> to other human beings.

(a) resilient (b) elastic
(c) defenceless (d) crude

(c) *(**c**)* 'Defenceless' is the correct synonym of 'vulnerable'. Both words mean 'open to attack or damage.'
Some other synonyms are - unprotected, unguarded, etc.

7. His language is political and <u>vitriolic</u>.

(a) imaginative (b) sprightly
(c) vivacious (d) abusive

(d) *(**d**)* 'Abusive' is the correct synonym of the word 'vitriolic'. Both words mean 'violent hate and anger expressed through severe criticism'.
Some other synonyms are - acrimonious, bitter, etc.

8. The Managing Director of the company declared that he is <u>broke</u> and there is a need to seek support from the government.

(a) bankrupt
(b) rich
(c) making profit
(d) having liabilities

(a) *(**a**)* 'Bankrupt' and 'broke' are synonyms. Both words mean 'having completely run out of money.'
Some other synonyms are - insolvent, penniless, etc.

9. He loves doing <u>nasty</u> things.

(a) nice (b) fastidious
(c) foul (d) finicky

(c) *(**c**)* 'Foul' is the correct synonym of 'nasty'. Both words mean 'something unpleasant or disagreeable'.
Some other synonyms are - awful, disgusting, etc.

10. He could not <u>muster</u> courage to speak the truth before his friend.

(a) injure (b) spoil
(c) gather (d) maim

(c) (**c**) 'Gather' is the correct synonym of 'muster'. Both words mean 'assemble or bring together'.
Some other synonyms are - gather, assemble, etc.

2019 (I)

11. Some people complain when they <u>encounter</u> a small misfortune in the course of their thoroughly happy life.

(a) run into (b) run away
(c) run down (d) run with

(a) *(**a**)* 'Run into' means the same as 'encounter'. Both words mean 'meet someone unexpectedly'.
Some other synonyms are-'collide with' or 'bump into.'

12. This world is full of <u>miseries</u>.

(a) indifferent love
(b) perfect happiness
(c) great suffering
(d) moderate sympathies

(c) *(**c**)* 'Great suffering' is a perfect synonym of word 'miseries'. 'Miseries' means 'a state or feeling of great physical or mental distress'.
Some other synonyms are-unhappiness, distress, etc.

13. A glance at a beautiful object gives us <u>delight</u>.

(a) wisdom (b) happiness
(c) purity (d) peace

(b) *(**b**)* 'Happiness' is the appropriate synonym of word 'delight'. Both words mean 'a feeling of great pleasure and satisfaction.'
Some other synonyms are-pleasure, joy, etc.

14. It is terrible for people to die of <u>starvation</u>.
(a) starch (b) staple
(c) plenty (d) hunger

(d) 'Hunger' is the correct synonym of word 'starvation'. 'Starvation' means 'the state of having no food for a long period, often causing death'.
Some other synonyms are-lack of food, famine, deprivation, etc.

15. The university has constituted a <u>grievance</u> redressal committee to look into the matter.
(a) depression (b) complaint
(c) abrasion (d) gratefulness

(b) 'Complaint' means the same as the word 'grievance'. 'Grievance' means 'an official statement of a complaint over something believed to be wrong or unfair'.
Some other synonyms are - grudge, grumble, etc.

16. Rakesh delivered a <u>slanderous</u> speech.
(a) abusive (b) praiseworthy
(c) moderate (d) inspiring

(a) 'Abusive' is similar in meaning to the word 'slanderous'. 'Slanderous' means 'a false spoken statement about someone that damages their reputation'.
Some other synonyms of 'slanderous' are – malicious, damaging, derogatory, etc.

17. Suddenly, the sky was darkened by a <u>gigantic</u> bird.
(a) winged (b) small
(c) tiny (d) enormous

(d) 'Enormous' means the same as 'gigantic'. The word 'gigantic' means 'something of very great size or extent'.
Some other synonyms of the word are – huge, vast, expansive, etc.

18. To <u>abolish</u> poverty would be to destroy the soil upon which mankind produces the virtues conducive to higher civilisation.
(a) detest (b) eradicate
(c) nurture (d) assimilate

(b) 'Eradicate' is similar word in meaning to the word 'abolish'. Both words mean 'to remove forever'.
Some other synonyms are – eliminate, wipe out, terminate, get rid of, etc.

19. The Arabs who are not in the cities live in the desert throughout the year, shifting from one <u>oasis</u> to another.
(a) sandbank (b) mound
(c) dune (d) spring

(d) 'Spring' is the most suitable synonym from the given options. 'Oasis' means 'a fertile spot in a desert where water is found'.
Some other synonyms can be– watering hole, watering place, etc.

20. The <u>various</u> facets of life can be found reflected in large city.
(a) several (b) similar
(c) valuable (d) singular

(a) 'Various' means 'several or different'. So, 'several' is a perfect synonym of word 'various'.
Some other synonyms are-varied, varying, etc.

2018 (II)

21. The Industrial Revolution saw a <u>massive</u> rise in the population of Europe.
(a) enormous (b) erroneous
(c) hazardous (d) perilous

(a) 'Enormous' is the nearest in meaning to the word 'massive' which means 'exceptionally large'.
Some other synonyms are-huge, vast, etc.

22. I had some deepest <u>convictions</u> reflected in my work.
(a) ideas and opinions
(b) firm beliefs
(c) prejudices
(d) biases

(b) 'Firm beliefs' is the nearest in meaning to the word 'convictions' which means 'firmly held beliefs or opinion'. Other options are not suitable here.
Some other synonyms are-faith, dogma, persuasion, etc.

23. This boy is very <u>timid</u>.
(a) courageous (b) shy
(c) clever (d) dull

(b) 'Shy' is nearest in meaning to the word 'timid', which means 'showing lack of courage or confidence'.
Some other synonyms are-afraid, diffident, etc.

24. My friend is as <u>stubborn</u> as a mule.
(a) observant (b) obnoxious
(c) obstinate (d) corpulent

(c) 'Obstinate' is the nearest in meaning to the word 'stubborn' which means 'a person who is determined not to change one's atitude'.
Some other synonyms are–adamant, fixed, etc.

25. His behaviour was deliberately <u>provocative</u>.
(a) exciting desire
(b) infuriating
(c) pitiable
(d) creating frustration

(b) 'Infuriating' is the nearest in meaning to the word 'provocative' which means 'causing anger'.
Some other synonyms are-aggravating, annoying, etc.

2018 (I)

26. I do not want you to lead a life of <u>sycophancy</u> as you did during the foreign rule.
(a) admiration
(b) love
(c) appreciation
(d) flattery

(d) A sycophant is one who praises people in authority in a way that is not sincere, usually in order to get some advantage of them. So, 'flattery' is the nearest in meaning to 'sycophancy'.
Some other synonyms are-exaltation, glorification, etc.

27. In India, it has become easy to <u>attack</u> cultural artefacts these days.
(a) beckon (b) assault
(c) belch (d) appreciate

(b) 'Assault' is the correct synonym of 'attack'. as it means 'a sudden attack'
Some other synonyms are-beat, strike at, thrash, etc.

28. A local court <u>granted</u> bail to the criminal on Thursday.
(a) confessed (b) donated
(c) allowed (d) yielded

(c) 'Allowed' is the correct synonym of 'granted'. Both words mean 'to give permission'.
Some other synonyms are-permit, entitle, etc.

29. The judge told that he would analyse the evidence and then <u>deliver</u> the verdict.
(a) liberate
(b) surrender
(c) transfer
(d) pronounce

(d) 'Pronounce' is the correct synonym of 'deliver'. 'Deliver' means 'to give or produce a speech or result'.
Some other synonyms are-convey, give, announce, etc.

30. The growth and development of the peasant movement was <u>indissolubly</u> linked with the national struggle for freedom.

(a) firmly (b) vaguely
(c) individually (d) steadily

(›) *(**a**)* In the given sentence, 'indissolubly' means 'incapable of being broken'. So, 'firmly' is the nearest in meaning to it.

Some other synonyms are-lasting, enduring, etc.

31. Weather conditions have been <u>improving</u> over the past few days.

(a) mending (b) amending
(c) becoming better (d) advancing

(›) *(**c**)* 'Becoming better' is the perfect synonym of 'improving' as 'improving' means 'to become or to make something better'.

Some other synonyms are– correcting, fixing, etc.

32. The confusion on the <u>interlocutor's</u> face was gratifying.

(a) government officer (b) party worker
(c) dialogist (d) revolutionary

(›) *(**c**)* An 'interlocutor' is the person who engages himself in dialogues. So, option (c) 'dialogist' is a suitable choice.
Some other synonyms are-interviewer, speaker, etc.

33. He spends his money <u>lavishly</u>.

(a) hesitatingly (b) generously
(c) foolishly (d) carefully

(›) *(**b**)* 'Lavishly' may also means 'to spend money extravagantly or wastefully'.
Some other synonyms are–abundantly, luxuriantly, etc.

So, option (b) 'generously' is a suitable choice. Other options do not match here. '

34. The government's new policies will come into force from the next <u>fiscal</u> year.

(a) calendar (b) academic
(c) financial (d) leap

(›) *(**c**)* 'Fiscal' means 'relating to money and finances'. So, option (c) 'financial' is nearest in meaning to it.
Some other synonyms are-commercial, budgetary, etc.

35. <u>Abundant</u> food was available for the soldiers in the mess.

(a) little (b) plentiful
(c) delicious (d) wholesome

(›) *(**b**)* 'Plentiful' is the correct synonym for the word 'abundant' which means 'in large quantity'.
Some other synonyms are-plenty, profuse, ample, etc.

2017 (II)

36. The discussion was <u>wound up</u> after a long fruitful exchange of view.

(a) postponed (b) cut short
(c) interrupted (d) concluded

(›) *(**d**)* 'Concluded' is the nearest to the meaning of 'wound up' as both mean 'finished or completed'.

Some other synonyms are-wrap up, ended, etc.

37. He was fully <u>alive to</u> the need for making adjustments.

(a) concerned about
(b) worried about
(c) aware of
(d) indifferent about

(›) *(**c**)* 'Aware of' will be the nearest in meaning of 'alive to' as both words mean 'able to notice something'. So, option (c) is a suitable choice.

38. The police officer tried to <u>intimidate</u> the witness but in vain.

(a) inform (b) reward
(c) frighten (d) persuade

(›) *(**c**)* 'Frighten' is the nearest in meaning to 'intimidate' as both mean 'to threaten'.

Some other synonyms are- scare, terrorise, etc.

39. We must adopt <u>drastic</u> measures to control population growth.

(a) simple (b) dramatic
(c) realistic (d) severe

(›) *(**d**)* 'Severe' is the nearest in meaning to the word 'drastic' as it means 'likely to have a strong or extreme effect'.

Some other synonyms are – extreme, serious, radical, etc.

40. He is extremely <u>meticulous</u> in his approach.

(a) simple (b) careful
(c) fair (d) reasonable

(›) *(**b**)* 'Careful' is the nearest in meaning to the word of 'meticulous' which means 'showing great attention to detail'.

Some other synonyms are-cautious, diligent, etc.

41. The experts' <u>minute</u> examination brought to light some important clues.

(a) quick
(b) detailed
(c) superficial
(d) prolonged

(›) *(**b**)* 'Detailed' will be the nearest in meaning to the word 'minute' as both mean 'taking the smallest points into consideration'.

Some other synonyms are– precise, minuscule, etc.

42. The decision of the Union Government to <u>repeal</u> the Urban Land Ceiling Act has been welcomed by all.

(a) suppress (b) amend
(c) cancel (d) withhold

(›) *(**c**)* 'Repeal' means 'to revoke or withdraw'. So, 'cancel' is its correct synonym. Some other synonyms are-abolition, annulment, revoke, etc.

43. This is his <u>maiden</u> appearance on the screen.

(a) first (b) last
(c) girlish (d) shy

(›) *(**a**)* 'Maiden' means 'first', so option (a) is correct here.

Some other synonyms are– initial, inaugural, etc.

44. At the end of the marathon everybody was <u>exhausted</u>.

(a) weakened (b) honoured
(c) satisfied (d) tired

(›) *(**d**)* 'Exhausted' means 'very tired'. So, option (d) 'tired' is the correct choice.

Some other synonyms are– weary, fatigued, etc.

45. He gave me a <u>counterfeit</u> coin.

(a) rare (b) fake
(c) unmatured (d) inferior

(›) *(**b**)* 'Fake' will be the nearest in meaning to the word 'counterfeit' as both mean 'a fraudulent imitation of something else'.

Some other synonyms are– bogus, copied, forged, etc.

2017 (I)

46. It is unwise to <u>sever</u> diplomatic relations with a neighbouring country over small matters.

(a) engage (b) estrange
(c) cut off (d) twist

(›) (*c*) 'Sever' means 'to cut off'. So, option (c) is the correct choice.

Some other synonyms are– disconnect, split, separate, etc.

47. Bad tendencies are to be <u>countered</u> by good ones until all that is evil disappears.

(a) opposed (b) balanced
(c) reduced (d) bypassed

(*a*) 'Countered' means 'to oppose'. So, option (a) 'opposed' is a suitable choice.
Some other synonyms are– confront, contravene, retaliate, etc.

48. The police fired <u>indiscriminately</u> at the crowd, killing many innocent women and children.
(a) continuously
(b) without distinguishing
(c) foolishly
(d) rapidly

(*b*) 'Indiscriminately' means 'without distinguishing or purposelessly.'
Some other synonyms are- blindly, aimlessly, etc.

49. Businessmen who lack <u>acumen</u> cannot be expected to be very successful.
(a) fairness (b) sharpness
(c) boldness (d) righteousness

(*b*) 'Acumen' means 'the ability to make good judgement and take quick decisions'. So, 'sharpness' is its correct synonym.
Some other synonyms are-ingenuity, insight, etc.

50. His <u>candid</u> opinions have won him many friends.
(a) kind (b) courteous
(c) generous (d) frank

(*d*) 'Candid' means 'honest and telling the truth, especially about something difficult or painful'. So, option (d) 'Frank' is the correct choice.
Some other synonyms are-outspoken, straight forward, etc.

2016 (II)

51. <u>Hospitality</u> is a virtue for which the natives of the East in general are highly admired.
(a) Duty of a doctor
(b) Generosity shown to guests
(c) Cleanliness in hospitals
(d) Kindness

(*b*) 'Hospitality' means 'generosity shown to guests'. Hence, option (b) is suitable usage and some other synonyms are amiability, friendliness and cordiality.

52. House rent in cities like Mumbai or Delhi has risen to <u>astronomical</u> figures beyond the reach of even high-salaried people.
(a) exorbitant (b) commercial
(c) planetary (d) illogical

(*a*) 'Astronomical' has the sense of 'going high'. The same sense is conveyed by the use of 'exorbitant'. Some other synonyms are– excessive, steep, expensive, etc.

53. <u>Appropriate</u> technology holds the key to a nation's development.
(a) Modern (b) Suitable
(c) Effective (d) Growing

(*b*) 'Appropriate' and 'suitable' are the suitable synonyms. 'Appropriate' means 'suitable or proper in circumstances'. Apt, pertinent are some other synonyms of the given word.

54. He was <u>enamoured</u> of his own golden voice.
(a) very fond of
(b) concerned with
(c) obsessed with
(d) imbued with

(*a*) 'Enamoured' means 'being very fond of '. So, option (a) is suitable usage.
Some other synonyms are-ardent, devoted, etc.

55. Some journalists are guilty of indulging in <u>yellow journalism</u>.
(a) mis-representation
(b) vulgarisation
(c) sensational reporting
(d) loud gestures

(*c*) 'Yellow journalism' has the usage of 'publishing sensational news in order to get in limelight.' So, option (c) is the correct choice.

2016 (I)

56. Suddenly there was a bright flash, followed by a <u>deafening</u> explosion.
(a) dangerous (b) terrifying
(c) mild (d) very loud

(*d*) 'Deafening' means 'extremely loud'. Therefore, option (d) 'very loud' is similar in meaning to the word 'deafening'. Some other synonyms are-blaring, noisy.

57. He showed <u>exemplary</u> courage during the crisis.
(a) usual (b) durable
(c) commendable (d) some

(*c*) 'Exemplary' means 'to provide a good example for others' and 'Commendable' means 'praise worthy'. Therefore, option (c) is most similar in meaning to the word 'exemplary'. Some other synonyms are-ideal, laudable.

58. When the new teacher entered the classroom, he found the pupils <u>restive</u>.
(a) at rest (b) idle
(c) quiet (d) impatient

(*d*) 'Restive' means 'unable to stay still or being controlled'. Therefore, option (d) 'impatient' is most similar in meaning to the word 'restive'.
Some other synonyms are-restless, uneasy are.

59. There is no <u>dearth</u> of talent in this country.
(a) scarcity (b) availability
(c) plenty (d) absence

(*a*) 'Dearth' means 'a lack or scarcity of something'. Therefore, option (a) 'scarcity' is most similar in meaning to the word 'dearth'.
Inadequacy, shortage are some other synonyms.

60. The servants <u>retired</u> to their quarters.
(a) entered (b) went away
(c) ran away (d) mobilised

(*b*) 'Retired' means 'to leave a place'. In the context of the sentence, option (b) 'went away' is most similar in meaning to the word 'retired'.

61. The navy gave <u>tactical</u> support to marines.
(a) sensitive (b) strategic
(c) immediate (d) expert

(*b*) 'Tactical' means 'careful planning in order to achieve something'. Therefore, option (b) 'strategic' is most similar in meaning to the word 'tactical'.
Some other synonyms are– planned, judicious, etc.

62. A genius tends to <u>deviate</u> from the routine way of thinking.
(a) dispute (b) disagree
(c) distinguish (d) differ

(*d*) 'Deviate' means 'depart from an established course'. Therefore, option (d) 'differ' is most similar to the word 'deviate'.
Other synonyms are-diverge, depart, etc.

63. He was greatly <u>debilitated</u> by an attack of influenza.
(a) depressed (b) weakened
(c) worried (d) defeated

(*b*) 'Debilitate' means 'to weaken as a result of an illness'. Therefore, option (b) is most similar to the word 'debilitated'. Some other synonyms are-incapacitate, disable.

64. His efforts at helping the poor are laudable.
(a) welcome (b) sincere
(c) good (d) praise worthy

(**d**) 'Laudable' means 'worth appreciation'. So, option (d) 'praise worthy' is the correct synonym. Commendable, creditable are some other synonyms.

65. His conduct brought him reproach from quarters.
(a) rebuke (b) sympathy
(c) indifference (d) remorse

(**a**) 'Reproach' means 'expressing blame or criticism'. Therefore, option (a) 'rebuke' is most similar to the word 'reproach.'

Some other synonyms are-admonition, discredit, etc.

Directions (Q. Nos. 66-84) *Each of the following items consists of a sentence followed by four words or group of words. Select the synonym of the word (occurring in the sentence in capital letters) as per the context.*

2015 (II)

66. Many of his acquaintances avoid him because he is so GARRULOUS.
(a) Unreasonable (b) Talkative
(c) Quarrelsome (d) Proud

(***b***) 'Garrulous' means 'excessively talkative', especially on trivial matters. So, option (b) 'talkative' is its correct synonym.

Some other synonyms are- loquacious, voluble, etc.

67. He bore the pain with great FORTITUDE.
(a) Resignation (b) Defiance
(c) Indifference (d) Forbearance

(***d***) 'Fortitude' means 'courage in pain or adversity'. So, option (d) 'Forbearance' is its correct synonym. Some other synonyms are–endurance, valour, etc.

68. He gave his TACIT approval to the proposition.
(a) Full (b) Loud
(c) Clean (d) Implied

(***d***) 'Tacit' means 'understood or implied without being stated'. So, option (d) 'implied' is its correct synonym.

Some other synonyms are-implicit, insinuated, etc.

69. Inspite of hard work, the farmers could only get a MEAGRE yield.
(a) Satisfactory (b) Scanty
(c) Plenty (d) Normal

(***b***) 'Meagre' and 'Scanty' both have a same meaning 'less in amount'. So, these are synonyms.

Some other synonyms are-inadequate, insufficient, etc.

70. He was EXHILARATED at the outcome of the election results.
(a) Satisfied (b) Surprised
(c) Disappointed (d) Overjoyed

(***d***) 'Exhilarated' and 'overjoyed' are the similar words as both have same meaning, i.e. 'extremely happy.'

Some other synonyms are-delighted, excited, etc.

71. He was FIRED for negligence on duty.
(a) Relieved of his job
(b) Scolded
(c) Rebuked (d) Attacked

(***a***) 'Fired' means 'dismissed from a job'. Hence, option (a) 'relieved of his job' is the correct choice.

Some other synonyms are– dismiss, discharge, etc.

2015 (I)

72. Democracy is not the standardising of everyone so as to OBLITERATE all peculiarity.
(a) Demolish (b) Extinguish
(c) Erase (d) Change

(***a***) 'Demolish' is most suitable synonym here in context of the sentence, as 'obliterate' and 'demolish' both mean 'destroy utterly'.

Some other synonyms are-annihilate, exterminate, etc.

73. Divine grace is truly INEFFABLE.
(a) That which cannot be rubbed out.
(b) Incapable of being understood.
(c) That which is too great to be expressed in words.
(d) Too powerful to be defeated.

(***c***) 'Ineffable' means 'too great to be expressed in words'. Hence, option (c) is correct choice. Some other synonyms are–inexpressible, indiscribable, etc.

74. The convocation address was very EDIFYING.
(a) Tedious (b) In need of editing
(c) Instructive (d) Exciting

(***c***) 'Instructive' as well as 'edifying' both mean 'providing intellectual instruction'. So, option (c) is the correct choice. Some other synonyms are– informative, enlightening, etc.

2014 (II)

75. We should always try to maintain and promote communal AMITY.
(a) Bondage (b) Contention
(c) Friendship (d) Understanding

(***c***) 'Amity' means 'peaceful and friendly relations'. So, option (c) 'friendship' is its correct synonym.

Some other synonyms are harmony, fellowship, etc.

76. Many species of animals have become EXTINCT during the last hundred years.
(a) Aggressive (b) Non-existent
(c) Scattered (d) Feeble

(***b***) 'Extinct' means 'not now existing'. So, option (b) 'Non-existent' is its correct synonym.

Some other synonyms are-vanished, wiped out, etc.

77. True religion does not require one to PROSELYTISE through guile or force.
(a) Translate (b) Hypnotise
(c) Attack (d) Convert

(***d***) 'Proselytise' means 'convert or attempt to convert from one religion, belief to another'. So, option (d) 'convert' is its correct synonym.

78. That the plan is both inhuman and PREPOSTEROUS needs on further proof.
(a) Heartless (b) Impractical
(c) Absurd (d) Abnormal

(***c***) 'Preposterous' means 'completely unreasonable and ridiculous'. So, option (c) 'absurd' is its correct synonym.

Some other synonyms are-ridiculous, insane, etc.

79. The attitude of the Western countries towards the third-world countries is rather CALLOUS to say the least.
(a) Passive (b) Unkind
(c) Cursed (d) Unfeeling

(***d***) 'Callous' means 'not caring about other people feelings or suffering'. The best synonym to the word is 'unfeeling' which means 'lacking sensitivity towards others'.

80. Whatever the VERDICT of history may be, Chaplin will occupy a unique place in its pages.

(a) Judgement
(b) Voice
(c) Outcome
(d) Prediction

› *(**a**)* 'Verdict' means 'an opinion or judgement'. So, option (a) 'Judgement' is it correct synonym.

Some other synonyms are-decision, conclusion, etc.

81. He CORROBORATED the statement of his brother.

(a) Confirmed
(b) Disproved
(c) Condemned
(d) Seconded

› *(**a**)* 'Corroborated' means 'confirm or give support to a statement or theory'. So, option (a) 'confirmed' is its correct synonym.

Some other synonyms are- verify, ratify, etc.

82. Whatever opinion he gives is SANE.

(a) Rational
(b) Obscure
(c) Wild
(d) Arrogant

› *(**a**)* 'Sane' means 'reasonable or sensible'. So, option (a) 'rational' is its correct synonym which means 'based on or in accordance with reason or logic'.

Some other synonyms are- practical, sensible, realistic, etc.

83. The story is too fantastic to be CREDIBLE.

(a) Believable (b) False
(c) Readable (d) Praise worthy

› *(**a**)* 'Credible' means 'able to be believed or trusted'. So, option (a) 'believable' is its correct synonym.
Some other synonyms are-trustworthy, reliable, etc.

84. Catching snakes can be HAZARDOUS for people untrained in the art.

(a) Tricky (b) Harmful
(c) Difficult (d) Dangerous

› *(**d**)* 'Hazardous' means 'involving risk or danger'. So, option (d) 'dangerous' is its correct synonym.

Some other synonyms are–risky, perilous, etc.

37

ANTONYMS

Directions (Q. Nos. 1-66) *Each item in this section consists of a sentence with an underlined word followed by four options. Select the option that is opposite in meaning to the underlined word.*

2019 (II)

1. His conversations are always absurd.

(a) farcical (b) foolish
(c) preposterous (d) rational

(d) The word 'absurd' means 'unreasonable or inappropriate'. Its antonym is 'Rational' which means 'in accordance with reason or logic.'

2. The politician was a dissident.

(a) Agitator
(b) Rebel
(c) Conformist
(d) Revolutionary

(c) The word 'dissident' means 'a person who opposes official policy'. Its antonym from the given option is 'conformist', which means 'someone who conforms to the accepted behaviour or established practices'.

3. He wrote an incisive article on corruption in politics.

(a) vague
(b) penetrating
(c) trenchant
(d) precise

(a) The word 'incisive' means 'something that is sharp or direct'. Its antonym is 'vague' which means 'uncertain or indistinct'.

4. Rakesh is an eccentric prodigal.

(a) extravagant (b) profligate
(c) wasteful (d) thrifty

(d) The word 'prodigal' means 'spending money or using resources freely and recklessly'. Its antonym is 'thrifty', which means 'using money and other resources carefully'.

5. Friendship is always reciprocal.

(a) mutual (b) one-sided
(c) shared (d) corresponding

(b) The word 'reciprocal' means 'mutual or complementary'. Its antonym is 'one-sided' meaning 'having or occurring on side of something only'.

6. The debate was highly stimulating.

(a) arousing (b) invigorating
(c) boring (d) stirring

(c) The word 'stimulating' means 'exciting or arousing'. Its antonym is 'boring' which means 'not interesting'.

7. Malaria is a widespread disease.

(a) endemic (b) pervasive
(c) common (d) rare

(a) The word 'widespread' means 'extensive or distributed over a large area'.Its antonym is 'endemic' which means 'native and restricted to a certain place'.

8. The bill received a vehement resistance from the opposition party in the Parliament.

(a) animated (b) apathetic
(c) fervent (d) vigorous

(b) The word 'vehement' means 'showing strong feelings or shown with great energy or force'. Its antonym is 'apathetic' which means 'showing no enthusiasm or concern'.

9. He has been obstinate since his childhood.

(a) stubborn (b) dogged
(c) amenable (d) rigid

(c) The word 'obstinate' means 'stubborn or determined'. Its antonym is 'amenable' which means 'responsive to suggestion'.

10. Mahatma Gandhi was one of the most prominent leaders of his times.

(a) renowned (b) distinguishable
(c) eminent (d) unknown

(d) The word 'prominent' means 'readily noticeable.' Its antonym is 'unknown' which means 'not known or familiar'.

2019 (I)

11. Ramesh is a very dubious character.

(a) shady
(b) suspicious
(c) trustworthy
(d) doubtful

(c) The word 'dubious' means 'of doubtful character'. Hence, its antonym is 'trustworthy' which means 'reliable or dependable'.

12. Do not indulge in unmindful activities, please.

(a) vigilant (b) careless
(c) stupid (d) fatuous

(a) The word 'unmindful' means 'paying no heed or being careless'. Its antonym is 'vigilant', which means 'watchful or observant, alert' etc.

13. He is suffering from a curable disease.

(a) remediable (b) treatable
(c) terminal (d) operable

(c) The word 'curable' means 'treatable or remediable'. Its antonym is 'terminal' which means 'incapable of being treated or untreatable'.

14. He was born on a very <u>auspicious</u> day.
(a) propitious (b) fortunate
(c) ominous (d) opportune

(c) 'Auspicious' means 'favourable or conducive to success'. Its antonym is 'ominous' which means 'something sinister or inauspicious or doomy, unfavourable'.

15. He had deeper <u>hostility</u> towards Mohan.
(a) animosity (b) belligerence
(c) malice (d) friendship

(d) 'The word 'hostility' means 'malice or unfriendliness or opposition'. Its antonym is 'friendship' which means 'close association or bond'.

16. His life is rather <u>monotonous</u>.
(a) exciting (b) dreary
(c) tedious (d) uneventful

(a) The word 'monotonous' means 'boring or unexciting'. Its antonym is 'exciting' which means 'thrilling or stimulating'.

17. Macbeth is a morally <u>repulsive</u> character.
(a) abominable (b) attractive
(c) obnoxious (d) ugly

(b) The word 'repulsive' means 'arousing intense distaste'. Its antonym is 'attractive' which means 'pleasing or agreeable'.

18. The <u>serene</u> beauty of Kashmir had a soothing effect on his mind.
(a) placid (b) pleasing
(c) tranquil (d) turbulent

(d) 'The word 'serene' means 'peaceful or calm'. Its antonym is 'turbulent' which means 'unstable, chaotic or stormy'.

19. Life is <u>transient</u> in nature.
(a) brief
(b) momentary
(c) eternal
(d) short-lived

(c) The word 'transient' means 'temporary or short lived'. Its antonym is 'eternal', which means 'everlasting or immortal, perpetual'.

20. Sohan is a <u>vain</u> person.
(a) modest
(b) arrogant
(c) conceited
(d) proud

(a) The word 'vain' means 'proud or arrogant'. Its antonym is 'modest' which means 'humble or unpretentious'.

2018 (II)

21. Too many cooks <u>spoil</u> the broth.
(a) tarnish (b) wreck
(c) embellish (d) upset

(c) The word 'spoil' means 'to diminish or destroy'. So, its correct antonym is 'Embellish' which means 'to improve or enhance'.

22. He is <u>biased</u> against the students from cities.
(a) open (b) prejudiced
(c) liked (d) impartial

(d) The word 'biased' means 'inclined to favour one over another'. So, its correct antonym is 'impartial' which means 'treating everyone equally'.

23. It is easy to be an <u>orthodox</u>.
(a) idolatrous (b) intelligent
(c) malignant (d) heterodox

(d) 'Orthodox' means 'following or conforming to the traditions'.
Its antonym is 'heterodox' which means 'not conforming with accepted standards or traditions'.

24. <u>Permit</u> me to present you with a book.
(a) allow (b) enclose
(c) prohibit (d) persuade

(c) 'Permit' means 'to allow' whereas 'prohibit' means 'to disallow or deny'. So, it is the antonym of 'permit'.

25. None but the <u>brave</u> deserves the fair.
(a) ugly (b) coward
(c) jealous (d) weak

(b) 'Brave' means 'courageous or ready to face danger'. Its correct antonym is 'coward' which means 'someone who lacks courage and turns away from danger'.

2018 (I)

26. The country's economy must be <u>geared to</u> wartime requirements.
(a) subordinated to (b) related to
(c) adjusted to (d) unlinked to

(d) The word 'geared to' means 'to adapt or adjust'. 'Unlinked to' is correct antonym of word 'geared' which means 'to disconnect the links of or to separate'.

27. Why does fire <u>attract</u> insects?
(a) discharge (b) destroy
(c) repel (d) remove

(c) The word 'attract' means 'to cause somebody to like something or somebody'. Its antonym is 'repel' which means 'to make somebody feel disgusted'.

28. The party was excellent, and I would like to thank all the people <u>concerned</u>.
(a) cared (b) attentive
(c) dependable (d) uninvolved

(d) 'Uninvolved' is correct antonym of 'concerned' which means 'not concerned or connected'.

29. He is very <u>serious</u> by temperament.
(a) grave (b) trivial (c) sober (d) stupid

(b) 'Trivial' is the correct antonym of 'Serious'. 'Serious' means 'demanding or characterised by careful consideration' whereas 'trivial' means 'of little value or importance'.

30. There are a few <u>miscellaneous</u> items to discuss in this meeting.
(a) pure (b) mixed
(c) homogenous (d) discordant

(c) 'Homogenous' is correct antonym of 'miscellaneous'. 'Miscellaneous' means 'of mixed type' whereas 'homogenous' means 'of the same kind'.

31. Due to the postal strike, the <u>outgoing</u> mail got delayed.
(a) urgent (b) incoming
(c) ordinary (d) speedy

(b) 'Incoming' is the correct antonym of 'outgoing'. 'Incoming' means 'in the process of coming in' whereas 'outgoing' means 'an instance of going out'.

32. He had a <u>fine</u> ear for music.
(a) small (b) close
(c) coarse (d) smooth

(c) 'Coarse' is the correct antonym of 'fine'. 'Fine' means 'made of small pieces' whereas 'coarse' means 'consisting of large pieces'.

33. There is no <u>likeness</u> between him and his brother.
(a) unlikeliness (b) unlikelihood
(c) dissimilarity (d) disaffinity

(c) 'Dissimilarity' is the correct antonym as 'likeness' means 'resemblance' and 'dissimilarity' means 'when there is nothing similar or resembling with the other'.

34. Cultural <u>diversity</u> in the working place is good for business.
(a) uniformity (b) conformity
(c) identity (d) similarity

(a) 'Diversity' means 'the condition of having or being composed of different types of people or things'. So, its opposite will be 'uniformity', which means 'the quality of being same or equal as all other members of a group'.

35. The company was liquidated within five years.
(a) bankrupt (b) closed down
(c) flourishing (d) privatised

(c) 'Liquidate' means 'to cause a business to close, so that its assets can be sold to pay its debts'. So, its opposite would be 'flourishing', which means 'to develop in a successful way'.

2017 (II)

36. My mother has been working hard for the last two weeks and she feels run down.
(a) morbid (b) energetic
(c) exhausted (d) emotional

(b) 'Run down' means 'tired or exhausted', so 'energetic' will be its opposite as it means 'full of energy'.

37. The President condemned the Act of violence during the celebration of the festival.
(a) reason (b) instigation
(c) restraint (d) sobriety

(c) 'Violence' means 'behaviour involving physical force intended to hurt', so 'restrain' will be opposite in meaning to it as it means 'the state of being calm and to control oneself'.

38. The students made a generous contribution to the flood relief fund.
(a) niggard (b) selfish
(c) spendthrift (d) indecent

(a) 'Generous' means 'big-hearted and helpful' while 'niggard' means 'ingenerous or miser'. So, option (a) is the correct answer.

39. He was just idle by temperament.
(a) employed (b) occupied
(c) industrious (d) happy

(c) 'Idle' means 'lazy or unemployed' while 'industrious' means 'diligent and hard working'. So, they both are antonyms.

40. Most of the decisions taken by the officer were unjust.
(a) serious (b) lenient
(c) correct (d) imbecile

(c) 'Unjust' means 'not fair or incorrect' so 'correct' will be its correct antonym.

41. He is a loving father and takes great delight in his children.
(a) revolt (b) dissatisfaction
(c) enjoyment (d) disgust

(d) 'Delight' means 'please (someone) greatly' while disgust means 'a feeling of strong disapproval aroused by something unpleasant or offensive'. So, they both are antonyms.

42. He was quite concerned about his son's career.
(a) unrelated (b) indifferent
(c) dispassionate (d) carefree

(b) 'Concerned' means 'worried or anxious' so 'indifferent' will be its antonym, which means 'unconcerned or having no particular interest or sympathy'.

43. They are confident of success.
(a) imprudent (b) impatient
(c) diffident (d) reluctant

(c) 'Confident' means 'feeling sure about something' while 'diffident' means 'modest or shy because of a lack of self-confidence'.

44. We carried on the search for the missing person.
(a) delayed (b) reconsidered
(c) broke up (d) called off

(d) 'Carried on' means 'to continue an activity or task' while 'called of' means 'to decide to stop something that is already happening'. So, option (d) is the correct choice.

45. This TV has many indigenous components.
(a) Indian (b) foreign
(c) unnatural (d) genuine

(b) 'Indigenous' means 'native' so 'foreign' will be the word opposite in meaning.

2017 (I)

46. My first lecture in the classroom was a fiasco.
(a) Success (b) Joy
(c) Fun (d) Disaster

(a) 'Fiasco' means 'an utter failure'. So, 'success' is its antonyms.

47. It was indeed arduous to cross streets in New York.
(a) Pleasant (b) Effortless
(c) Interesting (d) Risky

(b) 'Arduous' means 'difficult'. So, its antonym will be 'effortless', which means 'easy or simple'.

48. Unlike his brother, he is affable.
(a) Reserved (b) Gullible
(c) Irritable (d) Lovable

(a) 'Affable' means 'friendly'. So, its antonym will be 'reserved' which means 'shy and keep their feelings hidden.'

49. The birth of his child decidedly proved to be an auspicious event in his life.
(a) Precious (b) Ominous
(c) Useless (d) Unforgettable

(b) 'Auspicious' means ' suggesting a positive and successful future.' So, 'ominous' is its antonym as it means 'inauspicious'.

50. The witness corroborated word for word the statement of the victim.
(a) Accepted (b) Confirmed
(c) Denied (d) Repeated

(c) 'Corroborated' means 'to confirm a statement'. So, 'denied' is its correct antonym.

2016 (II)

51. The culprit was sentenced by the Court.
(a) acquitted (b) punished
(c) relieved (d) pardoned

(a) 'Sentenced' means 'declaration of the punishment for the crime' but the word 'acquitted' means 'to be free from all the charges.' So, option (a) is nearly opposite to the word 'sentenced'.

52. Thrifty as he is, he can well afford to live within his means.
(a) Careless (b) Instinctive
(c) Sentimental (d) Extravagant

(d) 'Thrifty' means 'one who spends with utmost care'. Hence, 'extravagant' is its antonym, which means 'to spend without consideration.'

53. Do not give him a responsible job, he is immature.
(a) thoughtful (b) cautious
(c) calculating (d) seasoned

(a) 'Immature' means 'not well developed in thought'. So, 'thoughtful' is its correct opposite.

54. I was prepared to show my hand provided he agreed to do the same.
(a) to yield (b) to shake hands
(c) to conceal my plan
(d) to lose my ground

(c) 'To show my hand' means 'to reveal'. So, its antonym will be 'to conceal my plan'.

55. Akbar the great was a sagacious ruler.
(a) haughty (b) cunning
(c) rude (d) unwise

(d) 'Sagacious' means 'full of wisdom'. So, its antonym will be 'unwise' which means 'not wise at all'.

2016 (I)

56. He handled the machine with deft fingers.
(a) delicate (b) sturdy
(c) quick (d) clumsy

(d) 'Deft' means 'skillful and quick'. 'Clumsy' means 'doing something without skill'. Therefore, option (d) is nearly opposite to 'deft'.

57. I was deeply affected by his urbane behaviour.
(a) rural (b) rude
(c) irrational (d) indifferent

(b) 'Urbane' means 'confident and well-mannered' and 'rude' means 'being impolite'. Therefore, option (b) is nearly opposite to 'urbane.'

58. His timidity proved costly.
(a) arrogance (b) boldness
(c) skilfulness (d) cunning

(b) 'Timidity' means 'being shy and nervous' and 'bold' means 'being brave confident'. Therefore, option (b) is nearly opposite to 'timidity'.

59. Arrangements were made to handle the mammoth gathering tactfully.
(a) significant (b) small
(c) unruly (d) noisy

(b) 'Mammoth' means 'extremely large'. Therefore, option (b) 'small' is nearly opposite to 'mammoth'.

60. He was engrossed in his work when I walked in.
(a) occupied (b) inattentive
(c) engaged (d) absent

(b) 'Engrossed' means 'involved or attentive'. Therefore, option (b) 'inattentive' is nearly opposite to 'engrossed'.

61. These are the main points of the preceding paragraph.
(a) following (b) previous
(c) first (d) last

(a) 'Preceding' means 'happening or coming before in order' and 'following' means 'next in order'. Therefore, option (a) is nearly opposite to 'preceding'.

62. He made a shrewd guess.
(a) clever (b) wild
(c) incorrect (d) discriminating

(c) 'Shrewd' means 'to be right or correct'. Therefore, option (c) 'incorrect' is nearly opposite to 'shrewd'.

63. He is suffering from a severe cough.
(a) violent (b) mild
(c) bad (d) continuous

(b) 'Severe' means 'harsh or serious'. 'Mild' means 'gentle, not serious or harsh'. Therefore, option (b) 'mild' is nearly opposite to 'severe'.

64. Cumulatively, the effect of these drugs quite bad.
(a) Individually (b) Obviously
(c) Clearly (d) Collectively

(a) 'Cumulative' means 'including everything or as a whole'. Therefore, option (a) 'individually' is nearly opposite to 'cumulative'.

65. He was conspicuous because of his colourful shirt.
(a) charming (b) ugly
(c) small (d) unnoticeable

(d) 'Conspicuous' means 'easy to see or notice'. Therefore, option (d) 'unnoticeable' is nearly opposite to 'conspicuous'. 'Unnoticeable' means 'not easily observed or noticed'.

66. He hates these continual arguments with friend.
(a) repeated (b) irrational
(c) occasional (d) regular

(c) 'Continual' means 'continuous or repetitive'. 'Occasional' means 'intermittent or done irregularly'. Therefore, option (c) 'occasional' is nearly opposite to 'continual'.

Directions (Q. Nos. 67-89) *Each of the following items consists of a sentence/word followed by four words or group of words. Select the antonym of the word occurring in the sentence in capital letters as per the context.*

2015 (II)

67. He CONCEALED his thoughts very cleverly.
(a) Emphasised
(b) Expressed
(c) Affirmed
(d) Revealed

(d) 'Concealed' means 'to hide'. So, its antonym will be 'reveal' that has a meaning 'to disclose'.

68. The proposal was DENOUNCED by one and all.
(a) Renounced (b) Recommended
(c) Announced (d) Commended

(b) 'Denounced' means 'to criticise'. So, 'recommended' is its antonym that means 'to praise'.

69. She was SKEPTICAL about the safety of the new drug.
(a) Doubtful (b) Certain
(c) Hopeful (d) Sanguine

(b) 'Skeptical' means 'doubtful', so, its correct antonym will be 'certain' which means 'fix or which cannot change'.

70. The answers to the question were COHERENT.
(a) Relaxed (b) Loose
(c) Consistent (d) Disconnected

(d) 'Coherent' means 'in connection to one another', so, 'disconnected' is opposite in meaning.

71. It was no ALTRUISTIC motive that prompted him to help her.
(a) Selfish (b) Inhuman
(c) Brutal (d) Wicked

(a) 'Altruistic' means 'not selfish', so its correct antonym is 'selfish'.

2015 (I)

72. John is always SHABBILY dressed.
(a) Decently (b) Beautifully
(c) Extravagantly (d) Scantily

(a) 'Shabbily' means 'torn' or 'dirty'. Its opposite will be 'decently' which means 'well sophisticated'.

73. The new manager of the bank is URBANE in his manners.
(a) Civilised (b) Slow
(c) Rude (d) Foolish

(c) 'Urbane' means 'polite', so 'rude' is the correct antonym of it as it means 'impolite'.

74. It is necessary to develop THRIFTY habits to be able to lead a comfortable life.
(a) Expensive (b) Extravagant
(c) Economical (d) Good

(b) 'Thrifty' means 'economical' or 'money saving'. Its opposite meaning word will be 'extravagant' which means 'spend much'.

75. Many people suffer because of PRIDE.

(a) Lowliness (b) Submission
(c) Humility (d) Obedience

(c) 'Pride' means 'arrogance' or 'haughtiness'. Here, its opposite meaning word should be 'humility' which means 'modest' or 'humble'.

2014 (II)

76. His short but pointed speech was APPLAUDED by all sections of the audience.

(a) Disapproved
(b) Misunderstood
(c) Praised
(d) Welcomed

(a) 'Applauded' means 'praised', so 'disapproved' is its correct antonym which means 'not praised' or 'criticised'.

77. In ancient India, scholars had no interest in political power or MATERIAL growth.

(a) Internal (b) Spiritual
(c) Psychic (d) Celestial

(b) 'Material' means 'earthly' or 'related to wealth', so 'spiritual' is the correct antonym which means 'divine'.

78. A FRIENDLY dog met us at the farm gate.

(a) Helpful (b) Understanding
(c) Quiet (d) Hostile

(d) 'Friendly' means 'supportive', so 'hostile' is the correct antonym which means 'unfriendly'.

79. He is extremely INTELLIGENT but proud.

(a) Dull (b) Weak
(c) Ignorant (d) Arrogant

(a) 'Intelligent' means 'smart', so 'dull' is its correct antonym which means 'stupid'.

80. The young leader was RELUCTANT to shoulder the responsibilities of the ministerial office.

(a) Wanting (b) Willing
(c) Anxious (d) Eager

(b) 'Reluctant' means 'unwilling', so 'willing' is the correct antonym which means 'ready'.

81. He ABANDONED his family.

(a) Supported (b) Encouraged
(c) Pleased (d) Saved

(a) 'Abandoned' means 'left completely, so 'supported', is its correct antonym which means 'defended'.

82. History ABOUNDS in instances of courage.

(a) Shines (b) Lacks
(c) Suffices (d) Fails

(b) 'Abounds' means 'excessive', so 'lacks' is the correct antonym which means 'deficit'.

83. ADVERSITY teaches man to be humble and self-reliant.

(a) Sincerity (b) Animosity
(c) Curiosity (d) Prosperity

(d) 'Adversity' means 'penury' or 'misery', so 'prosperity' is the correct antonym which means 'happiness' or 'wealth'.

84. Like poverty, AFFLUENCE can sometimes create its own problems.

(a) Indigence
(b) Opulence
(c) Sorrow
(d) Exuberance

(a) 'Affluence' means 'state of richness', so 'indigence' is the correct antonym which means 'extreme poverty'.

85. The habit of SQUANDERING money should not be encouraged.

(a) Discarding (b) Hoarding
(c) Donating (d) Stealing

(b) 'Squandering' means 'wasting', so 'hoarding' is its correct antonym which means 'to collecting'.

2014 (I)

86. FRAIL

(a) Strong (b) Big (c) Old (d) Weak

(a) 'Frail' means 'weak', so its antonym is 'strong', which means 'powerful'.

87. INDIGENOUS

(a) Genuine (b) Foreign
(c) Indigent (d) Indignant

(b) 'Indigenous' means 'native or belonging naturally to a place' and 'foreign' means 'of other countries or coming from outside', so they are opposite to each other.

88. POVERTY

(a) Prosperity (b) Adversary
(c) Adversity (d) Diversity

(a) 'Poverty' means 'being poor or the state of being poor' and 'prosperity' means 'prosperous state or riches'. So they are antonyms to each other.

89. WHOLESOME

(a) Complete (b) Unhealthy
(c) Incomprehensible (d) Few

(b) 'Wholesome' implies 'healthy or healthful', so its opposite is 'unhealthy'.

38

IDIOMS AND PHRASES

Directions (Q. Nos. 1-20) *Given below are some idioms/phrases followed by four alternative meanings to each. Choose the response (a), (b), (c) or (d) which is the most appropriate expression.*

2019 (II)

1. Be over the hill

(a) To be on a mountain top
(b) To travel in the jungle
(c) To be too old to do things
(d) To do something in the most complete way

(**c**) Idiom 'Be over the hill' means 'to be too old to do things'. Hence, option (c) is a suitable choice.

2. Bite your tongue

(a) To stop yourself from saying something because it would be better not to
(b) To bite off your tongue while eating
(c) To feel sorry
(d) To ask someone something that you want

(**a**) Idiom 'Bite your tongue' means 'to stop yourself from saying something because it would be better not to'. So option (a) is a suitable choice.

3. Turn a blind eye

(a) To run away
(b) To begin to be have in a more positive manner
(c) To change a situation
(d) To choose to ignore behaviour that you know is wrong

(**d**) The given idiom 'Turn a blind eye' means 'to ignore something that you know is wrong'. So, option (d) is the correct choice.

4. Yellow journalism

(a) Paid storytelling
(b) Government reports published on yellow coloured papers
(c) Writings in newspapers that try to influence people's opinion by using strong language and false information
(d) Journalism which agrees completely with the policies of the government and capitalists

(**c**) Idiom 'Yellow Journalism' means 'writing in newspapers that try to influence people's opinion by using strong language and false information'. So, option (c) is a suitable choice.

5. To be in seventh heaven

(a) To fly in the ail
(b) To be extremely happy
(c) To be completely alone
(d) To visit a place which is in the lit of the seven wonders of the world.

(**b**) Idiom 'To be in seventh heaven' means 'to be extremely happy'. So, option (b) is the correct choice.

2019 (I)

6. A dark horse

(a) A black coloured horse
(b) A person who wins a race or competition although no one expected him to
(c) A person who keeps secrets
(d) An ignorant person

(**b**) Idiom 'a dark horse' means 'an unknown and unexpected winner of a race or contest'. Hence, option (b) expresses the correct meaning of given idiom.

7. A show-stopper

(a) Someone who stops the show
(b) Someone who organises the show
(c) A performance that is extremely good
(d) A fashionable person

(**c**) Idiom 'A show-stopper' means 'a performance in a show that is extremely good'. So, option (c) expresses the correct meaning of the given idiom.

8. A jack of all trades

(a) Someone who has many skills
(b) A confident and not very serious young man
(c) Someone who has hit the jackpot
(d) a great businessman

(**a**) Idiom 'A jack of all trades' means 'someone who has many skills'. So, option (a) is a suitable choice.

9. Fight tooth and nail

(a) To quarrel with someone
(b) To attack someone with a lot of force
(c) To try hard to prevent something from happening
(d) To try very hard to achieve something

(**d**) Idiom 'Fight with tooth and nail' means 'to try very hard to achieve something'. So, option (d) is a suitable choice.

10. Fair and square

(a) in an honest way
(b) in a critical way
(c) neither very good nor very bad
(d) in a foolish way

(**a**) Idiom 'Fair and square' means 'in a honest way and according to the rules'. So, option (a) expresses the correct meaning of the given idiom.

2018 (II)

11. A red-letter day

(a) A trivial day
(b) A very important or significant day
(c) A day of bloodshed and violence
(d) A mourning day

(**b**) Idiom 'A red-letter day' means 'a very important or significant day'. Hence, option (b) is a suitable choice.

12. The gift of the gab

(a) Ability to speak easily and confidently
(b) Ability to spoil something
(c) Ability to sell things
(d) Gift from a sacred institution

(**a**) The given idiom 'Gift of the gab' means 'ability to speak easily and confidently'. So, option (a) expresses the meaning of given idiom.

13. Walk a tightrope

(a) To be forced to leave your job
(b) To be ready to fall
(c) To act very carefully
(d) To invite danger

(**c**) Phrase 'walk a tightrope' means 'to act very carefully'. So, option (c) is the correct choice.

14. To be in a fix

(a) To receive strong criticism
(b) To support oneself
(c) To fix problems
(d) To be in a difficult situation

(**d**) Idiom 'To be in a fix' means 'to be in difficult or dangerous situation'. So, option (d) expresses the correct meaning of the given idiom.

15. To fish in troubled waters

(a) To borrow money
(b) To steal belongings of
(c) To get benefit in bad situation
(d) To extend a helping hand

(**c**) Idiom 'To fish in trouble waters' means 'to get benefit from a difficult or bad situation'. So, option (c) expresses the correct meaning of the given idiom.

2018 (I)

16. Cry over spilt milk

(a) Complaining about a loss in the past
(b) Too much inquisitive about something
(c) When something is done badly to save money
(d) Dealing with a problem only in an emergency situation

(**a**) The given idiom 'Cry over Spilt milk' means 'to express regret about something that has already happened'. So, option (a) expresses the correct meaning of the given idiom.

17. Cut the mustard

(a) Prepare spices out of mustard seeds
(b) To come up to expectations
(c) Making absurd expectations
(d) Very enthusiastic

(**b**) Phrase 'Cut the mustard' means 'to succeed or to come up to expectations'. So, option (b) expresses the correct meaning of the given phrase.

18. Devil's advocate

(a) A dangerous person
(b) To present a counter argument
(c) Very argumentative person
(d) Creating an unpleasant situation

(**c**) Idiom 'Devil's advocate' means 'Someone who argues against something just for the sake of arguing'. So, option (c) 'very argumentative person' expresses the correct meaning of the given idiom.

19. Don't count your chickens before the eggs have hatched.

(a) If you are not good at something, better to avoid that
(b) Don't make plans for something that might not happen
(c) Not to come up to expectations
(d) Don't put all your resources in one possibility

(**b**) The given idiom means 'don't make plans for something that might not happen'. So, option (b) is the correct choice.

20. Give the benefit of doubt

(a) To be partial to someone
(b) To be judgemental
(c) Regard someone as innocent until proven otherwise
(d) Say something exactly right

(**c**) Idiom 'Give the benefit of doubt' means 'to regard someone as innocent until proven otherwise'. So, option (c) expresses the correct meaning of the given idiom.

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CLOZE TEST

2018 (II)

Directions (Q.Nos. 1-15) *In the following passages, there are some blank spaces with four words or groups of words given. Select whichever word or group of words you consider most appropriate for the blank space.*

Passage 1

The honeybee is a very unusual kind of insect. ..... **1.** (a) Unlike
(b) Similar to
(c) With
(d) Like
other insects which live alone, the honeybee lives as a/an..... **2.** (a) group
(b) individual
(c) member
(d) troop
of a community. These bees live together in what is known as a bee..... **3.** (a) house.
(b) home.
(c) army.
(d) colony.
The head of the bees is called the queen bee. She is ..... **4.** (a) heaviest
(b) heavier
(c) largest
(d) larger
than the rest of the bees. Her main task is to ..... **5.** (a) lay eggs.
(b) hatch
(c) make
(d) bear

1. ***(a)*** *'Unlike' is the suitable usage here.*
'Unlike' means 'different from or not similar to'.

2. ***(c)*** 'Member' will be used to fill the blank as 'member of a community' is the correct usage.

3. ***(d)*** The word 'colony' will be used here as 'bee colony' is the correct usage.

4. ***(d)*** 'Larger' is the suitable usage here as in the sentence comparison is made between 'queen bee' and rest of the bees.

5. ***(a)*** 'Lay' will be used here as 'lay eggs' is the correct usage.

2017 (II)

Passage 2

After this incident, I went to Nainital and returned after nearly a month. I had **6.** ..... taken **7.** .....my clothes when I saw Gangu standing
8. .....a new baby. He was **9.**.....with joy. Even Nanda
10. ..... not have **11.** ..... such joy **12.** ..... getting Krishna. His face had the same **13.** ..... that
14. ..... on the face of a **15.** ..... man after a full meal.

6. (a) hardly (b) barely
(c) merely (d) rarely

7. (a) out (b) away
(c) off (d) on

8. (a) by (b) near
(c) with (d) at

9. (a) jumping (b) bursting
(c) dancing (d) singing

10. (a) could (b) would
(c) should (d) ought

11. (a) experimented (b) shown
(c) felt (d) heard

12. (a) at (b) in
(c) on (d) into

13. (a) light (b) glow
(c) sense (d) hope

14. (a) comes (b) appears
(c) rises (d) shows

15. (a) starved (b) starving
(c) hungry (d) satisfied

6. *(**b**)* 'Barely' is the correct usage here. The word 'barely' means 'only just'.

7. *(**c**)* 'Off' is the correct usage here as 'taken off' is a phrasal verb which means 'to remove a piece of clothing'.

8. *(**c**)* Preposition 'with' will be used here as it is used to show togetherness.

9. *(**a**)* 'Jumping' is the suitable usage here as 'Jumping with joy' is an idiom meaning 'to be extremely happy'.

10. *(**a**)* 'Could' is the correct usage here as modal 'Could' is used to show ability.

11. *(**c**)* With the word 'joy', option (c) 'felt' is the correct usage as 'joy' is a feeling which can be felt. Other words are irrelevant here.

12. *(**c**)* Preposition 'on' will be used to fill the given blank as it indicates that something is already in the position.

13. *(**b**)* 'Glow' is the suitable choice in the context as face glows with some positive emotion.

14. *(**b**)* 'Appears' is the correct usage here.
'Appears' means 'to become visible or noticeable'.

15. *(**b**)* 'Starving' is the correct usage here as the sentence is in Present tense and 'satisfied', 'hungry' and 'starved' are not suitable for the blank.

Directions (Q. Nos. 16-33) *In the following passages, at certain points, you are given a choice of three words marked (a), (b) and (c), one of which fits to the meaning of the passage. Choose the best word out of the three.*

2015 (I)

Passage 3

After having slept for an hour, Bond decided to go out into the city and try to find his contact. After changing his suit, he carefully locked his room and stepped out of the hotel. No one was around. But he had walked only for a few minutes when it suddenly

16. (a) felt
(b) occurred
(c) happened

to him that he was being **17.** (a) followed
(b) shot
(c) looked

by someone. There was really no **18.** (a) reason
(b) cause
(c) evidence

for it except a very slight **19.** (a) weight
(b) sound
(c) hurting

of footsteps. He was now walking **20.** (a) from
(b) down
(c) over

the main street, which was crowded **21.** (a) with
(b) by
(c) in

people. He became extremely aware of the danger of people threatening him.

16. *(b)* 'Occurred' meaning 'suddenly realised' is the correct usage. 'Felt' or 'happened' are not suitable here.
17. *(a)* In the given context, the word 'followed' is the correct usage as in the passage, the author felt that someone is coming behind him.
18. *(c)* 'Evidence' meaning 'proof' will fit here as he could not provide any proof of being followed by someone.
19. *(b)* 'Slight sound' is correct usage. 'Weight' and 'hurting' do not make sense with 'slight' here.
20. *(b)* 'Down' is the correct usage here as 'walking down' is a phrase meaning 'walking along'.
21. *(a)* 'Crowded' is always followed by preposition 'with'. So, option (a) is correct here.

2014 (I)

Passage 4

Drobny defied the critics. He had played through the Wimbledon fortnight and reached the final. Could he win just one more match and take the crown or would be fail again at the last test and justify those who said that he lacked the little something that makes the champion? His opponent was the Australian Ken Rosewall, **22.** (a) that brilliant
(b) a
(c) the

youngster and already a master **23.** (a) with
(b) a
(c) that

worldwide experience behind him. Jaroslav Drobny, **24.** (a) a
(b) an
(c) the

son of a Czech carpenter, who **25.** (a) had
(b) a
(c) the

started the boy off at the **26.** (a) *age*
(b) stage
(c) career

of five with a wooden 'bat' in lieu **27.** (a) for
(b) of
(c) to

a racquet, went on **28.** (a) to
(b) a
(c) the

the famous Centre Court at Wimbledon **29.** (a) in
(b) on
(c) that

day with the full knowledge that this **30.** (a) can
(b) must
(c) would

be the game of his **31.** (a) life.
(b) age
(c) day

He had kept on trying, and **32.** (a) he
(b) one
(c) you

could keep on trying, but **33.** (a) games
(b) matches
(c) opportunities

would become fewer and fewer. He knew that he had to gain an advantage from the start, and he had this young rival set off against him.

22. *(**b**)* 'A' will be used as there is no previous reference to the person being talked about. So, the person is indefinite and hence, we will use the indefinite article 'a'.

23. *(**a**)* 'With' will be used. The word 'with' here implies that the person possessed experience.

24. *(**c**)* 'The' is the correct usage because it points out a particular person, Jaroslav Drobny, who is the son of a Czech carpenter.

25. *(**a**)* 'Had' is the correct choice because the use of Past Perfect Tense is appropriate in this sentence.

26. *(**a**)* The reference is to the age, i.e. the age of five. So, option (a) is correct.

27. *(**b**)* Preposition 'of' will be used here as phrase 'In lieu of' means 'in place of'.

28. *(**a**)* 'To' is the suitable usage here as 'Went on to' means 'proceeded to'.

29. *(**c**)* The reference is to a particular day, so 'that day' will be used.

30. *(**c**)* 'Would' should be used as it is the past equivalent of 'will'.

31. *(**a**)* The mentioned game is very crucial for Drobny, so 'game of his life' is the correct usage. Hence, option (a) is correct.

32. *(**a**)* The preceding sentence makes it clear that the pronoun 'he' is the suitable usage.

33. (**c**) With the passage of time, there will be less 'opportunities' to avail. So, option (c) 'opportunities' is the correct choice.

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COMPREHENSION

Directions (Passages 1-10) *In this section you have short passages.*
After the passage, you will find some items based on the passage. First read the passage and then answer the items based on it. You are required to select your answers based on the contents of the passage and opinion of the author only.

2019 (II)

Passage 1

Studies serve for delight, for ornament and for ability. Their chief use for delight, is in privateness and retiring; for ornament, is in discourse; and for ability, is in the judgement and disposition of business. For expert men can execute and perhaps judge of particulars, one by one; but the general counsels and the plots and marshalling of affairs, come best from those that are learned. To spend too much time in studies is sloth; to use them too much for ornament, is affectation; to make judgement wholly by their rules, is the humor of a scholar. They perfect nature and are perfected by experience: for natural abilities are like natural plants, that need pruning, by study; and studies themselves do give forth directions too much at large, except they be bounded in by experience. Crafty men condemn studies, simple men admire them and wise men use them; for they teach not their own use; but that is a wisdom without them and above them, won by observation. Read not to contradict and confute; nor to believe and take for granted; nor to find talk and discourse; but to weigh and consider.

1. According to the author, why should one study?

(a) To gather information about the world
(b) To pass the time in a creative way
(c) For pleasure, enhance capability and holistic growth
(d) To become only worldly wise

(›) (***c***) As stated in the passage, studies should be done for pleasure, enhancing capability and for holistic growth of a mind.

2. Why does the author not recommend too much of studies?

(a) Because it reflects idleness
(b) Because it requires too much money
(c) Because it is dangerous for the health of the mind
(d) Because it makes men crafty

(›) *(**a**)* The author does not recommend too much of studies as it can make a person lazy or idle.

3. According to the author, how can the studies be perfected?

(a) By reflection (b) By application
(c) By conversation (d) By experience

(›) *(**d**)* According to the author, studies can be perfected via experience.

4. What kind of human beings denounce studies?

(a) Innocent (b) Cunning
(c) Stupid (d) Wise

(›) *(**b**)* Crafty or Cunning human beings denounce studies.

5. According to the author, one should read to

(a) only argue (b) only believe
(c) evaluate and understand (d) preach and believe

(›) *(**c**)* As stated in the passage, a person should read to 'evaluate and understand'.

2019 (I)

Passage 2

I do not wish to suggest that because we were one nation, we had no differences, but it is submitted that our leading men travelled throughout India either on foot or in bullockcarts. They learned one another languages and there was no aloofness amongst them. What do you think could have been the intention of those farseeing ancestors of ours who established Setubandha (Rameshwar) in the South, Jagannath in the East and Haridwar in the North as places of pilgrimage? You will admit they were no fools. They knew that worship of God could have been performed just as well

at home. They taught us that those whose hearts were aglow with righteousness had the Ganges in their own homes. But they saw that India was one undivided land so made by nature. They, therefore, argued that it must be one nation. Arguing thus, they established holy places in various parts of India and fired the people with an idea of nationality in a manner unknown in other parts of the world. And we Indians are one as no two Englishmen are. Only you and I and others who consider ourselves civilised and superior persons imagine that we are many nations. It was after the advent of railways that we began to believe in distinctions and you are at liberty now to say that it is through the railways that we are beginning to abolish those distinctions.

An opium-eater may argue the advantage of opium-eating from the fact that he began to understand the evil of the opium habit after having eaten it. I would ask you to consider well what I had said on the railways.

1. According to the author, India

(a) has never been one nation
(b) has been an aggregate of several nations
(c) has always been one nation along with differences
(d) became a nation after the British came

*(**c**)* The author has stated in the passage that India has always been one nation along with diversities or differences.

2. Why did the great sages of India establish pilgrimages in the different corners of the country?

(a) Because they wanted to push people to travel to different places.
(b) Because they could observe the underlying unity of the country as made by nature.
(c) Because they themselves had travelled to these places.
(d) Because they wanted people to be religious everywhere.

*(**b**)* The great sages of India established pilgrimages in the different corners of the country because they could observe the underlying unity of the country as made by nature. For them, India was one undivided nation.

3. In the passage, the author's attitude towards the railways is

(a) critical
(b) sympathetic
(c) indifferent
(d) apathetic

*(**a**)* The author's attitude towards the railways is critical. According to the author, due to railways people have been able to see the diversities of nature, people, food, etc. But railways have also helped to unite people of different states.

4. What does the author mean when he says that ''Whose hearts were aglow with righteousness had the Ganges in their own homes''?

(a) One need not visit the Ganges to take holy bath
(b) The Ganges had been polluted, so one should bath at home
(c) One should take a holy dip in the Ganges to purify one's heart
(d) The purity of heart is superior to observance of any ritual

*(**d**)* According to the author, if one's heart is pure, there is no need to observe any rituals like going on pilgrimages or visiting different temples and other prayer structures. God resides in a pure heart.

5. The passage is written in a

(a) dialogic style (b) prescriptive style
(c) descriptive style (d) analytical style

*(**c**)* The passage is written in a descriptive style. In descriptive writing style, the author specifies an event, an object or a thing.

2018 (II)

Passage 3

"Now, ladies and gentlemen,' said the conjuror, 'having shown you that the cloth is absolutely empty. I will proceed to take from it a bowl of goldfish. Presto !"

All around the hall people were saying, "Oh, how wonderful ! How does he do it?"

But the Quick Man on the front seat said in a big whisper to the people near him, "He-had-it-up-his-sleeves."

Then the people nodded brightly at the Quick Man and said, 'Oh, of course'; and everybody whispered round the hall, "He-had-it-up-his-sleeves." "My next trick," said the conjuror, "is the famous Hindostanee rings. You will notice that the rings are apparently separate; at a blow they all join (clang, clang, clang)— Presto !"

There was a general buzz of stupefaction till the Quick Man was heard to whisper,

"He-must-have-had-another-lot-up-his- sleeves".

Again everybody nodded and whispered,

"The-rings-were-up-his-sleeves." The brow of the conjuror was clouded with a gathering frown.

"I will now," he continued, "show you a most amusing trick by which I am enabled to take any number of eggs from a hat. Will some gentleman kindly lend me his hat?' Ah, thank you—Presto !"

He extracted seventeen eggs and for thirty-five seconds the audience began to think that he was wonderful. Then the Quick Man whispered along the front bench,

"He-has-a-hen-up-his-sleeves," and all the people whispered it on. "He-has-a-lot-of-hens-up-his- sleeves".

The egg trick was ruined.

It went on like that all through. It transpired from the whispers of the Quick Man that the conjuror must have concealed up his sleeves, in addition to the rings, hens and fish, several packs of cards, a loaf of bread, a doll's cradle, a live guinea pig, a fifty-cent piece and a rocking chair.

1. ''The brow of the conjuror was clouded with a gathering frown.'' The sentence means that the conjuror

(a) was very pleased (b) was very sad
(c) was rather angry (d) was very afraid

(c) 'Was rather angry' is a correct option. All tricks of conjuror were ruined due to the Quick man's remarks.

2. ''The egg trick was ruined''. This means that
(a) eggs were all broken
(b) people were unconvinced
(c) conjuror was disappointed
(d) the trick could not be performed

› *(**b**)* 'The egg trick was ruined' means that people were unconvinced.

3. According to the Quick Man, the conjuror
(a) had everything bought for production
(b) produced things with the magic he knew
(c) had things in the large sleeves of his coat
(d) created an illusion of things with his magic

› *(**c**)* According to the Quick man, the conjuror had things in large sleeves of his coat.

4. The author believes that the Quick Man was really
(a) foolish
(b) clever
(c) wrong
(d) right

› *(**a**)* 'Foolish' as last sentence of the author shows that the Quick Man was not intelligent to presume that the conjuror must be having a live hen up his sleeve who on demand will give 17 eggs.

5. The conjuror extracted seventeen eggs from the hat of
(a) the Quick Man
(b) his own
(c) one gentleman from the audience
(d) None of the above

› *(**c**)* The conjuror extracted seventeen eggs from the hat of one gentleman from the audience.

Passage 4

Magda Good morning, Mrs. Smiles. It's wet, isn't it?
Mrs Smiles Yes, it is, isn't it? How are you today? All right? You haven't been studying too hard, have you? You look a bit pale.
Magda I don't, do I? Well, I haven't been out much lately. I don't like this weather a bit. Why, we haven't had a dry day for weeks, have we?
Mrs Smiles No, I don't think we have. Let's see, you'll be going back home next month, won't you?
Magda Oh no, I'm not going back yet. My sister's coming over first.
Mrs Smiles Oh, is she? You're looking forward to that, I expect. How long is it since you saw her?
Magda Nearly a year now. Yes, I am looking forward to it very much.
Mrs Smiles She's coming over here to study?
Magda Yes, that's right, I shan't go back home until she's settled down. I don't think I ought to, ought I?
Mrs Smiles Well, no, she's younger than you are, is she?
Magda Yes, she's only eighteen.

1. Magda had not been out much lately because
(a) she had not been keeping well
(b) she was busy with her studies
(c) the weather had been unpleasant
(d) her sister had come over to stay

› *(**c**)* As stated in the passage, Magda had not been out much lately because it has been raining continuously. It also implies that 'the weather has been unpleasant'.

2. Magda was not going back home yet because
(a) it had been raining heavily
(b) her sister was coming over
(c) her studies were not completed
(d) she was not feeling well

› *(**b**)* Magda was not going back home yet because her sister was coming over and she wants to help her in settling down.

3. Mrs. Smiles says, ''You're looking forward to that''. This means, Magda was …… her sister's arrival.
(a) expecting with pleasure (b) preparing hard for
(c) thinking greatly about (d) watching with joy

› *(**a**)* Magda was expecting with pleasure her sister's arrival as it has been a year since she met her sister.

4. Magda's sister was coming to
(a) visit her sister (b) help Mrs. Smiles
(c) settle down in England (d) pursue her studies

› *(**d**)* Magda's sister was coming to pursue her studies and shall not go back home until she is settled down.

5. Mrs. Smiles and Magda are
(a) classmates (b) school friends
(c) neighbours (d) sisters-in-law

› *(**c**)* From the conversation shown in the passage between Mrs. Smiles and Magda, it appears that both are neighbours.

2018 (I)

Passage 5

The rule of the road means that in order that the liberties of all may be preserved, the liberties of everybody must be curtailed. When the policeman, says, at a road-crossing steps into the middle of the road and puts out his hand, he is the symbol not of tyranny but of liberty. You have submitted to a curtailment of private liberty in order that you may enjoy a social order which makes your liberty a reality. We have both liberties to preserve — our individual liberty and our social liberty.
That is, we must have a judicious mixture of both. I shall not permit any authority to say that my child must go to this school or that, shall specialise in science or arts. These things are personal. But if I say that my child shall have no education at all, then society will firmly tell me that my child must have education whether I like it or not.

1. According to the author, the ''rule of the road'' implies

(a) the rule regulating the traffic on the road
(b) the principle on which a road is constructed to ensure safe traffic
(c) unrestricted freedom for all to lead a happy life
(d) restricted individual freedom to ensure freedom for all

*(**d**)* Rule of the road according to the passage means restricted individual freedom to ensure freedom for all.

2. The author thinks that when a policeman signals you to stop on a road-crossing, he is

(a) behaving in a whimsical manner
(b) interfering with your freedom to use the road
(c) protecting the liberty of all to use the road
(d) mischievously creating hurdles in your way for some personal motive

*(**c**)* According to the author, when a policeman signals us to stop on a road-crossing, he is protecting the liberty of all to use the road.

3. The author is of the view that we should

(a) have absolute individual liberty without any restrictions imposed by the society
(b) have everything, controlled by the society without any kind of individual liberty
(c) try to strike a sensible balance between our individual liberty and our social liberty
(d) have more of social liberty than individual liberty

*(**c**)* The author is of the view that we should try to strike a sensible balance between our individual liberty and our social liberty.

4. The author holds that

(a) educating or not educating his child is a matter of personal liberty
(b) educating or not educating his child is also a matter of social liberty
(c) choosing the school for his child is a matter of social liberty
(d) choosing the subject of study for his child is a matter of social liberty

*(**b**)* The author is of the opinion that educating or not educating his child is also a matter of social liberty.

5. The most suitable title of the passage would be

(a) The Policeman at a Road-Crossing
(b) The Laws of the Road
(c) Importance of Liberty
(d) Education of Children

*(**c**)* The most suitable title of the passage would be option (c). 'Importance of Liberty'.

Passage 6

My most interesting visitor comes at night, when the lights are still burning — a tiny bat who prefers to fly in through the open door and will use the window only if there is no alternative. His object in entering the house is to snap up the moths that cluster around the lamps. All the bats I have seen fly fairly high, keeping near the ceiling; but this particular bat flies in low, like a dive-bomber, zooming in and out of chair legs and under tables. Once, he passed straight between my legs. Has his radar gone wrong, I wondered, or is he just plain crazy?

1. Consider the following statements :

1. The tiny bat flew in low like a dive bomber.
2. The tiny bat like all bats keeps near the ceiling.
3. It has lost direction because its radar has gone wrong.
4. It wants to entertain the author with its skill in flying.

Which of the above statements may be assumed to be true from the information given in the passage?

(a) Only 1 (b) 1 and 3 (c) 2 and 4 (d) 3 and 4

*(**b**)* According to the given passage, the statements given in (1) and (3) may be assumed to be true. Statements (2) and (4) are not true.

2. The bat entered the room

(a) because there was no alternative
(b) to eat the moths round the lamps
(c) as it had gone mad
(d) as it preferred to fly in through the open door

*(**b**)* The bat entered the room to eat the moths round the lamps.

3. After comparing the habits of the tiny bat with those of other bats, the author was

(a) sure that this bat had lost its direction
(b) not sure of its preferences
(c) surprised to find that it was an expert flier
(d) unable to give the correct explanation for its behaviour

*(**d**)* The author was confused and hence was unable to give the correct explanation for the behaviour of the bat.

4. The author calls the tiny bat 'an interesting visitor'. This means

(a) the bat visits him at night
(b) the bat is interested in the moths
(c) this bat has peculiar qualities
(d) this bat surprises him by zooming in and out like a dive-bomber

*(**c**)* The tiny bat is called 'an interesting visitor' by the author as it has peculiar qualities.

5. What, according to you, can be the most suitable title for the passage?

(a) Someone visits me (b) Night of Mysteries
(c) My Nocturnal Visitor (d) A Funny Visitor

*(**c**)* 'My Nocturnal Visitor' is the most suitable title of the passage from the given options because the passage is about the 'bat' who is nocturnal. Nocturnal means awake and active at night and asleep during the day (used about animals and birds).

2017 (I)

Passage 7

We had just passed Tenali, where I roused myself in order to hear the name of the station. I was falling asleep again, a violent jolt shot me into the arms of somebody in the seat opposite. The engine with one wheel broken was lying across the track and beside it was the luggage van, likewise, derailed. Groaning, wheezing, gasping, sputtering in its death agony, the engine

was like a fallen horse which, snorting, trembling in every limb, its flanks heaving, its chest labouring, seems incapable of making the smallest effort to struggle on to its legs again.

1. The author had roused himself as he wanted to

(a) get off (b) meet someone
(c) know the name of the station (d) keep himself awake

(›) *(**c**)* The author had roused himself as he wanted to know the name of the station and suddenly a violent jolt shot him when he was falling asleep again.

2. The engine stopped because

(a) the driver had stopped it
(b) one of the wheels was broken
(c) there was a halting place
(d) there was no fuel

(›) *(**b**)* The engine stopped because one of the wheels of the engine was broken.

3. The engine is compared to a fallen horse because

(a) it was groaning like a fallen horse
(b) it was lying across the track
(c) it could make no effort to stand upright
(d) it had a broken wheel

(›) *(**a**)* The engine is compared to a fallen horse because it was groaning and snoring like a fallen horse.

Passage 8

I was beginning to tire a little now. I had been cutting steps continuously for two hours and Tenzing, too, was moving very slowly. As I chipped steps around still another corner, I wondered rather dully just how long we could keep it up. Our original zest had now quite gone and it was turning more into a grim struggle. I then realised that the ridge ahead, instead of still monotonously rising, now dropped sharply away, and far below I could see the North Pole and Rongbuk glacier. I looked upwards to see a narrow ridge running up to a snowy summing. A few more whacks of the ice-axe in the firm snow and we stood on top.

1. Which of the following is the most appropriate description of the area in which the narrator had been moving?

(a) A steep rocky slope
(b) A peak with a flight of steps
(c) A high peak covered with snow
(d) A long ridge

(›) *(**c**)* The area in which the narrator had been moving was a high peak covered with snow.

2. 'I had been cutting steps continuously for two hours' suggests that

(a) the climber was unskilled in the art of making steps
(b) he had to cut the steps out of snow before climbing up which was necessarily a slow process
(c) the slowness was caused by the fact that as soon as he cut the steps they were again covered with snow
(d) he has to work suspended from a ledge

(›) *(**b**)* The given statement suggests that he had to cut the steps out of snow before climbing up which was necessarily a slow process.

3. ''It was turning more into a grim struggle'' would mean

(a) that after the long exertion, he was feeling so weak that any further climbing meant a hard struggle with the forces of nature
(b) that Tenzing was making the task more difficult
(c) that strong cold wind was pushing him down from above and that he had to struggle with the forces of nature
(d) that he was in a mood to give up ultimately

(›) *(**a**)* ''It was turning more into a grim struggle'' would mean that after the long exertion, he was feeling so weak that any further climbing meant a hard struggle with the forces of nature.

2014 (I)

Passage 9

In a free country, the man who reaches the position of leader is usually one of outstanding character and ability. Moreover, it is usually possible to foresee that he will reach such a position, since early in life one can see his qualities of character. But this is not always true in the case of a dictator, often he reaches his position of power through chance, very often through the unhappy state of his country. It is possible that Hitler would never have become the leader of Germany if that country had not been driven to despair.

1. In a free country, one who becomes a leader

(a) must be of outstanding character and ability
(b) must show qualities of character from an early age
(c) is generally of a remarkable character and ability
(d) must see that his country is free from despair

(›) *(**c**)* Refer to the starting lines of the passage– In a free country, the man who reaches the position of leader is usually one of outstanding character and ability. It is not stated that he must be of outstanding character and ability or must show qualities at an early age. The word 'must' is nowhere used in the passage or in the sense as is given in the other three options.

2. Hitler became a leader because

(a) he exhibited leadership qualities
(b) Germany was a free country
(c) Germany was to despair
(d) Germans wanted a dictator

(›) *(**c**)* The last two lines of the passage make it clear that Hitler became the leader of Germany because of the atmosphere of despair, prevalent in that country.

3. The passage seems to suggest that

(a) despair sometimes leads to dictatorship
(b) Hitler was no leader
(c) a leader is chosen only by a free country
(d) a leader foresees his future position

(›) *(**a**)* Among the given options, option (a) appears to be the appropriate as the other options find no consonance in the context of the passage. Moreover, the answer can be traced from the lines "in the case of a dictator, often he reaches his position of power through chance, very often through the unhappy state of his country".

Passage 10

The dog is a very intelligent animal and it can be trained for many duties. Some dogs work with the police and the army as sentries and trackers and there are medals that are awarded to dogs for bravery in action. Other dogs act as seeing-eyes for the blind and some breeds are specially trained for rescue work among mountains and snow. Jobs like herding sheep and cattle and guarding homes are also done by them. Whenever man lives and works in the Arctic Circle, in the desert, in the jungle, in the moors or mountains, the dog is his faithful companion and help. A common mongrel may possess these qualities just as much as the aristocratic bloodhound.

1. The passage tells us that dogs

(a) can be trained to do different kinds of work
(b) are capable of difficult work
(c) are of different types
(d) alone are a great help to man

› *(**a**)* The first line of the passage explicitly states that "The dog is a very intelligent animal and it can be trained for many duties". The rest of the passage elaborates the statement made in the initial sentence by citing various ways in which dogs are trained to do different types of work.

2. Medals are awarded to some dogs for

(a) their brave deeds
(b) working with the police
(c) serving as sentries
(d) being able to perform difficult work

› *(**a**)* Refer to line 'there are medals that are awarded to dogs for bravery in action, which gives the correct answer, i.e. option (a).

3. The passage implies that

(a) only dogs are faithful but not man
(b) man would have felt helpless without dogs
(c) the dog has certain qualities that make it man's trustworthy companion
(d) dogs can do almost anything

› *(**c**)* In the passage, the dog is also referred to as the faithful companion and help of man. Hence, option (c) seems suitable among the given options.

4. The phrase 'aristocratic bloodhound' in the passage means

(a) the dog kept by wealthy people
(b) a pedigree dog
(c) a dog fond of blood
(d) any big dog

› *(**b**)* The phrase 'aristocratic bloodhound' means a pedigree dog.

GENERAL SCIENCE

41

PHYSICS

2019 (II)

1. A thin disc and a thin ring, both have mass M and radius R. Both rotate about axes through their centre of mass and are perpendicular to their surfaces at the same angular velocity. Which one of the following statements is correct?

(a) The ring has higher kinetic energy.
(b) The disc has higher kinetic energy.
(c) The ring and the disc both have the same kinetic energy.
(d) Kinetic energies of both the bodies are zero, since they are not in linear motion.

(a) Given, a thin disc and a thin ring have equal mass and radius.

Angular velocity of thin disc

= Angular velocity of thin ring

$$\omega_{ring} = \omega_{disc}$$

$\therefore$ Kinetic energy of ring

= Kinetic energy of disc

$$\frac{1}{2} I_{ring}\omega_{ring}^2 = \frac{1}{2} I_{disc}\omega_{disc}^2$$

$$\frac{1}{2}(MR^2)\,\omega_{ring}^2 = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega_{disc}^2$$

$$(\because I_{ring} = MR^2 \text{ or } I_{disc} = \frac{1}{2} MR^2)$$

$$\omega_{ring}^2 = \frac{\omega_{disc}^2}{2}$$

$$2\omega_{ring}^2 = \omega_{disc}^2$$

Hence, $KE_{ring} > KE_{disc}$

So, the ring has higher kinetic energy.

2. In which of the following phenomena do heat waves travel along a straight line with the speed of light?

(a) Thermal conduction
(b) Thermal convention
(c) Thermal radiation
(d) Both (a) and (c)

(c) Thermal radiations are heat waves (infrared waves) which are electromagnetic waves. Electromagnetic waves travel in straight line with the speed of light. It depends on the temperature of the body and nature of radiating surface of the body.

Hence, option (c) is correct.

3. Which one of the following statements regarding Ohm's law is not correct?

(a) Ohm's law is an assumption that current through a conductor is always directly proportional to the potential difference applied to it.
(b) A conducting device obeys Ohm's law when the resistance of a device is independent of magnitude and polarity of applied potential difference.
(c) A conducting material obeys Ohm's law when the resistance of material is independent of the magnitude and direction of applied electric field.
(d) All homogeneous materials obey Ohm's law irrespective of whether the field is within range or strong.

(d) Statement (d) is not correct, because in strong electric field, heating effect will dominant and here is deviations of curve, i.e. it does not obey Ohm's law.

4. When a ball bounces off the ground, which of the following changes suddenly?

(Assume no loss of energy to the floor)

(a) Its speed
(b) Its momentum
(c) Its kinetic energy
(d) Its potential energy

(b) When a ball bounces off the ground, there will be sudden change in the momentum of the ball.

- It includes the mass and the velocity of the object.
- It shows the sudden change of the velocity of the object according to its mass.
- If the mass is large, the change will be greater and if it is small, then the velocity will be lesser.

5. If an object moves at a non-zero constant acceleration for a certain interval of time, then the distance it covers in that time

(a) depends on its initial velocity
(b) is independent of its initial velocity
(c) increases linearly with time
(d) depends on its initial displacement

(a) If object moving with non-zero constant acceleration, then the displacement covered by it, is given by equation

$$s = ut + \frac{1}{2}at^2$$

where, s = displacement, u = initial velocity, a = acceleration and t = time.

Hence, from above equation, displacement is dependent to initial velocity.

Distance covered is a non-linear curve of time.

6. A solid disc and a solid sphere have the same mass and same radius. Which one has the higher moment of inertia about its centre of mass?

(a) The disc
(b) The sphere
(c) Both have the same moment of inertia
(d) The information provided is not sufficient to answer the question

(a) As we know, moment of the inertia

$$I_{disc} = \frac{1}{2} MR^2 \text{ and } I_{sphere} = \frac{2}{5} MR^2$$

$\because$ Given, $M_{disc} = M_{sphere}$ and

$R_{disc} = R_{sphere}$

So, $\frac{I_{disc}}{I_{sphere}} = \frac{\frac{1}{2}MR^2}{\frac{2}{5}MR^2} = \frac{5}{4}$ … (i)

Hence, from Eq. (i), we get

$I_{disc} > I_{sphere}$

So, the disc has higher moment of inertia.

7. Two substances of densities ρ_1 and ρ_2 are mixed in equal volume and their relative density is 4. When they are mixed in equal masses, relative density is 3. The values of ρ_1 and ρ_2 respectively, are

(a) 6, 2 (b) 3, 5
(c) 12, 4 (d) 9, 3

(a) When substances are mixed in equal volume.

Then, relative density,

$$4 = \frac{V\rho_1 + V\rho_2}{V + V}$$

[$\because$ volume(V) x density (ρ) = mass (M)]

$\Rightarrow \quad \rho_1 + \rho_2 = 8$ …(i)

Similarly, when substances are mixed in equal mass.

Relative density,

$$3 = \frac{M + M}{\frac{M}{\rho_1} + \frac{M}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \quad \text{… (ii)}$$

Hence, from Eqs. (i) and (ii), we get

$\rho_1 = 6$ and $\rho_2 = 2$

8. A car starts from Bengaluru, goes 50 km in a straight line towards South, immediately turns around and returns to Bengaluru. The time taken for this round trip is 2h. The magnitude of the average velocity of the car for this round trip

(a) is zero
(b) is 50 km/h
(c) is 25 km/h
(d) cannot be calculated without knowing acceleration

(a) According to question, a round trip is mentioned by a car. But as we know, for round trip the displacement is zero. So,

$$v_{av} = \frac{\text{displacement}}{\text{time}} = \frac{0}{2} = 0 \text{ ms}^{-1}$$

Hence, option (a) is correct.

9. "The sum of emf's and potential differences around a closed loop equals zero" is a consequence of

(a) Ohm's law
(b) conservation of charge
(c) conservation of momentum
(d) conservation of energy

(d) It is Kirchhoff's second law also known as loop's law.

i.e. $\Sigma E = iR$

It is based on conservation of energy, since work done in a closed path is zero.

10. A student measures certain lengths using a meter scale having least count equal to 1 mm.

Which one of the following measurement is more precise?

(a) 0.50 mm
(b) 29.07 mm
(c) 0.925 mm
(d) 910 mm

(c) Using instrument of higher precision improving experimental techniques, etc., we can reduce the least count error.

Here, option (c) 0.925 mm is more precise than other.

11. If the work done on the system or by the system is zero, then which one of the following statement for a gas kept at a certain temperature is correct?

(a) Change in internal energy of the system is equal to flow of the heat in or out of the system.
(b) Change in internal energy of the system is less than heat transferred.
(c) Change in internal energy of the system is more than the heat flow.
(d) Cannot be determined.

(a) For first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

where, ΔQ = heat, ΔU = change in internal energy and

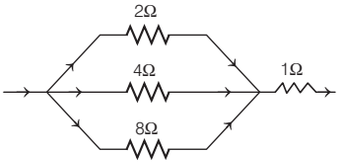
ΔW = work done.

Since, $\Delta W = 0$

So, $\Delta Q = \Delta U$

Hence, there is change in internal energy of the system is equal to flow of heat in or out of the system, option (a) is correct.

12. Consider the following part of an electric circuit.



The total electrical resistance in the given part of the electric circuit is

(a) $\frac{15}{8}\ \Omega$ (b) $\frac{15}{7}\ \Omega$
(c) 15 Ω (d) $\frac{17}{3}\ \Omega$

(b) There are three resistances (2Ω, 4Ω and 8Ω) are in parallel.

So, $\frac{1}{R_1} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \Rightarrow R_1 = \frac{8}{7}\Omega$

Now, R_1 is in series with 1 Ω resistance.

So, $R_{eq} = 1\ \Omega + R_1 = 1 + \frac{8}{7} = \frac{15}{7}\ \Omega$

Hence, option (b) is correct.

13. Which one of the following statements regarding lenses is not correct?

(a) A convex lens produces both real and virtual images.
(b) A concave lens produces both real and virtual images.
(c) A convex lens can produce images equal, greater and smaller than the size of the object.
(d) A concave lens always produces images smaller than the size of the object.

(b) A concave lens always formes virtual image, whereas convex lens forms both real and virtual images.

Hence, option (b) is incorrect.

14. 10 g of ice at – 10° C is mixed with 10 g of water at 0°C. The amount of heat required to raise the temperature of mixture to 10°C is

(a) 400 cal (b) 550 cal
(c) 1050 cal (d) 1200 cal

(c) Heat require by 10 g of water at – 10°C to 0°/C is

$H_1 = ms\Delta\theta$

$= 10 \times (0.50)\ (0 - (-10) = 50$ cal

Now, latent heat of melting of 10 g of ice

$H_2 = mL = 10 \times 80 = 800$ cal

Now, there is 20 g of water at 0°C, so heat required to reach the temperature of + 10° C.

$H_3 = ms\Delta\theta = 20 \times 1 \times (10 - 0) = 200$ cal

Total heat required,

$H = H_1 + H_2 + H_3$

$= 50 + 800 + 200$

$= 1050$ cal

15. Soaps clean surfaces on the principle of based on

(a) viscosity
(b) floatation
(c) elasticity
(d) surface tension

(d) Soaps clean surfaces on the principle of based on surface tension. Soap is a surface active agent or surfactant. It helps to reduce the surface tension of water and thereby speed up the cleaning process of soaps.

16. Water is heated with a coil of resistance R connected to domestic supply. The rise of temperature of water will depend on

1. supply voltage
2. current passing through the coil
3. time for which voltage is supplied

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) 1 and 2
(c) Only 1 (d) 2 and 3

(a) Heat generated by a resistance coil is given by

$$H = I^2Rt$$

From Ohm's law, $I = \frac{V}{R}$

So, $H = I\left(\frac{V}{R}\right)Rt = IVt$

Hence, option (a) is correct.

17. A pendulum clock is lifted to a height where the gravitational acceleration has a certain value of g. Another pendulum clock of same length, but of double the mass of the bob is lifted to another height where the gravitational acceleration is $\frac{g}{2}$.

The time period of the second pendulum would be (in terms of period T of the first pendulum)

(a) $\sqrt{2}\,T$ (b) $\frac{1}{\sqrt{2}}T$ (c) $2\sqrt{2}\,T$ (d) T

(a) Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots(i)$$

When acceleration due to gravity changed to $\frac{g}{2}$, then time period,

$$T' = 2\pi\sqrt{\frac{l}{g/2}} = \sqrt{2}\left(2\pi\sqrt{\frac{l}{g}}\right)$$

[from Eq. (i)]

$$\Rightarrow \quad T' = \sqrt{2}\,T$$

Hence, option (a) is correct.

18. The loudness of sound depends upon the

(a) velocity of sound waves in the medium
(b) amplitude of the sound waves
(c) frequency of the sound waves
(d) frequency and velocity of the sound waves

(b) Loudness of sound wave is depend on its amplitude, while sharpness of sound wave is depend on its frequency. Hence, option (b) is correct.

19. Two bodies of mass M each are placed R distance apart. In another system, two bodies of mass $2M$ each are placed $\frac{R}{2}$ distance apart. If F be the gravitational force between the bodies in the first system, then the gravitational force between the bodies in the second system will be

(a) 16 F (b) 1 F
(c) 4F (d) None of these

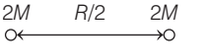
(a) **Situation I,**

Force between the masses

$$F = \frac{GMM}{R^2} = \frac{GM^2}{R^2} \quad \ldots(i)$$

Situation II,

Force between the masses



$$F' = \frac{G(2M)(2M)}{\left(\frac{R}{2}\right)^2} = \frac{16GM^2}{R^2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$F' = 16F$$

Hence, option (a) is correct.

20. Light rays move in straight lines. But through an optical fibre, they can move in any type of *zig-zag* path because

(a) the holes through the fibres are extremely fine
(b) light rays are absorbed at the entry end and relieved at the exit end of the fibre
(c) scattering of light occurs inside the fibre
(d) successive total internal reflections occur as a ray moves through the fibre

(d) The working of optical fibre is based on the total internal reflection. Its inner part is core of higher refractive index surrounded by another layer of glass of lower refractive index. It is surrounded by plastic jacket.

When light enters from the one end of the core and moves towards cladding, then total internal reflection takes place again and again and light propagate through it. Optical fibres are used in decorative table lamps.

21. Compared to audible sound waves, ultrasound waves have

(a) higher speed
(b) higher frequency
(c) Both (a) and (b)
(d) long wavelength

(b) Ultrasound waves have higher frequency than audible sound waves.

The audible range of sound for human beings is form 20 to 20,000 Hz. Children under the age of 5 and some animals such as dogs can hear upto 25,000 Hz. As people grow older, their ears become less sensitive to higher frequency.

Ultrasonic sound of frequency higher than 20,000 Hz, which cannot be heard by human beings.

22. A rigid body of mass 2 kg is dropped from a stationary balloon kept at a height of 50 m from the ground. The speed of the body when it just touches the ground and the total energy when it is dropped from the balloon are respectively.

(Take, acceleration due to gravity, $g = 9.8\ \text{m/s}^2$)

(a) 980 ms^{-1} and 980 J
(b) $\sqrt{980}$ ms^{-1} and $\sqrt{980}$ J
(c) 980 ms^{-1} and $\sqrt{980}$ J
(d) $\sqrt{980}$ ms^{-1} and 980 J

(d) Given, M = 2 kg and height = 50 m

Speed of body at the ground,

$$v^2 - u^2 = 2gH$$

$$v = \sqrt{2gH} \quad [\because u = 0]$$

$$= \sqrt{2 \times 9.8 \times 50}$$

$$\Rightarrow \quad v = \sqrt{980}\ \text{ms}^{-1}$$

Hence, the total energy = kinetic energy

$$\Rightarrow \quad \text{TE} = \frac{1}{2}Mv^2$$

$$= \frac{1}{2} \times 2 \times (\sqrt{980})^2 = 980\ \text{J}$$

Hence, option (d) is correct.

23. The temperature of a place on one sunny day is 113 in Fahrenheit scale. The Kelvin scale reading of this temperature will be

(a) 318 K (b) 45 K
(c) 62.8 K (d) 335.8 K

(a) Given, temperature,

$$T_f = 113\ \text{F}$$

As, we know

$$T_k = 273 + \frac{10}{18}(T_f - 32)$$

$$\Rightarrow \quad T_k = 273 + \frac{10}{18}(113 - 32)$$

$$\Rightarrow \quad T_k = 318\ \text{K}$$

24. Radiations coming from the sun are mostly in the form of
(a) light only
(b) light and long wavelength infrared
(c) light and short wavelength infrared
(d) Both short and long wavelength infrared

(*b*) The mode of transfer of heat from one body to another without any actual movement the particles involved and without heating the intervening medium, is called radiation. For radiation, there is no need of any medium because it is of electromagnetic nature.
So, radiation coming from the sun are mostly in form of light and long wavelength infrared.

25. Which of the following are the characteristics of electromagnetic waves?
1. They are elastic waves.
2. They can also move in vacuum.
3. They have electric and magnetic components which are mutually perpendicular.
4. They move with a speed equal to 3 lakh meters per second.

Select the correct answer using the codes given below.
(a) 1, 2, 3 and 4 (b) 1, 2 and 4
(c) 2 and 3 (d) 3 and 4

(*c*) Electromagnetic waves or EM waves are waves that are created as a result of vibrations between an electric field and a magnetic field. They are formed when an electric field comes in contact with a magnetic field. The electric field and magnetic field of an electromagnetic wave are perpendicular (at right angles) to each other. They are also perpendicular to the direction of the EM wave.
These waves travel with a constant velocity of $3.0 \times 10^8 m/s$ in vacuum. They are deflected neither by the electric field nor by the magnetic field.
Hence, statements 2 and 3 are correct.

2019 (I)

26. The light energy escaping from the sun can be spread by
(a) a shower of rain drops
(b) a plane mirror
(c) a convex lens
(d) a combination of a convex lens and a concave lens

(*a*) The light energy escaping from the sun can be spread by a shower of rain drops. The sunlight shines on a water droplet. As the light passes into the droplet, the light bends or refracts, a little because light travels slower in water than in air (because water is denser). Then, the light bounces off the back of water droplet and goes back the way it came, bending again as its speeds up when it exist the water droplet.
Hence, the light energy escaping from the sun can be spread. A disperse the light into VIBGYOR and spread it.

27. The correct sequence of energy transfer that occurs when an apple falls to the ground is
(a) gravitational potential energy → heat energy to air → kinetic energy → heat energy to ground and apple → sound energy
(b) gravitational potential energy → sound energy → kinetic energy → heat energy to air → heat energy to ground and apple
(c) gravitational potential energy → kinetic energy → heat energy to air → heat energy to ground and apple → sound energy
(d) gravitational potential energy → kinetic energy → sound energy → heat energy to air → heat energy to ground and apple

(*c*) When apple is on highest position, then it has only gravitational potential energy. When, it falls to the ground the first its gravitational potential energy starts converting into kinetic energy.
Due to air friction some kinetic energy losses and converts into heat energy to air. When apple falls on the ground, then its remaining energy converts into heat energy to ground and sound energy. Hence, option (c) is correct.

28. Which one of the following mineral is used as a fuel in nuclear power stations?
(a) Bauxite (b) Quartz
(c) Feldspar (d) Pitchblende

(*d*) Uraninite, formerly known as pitchblende is a radioactive uranium rich mineral which is used as a fuel in nuclear power stations. It is an ore with a chemical composition that is largely UO_2, but due to oxidation the mineral typically contains variable proportions of U_3O_8.

29. Which one of the following can charge an insulator?
(a) Current electricity
(b) Static electricity
(c) Magnetic field
(d) Gravitational field

(*b*) Static electricity can charge an insulator. Charging of insulator by induction process, static charge is required. A static electric charge can be created whenever two surfaces contact and separate at least one of the surfaces has a high resistance to electric current.

30. At 20°C, the speed of sound in water is approximately
(a) 330 m/s
(b) 800 m/s
(c) 1500 m/s
(d) 5000 m/s

(*c*) The speed of sound in water at 20°C is approximately 1500 m/s.

31. Let us consider a copper wire having radius r and length l. Let its resistance be R. If the radius of another copper wire is $2r$ and the length is $l/2$, then the resistance of this wire will be
(a) R (b) $2R$
(c) $R/4$ (d) $R/8$

(*d*) Given, length of copper wire = l and radius of copper wire = r
$\therefore$ Resistance,

$$R = \rho\frac{l}{A} = \rho \cdot \frac{l}{\pi r^2} \qquad (\because A = \pi r^2) \ldots (i)$$

For second wire,
length, $l_1 = \frac{l}{2}$ and radius, $r_1 = 2r$
Resistance of wire,

$$R_1 = \rho \cdot \frac{l_1}{\pi r_1^2} = \rho \cdot \frac{l/2}{\pi (2r)^2}$$
$$= \rho \cdot \frac{l}{2 \times \pi \cdot 4r^2} = \frac{1}{8} \cdot \rho \cdot \frac{l}{\pi r^2}$$
$$= \frac{1}{8} \cdot R \qquad \text{[from Eq. (i)]}$$
$$= \frac{R}{8}$$

32. Basic scientific principle behind a nuclear reactor is
(a) nuclear fusion
(b) controlled nuclear fusion
(c) uncontrolled nuclear fission
(d) controlled nuclear fission

(*d*) Basic scientific principle behind a nuclear reactor is controlled nuclear fission, in which a heavy atomic nucleus splits into two smaller nuclei and a large amount of energy is released in this process.

33. Who among the following has explained the phenomenon of photoelectric effect?
(a) Max Planck
(b) Albert Einstein
(c) Neils Bohr
(d) Ernest Rutherford

(*b*) The experiment of photoelectric effect is performed by Heinrich Hertz, but the phenomenon was explained by Albert Einstein. It is the phenomenon of ejection of electrons from the structure of a metal when light of suitable frequency strikes it.

34. Two metallic wires A and B are made using copper. The radius of wire A is r while its length is l. A DC voltage V is applied across the wire A, causing power dissipation P. The radius of wire B is $2r$ and its length is $2l$ and the same DC voltage V is applied across it causing power dissipation P_1. Which one of the following is the correct relationship between P and P_1?

(a) $P = 2P_1$ (b) $P = P_1/2$
(c) $P = 4P_1$ (d) $P = P_1$

(*b*) For metallic wire A,

resistance, $R_A = \rho \cdot \frac{l}{\pi r^2}$

Therefore, power dissipation in wire A, when a DC voltage V is applied,

$$P = \frac{V^2}{R_A} = \frac{V^2}{\rho \cdot \frac{l}{\pi r^2}} = \frac{V^2 \cdot \pi r^2}{\rho l}$$

$$P = \frac{\pi r^2 V^2}{\rho l} \quad \ldots(i)$$

For metallic wire B,

$l_B = 2l$, $r_B = 2r$, then resistance of wire B,

$$R_B = \rho \cdot \frac{l_B}{\pi r_B^{\,2}} = \rho \cdot \frac{2l}{\pi(2r)^2} = \rho \cdot \frac{2l}{4\pi r^2}$$

$\therefore$ Power dissipation in wire B,

$$P_1 = \frac{V^2}{R_B} = \frac{V^2}{\frac{\rho \cdot 2l}{4\pi r^2}} = \frac{2\pi r^2 V^2}{\rho l} = 2P$$

[from Eq. (i)]

$$\therefore \quad P = \frac{P_1}{2}$$

35. Consider the following statements about a solenoid.

1. The magnetic field strength in a solenoid depends upon the number of turns per unit length in the solenoid.
2. The magnetic field strength in a solenoid depends upon the current flowing in the wire of the solenoid.
3. The magnetic field strength in a solenoid depends upon the diameter of the solenoid.

Which of the statements given above are correct?

(a) 1, 2 and 3 (b) 1 and 3
(c) 2 and 3 (d) 1 and 2

(*d*) Magnetic field strength in a solenoid is given by

$$B = \mu_0 ni$$

where, μ_0 = permeability of free space,
n = number of turns per unit length
and i = current flowing through solenoid.

So, it is clear that magnetic field strength B in a solenoid depends on n and i only, it does not depends on diameter of solenoid.

Therefore, statements 1 and 2 are correct.

36. Light year is a unit of measurement of

(a) very large distances
(b) time interval in years
(c) amount of light received on earth in a year
(d) mass of atoms

(*a*) Light year is a unit of measurement of very large distances.

It is a distance travelled by light in vacuum in one year.

1 light year = 9.46×10^{15} m

37. The focal length of the objective lens of a telescope is 50 cm. If the magnification of the telescope is 25, then the focal length of the eyepiece is

(a) 12.5 cm (b) 5 cm
(c) 2 cm (d) 10 cm

(*c*) Given, focal length of objective lens, $f_o = 50$ cm and magnification of telescope $m = 25$

If f_e be the focal length of eyepiece, then

$$m = -\frac{f_o}{f_e}$$

$$\Rightarrow \quad 25 = -\frac{50}{f_e}$$

$$\Rightarrow \quad f_e = -\frac{50}{25} = -2 \text{ cm}$$

$$|f_e| = 2 \text{ cm}$$

38. Which one of the following force is non-central and non-conservative?

(a) Frictional force
(b) Electric force
(c) Gravitational force
(d) Mechanical force

(*a*) A force on body which does not always act towards a fixed point, is called non-central force.

The forces that do not store energy are called non-conservative forces.

Work done by non-conservative forces depends on the path of the body moving from one point to another point.

Therefore, frictional force is a non-central and non-conservative force.

39. Consider the following statements relating to Richter scale.

1. It was devised in 1935 by Charles F. Richter.
2. It describes the quantity of energy released by a single earthquake.
3. Richter scale has no upper limit.

Which of the statement(s) given above is/are correct?

(a) Only 1
(b) 1 and 2
(c) 2 and 3
(d) All of the above

(*b*) Richter scale is a scale of number used to tell the size of earthquakes. Charles Francis Richter developed the Richter scale in 1935.

This scale describes the quantity of energy released by a single earthquake. Numbers for the Richter scale are in range from 0 to 9.

So, here statements (1) and (2) are correct.

40. Suppose there are two planets, 1 and 2, having the same density, but their radii are R_1 and R_2 respectively, where $R_1 > R_2$. The accelerations due to gravity on the surface of these planets are related as

(a) $g_1 > g_2$ (b) $g_1 < g_2$
(c) $g_1 = g_2$
(d) Cannot say anything

(*a*) Acceleration due to gravity on the surface of first planet,

$$g_1 = \frac{GM_1}{R_1^{\,2}} \quad \ldots(i)$$

where, M_1 is the mass of first planet.

But, $M_1 = \frac{4}{3}\pi R_1^3 \cdot \rho_1$

($\because$ mass = volume × density)

where, ρ_1 = density of the first planet.

$$g_1 = \frac{G \cdot \frac{4}{3}\pi R_1^3 \cdot \rho_1}{R_1^{\,2}} \quad \text{[from Eq. (i)]}$$

$$g_1 = \frac{4}{3} G\pi\rho_1 R_1 \quad \ldots(ii)$$

Similarly, acceleration due to gravity on the surface of second planet,

$$g_2 = \frac{4}{3} G\pi\rho_2 R_2 \quad \ldots(iii)$$

$\therefore$ From Eqs. (ii) and (iii), we get

$$\frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2}$$

$$= \frac{R_1}{R_2} \quad (\because \rho_1 = \rho_2)$$

Since, $R_1 > R_2$, hence $g_1 > g_2$.

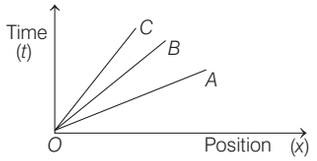
41. The sun is seen little before it rises and for a short while after it sets. This is because of
(a) total internal reflection
(b) atmospheric refraction
(c) apparent shift in the direction of sun
(d) dispersion

(***b***) The sun is seen little before it rises and for short while after it sets due to atmospheric refraction.

When light enters from vacuum to earth's atmosphere, it basically enters from rarer to denser medium and bends towards horizon.

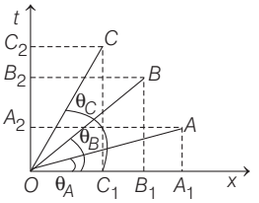
During sunrise, when the sun is just below the horizon, our atmosphere causes the light rays to bend and we see the sun early. Similarly, at sunset, the apparent position of the sun is visible to us and not the actual position due to the same bending of light rays effect.

42. The figure shown below gives the time (t) *versus* position (x) graph of three objects A, B and C. Which one of the following is the correct relation between their speeds v_A, v_B and v_C, respectively at any instant ($t > 0$)?



(a) $v_A < v_B < v_C$ (b) $v_A > v_B > v_C$
(c) $v_A = v_B = v_C \neq 0$
(d) $v_A = v_B = v_C = 0$

(***b***) From figure,



Velocity of object A,

$$v_A = \frac{OA_1}{AA_1} = \cot\theta_A$$

Velocity of object B,

$$v_B = \frac{OB_1}{BB_1} = \cot\theta_B$$

Velocity of object C,

$$v_C = \frac{OC_1}{CC_1} = \cot\theta_C$$

Since, $\theta_C > \theta_B > \theta_A$

$\Rightarrow \cot\theta_A > \cot\theta_B > \cot\theta_C$

$\Rightarrow v_A > v_B > v_C$

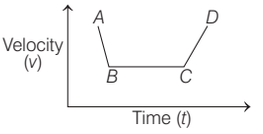
43. 1 dyne (a unit of force in CGS system) equals to
(a) 10^3 g-cm/s^2
(b) 10^{-3} g-cm/s^2
(c) 10^5 kg-cm/s^2
(d) 10^{-5} kg-m/s^2

(***d***) 1 dyne = 1 g-cm/s^2

$$= \frac{1\text{g}\cdot 1\text{cm}}{1\text{s}^2} = \frac{10^{-3}\text{kg}\cdot 10^{-2}\text{m}}{1\text{s}^2}$$

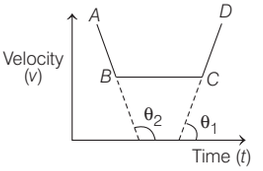
$$= 10^{-5} \text{ kg-m/s}^2$$

44. In the given velocity (v) *versus* time (t) graph, accelerated and decelerated motions are respectively represented by line segments



(a) CD and BC (b) BC and AB
(c) CD and AB (d) AB and CD

(***c***) The slope of velocity-time graph gives acceleration produced in the moving body.



From graph, slope of line segment $AB = \tan\theta_2$
which is negative because

$$90° < \theta_2 < 180°.$$

Therefore, line segment AB represents decelerated motion.

Slope of line segment $CD = \tan\theta_1$
which is positive because

$$0° < \theta_1 < 90°.$$

Therefore, line segment CD represents accelerated motion.

45. Which one of the following statement regarding a thermo flask is not correct?
(a) The walls of flask are separated by vacuum and made of glass which is a poor conductor of heat.
(b) The glass walls themselves have shiny surfaces.
(c) The surface of inner wall radiates good amount of heat and the surface of outer wall absorbs some of the heat that is radiated from the inner wall.
(d) The cork supports are poor conductors of heat.

(***c***) The thermo flask has the silver coating on the inner surface which prevents heat transfer by radiation. The vacuum between its double wall prevents heat moving by convection. The thinness of the glass walls stops heat entering or leaving the flask by conduction.

Hence, option (c) is not a correct statement regarding a thermo flask.

46. Black hole is a
(a) huge black star which has zero acceleration due to gravity on its surface
(b) star which has moderate acceleration due to gravity on its surface
(c) star which has collapsed into itself and has large acceleration due to gravity on its surface
(d) star which has collapsed into itself and has zero acceleration due to gravity on its surface

(***c***) A black hole is a star which has collapsed into itself and has large acceleration due to gravity on its surface.

A black hole forms when the center of a very massive star collapses in upon itself. The region in space of a black-hole, the pulling force of gravity is, so strong that light is not able to escape.

47. The formula for conversion between Fahrenheit and Celsius is $°\text{F} = X + (1.8 \times °\text{C})$.
(a) 32 (b) 22
(c) 98 (d) 42

(***a***) The formula for conversion between Fahrenheit and Celsius is

$$\frac{°\text{C}}{5} = \frac{°\text{F} - 32}{9}$$

$$\Rightarrow \quad \frac{9}{5}\,°\text{C} = °\text{F} - 32$$

$$\Rightarrow \quad 1.8\,°\text{C} = °\text{F} - 32$$

$$\Rightarrow \quad °\text{F} = 32 + 1.8\,°\text{C}$$

Comparing with given relation of

$$°\text{F} = X + (1.8 \times °\text{C})$$

We get, $X = 32$

48. When a beam of white light passes through a glass prism, the colour of light beam that deviates the least is
(a) blue
(b) red
(c) green
(d) violet

(***b***) The wavelength of red colour of light is maximum, therefore when a beam of white light passes through a glass prism, then the light of red colour deviates the least.

49. LIGO stands for
(a) Laser Interferometer Gravitational wave Observatory
(b) Light Interferometer Gravitational wave Observatory
(c) Light Induced Gravity Observatory
(d) Laser Induced Gaseous Optics

(*a*) LIGO stands for Laser Interferometer Gravitational wave Observatory.
It is a large scale physics experiment and observatory to detect cosmic gravitational waves and to develop gravitational waves observations as an astronomical tool.

50. A fuse wire must be
(a) conducting and of low melting point
(b) conducting and of high melting point
(c) insulator and of high melting point
(d) insulator and of low melting point

(*a*) Fuse wire should have high resistance and low melting point. It is because when high current pass through the circuit, at first, it will resist the high current.
A fuse wire protects our electric appliances from damaging due to sudden increase in electric current.

2018 (II)

Directions (Q. Nos. 51-52) *The following items consist of two statements, Statement I and Statement II. Examine these two statements carefully and select the correct answer using the codes given below.*
Codes
(a) Both the statements are true and Statement II is the correct explanation of Statement I.
(b) Both the statements are true but Statement II is not the correct explanation of Statement I.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

51. **Statement** I The pitch of sound wave depends upon its frequency. **Statement II** The loudness of the sound wave depends upon its amplitude.

(*b*) Pitch of sound wave depends upon its frequency because pitch of a sound is actually its property due to which we can differentiate between a shrill and grave sound.
The amplitude of a sound wave determines its loudness or volume.
So, option (b) is the correct answer.

52. **Statement** I Sound waves cannot propagate in vacuum. **Statement** II Sound waves are elastic waves and require a medium to propagate.

(*a*) Sound waves are elastic waves because their propagation depends on the material media, which possess elasticity and inertia. So, it cannot propagate in vacuum.
Hence, option (a) is the correct answer.

53. The frequency of ultrasonic waves is
(a) less than 20 Hz
(b) between 20 Hz and 2 kHz
(c) between 2 kHz and 20 kHz
(d) greater than 20 kHz

(*d*) Ultrasonic waves range above 20 kHz which is above the audible range of human ears.

54. The magnetic field strength of a current carrying wire at a particular distance from the axis of the wire
(a) depends upon the current in the wire
(b) depends upon the radius of the wire
(c) depends upon the temperature of the surroundings
(d) None of the above

(*a*) The magnetic field strength of a current carrying wire at a particular distance from the axis of the wire is given by the Biot-Savart's law.
Magnetic field, $dB \propto \frac{idl\sin\theta}{r^2}$
where, i = current in wire
and r = distance of the point from the wire.
$$dB \propto i$$
Hence, option (a) is correct.

55. A stainless steel chamber contains Ar gas at a temperature T and pressure p. The total number of Ar atoms in the chamber is n. Now, Ar gas in the chamber is replaced by CO_2 gas and the total number of CO_2 molecules in the chamber is $n/2$ at the same temperature T. The pressure in the chamber now is p'. Which one of the following relation holds true? (Both the gases behave as an ideal gas)
(a) $p' = p$
(b) $p' = 2p$
(c) $p' = p/2$
(d) $p' = p/4$

(*c*) For Ar gas $pV = nRT$
$$p = \frac{nRT}{V} \quad ...(i)$$
For CO_2 gas, $n' = \frac{n}{2}$, $p = p'$
$$p'V = n'RT \Rightarrow p' = \frac{n}{2}\frac{RT}{V}$$
From Eq. (i), we get
$$p' = \frac{p}{2}$$

56. Which one of the following is the correct relation between Å and nm?
(a) 1 nm = 10^{-1} Å (b) 1 nm = 10 Å
(c) 1 nm = 1 Å (d) 1 nm = 10^{-2} Å

(*b*) 1Å = 0.1 nm ⇒ 10Å = 1 nm

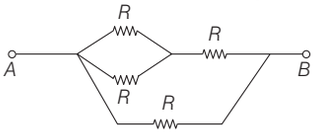
57. The full form of LED is
(a) Light Emitting Diode
(b) Light Emitting Device
(c) Light Enhancing Device
(d) Light Enhancing Diode

(*a*) The full form of LED is Light Emitting Diode. It is a heavily doped *p-n* junction diode which converts electrical energy into light energy.

58. If a free electron moves through a potential difference of 1 kV, then the energy gained by the electron is given by
(a) 1.6×10^{-19} J (b) 1.6×10^{-16} J
(c) 1×10^{-19} J (d) 1×10^{-16} J

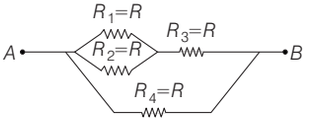
(*b*) Given, potential difference,
$V = 1\,\text{kV} = 1 \times 10^3\,\text{V}$
We know that, energy gained by electron $(E) = e \cdot V$
$= 1.6 \times 10^{-19} \times 10^3 = 1.6 \times 10^{-16}$ J

59. Consider the following circuit.



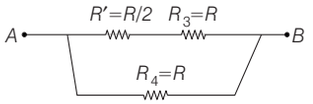
Which one of the following is the value of the resistance between points A and B in the circuit given above?
(a) $\frac{2}{5}R$ (b) $\frac{3}{5}R$ (c) $\frac{3}{2}R$ (d) $4R$

(*b*) According to question,



In the above circuit, R_1 and R_2 are connected in the parallel combination.
Hence, equivalent resistance,
$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} \Rightarrow R' = \frac{R}{2}$$

Now, circuit becomes



In the circuit, R' and R are connected in series combination. Hence,

$$R'' = R/2 + R = \frac{R + 2R}{2} \Rightarrow R'' = \frac{3R}{2}$$

Now R'' and R_4 are connected in parallel combination. Hence, equivalent resistance between A and B is

$$\frac{1}{R_{AB}} = \frac{1}{\frac{3R}{2}} + \frac{1}{R}$$

$$\frac{1}{R_{AB}} = \frac{2}{3R} + \frac{1}{R} \Rightarrow \frac{1}{R_{AB}} = \frac{2+3}{3R}$$

$$\frac{1}{R_{AB}} = \frac{5}{3R} \Rightarrow R_{AB} = \frac{3}{5}R$$

60. The absolute zero temperature is 0°K. In °C unit, which one of the following is the absolute zero temperature?

(a) 0°C (b) –100 °C
(c) –273.15 °C (d) –173.15 °C

(***b***) The absolute zero temperature is 0°K. So, the relation between Celsius and Kelvin scale is

$$\frac{C}{100} = \frac{K - 273.15}{100} \quad (\because \text{given, } K = 0)$$

So, $C = -273.15°C$

61. Consider the following statements about visible light, UV light and X-ray.

1. The wavelength of visible light is more than that of X-ray.
2. The energy of X-ray photons is higher than that of UV light photons.
3. The energy of UV light photons is less than that of visible light photons.

Which of the statement(s) given above is/are correct?

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) Only 1

(***b***) (1) The frequency of X-rays is greater than visible light. So, the wavelength of visible light is more than that of X-rays.

(2) We know that, $E = h\nu$
Energy (E) ∝ Frequency (ν),
X-rays have higher frequency, so it has higher energy than that of UV light photons.

(3) UV light have higher frequency, so it has higher energy than that of visible light.

Hence statements 1 and 2 both are correct but statement 3 is in correct.

62. The time period of oscillation of a simple pendulum having length L and mass of the bob m is given as T. If the length of the pendulum is increased to $4L$ and the mass of the bob is increased to 2 m, then which one of the following is the new time period of oscillation?

(a) T (b) $2T$ (c) $4T$ (d) $\frac{T}{2}$

(***b***) Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \ldots(i)$$

According to the question, the length of the pendulum increased to $4L$, i.e.

$$L' = 4L$$

New, time period of simple pendulum,

$$T' = 2\pi\sqrt{\frac{4L}{g}}$$

$$= 2 \times \left(2\pi\sqrt{\frac{L}{g}}\right)$$

From Eq. (i), we get

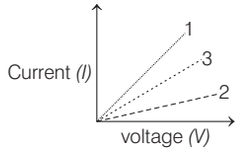
$$T' = 2T$$

63. The connecting cable of electrical appliances like electric iron, water heater or room heater contains three insulated copper wires of three different colours—Red, Green and Black. Which one of the following is the correct colour code?

(a) Red-live wire, Green-neutral wire and Black-ground wire
(b) Red-neutral wire, Green-ground wire and Black-live wire
(c) Red-live wire, Green-ground wire, Black-neutral wire
(d) Red-ground wire, Green-live wire and Black-neutral wire

(***c***) Red-live wire (brings the current)
Green-ground wire
and Black-neutral wire

64. The graph between current (I) and voltage (V) for three linear resistors 1, 2 and 3 is given below :



If R_1, R_2 and R_3 are the resistances of these resistors, then which one of the following is correct?

(a) $R_1 > R_2 > R_3$ (b) $R_1 < R_3 < R_2$
(c) $R_3 < R_1 < R_2$ (d) $R_3 > R_2 > R_1$

(***b***) We know that,

$$\text{Slope of graph} \propto \frac{1}{\text{Resistance}}$$

In the graph,
slope of 1 > slope of 3 > slope of 2
Hence, $R_1 < R_3 < R_2$.

65. Consider the following statements about a microscope and a telescope.

1. Both the eyepiece and the objective of a microscope are convex lenses.
2. The focal length of the objective of a telescope is larger than the focal length of its eyepiece.
3. The magnification of a telescope increases with the increase in focal length of its objective.
4. The magnification of a microscope increases with the increase in focal length of its objective.

Which of the statement(s) given above are correct?

(a) 1 and 3 (b) 1 and 4
(c) 2, 3 and 4 (d) 1, 2 and 3

(***d***) (1) Microscope consists of two convex lenses co-axially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.

(2) The focal length of the objective of a telescope is larger than the focal length of eyepiece because objective has much larger aperture than the eyepiece.

(3) Magnification of telescope is directly proportional to the focal length of objective (f_o). Hence, the focal length of telescope increases with the increase in focal length of its objective.

Hence, statements (1), (2) and (3) all are correct.

66. A planet has a mass M_1 and radius R_1. The value of acceleration due to gravity on its surface is g_1.
There is another planet 2, whose mass and radius both are two times that of the first planet. Which one of the following is the acceleration due to gravity on the surface of planet 2?

(a) g_1 (b) $2g_1$
(c) $g_1/2$ (d) $g_1/4$

(**c**) Acceleration due to gravity at planet is given by

$$g_1 = \frac{GM_1}{R_1^2} \quad \ldots(i)$$

For the second planet,

$$M_2 = 2M_1, R_2 = 2R_1$$

Now, acceleration due to gravity,

$$g_2 = \frac{G(2M_1)}{(2R_1)^2} \Rightarrow g_2 = \frac{1}{2} \times \frac{GM_1}{R_1^2}$$

From Eq. (i), we get

$$g_2 = \frac{g_1}{2}$$

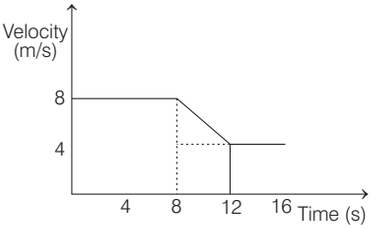
67. The wavelength of X-rays is of the order of

(a) 1 Å (b) 1 μm
(c) 1 mm (d) 1 cm

(**a**) The wavelength of X-rays is of the order of 1Å.

X-ray is a electromagnetic wave which is discovered by German professor Rontgen. It is used in surgery to detect the fracture, organs diseased, stones in the body.

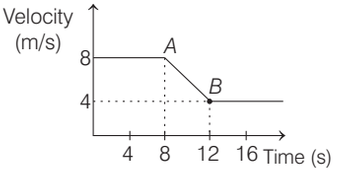
68. Consider the following velocity and time graph



Which one of the following is the value of average acceleration from 8 s to 12 s?

(a) 8 m/s^2 (b) 12 m/s^2
(c) 2 m/s^2 (d) –1 m/s^2

(**d**) From the figure,



Average acceleration from 8 s to 12 s, is given by

$$a_{av} = \frac{\text{Velocity at } B - \text{Velocity at } A}{\text{Time interval}}$$

$$= \frac{4-8}{(12-8)} = \frac{-4}{4} = -1\,\text{m/s}^2$$

69. If the focal length of a convex lens is 50 cm, then which one of the following is its power?

(a) +2 D (b) +0.02 D
(c) –0.5 D (d) +0.5 D

(**a**) Given, focal length of lens,

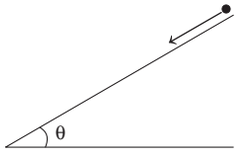
$$f = 50\,\text{cm} = 0.5\,\text{m}$$

We know that,

$$\text{Power of lens, } P = \frac{1}{\text{Focal length (m)}}$$

$$= \frac{1}{0.5} = +2\,\text{D}$$

70. A ball is released from rest and rolls down an inclined plane as shown in the following figure, requiring 4 s to cover a distance of 100 cm along the plane



Which one of the following is the correct value of angle θ that the plane makes with the horizontal? ($g = 1000$ cm/s^2)

(a) $\theta = \sin^{-1}(1/9.8)$ (b) $\theta = \sin^{-1}(1/20)$
(c) $\theta = \sin^{-1}(1/80)$ (d) $\theta = \sin^{-1}(1/100)$

(**c**) Given, initial velocity of ball, $u = 0$

distance, $s = 100$ cm and time, $t = 4$ s

We know that,

$$s = ut + \frac{1}{2}at^2$$

$$100 = 0 + \frac{1}{2} \times a \times (4)^2$$

$$a = \frac{100 \times 2}{4 \times 4} = \frac{25}{2}\,\text{cm/s}^2$$

Now, $a = g \sin\theta$

$$\sin\theta = \frac{a}{g} = \frac{25/2}{1000}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{80}\right)$$

71. The coefficient of areal expansion of a material is 1.6×10^{-5} K^{-1}. Which one of the following gives the value of coefficient of volume expansion of this material?

(a) 0.8×10^{-5} K^{-1}
(b) 2.4×10^{-5} K^{-1}
(c) 3.2×10^{-5} K^{-1}
(d) 4.8×10^{-5} K^{-1}

(**b**) We know that,

$$\frac{\text{Area expansion coefficient } (\beta)}{\text{Volumetric expansion coefficient } (\gamma)} = \frac{2}{3}$$

$$\gamma = \frac{3}{2} \times \beta, \text{ where } \beta = 1.6 \times 10^{-5}\text{K}^{-1}$$

$$\gamma = \frac{3}{2} \times 1.6 \times 10^{-5}\text{K}^{-1}$$

$$\gamma = 2.4 \times 10^{-5}\text{K}^{-1}$$

72. The refractive indices of two media are denoted by n_1 and n_2 and the velocities of light in these two media are respectively, v_1 and v_2. If n_2/n_1 is 1.5, then which one of the following statement is correct?

(a) v_1 is 1.5 times v_2
(b) v_2 is 1.5 times v_1
(c) v_1 is equal to v_2
(d) v_1 is 3 times v_2

(**a**) We know that,

$$\text{Refractive index} = \frac{\text{Velocity in vacuum } (c)}{\text{Velocity in medium } (v)}$$

$$\text{In first medium, } n_1 = \frac{c}{v_1} \quad \ldots(i)$$

$$\text{Similarly, } n_2 = \frac{c}{v_2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{n_2}{n_1} = \frac{v_1}{v_2}$$

$$1.5 = \frac{v_1}{v_2} \Rightarrow v_1 = 1.5v_2$$

Hence, option (a) is correct.

73. Which one of the following statement is correct for a plane mirror?

(a) Its focal length is zero.
(b) The size of the image of an object placed in front of the mirror is slightly less than that of the object.
(c) The image is virtual, erect and laterally inverted.
(d) Its focal length is 200 cm.

(**c**) In a plane mirror, image formed is virtual, erect and laterally inverted at the same distance behind the mirror as the object is in front of mirror. The focal length of plane mirror is infinite and power is zero.

74. An object is placed in front of a convex mirror. Which one of the following statement is correct?

(a) It will never form an inverted image.
(b) The image moves towards the focus when the object moves towards the mirror.
(c) Depending on the position of the object with respect to the mirror, the image can be inverted and real.
(d) The size of the image becomes larger than that of the object when the object is placed at a distance equal to half the focal length.

(**a**) The convex mirror will never form an inverted image.

Convex mirror always form virtual and erect image but the size of image is always smaller than size of object.

75. A circular coil of radius R having N number of turns carries a steady current I. The magnetic induction at the centre of the coil is 0.1 T. If the number of turns is doubled and the radius is halved, then which one of the following will be the correct value for the magnetic induction at the centre of the coil?

(a) 0.05 T (b) 0.2 T (c) 0.4 T (d) 0.8 T

(c) (*c*) Magnetic field at the centre of coil,

$$B = \frac{\mu_0 NI}{2r}$$

$$\Rightarrow \quad 0.1 = \frac{\mu_0 NI}{2r} \qquad \ldots(i)$$

According to the question,

$$N' = 2N, r' = \frac{r}{2}$$

Magnetic inductance at the centre of the coil,

$$B' = \frac{\mu_0 N' I}{2r'}$$

$$= \frac{\mu_0 (2N)(I)}{2 \times \frac{r}{2}}$$

$$\Rightarrow \quad B' = 4\left(\frac{\mu_0 NI}{2r}\right)$$

From Eqs. (i), we get

$$B' = 4 \times 0.1$$

$$\Rightarrow \quad B' = 0.4 \text{ T}$$

2018 (I)

76. Which one of the following statement is correct?

(a) Any energy transfer that does not involve temperature difference in some way is not heat.
(b) Any energy transfer always requires a temperature difference.
(c) On heating, the length and volume of the object remain exactly the same.
(d) Whenever there is a temperature difference, heat is the only way of energy transfer.

(a) (*a*) Practically, all the bodies around us either release or absorb energy often in the form of heat. The form of energy which is exchanged among various bodies or system on account of temperature difference is defined as heat.

On heating the matter, the energy of atoms increases and hence the average distance between them increases. This results in thermal expansion.

Hence, option (a) is the correct statement.

77. If T is the time period of an oscillating pendulum, then which one of the following statement is not correct?

(a) The motion repeats after time T only once.
(b) T is the least time after which motion repeats itself.
(c) The motion repeats itself after nT, where n is a positive integer.
(d) T remains the same only for small angular displacements.

(a) (*a*) Time period (T) is the time taken by the particle to complete one oscillation.

Time period of the simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

It is clear that the time period is independent of the mass of the pendulum. This formula is valid only when radius of point mass of pendulum is negligible compared to length of pendulum.

Here, motion of pendulum repeats itself after a fixed interval, i.e. it remains same only for small angular displacements (Q).

Hence, it is incorrect to say that this motion repeats after time T only once.

78. If an object moves with constant velocity, then which one of the following statement is not correct?

(a) Its motion is along a straight line.
(b) Its speed changes with time.
(c) Its acceleration is zero.
(d) Its displacement increases linearly with time.

(b) (*d*) Whenever an object moves with constant velocity, the motion is always in form of straight line, which is parallel to the X-axis. As velocity remains same, hence acceleration of the object is zero. Also, the speed of the object changes due to change in displacement, so than in distance.

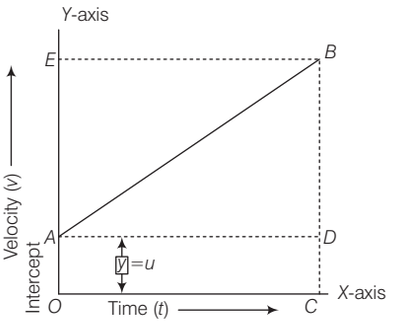
Hence, option (d) is incorrect because it is not necessary displacement increases linearly with time. It decreases linearly for constant velocity.

79. An object is moving with uniform acceleration a. Its initial velocity is u and after time t, its velocity is v. The equation of its motion is $v = u + at$. The velocity (along Y-axis)-time (along X-axis) graph will be a straight line

(a) passing through origin
(b) with X-intercept u
(c) with Y-intercept u
(d) with slope u

(c) (*c*) From the graph for the equation of its motion,

$v = (u + at)$



The object has an initial velocity u at point A and then its velocity changes at a uniform rate from A to B in time t. In other words, there is a uniform acceleration a from A to B and after time t its final velocity becomes v at B in the graph.

Hence, the slope of a velocity-time graph is equal to acceleration.

Thus, we can say that the velocity-time graph for the equation, $v = u + at$ will be straight line having slope equal to acceleration a with y intercept u.

Hence, option (c) is correct.

80. What is the net force experienced by a bar magnet placed in a uniform magnetic field?

(a) Zero
(b) Depends upon length of the magnet
(c) Never zero
(d) Depends upon temperature

(a) (*a*) The correct answer should be zero. When a bar magnet or magnetic dipole is placed in a uniform magnetic field, it will not experience only force, but it is bound to experience a torque.

This torque will align the dipole moment of the bar magnet to align along the direction of magnetic field. However, if a bar magnet is placed in a non-uniform magnetic field, then it will be subjected to a force and torque.

81. Which one of the following has maximum inertia?

(a) An atom (b) A molecule
(c) A one rupee coin (d) A cricket ball

(d) (*d*) Inertia means resistance or opposition offered by the body to change its state. Mass of a body is the measurement of its inertia. A body with greater mass shows greater inertia, i.e. it is more difficult to change its state of rest or uniform motion as compared to that of a body having small mass.

Hence, in the given option a cricket ball has maximum mass, so it has maximum inertia.

82. Which one of the following is the value of 1 kWh of energy converted into joules?
(a) 1.8×10^6 J
(b) 3.6×10^6 J
(c) 6.0×10^6 J
(d) 7.2×10^6 J

(*b*) 1 kWh is the unit of electrical power.
1 kWh = 1000 W × 3600 s
(∵ 1 J = 1 W × 1 s)
$= 3.6 \times 10^6$ J

83. Which one of the following statement about gravitational force is not correct?
(a) It is experienced by all bodies in the universe.
(b) It is a dominant force between celestial bodies.
(c) It is a negligible force for atoms.
(d) It is same for all pairs of bodies in our universe.

(*d*) Gravitational force is defined as a non-contact force of attraction between any two bodies in universe (no matter how far the bodies are).
It is experienced by all bodies in the universe. The celestial bodies experienced these forces, but it is not same for all pairs of bodies in our universe.
So, option (d) is not a correct statement.

84. Whether an object will float or sink in a liquid, depends on
(a) mass of the object only
(b) mass of the object and density of liquid only
(c) difference in the densities of the object and liquid
(d) mass and shape of the object only

(*c*) Whether an object sinks or floats depends on the density of the object and the fluid in which it is immersed.
An object that is denser than a fluid will sink in the fluid while an object that is less dense will float. A floating object is said to be buoyant.
The classical Greek inventor Archimedes' was first to understand that buoyancy is a force.
Archimedes' principle states that any object immersed in or floating in a fluid is buoyed up by a force equal to the weight of displaced fluid.

85. Which one of the following device is non-ohmic?
(a) Conducting copper coil
(b) Electric heating coil
(c) Semi-conductor diode
(d) Rheostat

(*c*) According to Ohm's law, the electric current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided the physical condition (such as temperature) remains unchanged.
Hence, the device which follows Ohm's law for all voltage across it is called as an ohmic device. For example, a wire rheostat conducting copper coil, electric heating coil, etc.
The device that does not follow Ohm's law is known as non-ohmic device. For example, semi-conductor diode.

86. Which one of the following is the natural phenomenon based on a simple periscope works?
(a) Reflection of light
(b) Refraction of light
(c) Dispersion of light
(d) Total internal reflection of light

(*a*) Periscope is an optical instrument which is based on the phenomenon of reflection of light. Light is reflected from the first mirror into the second one at 45° angle which is then reflected to the eye of the viewer.

87. Which one of the following frequency range is sensitive to human ears?
(a) 0-200 Hz (b) 20-20000 Hz
(c) 200-20000 Hz (d) 2000-20000 Hz

(*b*) 20-20000 Hz is the frequency range which is sensitive to human ears. The audible frequency range to which human ears can respond is 20-20000 Hz. The sound frequency beyond this limit may damage ear drums and cause hearing disabilities in humans.

88. Which of the following statement(s) about a fluid at rest in a cup is/are correct?
1. Pressure is same at all the points in the fluid.
2. Pressure is exerted on the walls.
3. Pressure exists everywhere in the fluid.
(a) 1 and 2 (b) 2 and 3
(c) Only 1 (d) 1, 2 and 3

(*d*) When a fluid is at rest in a cup then pressure is same at all the points in the fluid, pressure is exerted on the walls and pressure exists everywhere in the fluid.
The pressure at a point within a volume of fluid means that the point to be surrounded by a small container by the area of the container.
Hence, all the statements are correct

89. Which one of the following device is used to measure atmospheric pressure?
(a) Ammeter (b) Barometer
(c) Potentiometer (d) Lactometer

(*b*) **Barometer** is used to measure the atmospheric pressure. The function of a barometer is to know whether or not atmospheric pressure is rising or falling.
Ammeter is a device which is used to measure electric current in amperes. **Potentiometer** is used to measure the emf of a cell. **Lactometer** is used to measure the purity of milk.

90. Which of the following statement(s) about latent heat for a given substance is/are correct?
1. It is fixed at a given temperature.
2. It depends upon the temperature and volume.
3. It is independent of temperature and volume.
4. It depends on the temperature but independent of volume.
(a) Only 2 (b) 1 and 3
(c) Only 4 (d) 1 and 4

(*b*) The amount of heat per unit mass transferred during change of state of the substance is called latent heat of the substance. It is always fixed at a given temperature and does not change with temperature and volume of the substance, i.e. it is independent of temperature and volume.
e.g. Latent heat of ice is 3.34×10^5 J/kg.
So, option (b) is the correct answer.

91. Which one of the following statement about the mass of a body is correct?
(a) It changes from one place to another.
(b) It is same everywhere.
(c) It depends on its shape.
(d) It does not depend on its temperature.

(*b*) Mass is a quantity of matter present inside the body which measures the inertia of a body. The mass of the body cannot be zero. Irrespective of the position of body in the universe, mass of the body always remains constant everywhere, which is independent of its temperature.

92. A ball balanced on a vertical rod is an example of
(a) stable equilibrium
(b) unstable equilibrium
(c) neutral equilibrium
(d) perfect equilibrium

(*b*) When the centre of gravity of a body lies below point of suspension or support, the body is said to be in stable equilibrium. When the centre of gravity of a body lies above the point of

suspension or support, the body is said to be in unstable equilibrium.

A balanced ball on a vertical rod is an example of unstable equilibrium because when the rod is slightly disturbed its centre of gravity is lowered.

The line of action of its weight lies outside the base of the rod. Then, the torque due to weight, toppled it down.

When the centre of gravity of a body lies at the point of suspension, the body is said to be inventral equilibrium.

93. Which of the following statement(s) about specific heat of a body is/are correct?

1. It depends upon mass and shape of the body.
2. It is independent of mass and shape of the body.
3. It depends only upon the temperature of the body.

(a) Only 1
(b) 2 and 3
(c) 1 and 3
(d) Only 2

› (***a***) Every substance has a unique value for the amount of heat absorbed or rejected to change the temperature of unit mass of it by one unit. This quantity is referred as the specific heat capacity of a body.

It depends on the nature as well as mass of the substance and it changes in temperature.

Hence, option (a) is correct.

94. Which one of the following is an example of the force of gravity of the earth acting on a vibrating pendulum bob?

(a) Applied force
(b) Frictional force
(c) Restoring force
(d) Virtual force

› (***c***) An object that is vibrating is acted upon by a restoring force. The restoring force causes the vibrating object to slow down as it moves away from the equilibrium position and to speed up as it approaches the equilibrium position.

There is the force of gravity that acts downward upon the bob and the tension force results from string pulling upon the bob of the pendulum.

95. Which one of the following statement about the refractive index of a material medium with respect to air is correct?

(a) It can be either positive or negative.
(b) It can have zero value.
(c) It is unity for all materials.
(d) It is always greater than one.

› (***d***) Refractive index of a material is given by the ratio of speed of light in vacuum to the speed of light in any medium. Refractive index is always greater than one because by the definition of refractive index, (n) of a medium (e.g. glass) is the ratio of the speed of light (c), in a vacuum to the speed in the medium v and speed of light is maximum in vacuum, which gives the value of n greater than one.

96. Which one of the following statement about magnetic field lines is not correct?

(a) They can emanate from a point.
(b) They do not cross each other.
(c) Field lines between two poles cannot be precisely straight lines at the ends.
(d) There are no field lines within a bar magnet.

› (***c***) The magnetic field lines is defined as the path along which the compass needless are aligned. They cannot intersect i.e. do not cross each other.

Tangent drawn at any point gives the direction of magnetic field. Outside a magnet, they are directed from north to south pole and inside a magnet they are directed from south to north. These lines are always normal to the surface, which are precisely straight at the ends. Hence, statement (c) is not correct.

97. Two convex lenses with power 2 D are kept in contact with each other. The focal length of the combined lens system is

(a) 0.10 m (b) 2 m (c) 4 m (d) 0·25 m

› (***d***) Here, $P_1 = P_2 = +2$ D

Using the formula, total power,

$$P = P_1 + P_2 = 2 + 2 = +4\text{D}$$

$\therefore$ Focal length, $f = \frac{1}{P} = \frac{1}{4} = 0.25$ m

98. Which of the following statement(s) about electromagnetic waves, sound waves and water waves is/are correct?

1. They exhibit reflection.
2. They carry energy.
3. They exert pressure.
4. They can travel in vacuum.

(a) 1, 2 and 3 (b) 2 and 4
(c) 1 and 3 (d) Only 1

› (***a***) Electromagnetic waves, sound waves and water waves exhibit reflection, carry energy (EM waves), exert pressure (water waves), but sound waves cannot travel in vacuum.

Hence, statements 1, 2 and 3 are correct but 4 is incorrect.

99. Thermal capacity of a body depends on the

(a) mass of the body only
(b) mass and shape of the body only
(c) density of the body
(d) mass, shape and temperature of the body

› (***d***) Thermal capacity of a body can be defined as the amount of heat required to change the temperature of an object by a certain degree. It depends upon the mass, shape and temperature of the body.

100. Which one of the following metal is used in the filaments of photoelectric cells that converts light energy into electric energy?

(a) Tungsten (b) Copper
(c) Rubidium (d) Aluminium

› (***a***) Tungsten is used in the filament of photoelectric cells that converts light energy into electric energy. Main type of these cells are photo voltaic cell, photo emissive cell, photo conductive cell. Electrons are obtained from the filament by thermionic emission and are accelerated to the anode by a potential difference ($\sim$ 100 kV).

2017 (II)

101. Water boils at a lower temperature at high altitudes, because

(a) the air pressure is less
(b) outside temperature is less
(c) latent heat is less
(d) None of the above

› (***a***) Water boils at a lower temperature at high altitudes, because the air pressure is less. When atmospheric pressure is lower, it takes less energy/ heat to bring water to the boiling point, which means water will boil at a lower temperature at high altitudes.

102. Concave mirror is used in headlights of vehicles, because it

(a) focuses light from the bulb onto nearby vehicles
(b) sends parallel rays
(c) fits well into the shape of the headlight
(d) is cheaper than other mirrors

› (***b***) When an object is kept at the focus of a concave mirror, the light rays after reflection from it travels parallel to each other.

Therefore, the bulb of the headlight is kept at the focus of the concave mirror, so it allows the light to spread out (sends parallel) to infinity (longer distance).

103. If some object is weighed when submerged in water, then what will happen to its weight compared to its weight in air?
(a) Increases
(b) Decreases
(c) Remains exactly the same
(d) Increases or decreases cannot be predicted
(d) Greater than 20 kHz

(***b***) If some object is weighed when submerged in water, then weight of object will decrease as compared to its weight in air. This is explained by Archimedes' principle. According to this, if an object is immersed in a fluid, then it experiences an apparent loss of weight which is equal to the weight of fluid displaced.

104. Which one of the following statements about a satellite orbiting around the earth is correct?
(a) Satellite is kept in orbit by remote control from ground station.
(b) Satellite is kept in orbit by retro-rocket and solar energy keeps it moving around the earth.
(c) Satellite requires energy from solar panels and solid fuels for orbiting.
(d) Satellite does not required any energy for orbiting.

(***d***) Satellite does not require any energy for orbiting. It requires energy to be transferred to its orbit only.
Hence, other options are incorrect.

105. Which one of the following statements about energy is correct?
(a) Energy can be created as well as destroyed.
(b) Energy can be created but not destroyed.
(c) Energy can neither be created nor destroyed.
(d) Energy cannot be created but can be destroyed.

(***c***) The first law of thermodynamics also known as law of conservation of energy, stated that energy can neither be created nor destroyed. It can only be transformed from one form to another.

106. Step-up transformers are used for
(a) increasing electrical power
(b) decreasing electrical power
(c) decreasing voltage
(d) increasing voltage

(***d***) In a step-up transformer, number of turns in secondary winding is more than that of primary winding. As the voltage in a winding is directly proportional to the number of turns. Therefore, step-up transformers are used for increasing the voltage from primary to secondary winding.

107. Which one among the following waves carries the maximum energy per photon?
(a) X-rays (b) Radio waves
(c) Light waves (d) Microwaves

(***a***) Maximum energy per photon is directly proportional to the frequency associated with the wave. As, amongst the given options, X-rays have the highest frequency (10^{19} Hz). Therefore, X-rays carry the maximum energy per photon.

108. A rainbow is produced due to which one of the following phenomena?
(a) Dispersion of light
(b) Interference of light
(c) Diffraction of light
(d) Scattering of light by atmospheric dust

(***a***) A rainbow is a natural phenomenon caused by reflection, refraction and dispersion of light in water droplets after rain. This results in a spectrum of light appearing in the sky.

109. Bats detect obstacles in their path by receiving the reflected
(a) infrasonic waves (b) ultrasonic waves
(c) radio waves (d) microwaves

(***b***) Bats are one of the few mammals that can use sound to navigate this is called echolocation. Bats produce echolocation sounds, which are usually ultrasonic, ranging in frequency from 20 kHz to 200 kHz.
The ears and brain cells in bats are especially tuned to the frequencies of the sounds they emit and the echoes that result.

110. The statement that 'heat cannot flow by itself from a body at a lower temperature to a body at a higher temperature', is known as
(a) zeroth law of thermodynamics
(b) first law of thermodynamics
(c) second law of thermodynamics
(d) third law of thermodynamics

(***c***) It is impossible for a cyclic process to transfer heat from a body at lower temperature to one at a higher temperature without converting some heat to work. This is the statement of second law of thermodynamics.

111. Which one of the following wave does not belong to the category of the other three?
(a) X-rays (b) Microwaves
(c) Radiowaves (d) Sound waves

(***d***) Sound waves are longitudinal waves and they require material medium to propagate while X-rays, microwaves and radiowaves are electromagnetic waves and they don't require any material medium to propagate.

112. Which one of the following statements is not correct?
(a) Human eye is a refracting system containing a diverging lens.
(b) The retina of the human eye contains millions of light sensitive cells, called rods and cones, which convert the light into electrical messages.
(c) Every image that is focussed on the retina is upside down.
(d) We need both eyes to judge the relative positions of objects accurately.

(***a***) Human eye is an optical instrument. It reacts light to provide a three-dimensional moving image. Due to which we are able to see things around us. It is a refracting system containing a converging lens.

113. Which one of the following statements is not correct?
(a) Ultrasonic waves cannot get reflected, refracted or absorbed.
(b) Ultrasonic waves are used to detect the presence of defects like cracks, porsity, etc. in the internal structure of common structure materials.
(c) Ultrasonic waves can be used for making holes in very hard materials like diamond.
(d) Ultrasonic waves cannot travel through vacuum.

(***a***) The statement 'ultrasonic waves cannot get reflected, refracted or absorbed' is not correct as bats navigate in dark with the help of reflection of ultrasonic waves.

114. An object moves in a circular path with a constant speed. Which one of the following statements is correct?
(a) The centripetal acceleration of the object is smaller for a gentle curve (i.e. curve of larger radius) than that for a sharp curve (i.e. curve of smaller radius).
(b) The centripetal acceleration is greater for a gentle curve than that for a sharp curve.
(c) The centripetal acceleration is the same for both the gentle and sharp curves.
(d) The centripetal acceleration causes the object to slow down.

(***a***) Centripetal acceleration is equal to the linear acceleration by the radius of the curve. Therefore, it is smaller for a curve with larger radii than the curve with smaller radii.

115. The force acting on a particle of mass m moving along the X-axis is given by $F(x) = Ax^2 - Bx$. Which one of the following is the potential energy of the particle?

(a) $2Ax - B$ (b) $-\frac{x^2}{6}(2Ax - 3B)$
(c) $Ax^3 - Bx^2$ (d) Zero

(***b***) Potential energy,

$$U(x) = -\int_{x_0}^{x} F(x)\,dx$$

(For one-dimension motion)

Given, $F(x) = Ax^2 - Bx$

$$= -\int_0^x (Ax^2 - Bx)dx$$

$$= -\left[\frac{Ax^3}{3} - \frac{Bx^2}{2}\right] = -\left[\frac{2Ax^3 - 3Bx^2}{6}\right]$$

$$= -\frac{x^2}{6}[2Ax - 3B]$$

116. The symbol of SI unit of inductance is H. It stands for

(a) Holm (b) Halogen
(c) Henry (d) Hertz

(***c***) H stands for Henry. Here, 1 henry is equal to one kilogram metre square per second square per ampere square i.e.

$\therefore$ 1 H = 1 kg $m^2s^{-2}A^{-2}$

Hertz is SI unit of frequency.

117. In a vacuum, a five-rupee coin, a feather of sparrow bird and a mango are dropped simultaneously from the same height. The time taken by them to reach the bottom is t_1, t_2 and t_3, respectively. In this situation, we will observe that

(a) $t_1 > t_2 > t_3$ (b) $t_1 > t_3 > t_2$
(c) $t_3 > t_1 > t_2$ (d) $t_1 = t_2 = t_3$

(***d***) We know that,

$$v = u + gt, t = \frac{v - u}{g}$$

where, $v = 0$, u and g are same in all the three conditions.

So, $t_1 = t_2 = t_3$ (as there is zero air resistance)

118. Electron emission from a metallic surface by application of light is known as

(a) thermionic emission
(b) photo electric emission
(c) high field emission
(d) auto electronic emission

(***b***) Photo electric effect is the phenomenon of the emission of the electrons from the surface of the metal, when the radiations (light rays) of suitable frequency fall on it.

119. How long does light take to reach the earth from the sun?

(a) About 4 min (b) About 8 min
(c) About 24 min (d) About 24 h

(***b***) Distance of earth from sun
= 150 million kilometre
or 149600000 km

Speed of light
$= 3.0 \times 10^8$ m/s $= 3 \times 10^5$ km/s

Thus, the time that light takes to reach the earth from the sun $= \frac{149600000}{3 \times 10^5}$

= 498.66 s or 8 min 20 sec (approx.)

120. Radioactivity is measured by

(a) GM counter (b) polarimeter
(c) calorimeter (d) colorimeter

(***a***) Geiger-Muller (GM) counter is a device which contains a Geiger counter which is used for measuring ionising radiations. Therefore, radioactivity is measured by a **GM counter**.
A **polarimeter** is a scientific instrument used to measure the angle of rotation caused by passing polarised light through an optically active substance.
A **calorimeter** is a device used to measure the heat flow of a chemical reaction or physical change.
A **colorimeter** is a light sensitive device used for measuring the transmittance and absorbance of light passing through a liquid sample.

121. The mirrors used as rear-view mirrors in vehicles are

(a) concave (b) convex
(c) cylindrical (d) plane

(***b***) Rear view mirrors are convex mirrors because these mirrors provide a wider viewing angle than any other mirrors.

122. Which one of the following waves is used for detecting forgery in currency notes?

(a) Ultraviolet waves
(b) Infrared waves
(c) Radio waves
(d) Microwaves

(***a***) Ultraviolet waves are used for detecting forgery in currency notes. There are certain chemicals in the paper of the real bank notes which absorb ultraviolet light and emit the energy as blue light. These paper notes have invisible markings that only shows up using ultraviolet light.

123. The majority charge carriers in a p-type semiconductor are

(a) free electrons
(b) conduction electrons
(c) ions
(d) holes

(***d***) Silicon and germanium doped with electron deficit impurities are called p-type semiconductors. When Si or Ge is doped with group 13 elements like B or Al which contains only 3 valence electrons in the place of fourth electron, a hole is created.
This hole in turn is filled up by an electron from some other site and so on. The motion of charges in this case is more conveniently thought of as the motion of the holes. Therefore, the majority charge carriers are holes.

124. The ionisation energy of hydrogen atom in the ground state is

(a) 13.6 MeV
(b) 13.6 eV
(c) 13.6 J
(d) zero

(***b***) The ionisation energy, $E = \frac{13.6Z^2}{n^2}$eV

For hydrogen atom, $Z = 1$ and for ground state, $n = 1$

$\therefore$ Energy, $E = \frac{13.6}{1}$eV = 13.6 eV

Hence, the ionisation energy of hydrogen atom in the ground state is 13.6 eV.

2017 (I)

125. Which one of the following devices changes low voltage alternating current to high voltage alternating current and *vice-versa?*

(a) Generator
(b) Motor
(c) Transformer
(d) Vibrator

(***c***) Transformer is an electronic device which changes low voltage to high voltage alternating current and *vice-versa.* This change of voltage is obtained by two types of transformers
(i) Step-up transformer
(ii) Step-down transformer
In step-up transformer, the number of turns in secondary coil $N_S > N_P$ (Number of turns in primary coil), which increases the voltage.
Similarly in step-down transformer $N_S < N_P$, which decreases the voltage.

126. An optical illusion which occurs mainly in deserts during hot summer is based on the principle of
(a) reflection
(b) interference
(c) dispersion
(d) total internal reflection

(*d*) An optical illusion in deserts is based on the phenomenon of Total Internal Reflection (TIR) of light.

It is the phenomenon in which when light rays incident on a refracting medium at an angle greater than the critical angle, whole of the light is reflected back to the same medium. Therefore, when the layers of air close to the ground have varying temperature with hottest layers near the ground, light from a distant tree may undergo total internal reflection and the apparent image of the tree may create an illusion to the observer.

127. At which place earth's magnetic field becomes horizontal?
(a) Magnetic meridian
(b) Magnetic equator
(c) Geographical pole
(d) Tropic of cancer

(*b*) The magnetic equator is defined as the line around the earth's, where the magnetic field is horizontal or parallel to the earth's surface. It is a line connecting all points on the earth's surface at which a magnetic needle balances horizontally without dipping.

128. The speed of a car travelling on a straight road is listed below at successive intervals of 1 s.

Time (s)	0	1	2	3	4
Speed (m/s)	0	2	4	6	8

Which of the following is/are correct? The car travels
1. with a uniform acceleration of 2 m/s^2.
2. 16 m in 4 s.
3. with an average speed of 4 m/s.
(a) Only 1 (b) 2 and 3
(c) 1 and 2 (d) All of these

(*a*) From above table, it is clear that every second, the speed is increased by 2 m/s. So, the car is moving with uniform acceleration of 2 m/s^2.

129. The speed of a body that has mach number more than 1 is
(a) supersonic (b) subsonic
(c) 300 m/s (d) about 10 m/s

(*a*) Mach number is the ratio of the speed of a body to the speed of sound in the surrounding medium. As mach number is more than 1, speed of the body is more than the speed of sound, so it will be supersonic.

130. Rutherford's alpha-particle scattering experiment was responsible for the discovery of
(a) electron (b) proton
(c) nucleus (d) helium

(*c*) Rutherford's alpha-particle scattering experiment was responsible for the discovery of nucleus. He proposed that there is a positively charged spherical centre in an atom, called the nucleus. Nearly, all the mass of an atom resides in the nucleus.

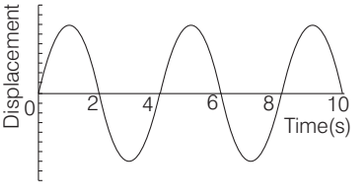
131. Which one of the following statements is not correct?
(a) In the conduction mode of transference of heat, the molecules of solid pass heat from one molecule to another without moving from their positions.
(b) The amount of heat required to raise the temperature of a substance is called its specific heat capacity.
(c) The process of heat transfer in liquids and gases is through convection mode.
(d) The process of heat transfer from a body at higher temperature to a body at lower temperature without heating the space between them is known as radiation.

(*b*) The amount of heat required to raise the temperature of a substance is called its specific heat capacity is not correct statement, because specific heat capacity represents the amount of heat required to raise the temperature of 1 kg of a body by 1° C.

132. The amount of heat required to change a liquid to gaseous state without any change in temperature is known as
(a) specific heat capacity
(b) mechanical equivalent of heat
(c) latent heat of vaporisation
(d) quenching

(*c*) The amount of heat required to change a liquid to gaseous state without any change in temperature is known as latent heat of vaporisation.

133. The following figure shows displacement *versus* time curve for a particle executing simple harmonic motion.



Which one of the following statements is correct?
(a) Phase of the oscillating particle is same at t = 1 s and t = 3 s.
(b) Phase of the oscillating particle is same at t = 2 s and t = 8 s.
(c) Phase of the oscillating particle is same at t = 3 s and t = 7 s.
(d) Phase of the oscillating particle is same at t = 4 s and t = 10 s.

(*c*) From the above graph of particle, it is clear that the phase of oscillating particle is same at time t = 3 s and t = 7 s.

134. Match List I with List II and select the correct answer using the codes given below the lists.

List I (Disease)	**List II** (Remedy)
A. Hypermetropia	1. Concave lens
B. Presbyopia	2. Bifocal lens
C. Myopia	3. Surgery
D. Cataract	4. Convex lens

Codes

	A	B	C	D
(a)	4	2	1	3
(b)	4	1	2	3
(c)	3	1	2	4
(d)	3	2	1	4

(*a*) **Hypermetropia** In this defect, the person cannot see near object clearly and image is formed behind the retina. This defect can be corrected by using convex lens.

Presbyopia In old age, the power of accommodation of the eye lens decrease, therefore neither near nor distant objects are clearly seen. It is corrected by using bifocal lens.

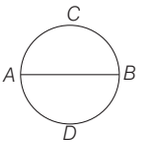
Myopia In this defect, the person cannot see the distant object clearly and image is formed before the retina. This defect can be corrected by using concave lens.

Cataract In this defect, an opaque white membrane is developed on cornea due to which a person losses power of vision partially or completely. This defect can be removed by removing this membrane through surgery.

So, option (a) is the correct answer.

135. A circular coil of single turn has a resistance of 20 Ω. Which one of the following is the correct value for the resistance between the ends of any diameter of the coil?
(a) 5 Ω
(b) 10 Ω
(c) 20 Ω
(d) 40 Ω

(***a***) The resistance of single turn of coil is 20 Ω.



The resistance of half semicircle *ACB* and *ADB* will be 10 Ω each.

Diagonally, they are in parallel, so the equivalent resistance,

$$\frac{1}{R_{AB}} = \frac{1}{R_{ACB}} + \frac{1}{R_{ADB}}$$

$$= \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$$\Rightarrow \quad R_{AB} = 5\,\Omega$$

136. In a solenoid, the current flowing through the wire is I and number of turns per unit length is n. This gives a magnetic field B inside the solenoid. If number of turns per unit length is increased to $2n$, what will be the value of magnetic field in the solenoid?

(a) B (b) $2B$ (c) $\frac{B}{2}$ (d) $\frac{B}{4}$

(***b***) The magnetic field inside the solenoid,

$$B = \mu_0 nI, \quad \text{so } B \propto n$$

When the number of turns per unit length is increased to $2n$, then the field

$$B' \propto 2n$$

$$\therefore \quad B' = 2B$$

137. Which one of the following statements is correct about the magnification of an optical microscope?

(a) Magnification increases with the increase in focal length of eyepiece.
(b) Magnification increases with the increase in focal length of objective.
(c) Magnification does not depend upon the focal length of eyepiece.
(d) Magnification decreases with the increase in focal length of eyepiece.

(***d***) The magnification of compound microscope is given by the following formula, $m = \left(1 - \frac{v_o}{f_o}\right)\left(1 + \frac{D}{f_e}\right)$

where,

v_o = object distance,
f_o = focal length of objective,
f_e = focal length of eyepiece

and D = least distance of distant vision.

$$m \propto \frac{1}{f_e}$$

∴ Magnification will decrease with the increase in the focal length of the eyepiece.

138. Colour vision in human eyes is the function of photoreceptor cells named

(a) rods (b) cones (c) blind spot (d) fovea

(***b***) Day light vision (photopic) or colour vision in human eyes is the function of photoreceptor cells named **cones**. Three types of cone cells present in human eye that contain photoreceptive pigments for red, green and blue lights.

Rods are the other category of photoreceptor cells for twilight vision (scotopic vision).

Blind spot is the point at which the optic nerve (transmits visual information from retina to the brain) leaves the eye. It contains no rods and cones, so an image formed at this point is not sent to the brain.

Fovea is a tiny pit located in the macula of the retina that provides the clearest vision of all.

139. The radii of curvature of the faces of a double convex lens are 10 cm and 20 cm. The refractive index of the glass is 1.5. What is the power of this lens?

(a) + 7.5 D (b) – 7.5 D
(c) + 2.5 D (d) + 5.0 D

(***a***) From lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $\mu = 1.5$, $R_1 = 10$ cm,

$R_2 = -20$ cm

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10} - \frac{1}{(-20)}\right)$$

$$= 0.5\left(\frac{1}{10} + \frac{1}{20}\right) = 0.5\left(\frac{3}{20}\right)$$

$$\Rightarrow \quad f = \frac{200}{15}\text{ cm}$$

∴ Power of lens,

$$P = \frac{100}{\text{Focal length (in cm)}}$$

$$= \frac{15 \times 100}{200} = +7.5\text{ D}$$

140. The time period of a simple pendulum made by using a thin copper wire of length L is T. Suppose the temperature of the room in which this simple pendulum is placed increases by 30°C. What will be the effect on the time period of the pendulum?

(a) T will increase slightly
(b) T will remain the same
(c) T will decrease slightly
(d) T will become more than 2 times

(***a***) The time period of simple pendulum is given by expression

$T = 2\pi\sqrt{\frac{L}{g}}$, where L = length of wire.

Also, when the temperature is changed, the length is given as

$$L_t = L_0(1 + \alpha\,\Delta t)$$

So, $L_t \propto \Delta t$

where, Δt is increase in temperature.

∴ As the temperature increases, the length also increases.

As, $T' = 2\pi\sqrt{\frac{L_t}{g}} \Rightarrow T' \propto L_t^{\frac{1}{2}}$

∴ T will increase slightly.

141. Which one of the following physical quantities has the same unit as that of pressure?

(a) Angular momentum (b) Stress
(c) Strain (d) Work

(***b***) Pressure,

$$= \frac{\text{Force}}{\text{Area}} = \frac{\text{Newton}}{\text{Metre}^2}$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

∴ Unit of stress $= \frac{\text{N}}{\text{m}^2}$

The unit of work is joule and strain is unit less.

142. Which one of the following statements is correct with regard to the material of electrical insulators?

(a) They contain no electrons.
(b) Electrons do not flow easily through them.
(c) They are crystals.
(d) They have more number of electrons than the protons on their surface.

(***b***) In insulators, the number of electrons are very less and they do not flow easily through them, very little electric current will flow through it under the influence of an electric field. This contrasts with other materials, semiconductors and conductors, which conduct electric current more easily.

143. Which one of the following physical quantities does not affect the resistance of a cylindrical resistor?

(a) The current through it.
(b) Its length.
(c) The resistivity of the material used in the resistor.
(d) The area of cross-section of the cylinder.

(***a***) The resistance of a resistor is given by $R = \rho\frac{l}{A}$, i.e. R depends on length (l), area of cross-section (A) and nature of material (ρ).

Hence, current does not affect the resistance of a cylindrical resistor.

144. Which one of the following is the correct relation between frequency f and angular frequency ω ?

(a) $f = \pi\omega$ (b) $\omega = 2\pi f$

(c) $f = 2\omega/\pi$ (d) $f = 2\pi\omega$

(b) (***b***) The relation between frequency (f) and angular frequency (ω) is

$$\omega = 2\pi f$$

The SI unit of angular frequency is radian/second.

145. A Kelvin thermometer and a Fahrenheit thermometer both give the same reading for a certain sample. What would be the corresponding reading in a Celsius thermometer?

(a) 574 (b) 301 (c) 273 (d) 232

(b) (***b***) Let the common temperature in Kelvin and Fahrenheit is X, then

$$\frac{F-32}{180} = \frac{K-273}{100}$$

$$\Rightarrow \frac{X-32}{180} = \frac{X-273}{100}$$

$$\Rightarrow \frac{X-32}{9} = \frac{X-273}{5}$$

On solving, $X = 574$

In Celsius thermometer,

$$K = C + 273$$

$$\Rightarrow C = K - 273$$

$$= 574 - 273 = 301$$

146. If the potential difference applied to an X-ray tube is doubled while keeping the separation between the filament and the target as same, what will happen to the cut-off wavelength?

(a) Will remain same

(b) Will be doubled

(c) Will be halved

(d) Will be four times of the original wavelength

(c) (***c***) The cut-off wavelength of X-ray,

$$\lambda_{min} = \frac{12375}{V}\,\text{Å} \qquad \left[\because \lambda_{min} \propto \frac{1}{V}\right]$$

According to question, if the potential difference is doubled λ_{min} will be halved.

$$\lambda'_{min} \propto \frac{1}{2V}$$

$$\therefore \quad \lambda'_{min} = \frac{1}{2}\lambda_{min}$$

147. Which one of the following statements is true for the relation, $F = \dfrac{Gm_1m_2}{r^2}$?

(All symbols have their usual meanings.)

(a) The quantity G depends on the local value of g, acceleration due to gravity.

(b) The quantity G is greatest at the surface of the earth.

(c) The quantity G is smallest at the surface of the earth.

(d) The quantity G is a universal constant.

(d) (***d***) In relation, $F = \dfrac{Gm_1\,m_2}{r^2}$ the force between two bodies, G is universal constant. It does not depend upon the constitution of bodies or any other factor. Its value is 6.67×10^{-11}N -m^2/kg^2.

148. Why is it difficult to measure the coefficient of expansion of a liquid than solid?

(a) Liquids tend to evaporate at all temperatures

(b) Liquids conduct more heat

(c) Liquids expand too much when heated

(d) Their containers also expand when heated

(d) (***d***) Liquids can be heated only by putting them in a container. So, when heat is provided to the liquid, the container also expands.

149. If the absolute refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively, what will be the ratio of velocity of light in glass and water?

(a) 3 : 4 (b) 4 : 3 (c) 8 : 7 (d) 8 : 9

(d) (***d***) Given, $\mu_g = 3/2$ and $\mu_w = 4/3$

$$\because \quad \mu_g = \frac{c}{v_g}, \mu_w = \frac{c}{v_w}$$

where, v_g = velocity of light in glass

and v_w = velocity of light in water.

$$\Rightarrow \quad \frac{\mu_g}{\mu_w} = \frac{v_w}{v_g} \quad \Rightarrow \quad \frac{3/2}{4/3} = \frac{v_w}{v_g}$$

$$\frac{9}{8} = \frac{v_w}{v_g} \quad \text{or} \quad \frac{v_g}{v_w} = \frac{8}{9}$$

$$\Rightarrow v_g : v_w = 8 : 9$$

150. A positive charge + q is placed at the centre of a hollow metallic sphere of inner radius a and outer radius b. The electric field at a distance r from the centre is denoted by E. In this regard, which one of the following statements is correct?

(a) $E = 0$ for $a < r < b$.

(b) $E = 0$ for $r < a$.

(c) $E = \dfrac{q}{4\pi\varepsilon_0 r}$ for $a < r < b$.

(d) $E = \dfrac{q}{4\pi\varepsilon_0 a}$ for $r < a$.

(b) (***b***) The electric field inside a hollow metallic sphere is zero at every point because from Gauss' law, all the charges must reside on the outer surface of the conductor (sphere), hence no charge is present inside the sphere, i.e. $E = 0$, for $r < a$.

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151. Which one of the following statements is not correct?

(a) The longest wavelength of light visible to human eye is about 700 nm.

(b) The shortest wavelength of light visible to human eye is about 400 nm.

(c) The wavelength of gamma rays is longer than that of X-rays.

(d) The ability of a telescope to form separable images of close objects is called its resolving power.

(c) (***c***) Visible light has wavelengths roughly in the range 380 nm to 780 nm. The gamma ray region and the X-ray region overlap considerably. On the average, wavelengths of gamma rays are shorter than that of X-rays. The wavelengths of X-rays are about 10^{-8} m and of gamma rays 10^{-10} m.

152. If the image of an object formed by a concave mirror is virtual, erect and magnified, then the object is placed

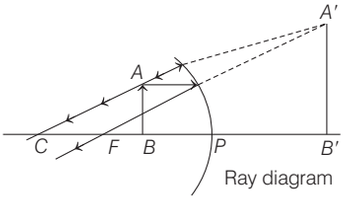
(a) at the principal focus

(b) at the centre of curvature

(c) beyond the centre of curvature

(d) between the pole of the mirror and the principal focus

(d) (***d***) If the image of an object formed by a concave mirror is virtual, erect and magnified, then the object is placed between the pole and focus and the image will formed behind the mirror.



Ray diagram

153. When three resistors, each having resistance r, are connected in parallel, their resultant resistance is x. If these three resistances are connected in series, then total resistance will be

(a) $3x$ (b) $3rx$ (c) $9x$ (d) $\frac{3}{x}$

› (**c**) According to question,

In parallel combination,

$$\frac{1}{r}+\frac{1}{r}+\frac{1}{r}=\frac{1}{x}$$

$$\Rightarrow \quad \frac{3}{r}=\frac{1}{x}$$

$$\Rightarrow \quad r = 3x$$

If the same resistances are connected in series, then $R = r_1 + r_2 + r_3$

$$R = 3x + 3x + 3x$$

$$R = 9x$$

154. The property of electric current which is applicable in the fuse wire is

(a) chemical effect of current
(b) magnetic effect of current
(c) heating effect of current
(d) optical property of current

› (**c**) Fuse is used as a safety device in household circuits and is based on heating effect of current.

It is connected in series with the main supply. A fuse consists of an alloy of lead and tin which has appropriate melting point. This helps to protect the other circuit elements from hazards caused by heavy current.

155. Along a streamline flow of fluid,

(a) the velocity of all fluid particles at a given instant is constant
(b) the speed of a fluid particle remains constant
(c) the velocity of all fluid particles crossing a given position is constant
(d) the velocity of a fluid particle remains constant

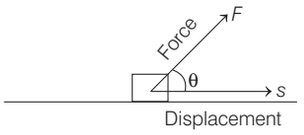
› (**c**) If every particle that passes through a particular point, moves along exactly the same smooth path followed by previous particles passing that point. This path is called streamline flow. In this flow, the velocity of all fluid particle crossing a given position is constant.

156. How is the kinetic energy of a moving object affected if the net work done on it is positive?

(a) Decreases
(b) Increases
(c) Remains constant
(d) Becomes zero

› (**b**) When force F acts on an object in such a direction that it makes an angle with the direction of displacement, then

$$W = Fs\cos\theta$$



When force and displacement are in the same direction, then work done will be positive.

From work-energy theorem,

Work done = Change in kinetic energy

$W_{net} = \Delta$ KE = $KE_f - KE_i$

Net positive work done : Increase in KE.

Net negative work done : Decrease in KE.

157. A particle is executing simple harmonic motion. Which one of the following statements about the acceleration of the oscillating particle is true?

(a) It is always in the opposite direction to velocity.
(b) It is proportional to the frequency of oscillation.
(c) It is minimum when the speed is maximum.
(d) It decreases as the potential energy increases.

› (**c**) In a simple harmonic motion (SHM), a particle moves to and fro repeatedly or an straight or nearly straight path about a mean position. When the speed of particle is maximum, its acceleration will be minimum.

158. Which one of the following four particles, whose displacement x and acceleration a_x are related as follows, is executing simple harmonic motion?

(a) $a_x = +3x$ (b) $a_x = +3x^2$
(c) $a_x = -3x^2$ (d) $a_x = -3x$

› (**d**) Displacement of the particles in SHM,

$$x = A_x\sin(\omega t + \phi)$$

Then, for SHM, acceleration of the particles,

$$a_x = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) \qquad \left(\because \frac{dv}{dt} = \frac{dx}{dt}\right)$$

$$a_x = -A_x\omega^2\sin(\omega t + \phi)$$

$$[\because x = A_x\sin(\omega t + \phi)]$$

$$a_x = -\omega^2 x$$

$$\therefore \quad a_x = -3x$$

159. If we plot a graph between volume V and inverse of pressure p (i.e. $\frac{1}{p}$) for an ideal gas at constant temperature T, the curve so obtained is

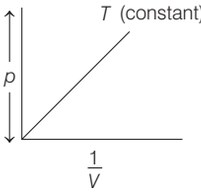
(a) straight line (b) circle
(c) parabola (d) hyperbola

› (**a**) At a given temperature, the volume of a given mass of a gas is inversely proportional to its pressure. This is the statement of Boyle's law.

i.e. $V \propto \frac{1}{p}$

$$pV = \text{constant}$$

If temperature T remains constant, the process is isothermal. Graph between p and V at constant temperature is isothermal and is an equilateral hyperbola. By plotting p *versus* $1/V$, this hyperbola is converted into a straight line.



160. Which one of the following statements is correct?

(a) The speed of sound waves in a medium depends upon the elastic property of the medium, but not on an inertia property.
(b) The speed of sound waves in a medium depends upon the inertia property of the medium, but not on an elastic property.
(c) The speed of sound waves in a medium depends neither on its elastic property nor on its inertia property.
(d) The speed of sound waves in a medium depends on both elastic and inertia properties of the medium.

› (**d**) The speed of sound depends on the elasticity and density of the medium through which it is travelling. In general, sound travel faster in liquids than in gases and faster in solids than in liquids. The greater the elasticity and the lower the density, the faster sound travels in the medium.

161. Which one of the following statements is not correct?

(a) Pitch of a sound is its characteristic by which we can generally differentiate between a male voice and a female voice.
(b) The loudness of sound is related to its frequency.
(c) A musical sound has certain well defined frequencies, which are generally harmonics of a fundamental frequency.
(d) The timbre of a particular musical sound is related to the waveform of the sound wave.

› (**b**) The loudness of a sound depends upon its amplitude. The amplitude of the sound wave depends upon the force with which an object is made to vibrate.

162. A particle executes linear simple harmonic motion with amplitude of 2 cm. When the particle is at 1 cm from the mean position, the magnitudes of the velocity and the acceleration are equal. Then, its time period is

(a) $\frac{2\pi}{\sqrt{3}}$ s (b) $\frac{\sqrt{3}}{2\pi}$ s (c) $\frac{\sqrt{3}}{\pi}$ s (d) $\frac{1}{2\pi\sqrt{3}}$ s

(**a**) The magnitude of velocity and acceleration of particle in SHM when displaced by y, is $\omega\sqrt{(A^2 - y^2)}$ and $\omega^2 y$, respectively.

Equating it, $\omega\sqrt{(A^2 - y^2)} = \omega^2 y$

$$\omega = \frac{\sqrt{A^2 - y^2}}{y} = \sqrt{(2)^2 - (1)^2}$$

$$= \sqrt{4 - 1} = \sqrt{3}$$

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$ s

163. When a ray of light enters a glass slab, then

(a) only the frequency change
(b) frequency and velocity change
(c) frequency does not change
(d) frequency and wavelength change

(**c**) As the light enters from rarer medium to the denser medium, the speed of light decreases but the frequency remains the same. The relation between speed of light, wavelength and the frequency is given as

$$v = n\lambda$$

where, v is the speed of light, n is the frequency of light and λ is the wavelength of light.

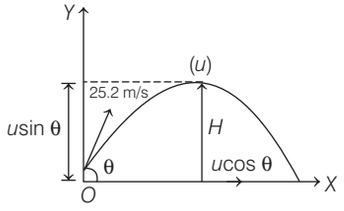
$$\therefore \quad n = \frac{v}{\lambda}$$

In denser medium, n remains constant and v decreases. Therefore, λ also decreases to keep the frequency constant.

164. A ball is thrown vertically upward from the ground with a speed of 25.2 m/s. The ball will reach the highest point of its journey in

(a) 5.14 s (b) 3.57 s
(c) 2.57 s (d) 1.29 s

(**c**) The ball will reach the highest point in time t_a,



$$t_a = \frac{u\sin\theta}{g} = \frac{25.2\sin 90°}{9.8}$$

$$(\because \theta = 90° \text{ and } g = 9.8$$

$$= \frac{25.2 \times 1}{9.8} = 2.57 \text{ s}$$

165. When sound waves are propagated through a medium, the physical quantity/quantities transmitted is/are

(a) matter only
(b) energy only
(c) energy and matter only
(d) energy, momentum and matter

(**b**) The sound waves are propagated when an object is set into vibration. The air particles move to and fro about its average position along an axis.

Thus, only energy is transmitted with it.

166. Pressure is a scalar quantity because

(a) it is the ratio of force to area and both force and area are vectors
(b) it is the ratio of magnitude of force to area
(c) it is the ratio of component of force (normal to area) to area
(d) None of the above

(**a**) Pressure $(p) = \frac{\text{Force}\,(F)}{\text{Area}\,(A)}$

It is the ratio of magnitude of force to area. Force and area, both are vector quantities and ratio of two vector quantities will always be a scalar quantity.

The SI unit of pressure is N/m^2 or pascal.

167. A person is unable to read a newspaper without his glasses. He is most probably suffering from

(a) myopia (b) presbyopia
(c) astigmatism (d) hypermetropia

(**b**) A person is unable to read a newspaper without his glasses because he is probably suffering from **presbyopia**. It is found in old age people. In this defect, one cannot read comfortably and clearly. For most old age people, the near point gradually recedes away. This defect can be corrected by using bifocal lenses.

168. The free fall acceleration g increases as one proceeds at sea level, from the equator towards either pole. The reason is

(a) earth is a sphere with same density everywhere
(b) earth is a sphere with different density at the polar regions than in the equatorial regions
(c) earth is approximately an ellipsoid having its equatorial radius greater than its polar radius by 21 km
(d) earth is approximately an ellipsoid having its equatorial radius smaller than its polar radius by 21 km.

(**c**) Earth is not a sphere. Earth is approximately an ellipsoid, flattened at the pole and bulging at the equator. Its equatorial radius is greater than its polar radius by 21 km. Thus, a point at the pole is closer to the dense core of earth than a point on the equator. This is one reason, the free-fall acceleration g increases as one proceeds at sea level from the equator towards either the pole.

169. Which one of the following statement is correct?

(a) The measurement of mass taken by a spring weighing balance is correct at the place where the spring balance is calibrated for.
(b) The measurement of mass taken by a spring weighing balance is correct at all places.
(c) The measurement of mass taken by a spring weighing balance is correct at the places where the acceleration due to gravity is same with the place where the spring balance is calibrated for.
(d) A spring balance cannot be used to measure mass at any place.

(**d**) A spring balance cannot be used to measure mass at any place because it measures only weight, which varies at different places, whereas mass remains constant.

170. Which one of the following is not a contact force?

(a) Push force
(b) Gravitational force
(c) Frictional force (d) Strain force

(**b**) Contact forces are those forces that cause a change in another object because the objects are physically touching one another.
Gravitational force is not an example of a contact force.

171. When a force of 1 N acts on a mass of 1 kg, which is able to move freely, the object moves in the direction of force with a/an

(a) speed of 1 km/s
(b) acceleration of 1 m/s^2
(c) speed of 1 m/s
(d) acceleration of 1 km/s^2

(**b**) According to the question,

$$F = 1\,\text{N} \quad \text{and} \quad m = 1\,\text{kg}$$

Newton's second law of motion,

$$F = ma$$

$$1\,\text{N} = 1\,\text{kg} \times a \Rightarrow a = 1\,\text{m/s}^2$$

172. Which one of the following statement is not correct?

(a) The SI unit of charge is ampere-second.

(b) Debye is the unit of dipole moment.

(c) Resistivity of a wire of length l and area of cross-section a depends upon both l and a.

(d) The kinetic energy of an electron of mass m kilogram and charge e coulomb, when accelerated through a potential difference of V volt is eV joule.

(c) $\because$ Resistivity, $\rho = \frac{Ra}{l}$

Hence, resistivity of a wire depends on its material length l and area of cross-section a.

Hence, option (c) is not correct.

173. Two balls, A and B are thrown simultaneously, A vertically upward with a speed of 20 m/s from the ground and B vertically downward from a height of 40 m with the same speed and along the same line of motion. At what points do the two balls collide by taking acceleration due to gravity as 9.8 m/s^2?

(a) The balls will collide after 3 s at a height of 30.2 m from the ground

(b) The balls will collide after 2 s at a height of 20.1 m from the ground

(c) The balls will collide after 1 s at a height of 15.1 m from the ground

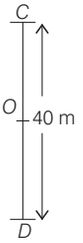
(d) The balls will collide after 5 s at a height of 20 m from the ground

(c) Given, speed, $u = 20$ m/s, height, $h = 40$ m and acceleration due to gravity $g = 9.8$ m/s^2

Then, from equation of motion,

$$s = ut + \frac{1}{2}at^2$$

Let, on point O after time t, O and C meet



$$OC = 20t + \frac{1}{2}gt^2 \quad \ldots(i)$$

and $$OD = 20t - \frac{1}{2}gt^2 \quad \ldots(ii)$$

$$OC + OD = CD = 40$$

Adding Eqs. (i) and (ii), we get

$$40t = 40 \Rightarrow t = 1\text{ s}$$

$$\therefore \quad OD = 20 \times 1 - \frac{1}{2} \times 9.8 \times 1 \times 1$$

$$= 15.1\text{ m}$$

This shows that balls will collide after 1s at a height of 15.1 m from the ground.

174. Which of the following item is used in the household wirings to prevent accidental fire in case of short circuit?

(a) Insulated wire

(b) Plastic switches

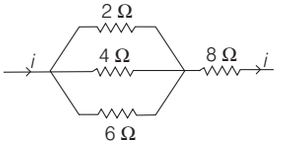
(c) Non-metallic coating on conducting wires

(d) Electric fuse

(d) An electrical fuse is a simple device used to interrupted an electrical circuit during over current condition due to short circuit and/or overload. An electrical fuse operates on the principle of heating effect of electrical current. When an over-current or short circuit condition occurs, which is the same as saying that the magnitude of current flowing in the circuit becomes higher than the rating of the fuse employed, its internal 'fuse wire' gets heated up to such an extent that it melts and breaks apart, which cuts off the supply of current to the circuit downstream. When this happens, the fuse is said to have 'blown'.

2016 (I)

175. Consider the following circuit

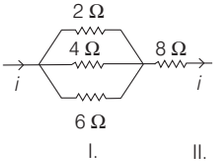


The equivalent resistance of the circuit will be

(a) 12 Ω (b) $8\frac{11}{12}$ Ω

(c) $9\frac{1}{11}$ Ω (d) $\frac{24}{25}$ Ω

(c) Consider the given circuit



I. II.

This circuit consists of two sections, I and II. In first section, three resistors are in parallel connection, therefore equivalent resistance is

$$\frac{1}{R_1} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12}$$

$$\Rightarrow \quad R_1 = \frac{12}{11}\ \Omega$$

This resistance is in series with the resistance 8 Ω of section II.

$\therefore$ Equivalent resistance of the circuit,

$R_{eq} = R_1 + 8$

$$= \frac{12}{11} + 8 = \frac{100}{11} = 9\frac{1}{11}\Omega$$

176. A racing car accelerates on a straight line from rest to a speed of 50 m/s in 25 s. Assuming uniform acceleration of the throughout, the distance covered in this will be

(a) 625 m (b) 1250 m

(c) 2500 m (d) 50 m

(a) Acceleration of the car is uniform. Therefore, according to first equation of motion

$$v = u + at$$

where, v = final velocity of the car = 50 m/s,

a = acceleration of the car,

t = time interval = 25 s

and u = initial velocity = 0.

$\therefore \quad 50 = 0 + a \times 25$

$\Rightarrow \quad a = 2\text{ m/s}^2$

Now, applying second equation of motion, we get

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 2 \times 25 \times 25$$

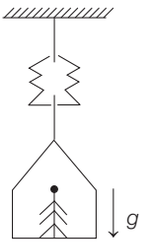
$$= 625\text{ m}$$

177. A man weighing 70 kg is coming down in lift. If the cable of the lift breaks sudden, the weight of the man would become

(a) 70 kg (b) 35 kg

(c) 140 kg (d) zero

(d) Consider the mass of the man in the lift is m when cable breaks as shown in the diagram.



Now, the man in the lift is in free fall. Therefore, his acceleration is same as acceleration due to gravity 'g'. Due to this acceleration pseudo force as the man is mg upward. Therefore, weight of the man,

$$w = mg - mg = 0$$

178. A given conductor carrying a current of 1 ampere. It produces an amount of heat equal to 2000 J. If the current through the conductor is doubled, then the amount of heat produced will be

(a) 2000 J (b) 4000 J
(c) 8000 J (d) 1000 J

(c) (**c**) Let resistance of the coil is R and measuring heat produced for time t seconds.

$\therefore \quad H = I^2RT = (1)^2 (R)(t) \Rightarrow 2000\ \text{J} = Rt$

Similarly, when current is doubled, the amount of heat produced is

$$H' = (I')^2Rt = (2^2)(Rt) = 4Rt$$
$$= 4 \times 2000\ \text{J} = 8000\ \text{J}$$
$$(\because Rt = 2000\ \text{J})$$

179. A lady is standing in front of a plane mirror at a distance of 1 m from it. She walks 60 cm towards the mirror. The distance of her image now from herself (ignoring the thickness of the mirror) is

(a) 40 cm (b) 60 cm
(c) 80 cm (d) 120 cm

(c) (**c**) Initial distance of the lady from the mirror is

$$x_1 = 1\ \text{m} = 100\ \text{cm}$$

After moving 60 cm towards the mirror, new distance, $x_2 = 100 - 60 = 40\ \text{cm}$

$\therefore$ Distance of her image from herself,

$$d = x_2 + x_2 = 2x_2 = 80\ \text{cm}$$

180. The brightness of a star depends on its

(a) size and temperature
(b) size and distance from the earth
(c) size, temperature and mass
(d) size, temperature and distance from the earth

(c) (**a**) According to Stefan's law, rate of loss of heat energy in the form of radiation (brightness) of a star depends upon
(i) nature of surface of radiant object,
(ii) surface area (size) of the star and
(iii) temperature of the star.

181. A glass vessel is filled with water up to the brim and a lid is fixed to it tightly. Then it is kept inside a freezer for hours. What is expected to happen?

(a) The water freezes to ice and the level of ice comes down.
(b) The water in the glass vessel summer freezes to ice.
(c) The glass vessel breaks due to expansion as water freezes to ice.
(d) The water does not freeze at all.

(c) (**c**) Water shows anomalous behaviour in expansion during temperature range of 4°C to 0°C. When temperature falls below 4°C, water expands. As lid is fixed tightly, stress will develop in the bottle due to this anomalous expansion which breaks the glass when water is converted into ice that is volume is maximum.

182. A simple circuit contains a 12 V battery and bulb having 24 Ω resistance. When turn on the switch, the ammeter connected to the circuit would read

(a) 0.5 A (b) 2 A (c) 4 A (d) 5 A

(c) (**a**) Given, emf of the battery, $E = 12$ V and resistance of the bulb, $R = 24\ \Omega$ where, switch is turned on, current through the bulb is

$$I = \frac{V}{R} = \frac{12}{24} = 0.5\ \text{A}$$

Hence, reading of ammeter is 0.5 A.

183. Three resistors with magnitudes 2, 4 and 8 Ω are connected in parallel, then equivalent resistance of the system would be

(a) less than 2 Ω
(b) more than 2 Ω but less than 4 Ω
(c) 4 Ω
(d) 14 Ω

(c) (**a**) Equivalent resistance of three resistors of resistances 2, 4 and 8 Ω,

$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4+2+1}{8} = \frac{7}{8}$$

$$\therefore \quad R_{eq} = \frac{8}{7}\ \Omega = 1.142\ \Omega$$

Therefore, equivalent resistance is less than 2 Ω.

184. Two bodies A and B are moving with equal velocities. The mass of B is doubled that of A. In this context, which one of the following statement is correct?

(a) Momentum of B will be double that of A.
(b) Momentum of A will be double that of B.
(c) Momentum of B will be four times that of A.
(d) Momentum of both A and B will be equal.

(c) (**a**) Let magnitude of the velocity of each body is v.

$\therefore$ Momentum of A, $p_A = m_A v$

Similarly, momentum of B,

$$p_B = m_B v$$

$$\therefore \quad \frac{p_A}{p_B} = \frac{m_A}{m_B} = \frac{m_A}{2m_A} \qquad (\because m_B = 2m_A)$$

$$\Rightarrow \quad p_B = 2p_A$$

185. The SI unit of acceleration is

(a) ms^{-1} (b) ms^{-2}
(c) cms^{-2} (d) kms^{-2}

(c) (**b**) Rate of change in velocity is called acceleration.

$\therefore$ Acceleration,

$$a = \frac{\text{Change in velocity (m/s)}}{\text{Time interval (s)}}$$

Therefore, SI unit of acceleration is m/s^2.

186. An object is placed at the centre of curvature of a concave mirror of focal length 16 cm. If the object is shifted by 8 cm towards the focus, then the nature of the image would be

(a) real and magnified
(b) virtual and magnified
(c) real and reduced
(d) virtual and reduced

(c) (**a**) Focal length of the concave mirror, $f = -16\ \text{cm}$

$\therefore$ Magnitude of radius of curvature of the mirror, $|R| = |2f| = 32\ \text{cm}$

When object is shifted towards the focus, new distance of the object

$$u = -(32 - 8)\ \text{cm} = -24\ \text{cm}$$

Applying mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{24} = \frac{1}{-16}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{24} - \frac{1}{16}$$

$$= \frac{16 - 24}{24 \times 16} = \frac{-8}{24 \times 16}$$

$$\Rightarrow \quad v = -48\ \text{cm}$$

$\therefore$ Magnification, $m = -\frac{v}{u} = \frac{-(-48)}{-24} = -2$

As magnification is negative and having magnitude more than one, image will be real and magnified.

187. A pencil is placed upright at a distance 10 cm from a convex lens of focal length 15 cm. The nature of the image of the pencil will be

(a) real, inverted and magnified
(b) real, erect and magnified
(c) virtual, erect and reduced
(d) virtual, erect and magnified

(c) (**c**) It is given that,

Focal length of the convex lens, $f = 15\ \text{cm}$

Distance of pencil from the pole, $u = -10\ \text{cm}$

Applying mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{10} = \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} + \frac{1}{15} = \frac{25}{150}$$

$\Rightarrow \quad v = 6$ cm

$\therefore$ Magnification, $m = -\frac{v}{u} = \frac{-6}{-10} = 0.6$

As magnification is positive and less than one, so image will be virtual, erect and diminished (reduced).

188. Which one of the following is a conventional energy source?

(a) Tidal energy
(b) Geothermal energy
(c) Solar energy
(d) Bio-mass energy

(*d*) **Bio-mass energy** covers entire life of our planet. This type of energy has been used, since the cave-men discovered fire. Bio-mass fuel comes from things that were once living. e.g. Wood product, dried vegetation, crop residue, aquatic plants, etc.

Tidal energy This is the form of hydropower that converts the energy obtained from tides into useful forms of power. It is non-conventional energy.

Geothermal energy This is heat that comes from the sub-surface of the earth. It is contained in the rocks and fluid beneath the earth's crust.

Solar energy It is a non-conventional source of energy. It refers to capturing the energy from the sun and subsequently converting it into electricity. It is simply the light and heat that come from the sun.

189. A container is first filled with water and then the entire water is replaced by mercury. Mercury has a density of 13.6×10^3 kg/m^3. If X is the weight of the water and Y is the weight of the mercury, then

(a) $X = Y$ (b) $X = 13.6Y$
(c) $Y = 13.6X$ (d) None of these

(*c*) Let volume of the container is V.

Density of mercury,
$\rho_m = 13.6 \times 10^3$ kg /m^3

We know that, density of water,
$\rho_w = 10^3$ kg /m^3

Volume of container remains same in both the cases.

$\therefore \quad \frac{X}{\rho_w} = \frac{Y}{\rho_m}$

$\Rightarrow \quad Y = \left(\frac{\rho_m}{\rho_w}\right)X$

$\Rightarrow \quad Y = \left(\frac{13.6 \times 10^3}{10^3}\right)X = 13.6X$

190. Density of water is

(a) maximum at 0°C
(b) minimum at 0°C
(c) maximum at 4°C
(d) minimum at – 4°C

(*c*) Water shows anomalous behaviour between temperatures 0°C and 4°C. It expands when temperature goes below 4°C.

Therefore, density decreases below 4°C. Also, it expands when temperature goes above 4°C. Therefore, density of water is maximum (1000 kg m^{-3}) at 4°C because volume of water is minimum at 4°C.

191. Suppose the force of gravitation between two bodies of equal masses is F. If each mass is doubled keeping the distance of separation between them unchanged, then the force would become

(a) F (b) $2F$
(c) $4F$ (d) $\frac{1}{4}F$

(*c*) Let mass of each body is m and separation between the bodies is r. Gravitational force between the bodies,

$$F = \frac{Gm^2}{r^2}$$

After doubling the mass, gravitational force becomes,

$$F' = \frac{G(2m)^2}{r^2} = \frac{4Gm^2}{r^2} = 4F$$

192. A body has a free fall from a height of 20 m. After falling through a distance of 5 m, then body would

(a) lose one-fourth of its total energy
(b) lose one-fourth of its potential energy
(c) gain one-fourth of its potential energy
(d) gain three-fourth of its total energy

(*b*) Initial gravitational potential energy of the body,

$U_1 = mgh = 20mg \quad (\because h = 20$ m)

$\therefore$ Initial total energy of the body,

$E = U_1 + K_1 = U_1 + 0 = U_1 = 20mg$

After falling through 5 m, then the new potential energy is

$U_2 = mgh' = mg(20 - 5) = 15mg$

$\therefore$ Loss in potential energy,

$\Delta U = U_1 - U_2 = 20mg - 15mg$
$= 5mg$

$\therefore$ Ratio of potential energy

$= \frac{U}{\Delta U} = \frac{20}{5} = \frac{4}{1}$

Hence, body would lose one-fourth of its potential energy.

193. Mass of a particular amount of substance

1. is the amount of matter present in it
2. does not vary from place to place
3. changes with change in gravitational force

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) Only 1

(*b*) Mass of a particular amount of substance is the amount of matter present in it. It is the inherent property of the substance and does not depend on location of the substance.

Therefore, mass of the substance is independent of gravitational force between them.

194. The impulse on a particle due to a force acting on it during a given time interval is equal to the change in its

(a) force
(b) momentum
(c) work done
(d) energy

(*b*) According to Newton's second law of motion,

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a}$$

where, $\mathbf{F}$ = applied external force,
m = mass of the particle
and $\mathbf{a}$ = acceleration of the particle.

$\therefore$ Impulse, $\mathbf{I} = \mathbf{F} \cdot dt = d\mathbf{p}$ = change in momentum

195. Which one of the following statement with regarding to expansion of materials due to heating is not correct?

(a) As ice melts, it expands uniformly upto 4°C.
(b) Mercury thermometer works using the principle of expansion due to heating.
(c) Small gap is kept between two rails to allow for expansion due to heating.
(d) The length of metallic wire increases when its temperature is increased.

(*a*) Generally, substances expand on heating and contract on cooling. Now, if temperature of certain amount of water is increased from 0°C to 100°C, its volume decreases between 0°C (ice) to 4°C and increases from 4°C to 100°C.

Therefore, during 0°C to 4°C, anomalous behaviour of water is observed, i.e. as ice melts, it expands uniformly upto 4°C.

196. Which one of the following is not a form of stored energy?

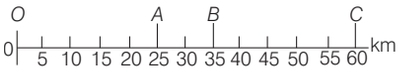
(a) Nuclear energy
(b) Potential energy
(c) Electrical energy
(d) Chemical energy

(*a*) Potential energy of a system is the stored energy associated with the configuration of the system. Electrical energy of a system is also stored energy in the form of electrostatic potential energy.

Similarly, chemical energy is stored in the form of chemical potential. Potential energy associated with the chemical bonds, but nuclear energy is obtained by breaking a heavy nucleus into light nucleus of comparable masses. It can also be obtained by the formation of a heavy nucleus due to fusion of two light nuclei.

Therefore, it is not a form of stored energy.

197. The motion of a car along a straight path is shown by the following figure



The car starts from *O* and reaches at *A*, *B* and *C* at different instants of time. During its motion from *O* to *C* and back to *B*, the distance covered and the magnitude of the displacement are respectively

(a) 25 km and 60 km
(b) 95 km and 35 km
(c) 60 km and 25 km
(d) 85 km and 35 km

(***d***) During motion from *O* to *C* and back to *B*, distance covered,

d = actual path taken
$= OC + BC = 60\text{ km} + (60 - 35)\text{ km}$
$= 85\text{ km}$

Similarly, displacement,

x = least distance between *O* and *B*
$= OB = 35\text{ km}$

2015 (II)

198. Which one of the following statement is correct?

(a) The image formed by a concave mirror for an object lying at infinity is at the principal focus, highly diminished, real and inverted.
(b) A ray of light parallel to the principal axis after reflection from a concave mirror appears to diverge from the principal focus of the mirror.
(c) The focal length of a spherical mirror is double of its radius of curvature.
(d) A ray of light travelling from a rarer medium to a denser medium bends away from the normal.

(***a***) When an object is placed at infinity in front of a concave mirror, it will form a highly diminished, real and inverted image at focus of the mirror. Focal length of a spherical mirror is half of its radius of curvature. A ray of light coming parallel to the principal axis converges to focus of the spherical mirror after reflection from this mirror. During refraction, a ray of light travelling from a rarer medium to a denser medium bends towards the normal.

199. Which one of the following statement is not correct?

(a) In steady flow of a liquid, the velocity of liquid particles reaching at a particular point is the same at all points.
(b) Steady flow is also called streamlined flow.
(c) In steady flow, each particle may not follow the same path as taken by a previous particle passing through that point.
(d) Two streamlines cannot intersect with each other.

(***c***) In steady flow, the velocity of fluid particles reaching at a particular point is the same at all time. Thus, each particle follows the same path as taken by a previous particle passing through that point. Therefore, two streamlines cannot intersect with each other.

200. A brass ball is tied to a thin wire and swung, so as to move uniformly in a horizontal circle. Which of the following statement(s) in this regard is/are true?

1. The ball moves with constant velocity.
2. The ball moves with constant speed.
3. The ball moves with constant acceleration.
4. The magnitude of the acceleration of the ball is constant.

Select the correct answer using the codes given below

(a) Only 1 (b) 1 and 3
(c) 1, 2 and 4 (d) 2 and 4

(***d***) Since, the direction of the ball changes continuously during motion in horizontal circle, so the magnitude of velocity, i.e. speed and magnitude of acceleration remains constant. But their direction changes continuously.

201. Two long wires each carrying a DC current in the same direction are placed close to each other. Which one of the following statement is correct?

(a) The wires will attract with each other.
(b) The wires will repel with each other.
(c) There will be no force between the wires.
(d) There will be a force between the wires only at the moment when the current is switched ON or OFF.

(***a***) When two wires carry current I_1 and I_2 in the same direction and separated by a distance d.

The force of attraction between the wires is

$$F = \frac{\mu_0}{2\pi d} I_1 I_2 L$$

where, L is the length of the wire.

202. The absolute zero, i.e. temperature below which is not achievable, is about

(a) 0°C (b) – 275 °C
(c) – 273 °C (d) – 300 °C

(***c***) Absolute zero is a temperature at which a thermodynamic system has the lowest internal energy. It corresponds to – 273.15°C on the Celsius scale and to – 459.67°F on Fahrenheit scale.

203. Lightning conductors are used to protect building from lightning strikes. Which of the following statement(s) is/are true about lightning conductors?

1. Lightning conductors create an electric field at its top, so that lightning strikes it preferentially.
2. Lightning conductors reduce the effect of the strike by uniformly distributing the charge (current) over the surface of the building.
3. Lightning conductors take all charges (current) to deep down in the earth.
4. Lightning conductors must be installed at a place taller than the building.

Select the correct answer using the codes given below

(a) 1 and 2 (b) 3 and 4
(c) 1, 3 and 4 (d) Only 4

(***b***) Since, lightning conductors are at lower potential (as its wire is buried deep inside the earth), it attracts lightning (which is at the higher potential) and send them to deep down in the earth. Lightning conductors must be installed at a place taller than the building.

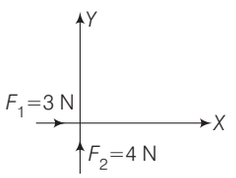
204. The silvering in thermo flask is done to avoid heat transfer by
(a) convection (b) conduction
(c) Both (a) and (b) (d) radiation

(c) *(**c**)* The silvering of inner wall of a thermo flask is done to prevent heat transfer through radiations.
The vacuum space between the walls prevents heat transfer through conduction and convection.

205. Conservation of momentum in a collision between particles can be understood on the basis of
(a) Newton's first law of motion
(b) Newton's second law of motion
(c) Both Newton's second law of motion and Newton's third law of motion
(d) conservation of energy

(c) *(**c**)* Newton's second law states that, the rate of change of momentum is proportional to the force applied and Newton's third law also states about the action and reaction forces. Thus, we can say that principle of conservation of momentum can be understood by both of these laws.
Newton's first law of motion states about inertia. Conservation of energy gives idea about the conversion between potential and kinetic energy.

206. Two forces, one of 3 N and another of 4 N are applied on a standard 1 kg body, placed on a horizontal and frictionless surface, simultaneously along the *X*-axis and the *Y*-axis, respectively, as shown below.



The magnitude of the resultant acceleration is
(a) 7 m/s^2 (b) 1 m/s^2
(c) 5 m/s^2 (d) $\sqrt{7}$ m/s^2

(c) *(**c**)* As two forces are perpendicular to each other, so resultant force is given by

$$F_{net} = \sqrt{F_1^2 + F_2^2}$$
$$= \sqrt{(3)^2 + (4)^2}$$
$$= \sqrt{25} = 5 \text{ N}$$

Now, from second law of Newton,
$F = ma$

Resultant acceleration, $a_{net} = \frac{F_{net}}{m} = \frac{5}{1}$
$= 5$ m/s^2

207. Magnetic meridian is an imaginary
(a) line along North-South
(b) point
(c) vertical plane
(d) horizontal plane

(c) *(**c**)* The vertical plane passing through the axis of a freely suspended magnet is called magnetic meridian. It is an imaginary vertical plane joining the magnetic North and South pole.

208. In SI unit of force 'newton' (N) is given by (where, 'kg' stands for 'kilogram' 'm' stands for 'metre' and 's' stands for 'second')
(a) 1 N = 2 kg-ms^{-2} (b) 1 N = 1 kg-ms^{-2}
(c) 1 N = 4 kg-ms^{-2} (d) 1 N = 3 kg-ms^{-2}

(c) *(**b**)* As we know, $F = ma$

When mass (m) is taken as 1kg and acceleration (a) is taken as 1 m/s^2, the force will be 1 N.
So, 1 N = 1 kg-ms^{-2}.

209. The acceleration due to gravity 'g' for objects on or near the surface of earth is related to the universal gravitational constant 'G' as ('M' is the mass of the earth and 'R' is its radius)
(a) $G = \frac{gM}{R^2}$ (b) $g = \frac{GM}{R^2}$
(c) $M = \frac{gG}{R^2}$ (d) $R = \frac{gG}{M^2}$

(c) *(**b**)* The acceleration due to gravity can be derived from law of gravitation.
Gravitational force between mass of earth *(M)* and a body of mass (m) is given by

$$F = \frac{GMm}{R^2} \quad \text{...(i)}$$

where, R = radius of earth.
Again, we know earth attracts anybody with the force,

$$F = mg \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{GMm}{R^2} = mg \quad \text{or} \quad g = \frac{GM}{R^2}$$

210. The loudness of sound is related to
(a) its frequency (b) its amplitude
(c) its speed (d) its pitch

(c) *(**b**)* The amplitude of a sound wave determines its loudness. A larger amplitude means a louder sound and a smaller amplitude means a softer sound. Pitch and frequency are related with shrillness of sound.

211. X-rays are electromagnetic radiation whose wavelengths are of the order of
(a) 1 m (b) 10^{-1} m
(c) 10^{-5} m (d) 10^{-10} m

(c) (**d**) Mostly X-rays have a wavelength ranging from 0.01 nm to 10 nm.
$\because$ 1 nm = 10^{-9} m
$\therefore$ 0.1 nm = 10^{-10} m

212. In case of a compound microscope, which of the following statements is/are correct?
1. The focal length of the eyepiece is larger than the focal length of the objective.
2. The focal length of the eyepiece is smaller than the focal length of the objective.
3. The image produced in a normal optical microscope is real.
4. The image produced in a normal optical microscope is virtual.

Select the correct answer using the codes given below
(a) Only 1
(b) 1 and 4
(c) 2 and 3
(d) 2 and 4

(c) *(**b**)* The focal length of the eyepiece is larger than the focal length of the objective and image produced in normal optical microscope is virtual. Focal length of objective is kept smaller to form a greatly enlarged image. This image is then viewed through eyepiece.

213. Which one of the following statements is not correct?
(a) The radius of curvature of a concave mirror is twice its focal length.
(b) Power of a convex lens is negative and that of a concave lens is positive.
(c) The radius of curvature of a plane mirror is infinity.
(d) When a ray of light passes from an optically denser medium to an optically rarer medium, an angle of refraction is greater than the corresponding angle of incidence.

(c) *(**b**)* Power of lens is given by $\frac{1}{f(\text{m})}$

For a concave lens 'f' is negative, so power will be negative and focal length of convex lens is positive, so its power will be positive.

214. Which one of the following statements is not correct?
(a) If the velocity and acceleration have opposite sign, then the object is slowing down.
(b) If the velocity is zero at an instant, then the acceleration should also be zero at that instant.
(c) If the velocity is zero for a time interval, then the acceleration is zero at any instant within the time interval.
(d) If the position and velocity have opposite sign, then the object is moving towards the origin.

(b) During retarded motion, velocity and acceleration are in opposite directions. Therefore, we may have a situation, where velocity is zero but acceleration is not zero. e.g. In a spring block system velocity is zero at extreme position but acceleration is non-zero (maximum).

215. Three equal resistances when combined in series are equivalent to 90 Ω. Their equivalent resistance when combined in parallel will be
(a) 10 Ω (b) 30 Ω
(c) 270 Ω (d) 810 Ω

(a) Let each resistance be R.
According to the question,

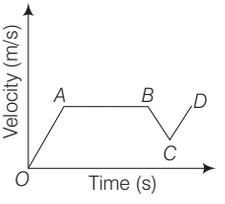
$$R + R + R = 90\ \Omega$$

$$3R = 90\ \Omega \Rightarrow R = 30\ \Omega$$

where, these resistors are combined in parallel, then equivalent resistance is

$$R_{eq} = \frac{R}{3} = \frac{30}{3} = 10\ \Omega$$

216. The following figure represents the velocity-time graph of a moving car on a road.



Which segment of the graph represents the retardation?
(a) *AB* (b) *BC*
(c) *CD* (d) None of these

(b) Rate of decrease of velocity with time is called retardation. Therefore, during retardation velocity of the car should decreases with time. Pitch and frequency are related with shrillness of sound.
Thus, segment *BC* represents retardation as velocity of the car decreases with time.

217. Which one of the following statements is not correct?
(a) The Kelvin scale of temperature is called the absolute scale.
(b) Visible light radiation has wavelength range of 400-700 nm.
(c) The capacity to do work is called power.
(d) The wavelength of γ-rays is less than the wavelength of X-rays.

(c) Capacity of doing work is called energy and the rate of doing work is called power.
Wavelength of X-rays is in the order of 10^{-10}m or 1 Å. Wavelength of γ-rays is in the order of 10^{-12}m. Therefore, the wavelength of γ-rays is less than the wavelength of X-rays.

218. A man is sitting in a train which is moving with a velocity of 60 km/h. His speed with respect to the train is
(a) $\frac{10}{3}$ m/s (b) 60 m/s
(c) infinite (d) zero

(d) According to the concept of relative velocity, both the train and man moves with the velocity of 60 km/h.
So, velocity of man w.r.t. train will be velocity of man – velocity of train,
i.e. 60 – 60 = 0 (zero)

219. Which one of the following is the SI unit of the thermal conductivity of a material?
(a) $Wm^{-1}K^{-1}$ (b) Wm/K
(c) Wm^{-1}/K^{-1} (d) $Js^{-1}m^{-1}K$

(a) Thermal conductivity of a material is the amount of heat required per second to increase the temperature of unit length of the material having unit cross-section through 1°C or 1 K.
Therefore, unit of thermal conductivity is $W\,m^{-1}\,K^{-1}$.

220. Which one of the following statements is not correct?
(a) Conduction can occur easily in solids, less easily in liquids but hardly at all in gases.
(b) Heat energy is carried by moving particles in a convection current.
(c) Heat energy is carried by electromagnetic waves in radiation.
(d) The temperature at which a solid changes into a liquid is called the boiling point.

(d) Melting is a process in which solid changes to liquid or phase change takes place. During melting temperature remains constant. This temperature at which a solid changes into a liquid is called the melting point.

221. The resistance of a wire of length l and area of cross-section a is x Ω. If the wire is stretched to double its length, then its resistance would become
(a) $2x$ Ω
(b) $0.5x$ Ω
(c) $4x$ Ω
(d) $6x$ Ω

(c) According to the question, resistance of wire is given by

$$x = \frac{\rho l}{a}$$

(where, ρ = resistivity, l = length and a = area of cross-section)
If length is doubled, its area of cross-section decreases but volume remains constant.
i.e. $l \times a = 2l \times A'$
(here, A' = new area)

$$\Rightarrow \quad A' = \frac{a}{2}$$

Now, new resistance,

$$R = \frac{\rho\, 2l}{a/2} = \frac{4\,\rho l}{a}$$

As $\frac{\rho l}{a} = x\ \Omega$

So, $R = 4x\ \Omega$

2015 (I)

Directions (Q. Nos. 222-225) *The following four items consist of two statements, Statement* I *and Statement* II. *Examine these two statements carefully and select the correct answer using the codes given below.*

Codes
(a) Both the statements are individually true and Statement II is the correct explanation of Statement I.
(b) Both the statements are individually true but Statement II is not the correct explanation of Statement I.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

222. **Statement I** Diamond is very bright.
Statement II Diamond has very low refractive index.

(c) Diamond possesses high refractive index when light enters into it, suffers multiple total internal reflections due to which it is very bright (unusually brilliance).
So, option (c) is the correct answer.

223. **Statement I** A body weighs less on a hill top than on the earth's surface even though its mass remains unchanged.
Statement II The acceleration due to gravity of the earth decreases with height.

(a) Weight = Mass × Acceleration due to gravity

i.e. $w = mg$

Since, the acceleration due to gravity of the earth decreases with height, so a body weighs less on a hill top than surface.

So, option (a) is the correct answer.

224. **Statement I** While putting clothes for drying up, we spread them out.
Statement II The rate of evaporation increases with an increase in surface area.

(a) While putting clothes for drying up, we spread them out because the rate of evaporation increases with an increase in surface area.

So, option (a) is the correct answer.

225. **Statement I** Due to diffused or irregular reflection of light, a closed room gets light even if no direct sunlight falls inside the room.
Statement II Irregular reflection, where the reflected rays are not parallel, does not follow the laws of reflection.

(c) Due to diffused or irregular reflection of light, a closed room gets light even if no direct sunlight falls inside the room because diffuse reflection of light is not due to the failure of the laws of reflection.

226. Ultrasonic waves of frequency 3×10^5 Hz are passed through a medium, where speed of sound is 10 times that in air (speed of sound in air is 300 m/s). The wavelength of this wave in that medium will be of the order of

(a) 1 cm (b) 10 cm
(c) 100 cm (d) 0.1 cm

(d) The velocity of sound, $v = n\lambda$

$$\text{or } \lambda = \frac{v}{n} = \frac{300}{3 \times 10^5}$$

$$= 100 \times 10^{-5} = 10^{-3} \text{ m}$$

$$= 10^{-3} \times 100 \text{ cm} = 0.1 \text{ cm}$$

$\therefore$ The wavelength of wave, $\lambda = 0.1$ cm

227. If radius of the earth were to shrink by 1%, then its mass remaining the same, g would decrease by nearly

(a) 1% (b) 2%
(c) 3% (d) 4%

(b) Acceleration due to gravity (g) at the surface of earth is given by

$$g = \frac{GM}{R^2}$$

where, R = radius of the earth,
M = mass of the earth
and G = gravitational constant.

$$\therefore \quad \frac{\Delta g}{g} = \frac{2\Delta R}{R}$$

$$= 2 \times (-1\%)$$

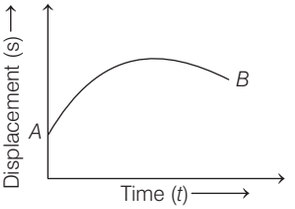
$$= -2\%$$

Therefore, g would decrease by nearly 2%.

228. The displacement-time *(s-t)* graph of a particle acted upon by a constant force is

(a) a straight line
(b) a circle
(c) a parabola
(d) any curve depending upon initial conditions

(c) It is given that acted force is constant.
Therefore, acceleration of the particle will also be constant, say it is a.



From second equation of motion, displacement of the particle at any time t is

$$s = ut + \frac{1}{2}at^2$$

where, u = initial speed of the particle.
Clearly, s-t graph will be a parabola.

229. Which one of the following is not a result of surface tension?

(a) Nearly spherical drop of rain
(b) Capillary rise
(c) Removal of dirt by soap or detergent
(d) Flow of a liquid

(d) Flow of a liquid is not a result of surface tension because it takes place due to pressure difference between two points.

230. If a charged particle ($+q$) is projected with certain velocity parallel to the magnetic field, then it will

(a) trace helical path
(b) trace circular path
(c) continue its motion without any change
(d) come to rest instantly

(c) Force on the charged particle,
$F = qvB \sin\theta$

where, B = intensity of magnetic field
and θ = angle between velocity v and magnetic field B.

When the charged particle ($+q$) is projected with certain velocity parallel to the magnetic field, then it will continue its motion without any change in velocity. If $\theta = 0°$, then $F = 0$.

231. Optical fibres, though bent in any manner, allows light to pass through. What is the inference that one can draw from it?

(a) The concept that light travels in straight path is wrong.
(b) Light can flow through the optical fibres.
(c) Light can travel through the fibres because of their ductility.
(d) Light can travel through the fibres due to multiple total internal reflection.

(d) Optical fibres work on the principle of total internal reflection. Therefore, light can travel through the fibres due to multiple total internal reflection. But still we can say that light travels in straight line and it is independent of ductility of the optical fibres.

232. Which one among the following happens when a swing rises to a certain height from its rest position?

(a) Its potential energy decreases while kinetic energy increases.
(b) Its kinetic energy decreases while potential energy increases.
(c) Both potential and kinetic energy decreases.
(d) Both potential and kinetic energy increases.

(b) Mechanical energy of the swing remains constant. When the swing rises to a certain height, its potential energy increases. As mechanical energy is some of potential energy and kinetic energy and potential energy of the swing is increasing, its kinetic energy must be decreases.

233. Match List I with List II and select the correct answer using the codes given below the lists.

List I (Physical quantity)	List II (Unit)
A. Distance	1. Mole
B. Amount of material	2. Coulomb
C. Amount of electrical charge	3. Light year
D. Energy	4. Watt-hour

Codes

	A	B	C	D
(a)	3	1	2	4
(b)	3	2	1	4
(c)	4	2	1	3
(d)	4	1	2	3

(a) Amount of a substance (material) is measured in mole.
Watt-hour (W-h) or kW-h measures energy consumed through an electrical system.
Light year indicates the distance travelled by light in one year.
Coulomb is the unit of electrical charge.

234. A ray of light when refracted suffers change in velocity. In this context, which one among the following statements is correct?

(a) Velocity increases as the ray passes from a rarer to a denser medium.
(b) Velocity decreases as the ray passes from a denser to a rarer medium.
(c) Velocity decreases as the ray passes from a rarer to a denser medium.
(d) Change of velocity does not depend on the nature of medium.

(c) When light goes from one medium to another, it is called refraction.

Velocity of light in any medium,

$$v = \frac{\text{Velocity of light in vacuum } (c)}{\text{Refractive index of the medium } (\mu)}$$

For denser medium $\mu_d > \mu_r$ (rarer medium).

$\therefore \quad v_d < v_r$

Therefore, velocity decreases as the ray passes from a rarer to a denser medium.

235. An object is placed 10 cm in front of a convex lens of focal length 15 cm. The image produced will be

(a) real and magnified
(b) virtual and magnified
(c) virtual and reduced in size
(d) real and reduced in size

(b) Lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Here, $f = +15$ cm, $u = -10$ cm

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{15} - \frac{1}{10}$$

or $\quad \frac{1}{v} = \frac{2-3}{30}$ or $\frac{1}{v} = \frac{-1}{30}$

$v = -30$ cm

Now, we can write

Magnification,

$$m = \frac{\text{Height of image } (h_i)}{\text{Height of object } (h_o)} = \frac{v}{u}$$

$$\Rightarrow \quad m = \frac{-30}{-10} = 3$$

$\because \quad m > 0$ and $m > 1$

$\therefore$ So, the image is virtual and magnified.

236. A person stands on his two feet over a surface and experiences a pressure p. Now, the person stands on only one foot. He would experience a pressure of magnitude

(a) $4p$ (b) p (c) $\frac{1}{2}p$ (d) $2p$

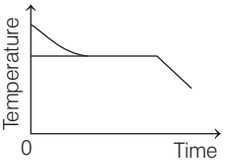
(d) Let the area of one foot = A

$\therefore$ Pressure due to two feet, $p = \frac{F}{2A}$

$\Rightarrow$ Pressure due to one foot, $p' = \frac{F}{A}$

Therefore, $p' = 2p$.

237. A solid is melted and allowed to cool and solidify again. The temperature is measured at equal intervals of time. The graph alongside shows the change of temperature with time.



The part of the curve that is practically horizontal is due to

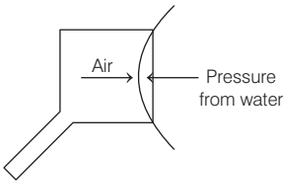
(a) latent heat given away by the liquid
(b) specific heat given away by the liquid
(c) thermal capacity changes with time keeping temperature constant
(d) change in density during transformation

(a) During melting, temperature of solid remains constant. Also heat required for melting is proportional to latent heat of the solid. Therefore, the part of the curve that is practically horizontal is due to latent heat of the solid.

238. A deep sea diver may hurt his ear drum during diving because of

(a) lack of oxygen
(b) high atmospheric pressure
(c) high water pressure
(d) All of the above

(c) In normal situations, the air inside your ear is at the same pressure as the air outside your ear. This means that the force on the two sides of the ear drum are the same and you are happy. In this case, there is water on the outside of this ear drum, but air on the inside. If you don't do anything, the pressure inside will still be at atmospheric pressure.



However, on the outside, the pressure will be greater. This means that the force from the inside air will not cancel with the pressure from the outside. Your ear drum can't move, so it stretches like a spring to produce a net force of zero. This stretching of the ear drum hurts.

239. When you walk on a woolen carpet and bring your finger near the metallic handle of a door, an electric shock is produced. This is because

(a) charge is transferred from your body to the handle
(b) a chemical reaction occurs when you touch the handle
(c) the temperature of the human body is higher than that of the handle
(d) the human body and the handle arrive at thermal equilibrium by the process

(a) When you walk on a woolen carpet, electrons get transferred to the carpet through your shoes. With every step you become more and more positively charged. This charge distributes itself over your body. Now when you bring your finger near the metallic handle of a door, an electric shock is produced because electron is transferred from the handle to your body.

240. The product of conductivity and resistivity of a conductor

(a) depends on pressure applied
(b) depends on current flowing through conductor
(c) is the same for all conductors
(d) varies from conductor-to-conductor

(c) As we know that,

$$\text{Resistivity} = \frac{1}{\text{Conductivity}}$$

or Resistivity × Conductivity = 1

Therefore, the product of conductivity and resistivity of a conductor is same for all conductors and it is independent of pressure applied or current flowing through the conductor.

241. A spring can be used to determine the mass m of an object in two ways : I by measuring the extension in the spring due to the object and II by measuring the oscillation period for the given mass.

Which of these methods can be used in a space-station orbiting the earth?

(a) Both I and II
(b) Only the extension method
(c) Only the oscillation method
(d) Neither I nor II

› ***(c)*** Inside the space station orbiting the earth, each body is weightless and if a body is hanged with the string no tension is produced in the spring. Time period of spring block system is independent of weight of the object. It only depends upon spring constant and mass.

Therefore, by measuring the period of oscillation of the mass, we can reduce the mass.

242. The sun is observed to be reddish when it is near the horizon, i.e. in the morning and the evening. This is because

(a) red light is least scattered by atmosphere
(b) red light is most scattered by atmosphere
(c) it is the colour of the sun in the morning and evening
(d) the earth's atmosphere emits red light

› ***(a)*** The sun is observed to be reddish when it is near the horizon because red light is least scattered by atmosphere and dominant part of light received from the sun is red.

243. Thermal conductivity of aluminium, copper and stainless steel increases in the order

(a) Copper < Aluminium < Stainless steel
(b) Stainless steel < Aluminium < Copper
(c) Aluminium < Copper < Stainless steel
(d) Copper < Stainless steel < Aluminium

› ***(b)*** The thermal conductivity of copper is maximum and that of stainless steel (alloy) is minimum. So, the order of thermal conductivity is
Stainless steel < Aluminium < Copper

244. The radius of the moon is about one-fourth that of the earth and acceleration due to gravity on the moon is about one-sixth that on the earth. From this, we can conclude that the ratio of the mass of the earth to the mass of the moon is about

(a) 10 (b) 100
(c) 1000 (d) 10000

› ***(b)*** The mass of the earth, $M_e = \frac{gR_e^2}{G}$

The mass of the moon,

$$M_m = \frac{g' R_m^2}{G}$$

$$\therefore \quad \frac{M_e}{M_m} = \frac{gR^2}{g' R_m^2} = \frac{g}{g/6} \times \frac{R^2}{\left(\frac{R}{4}\right)^2}$$

$$\Rightarrow \quad \frac{M_e}{M_m} = 96 \approx 100$$

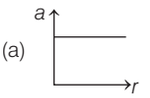
2014 (II)

245. Light waves are

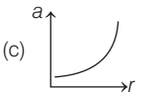
(a) electromechanical waves
(b) electromagnetic waves
(c) electrooptical waves
(d) magnetooptical waves

› ***(b)*** Light waves are electromagnetic waves. In this wave, electric and magnetic fields are always perpendicular to the direction of travel of the wave.

246. A particle is moving in a circular path of radius r at a constant speed v. Which one of the following graphs correctly represents its acceleration a?

(a) 

(b) 

(c) 

(d) 

› (***d***) The given condition resembles with uniform circular motion. In this case, the centripetal acceleration $a_c = \frac{v^2}{r}$ also $|\mathbf{v}|$ is constant.

$\therefore\ a_c \propto \frac{1}{r}$.

Hence, the graph between a and r is

247. Which one of the following circuit elements is an active component?

(a) Resistor
(b) Transistor
(c) Inductor
(d) Capacitor

› ***(b)*** Amplifying components are called active components like transistors, tunnel diodes, vacuum tubes, etc.
Passive components include two terminal components like resistors, capacitors, inductors, etc.

248. Fahrenheit and Celsius are the two scales used for measuring temperature. If the numerical value of a temperature recorded in both the scales is found to be same, then what is the temperature?

(a) – 40° (b) + 40°
(c) + 72° (d) – 72°

› ***(a)*** Relation between Fahrenheit and Celsius scale is

$$\frac{C - 0°}{100} = \frac{F - 32}{180}$$

Given that, numerical value of a temperature recorded in both the scales are same.

$$°C = F = x \quad \text{(say)}$$

$$\Rightarrow \quad \frac{x}{100} = \frac{x - 32}{180}$$

$$\Rightarrow \quad 180x = 100x - 3200$$

$$\Rightarrow \quad 80x = -3200$$

$$\Rightarrow \quad x = -\frac{3200}{80}$$

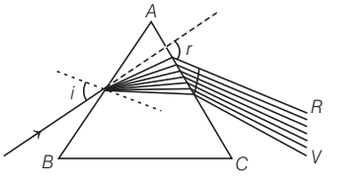
$$\Rightarrow \quad x = -40°$$

249. Which one of the following processes explains the splitting of a beam of white light into its constituent colours?

(a) Dispersion (b) Reflection
(c) Diffraction (d) Polarisation

› ***(a)*** The phenomenon of splitting of white light (sunlight) into its constituent colours is called dispersion.

Dispersion of white light when it passes through a prism, is shown as below



Reflection of light is associated with returning of light in the same medium after striking a surface.

Diffraction is bending of light due to obstacles of size comparable with the wavelength of light.

Polarisation involves the phenomenon of restricting the vibration or propagation of light to a particular direction.

250. Two bodies A and B having masses m and $4\,m$ respectively, are moving with equal linear momentum. The ratio of kinetic energies between A and B is

(a) 1 : 4 (b) 4 : 1
(c) 1 : 1 (d) 1 : 2

(b) The relation between linear momentum p and kinetic energy K is

$$K = \frac{p^2}{2m} \Rightarrow K_A = \frac{p_A^2}{2m} \text{ and } K_B = \frac{p_B^2}{2(4\,m)}$$

$$\therefore \quad \frac{K_A}{K_B} = \frac{p_A^2}{2m} \times \frac{8\,m}{p_B^2} \quad (\text{given, } p_A = p_B)$$

$$\text{or} \quad \frac{K_A}{K_B} = \frac{4}{1} = 4 : 1$$

251. Optical glass used in the construction of spectacles is made by

(a) flint glass (b) crookes glass
(c) quartz glass (d) hard glass

(a) Optical glass used in the construction of spectacles is made by flint glass. The reason behind using flint glasses are higher refractive index of these glasses than crown glasses. Due to higher refractive index, dispersive power is high which is useful for correction of chromatic aberration.

252. A force applied on a body is represented as $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ and accelerates it at 1 m/s^2. The mass of the body is

(a) 10 kg (b) $10\sqrt{2}$ kg
(c) $2\sqrt{10}$ kg (d) 8 kg

(b) Given, $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$

$\therefore$ Resultant force,

$$|\mathbf{F}| = \sqrt{(6)^2 + (-8)^2 + (10)^2}$$

$$= \sqrt{36 + 64 + 100} = 10\sqrt{2} \text{ N}$$

Also, $|\mathbf{a}| = 1\,\text{m/s}^2$

Using $F = ma$

$\Rightarrow \quad 10\sqrt{2} = m \times 1 \Rightarrow m = 10\sqrt{2}$ kg

253. A force $\mathbf{F}$, acting on an electric charge q, in presence of an electromagnetic field, moves the charge parallel to the magnetic field with velocity $\mathbf{v}$. Then, $\mathbf{F}$ is equal to (where, $\mathbf{E}$ and $\mathbf{B}$ are electric field and magnetic field, respectively)

(a) $q\,\mathbf{E}$ (b) $q\,(\mathbf{v} \times \mathbf{B})$
(c) $q\,(\mathbf{v} \times \mathbf{E})$ (d) $q\,\mathbf{B}$

(a) Force on the charge q due to electric field $\mathbf{E}$,

$$\mathbf{F_E} = q\,\mathbf{E} \quad \overset{\mathbf{v}}{\longrightarrow} \quad \overset{\mathbf{B}}{\longrightarrow}$$

and force on the charge q due to magnetic field $\mathbf{B}$,

$$\mathbf{F_B} = q\,(\mathbf{v} \times \mathbf{B})$$

$$\Rightarrow \quad |\mathbf{F_B}| = q\,\mathbf{v}B\sin\theta$$

In the given case, $\mathbf{v} \,||\, \mathbf{B}$

$$\Rightarrow \quad \theta = 0°$$

$$\Rightarrow \quad |\mathbf{F_B}| = 0$$

Hence, the force $\mathbf{F}$ acting on the charge q is equal to

$$\Rightarrow \quad \mathbf{F} = \mathbf{F_E} = q\mathbf{E}$$

254. Which of the following are the correct parameters for the **common domestic power supply** in India?

(a) 220 V, 110 Hz (b) 220 V, 50 Hz
(c) 110 V, 220 Hz (d) 110 V, 50 Hz

(b) These were initial standards set up by Indian experts. In some countries these standards are 110 V, 50 Hz. The selection of these standard does not affect much to the electrical distribution system.

It may be differentiated little in terms of cost and electrical shocking less cost and more electrical shock is associated with 220 V, 50 Hz as comparison with 110 V, 50 Hz.

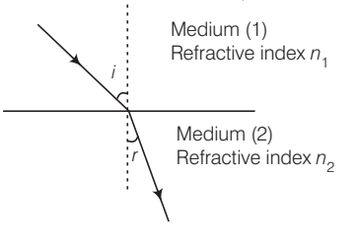
255. A ray of light travels from a medium of refractive index n_1 to a medium of refractive index n_2. If angle of incidence is i and angle of refraction is r, then $\dfrac{\sin i}{\sin r}$ is equal to

(a) n_1 (b) n_2
(c) $\dfrac{n_2}{n_1}$ (d) $\dfrac{n_1}{n_2}$

(c) According to Snell's law,

$$n_1 \times \sin i = n_2 \times \sin r$$

$$\Rightarrow \quad \frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$



256. Light waves projected on oil surface show seven colours due to the phenomenon of

(a) polarisatio
(b) refraction
(c) reflection
(d) interference

(d) The oil surface acts as thin film. When white light is projected on the oil surface, same part of the incident light gets reflected back and some parts gets refracted in the film. If we observe reflected or refracted light, then some colour of light interfere to produce maxima and some colour produces minima.

Therefore, we obtain coloured patches of light due to the interference.

Directions (Q. Nos. 257-258) *The following two questions consist of two statements,* **Statement I** *and* **Statement II**. *Examine these two statements carefully and select the correct answer using the codes given below.*

Codes

(a) Both the statements are individually true and Statement II is the correct explanation of Statement I.
(b) Both the statements are individually true but Statement II is not the correct explanation of Statement I.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

257. **Statement I** It is not necessary that every bar magnet has one North pole and one South pole.
Statement II Magnetic poles occur in pair.

(d) Magnetic poles always occur in pair. They cannot be dissociated. Even a smallest piece will exhibit its own North pole and South pole. When a bar magnet is brought near another bar magnet then like poles always repel each other and unlike poles always attract each other.

The earth itself behaves as a huge bar magnet in which the North pole is towards geographical South and South pole is towards geographical North pole.

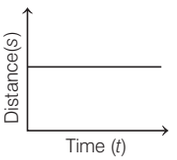
So, option (a) is the correct answer.

258. **Statement I** A body moving in a circular path is acted upon by the centripetal force.
Statement II Centripetal force acting on the body is doing work to keep it rotating in the circular path.

(c) *(**c**)* When a body moves in a circular path, it has an acceleration towards the centre. The inward force on the body moving in a circular path is called centripetal force.
Also, as the instantaneous centripetal force acting on the body is always perpendicular to the direction of motion, hence the work done is always zero.
So, option (c) is the correct answer.

259. If the motion of an object is represented by a straight line parallel to the time-axis in a distance-time graph, then the object undergoes
(a) an accelerated motion
(b) a decelerated motion
(c) a uniform non-zero velocity motion
(d) a zero velocity motion

(d) *(**d**)* It is clear from the graph that the position of the object is not changing with the change in time, hence the object is at rest or in other words we can say that it is a zero velocity motion.



260. Kerosene oil rises in a wick of lantern because of
(a) buoyancy of air
(b) diffusion of the oil through the wick
(c) capillary action in the wick
(d) gravitation pull of the wick

(c) *(**c**)* This capillary action of the wick is associated with surface tension of kerosene oil. Due to this, upward force acts on the surface of kerosene oil which lifts up the oil in the wick, and level of oil rises.

261. The working of a microwave oven involves
(a) absorption of microwaves by matter
(b) reception of microwaves by optical fibre
(c) microwave amplification by stimulated emission of radiation
(d) transmission of microwaves through a metal

(a) *(**a**)* A microwave oven (commonly reffered to as a microwave) is an electric oven that heats and cooks food by exposing it to electromagnetic radiation in the microwave frequency range. The working of a microwave absorption of microwaves by matter.

262. Two cars A and B have masses m_A and m_B respectively, with $m_A > m_B$. Both the cars are moving in the same direction with equal kinetic energy. If equal braking force is applied on both, then before coming to rest
(a) A will cover a greater distance
(b) B will cover a greater distance
(c) Both will cover the same distance
(d) distance covered by them will depend on their respective velocities

(c) *(**c**)* Applying third equation of motion for the cars, we get

$$v^2 = u^2 - 2as$$

where, v = final velocity = 0,
u = initial velocity,
a = retardation
and s = distance covered before coming to rest.

$$\therefore \quad 0 = u^2 - 2as$$

$$\Rightarrow \quad s = \frac{u^2}{2a} = \frac{mu^2}{2ma} = \frac{\frac{1}{2}mu^2}{ma} = \frac{K}{F}$$

where, K = initial kinetic energy of cars and F = braking force applied on the cars.

$$\therefore (s)_A = \left(\frac{K}{F}\right)_A = \left(\frac{K}{F}\right)_B = s_B$$

$\because K_A = K_B = K$ or $F_A = F_B = F$
Hence, both the cars will cover the same distance before coming to rest.

263. If the length of the equator is about 40000 km and the velocity of rotation is about 1700 km/ h, what would be the velocity of rotation at the pole?
(a) Zero (b) 850 km/h
(c) 1700 km/h (d) 3400 km/h

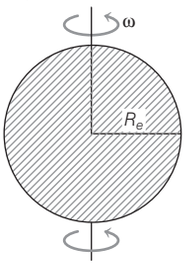
(a) *(**a**)* Using $v_e = R_e\omega$

$$\Rightarrow \quad 1700 = 20000 \times \omega$$

$$\text{or } \omega = \frac{1700}{20000} = \frac{17}{200} \text{ rotation/h}$$

Velocity of rotation at pole,

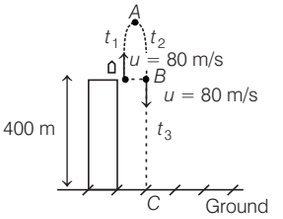
$$v_p = R_p\omega$$



As $R_p = 0 \Rightarrow v_p = 0$

264. A bullet is fired vertically up from a 400 m tall tower with a speed 80 m/s. If g is taken as 10 m/s^2, the time taken by the bullet to reach the ground will be
(a) 8 s (b) 16 s (c) 20 s (d) 24 s

(c) *(**c**)* Total time, $T = t_1 + t_2 + t_3$
Given, u = 80 m/s and g = 10 m/s^2



Velocity of the bullet at point A,

$$v_A = 0 \text{ m/s}, \; a = -10 \text{ m/s}^2$$

$$v = u + at$$

$$\Rightarrow \quad 0 = 80 - 10t_1 \quad \text{or} \quad t_1 = 8 \text{ s}$$

$$\Rightarrow \quad t_2 = t_1$$

[as there is no air resistance]

Calculation for t_3

Velocity of the bullet at point B,

$$v_B = u = 80 \text{ m/s}, \; h = 400 \text{ m}$$

Using $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad 400 = 80 \times t_3 + \frac{1}{2} \times 10 \times t_3^2$$

$$\Rightarrow \quad 5t_3^2 + 80t_3 - 400 = 0$$

$$\Rightarrow \quad t_3^2 + 16t_3 - 80 = 0$$

$$\Rightarrow \quad t_3(t_3 + 20) - 4(t_3 + 20) = 0$$

$$\Rightarrow \quad t_3 = -20 \text{ s or } 4 \text{ s}$$

Time cannot be negative hence, t_3 = 4 s
Total time, $T = 8 + 8 + 4 = 20$ s

265. A cyclotron accelerates particles of mass m and charge q. The energy of particles emerging is proportional to
(a) q^2/m (b) q/m^2 (c) q^2/m^2 (d) q

(a) *(**a**)* For cyclotron,

$$qvB = \frac{mv^2}{r} \text{ or } \frac{qBr}{m} = v$$

$$\Rightarrow \quad \text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{q^2B^2r^2}{m^2} = \frac{q^2B^2r^2}{2m}$$

$$\Rightarrow \quad \text{KE} \propto \frac{q^2}{m}$$

266. The electric field inside a perfectly conducting hollow object is
(a) 4π (b) infinite
(c) zero
(d) dependent upon the shape of the object

(c) *(**c**)* The electric field inside a perfectly conducting hollow object is zero, as there is no change inside the object

and from Gauss' law $\int E \cdot dS = \frac{q_{inside}}{\varepsilon_0}$

For any Gaussian surface inside the object, $q_{inside} = 0$.

$\Rightarrow \quad E_{inside} = 0$

267. The densities of three liquids are D, $2D$ and $3D$. What will be the density of the resulting mixture, if equal volumes of the three liquids are mixed?

(a) $6D$ (b) $1.4D$ (c) $2D$ (d) $3D$

(c) *(**c**)* The answer to this question can be found by calculating weighted average of the three liquids.

Let's assume that 1L each of liquids is mixed.

Therefore, density of resulting mixture is

$$= \frac{1D(1) + 2D(1) + 3D(1)}{1 + 1 + 1}$$

$$= \frac{(1 + 2 + 3)D}{3} = \frac{6D}{3} = 2D$$

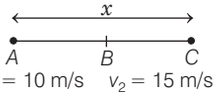
Therefore, the resulting mixture would have a density of $2D$.

268. A particle is moving with uniform acceleration along a straight line ABC, where $AB = BC$. The average velocity of the particle from A to B is 10 m/s and from B to C is 15 m/s. The average velocity for the whole journey from A to C (in m/s) is

(a) 12 (b) 12.5 (c) 13 (d) 13.5

(a) *(**a**)* It is given that, $AB = BC$

For A to B, $t_{AB} = \frac{x}{2 \times 10} = \frac{x}{20}$ m/s



$v_1 = 10$ m/s $\quad v_2 = 15$ m/s

and B to C,

$$t_{BC} = \frac{x}{2 \times 15} = \frac{x}{30} \text{ m/s}$$

Average velocity of whole journey,

$$v_{av} = \frac{x}{T} \quad \text{or} \quad v_{av} = \frac{x}{t_{AB} + t_{BC}}$$

$$= \frac{x}{\frac{x}{20} + \frac{x}{30}}$$

or $\quad v_{av} = \frac{20 \times 30}{20 + 30}$

$$= \frac{600}{50} = 12 \text{ m/s}$$

269. The dimension of 'impulse' is the same as that of

(a) pressure
(b) angular momentum
(c) work
(d) linear momentum

(d) *(**d**)* We know that, impulse = force × time interval

Also, $\quad F = \frac{\Delta p}{\Delta t}$

$\Rightarrow \quad F\Delta t = \Delta p$

$\therefore$ Impulse = Δp = Change in momentum

Hence, the dimension of impulse is same as linear momentum.

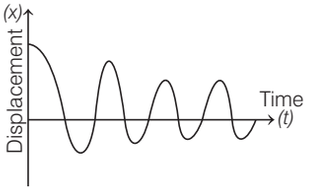
2014 (I)

270. Bernoulli's principle is based on which of among the following laws?

(a) Conservation of mass
(b) Conservation of momentum
(c) Conservation of angular momentum
(d) Conservation of energy

(d) *(**d**)* Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in static pressure or a decrease in the fluid's potential energy. According to its theorem, pressure energy + kinetic energy + potential energy = constant.

271. The displacement (x)-time (t) graph given below approximately represents the motion of a



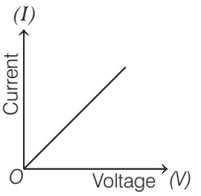
(a) simple pendulum placed in vacuum
(b) simple pendulum immersed in water
(c) simple pendulum placed in outer space
(d) point mass moving in air

(b) *(**b**)* When a simple pendulum immersed in water, then its oscillations are damped and its wave is cosine wave.

Therefore, given graph can approximately represent motion of a simple pendulum immersed in water. A simple pendulum placed in vacuum or in outer space oscillates without damping.

A point mass moving in air cannot have oscillatory motion.

272. The current (I)-voltage (V) plot of a certain electronic device is given below.



The device is

(a) a semiconductor
(b) a conductor which obeys Ohm's law
(c) a superconductor
(d) an insulator

(b) *(**b**)* According to Ohm's law, graph between voltage and current through a conductor is a straight line. Therefore, above I-V curve corresponds to a conductor. An insulator, semiconductor and a superconductor do not obey Ohm's law. Thus, their I-V curve is not a straight line.

273. The temperature of water at the bottom of a lake whose upper surface has frozen to ice would be around

(a) –10°C (b) 0°C (c) 4°C (d) –4°C

(c) *(**c**)* When temperature of water at upper surface of a lake decreases from 4° C, its density decreases. As temperature decreases farther water at upper surface remains there and water at the bottom remains at the bottom.

Therefore, temperature of water at upper surface goes down to 0°C and it starts freezing. As ice acts as insulator for heat with increasing thickness, water at bottom remains at 4°C.

274. Two conducting wires A and B are made of same material. If the length of B is twice that of A and the radius of circular cross-section of A is twice that of B, then their resistances R_A and R_B are in the ratio

(a) 2 : 1 (b) 1 : 2
(c) 1 : 8 (d) 1 : 4

(c) *(**c**)* Given that, $l_B = 2l_A$

and $\quad r_A = 2r_B$

Now, from $\quad R = \rho \frac{l}{A}$

$\Rightarrow \quad \frac{R_A}{R_B} = \frac{l_A}{l_B} \times \frac{r_B^2}{r_A^2}$

According to the question, $l_B = 2l_A$ and $r_A = 2r_B$.

$\therefore \quad \frac{R_A}{R_B} = \frac{l_A}{2l_A} \times \frac{r_B^2}{4r_B^2} = 1 : 8$

275. Consider the following statements:
A real image
1. can be formed on a screen
2. is always magnified and inverted
Which of the statements given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(c)*** A real image can be formed on a screen. It is always inverted. But it may be diminished or magnified.
Size of the image depends upon the distance of object and image from the optical system. If object distance is less and image distance is more we are getting magnified images. So, option (c) is the correct answer.

276. During short-circuiting, the current flowing in the electrical circuit
(a) reduces substantially
(b) does not change
(c) increases instantaneously
(d) varies continuously

› ***(a)*** A short-circuited path offers very low resistance or almost zero resistance to the flow of current. Therefore, current follows the short-circuited path and current through the mains electrical circuit reduces substantially.

277. Van de Graaff generator is used for
(a) accelerating charged particles
(b) generating large currents
(c) generating electric field
(d) generating high frequency voltage

› ***(d)*** Van de Graaff generator is used for generating high frequency voltage.
A Van de Graaff generator is an electrostatic generation which uses a moving belt to accumulate very high amounts of electrical potential on a hollow metal globe on the top of the stand. It was invented by American Physicist Robert J. Van de Graaff in 1929.

278. **Statement I** The acceleration due to gravity decreases with increase in height from the surface of the earth.
Statement II The acceleration due to gravity is inversely proportional to the square of the distance from the centre of the earth.
Codes
(a) Both the statements are individually true and Statement II is the correct explanation of Statement I
(b) Both the statements are individually true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

› ***(a)*** Acceleration due to gravity at a height h above the surface of the earth is $g_h = \frac{GM}{(R + h)^2}$.
where, symbols have their usual meanings.
Clearly, when h increases, g_h decreases. It is inversely proportional to the square of the distance from the centre of the earth that is $(R + h)^2$.
So, option (a) is the correct answer.

279. The pressure of an ideal gas undergoing isothermal change is increased by 10%. The volume of the gas must decrease by about
(a) 0.1% (b) 9% (c) 10% (d) 0.9%

› ***(b)*** Given that, $\Delta p = \frac{10}{100} \times p = 0.1p$

Now, $p' = (p + \Delta p) = 1.1p$
For isothermal change, $v'p' = vp$

$$\Rightarrow \quad v' = \frac{vp}{p'} = \frac{vp}{1.1p} = \frac{10v}{11}$$

$$\Delta v\,(\text{in}\%) = \left(\frac{v - v'}{v}\right) \times 100$$

$$= \left(1 - \frac{10}{11}\right) \times 100 = 9\%$$

280. A passenger in a moving train tosses a coin upward which falls behind him. It implies that the motion of the train is
(a) accelerated
(b) uniform
(c) retarded
(d) along the circular tracks

› ***(a)*** The tossed coin will have the same velocity as that of the passenger. If passenger moves with uniform velocity, then the coin will always be just above the passenger.
If passenger is accelerated, then the passenger will move forward and coin falls behind. Similarly, if passenger is in retarded motion coin will fall forward. When passenger moves along a circular track, the coin will move off the track tangentially, if tossed. Therefore, passenger is accelerated.

281. In optical instruments, the lenses are used to form image by the phenomenon of
(a) reflection
(b) refraction
(c) scattering
(d) diffusion

› ***(b)*** In optical instruments like microscope and telescope, two lenses are used. One of the lenses is called objective and the other is called eyepiece. Both these lenses form images by the phenomenon of refraction.

282. A semiconducting device is connected in a series circuit with a battery and a resistance. Current is found to pass through the circuit. If the polarity of the battery is reversed, then the current drops to zero. The device may be
(a) p-type semiconductor
(b) n-type semiconductor
(c) an intrinsic semiconductor
(d) p-n junction

› ***(d)*** Given characteristics of current through a semiconducting device is in accordance with a diode or p-n junction. In one polarity of the battery p-n junction is forward biased and current passes through the circuit. If the polarity of the battery is reversed, p-n junction becomes reverse biased and current drops to zero.

283. If the distance s covered by a moving car in rectilinear motion with a speed v in time t is given by $s = vt$, then the car undergoes
(a) a uniform acceleration
(b) a non-uniform acceleration
(c) a uniform velocity
(d) a non-uniform velocity

› ***(c)*** Let a car moves with a uniform speed v and covers a distance s in a time interval t.
The speed (v) and distance (s) are related as

$$v = \frac{\text{Distance covered}}{\text{Time taken}}$$

$$= \frac{s}{t}$$

$$\Rightarrow \quad s = vt$$

As car is in rectilinear motion and its speed is uniform, the rate of increase of velocity that is acceleration is zero.

284. The phenomenon of electromagnetic induction implies a production of induced
(a) resistance in a coil when the magnetic field changes with time
(b) current in a coil when an electric field changes with time
(c) current in a coil when a magnetic field changes with time
(d) voltage in a coil when an electric field changes with time

(c) *(**c**)* When magnetic field associated with a coil changes, magnetic flux associated with the coil also changes. This change of magnetic flux induces an emf in the coil which is called electromagnetic induction. This emf results a current through the coil.

285. If speed of light in air is 3×10^8 m/s, then the speed of light in glass (with refractive index 1.5) would be
(a) 2×10^8 m/s
(b) 4.5×10^8 m/s
(c) 3×10^8 m/s
(d) 1.5×10^8 m/s

(a) *(**a**)* Given that, $c = 3\times10^8$ m/s, $\mu = 1.5$

Then, the speed of light in glass,

$$v_g = \frac{c}{\mu}$$
$$= \frac{3\times10^8}{1.5}$$
$$= 2\times10^8 \text{ m/s}$$

286. While looking at an image formed by a convex lens (one half of the lens is covered with a black paper), which one of the following will happen to the image?
(a) Half of the image will be visible.
(b) Intensity of the image will be diminished.
(c) Image will be inverted now.
(d) One can see an image of smaller size.

(b) *(**b**)* When half of the lens is covered with a black paper. Light passes through remaining half part of the lens only. Therefore, intensity of the image will redue to half of the initial intensity, i.e. intensity of the image will be diminished. Position on size of the image remains unaffected.

287. A balloon filled up with gas would only go up in air it is filled up with
(a) a gas whose density is lower than air
(b) a gas whose density is higher than air
(c) cold air
(d) water vapour

(a) *(**a**)* The balloon will go up, if it is filled up with a gas whose density is lower than air. Cold air and water vapour are having densities higher than air.

Therefore weight of balloon will be more in comparison with buoyant force (lifting force) which causes the balloon to go down instead of rising up.

288. Planet A has double the radius than that of planet B. If the mass of planet A is 4 times heavier than the mass of planet B, then which of the following statements regarding weight of an object is correct?
(a) Heavier on planet A than on planet B.
(b) Heavier on planet B than on planet A.
(c) Same on both the planets.
(d) Cannot be measured on planet B.

(c) *(**c**)* Let mass of the object is m.
Weight of the object on planet A is

$$w_A = \frac{GM_A m}{R_A^2}$$

where, M_A and R_A are mass and radius of planet A, respectively.

Similarly, weight on planet B is

$$w_B = \frac{GM_B m}{R_B^2}$$

$$\therefore \quad \frac{w_A}{w_B} = \frac{M_A}{M_B}\times\left(\frac{R_B}{R_A}\right)^2 = 4\times\left(\frac{1}{2}\right)^2 = 1$$

$$\Rightarrow \quad w_A = w_B$$
$$(\because M_A = 4M_B \text{ and } R_A = 2R_B)$$

289. Sound waves are similar to the waves
(a) of laser light passing through air
(b) generated in a stretched wire by hitting or plucking the wire
(c) generated in a pipe filled with air by moving the piston attached to the pipe up and down
(d) generated by the mobile phone towers

(c) *(**c**)* Sound waves are similar to the waves generated in a pipe filled with air by moving the piston attached to the pipe up and down. Sound waves are longitudinal waves. Waves generated in the pipe filled with a piston are also longitudinal wave associated with laser light, stretched wire and mobile phones are of transverse nature.

290. A sound wave has frequency of 2 kHz and wavelength of 35 cm. If an observer is 1.4 km away from the source, then after what time interval could the observer hear the sound?
(a) 2 s (b) 20 s
(c) 0.5 s (d) 4 s

(a) *(**a**)* Given, $f = 2$ kHz $= 2\times10^3$ Hz

$\lambda = 35$ cm $= 35\times10^{-2}$ m

$d = 1.4$ km $= 1.4\times10^3$ m

$\therefore \quad v = f\lambda = 2\times10^3\times35\times10^{-2}$
$= 700$ m/s

$$t = \frac{d}{v} = \frac{1.4\times10^3}{700}$$
$$= 2\text{ s}$$

291. Which one among the following waves bats use to detect the obstacles in their flying path?
(a) Infrared waves
(b) Electromagnetic waves
(c) Ultrasonic waves
(d) Radio waves

(c) *(**c**)* Ultrasonic waves are used by bat to detect the obstacles in their flying path. It is an oscillating sound pressure wave with a frequency greater than the upper limit of the human hearing range.

292. A fuse is used in an electric circuit to
(a) break the circuit when excessive current flows through the circuit
(b) break the circuit when power gets off
(c) indicate if the current is flowing uninterrupted
(d) complete the circuit for flow of current

(a) *(**a**)* A fuse is a metal wire or strip of low resistance that melts when too much current flows through it. It is used in an electric circuit to break the circuit when excessive current flows through the circuit.

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CHEMISTRY

2019 (II)

1. Let there be an object having some chemicals in it. It starts moving with a uniform velocity v and a chemical reaction starts happening. In this case, which of the following statement(s) is/are correct?

1. Chemical reactions happening in the system cannot change the velocity (v) of the centre of mass of the object.
2. Chemical reactions happening in the system cannot change kinetic energy of the particles inside with respect to the centre of mass of object.

Select the correct answer using the code given below:

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) A chemical reaction is an internal process, so there is no change in momentum and mass (since, mass is conserved in a chemical reaction).
But in a chemical reaction, the kinetic energy of the particle is changes with respect to centre of mass of whole body.
Hence, statement 1 is correct statement 2 is incorrect.

2. Which one of the following compounds does not exhibit a different oxidation number of the same element?

(a) Pb_3O_4 (b) Fe_3O_4 (c) Fe_2O_3 (d) Mn_3O_4

(***c***) Among the given species/ compounds, only Fe_2O_3 does not exhibit different oxidation number of Fe, in this species can be calculated as :
Let x be the O.N. of Fe.

$\therefore \quad 2x + 3(-2) = 0$

$x = +3$

Hence, Fe is in only + 3 oxidation state in Fe_2O_3, whereas in rest of the species, the same metal exhibits different oxidation states.

3. Which one of the following statements about glass is not correct?

(a) Glass is often said to be a supercooled liquid.
(b) Glass has no definite melting point.
(c) Soda glass is harder than pyrex glass.
(d) Boron is present in pyrex glass.

(***c***) Pyrex glass is harder than soda lime glass. This is because, pyrex glass, (a particular blend of borosilicate glass) has a higher proportion of silicone dioxide than soda lime glass. Due to which, pyrex glass does not expand on heating, while soda lime glass is not likely to be directly and strongly heated.
Thus, the high heating tolerance of pyrex glass makes it harder than soda lime glass.
Hence, option (c) is incorrect.
Rest of the given statements are correct.

4. Which one of the following statements is correct?

(a) Both boiling and evaporation are surface phenomena.
(b) Boiling is a surface phenomenon, but evaporation is a bulk phenomenon.
(c) Both boiling and evaporation are bulk phenomena.
(d) Boiling is a bulk phenomenon, but evaporation is a surface phenomenon.

(***d***) Boiling is a bulk phenomenon, but evaporation is a surface phenomenon is the correct statement. This is because, during evaporation, the high energy molecules from the liquid go into the gaseous phase only through the surface.
While, in boiling, the vaporisation occurs throughout the bulk of the liquid and the vapours expand freely into the surrounding.

5. Which one of the following statements about diamond and graphite is not correct?

(a) Diamond has a tetrahedral structure, whereas graphite has a hexagonal planar structure.
(b) Both physical and chemical properties of diamond and graphite are different.
(c) Graphite is soft, but diamond is hard.
(d) Graphite is a good conductor of electricity, while diamond is not.

(***b***) Statement given in option (b) is incorrect. The correct statement is, the physical properties of both diamond and graphite are same. But, as the both diamond and graphite are allotropes of same element (carbon).
Thus, they both exhibits similar chemical properties.
So, option (b) is the correct answer.

6. The raw materials used for the manufacture of Portland cement are

(a) lime, silica and sulphur dioxide
(b) lime, silica and carbon dioxide
(c) lime, silica and alumina
(d) lime, silica and boric acid

(***c***) The raw materials used for the manufacture of Portland cement are lime, clay (provides alumina and silica) and gypsum.
Thus, the correct option is (c).

7. Which one of the following statements about water is not true?

(a) Hydrogen bonds are present in liquid water.
(b) Water has a high boiling point.
(c) Water has a high heat of fusion.
(d) Water is a non-polar molecule.

(***d***) The option (d) contains incorrect statement. Water is a polar molecule in which central oxygen atom is sp^3-hybridised.

Out of four sp^3-hybrid orbit, two for σ bond with H-atom and rest two are occupied by lone pair of electrons.

Thus, due to V-shaped structure of H_2O is a polar molecule (μ = 1840).

8. A sample of 'soft soap' contains
(a) caesium (b) potassium
(c) calcium (d) magnesium

(***b***) Soaps are the sodium or potassium salts of long chain fatty acid (*R*COONa). e.g. stearic acid ($C_{17}H_{35}COOH$), oleic acid ($C_{17}H_{33}COOH$) and palmitic acid ($C_{15}H_{31}COOH$).

Therefore, a sample of soft soap contains sodium or potassium.

Thus, the option (b) is correct.

9. Which one of the following does not represent the salt, calcium carbonate?
(a) Lime water (b) Limestone
(c) Chalk (d) Marble

(***a***) Calcium carbonate is also known as limestone and has a molecular formula is $CaCO_3$. It is found in nature in the form of chalk, marble etc. Thus, limestone. chalk and marble represents calcium carbonate.

Whereas, lime water is the aqueous solution of $Ca(OH)_2$.

Thus, the correct option is (a).

10. Which one of the following substances is not a mixture?
(a) Tin (b) Sea water
(c) Soil (d) Air

(***a***) A mixture is a type of matter which has variable composition throughout.

Among the given options, only Sn(tin) is not a mixture, while rest are the examples of mixture. Sn is an element which contains only one kind of particles or atoms.

11. Which one of the following is termed as 'dry ice'?
(a) Ice present in ice-cream
(b) Solid water at Antarctica
(c) Solid state of carbon dioxide
(d) Solid water of ionosphere

(***c***) When liquefied CO_2 is allowed to expand rapidly, it gets converted in the form of solid CO_2 which is also called dry ice. Thus, dry ice is solid state of the carbon dioxide.

12. Net movement of water from a dilute to a concentrated solution through a selectively permeable membrane is called
(a) Diffusion (b) Dispersion
(c) Osmosis (d) Absorption

(***c***) The net movement of water from a dilute to concentrated solution through a selectively permeable membrane is called osmosis. This term is used specifically to refers to the diffusions of water across semi or selectively permeable membrane.

Water moves from a region of higher concentration to a region of lower concentration until an equilibrium is reached.

13. Which one of the following statements regarding cathode rays is not correct?
(a) Cathode ray particles are electrons.
(b) Cathode ray particles start from anode and move towards cathode.
(c) In the absence of electrical and magnetic fields, cathode rays travel in straight lines.
(d) Television picture tubes are cathode ray tubes.

(***b***) Cathode rays start from cathode and move towards anode, because they are made of negatively charged electrons. Hence, option (b) is incorrect statement.

14. A very large volume of hydrogen can be accommodated by making
(a) non-metallic hydrides
(b) hydrogen peroxide
(c) non-stoichiometric hydrides
(d) alkali metal hydrides

(***c***) A very large volume of hydrogen can be accommodated by making non-stoichiometric hydrides.
Such hydrides are formed by some of the metals (e.g. Pd, Pt).

This property of hydrogen has high potential for hydrogen storage .

15. Which one of the following is not a monoatomic element?
(a) Copper (b) Helium
(c) Iodine (d) Barium

(***c***) Among the given species, only iodine is not a monoatomic element. It exists in the form of I_2 (diatomic), while rest occur as monoatomic element.

16. Which one of the following represents the correct order of electron releasing tendency of metals?
(a) Zn > Cu > Ag (b) Ag > Cu > Zn
(c) Cu > Zn > Ag (d) Cu > Ag > Zn

(***a***) The correct order of electron releasing tendency of given metals is

Zn> Cu > Ag.

This can be explained on the basis of standard reduction potentials of given elements. More negative be the E°_{red} value, greater is the tendency of element to release electron.

Zn has highly negative E°_{red} value, while Cu has least negative E°_{red} value.

Thus, the correct order is

Zn > Cu > Ag

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17. Which one of the following energy is stored in the links between the atoms?
(a) Nuclear energy
(b) Chemical energy
(c) Potential energy
(d) Thermal energy

(***b***) **Chemical energy** is stored in the bonds of atoms and molecules.
Nuclear energy is stored in the nucleus of an atom.
Potential energy is the energy of position or gravitational potential energy.
Thermal energy or heat is the vibration or movement of atoms and molecules in substances.

18. Which one of the following minerals is used as a fuel in nuclear power stations?
(a) Bauxite (b) Quartz
(c) Feldspar (d) Pitch-blende

(***d***) Pitch-blende is a radioactive uranium rich mineral which is used as a fuel in nuclear power stations.

It has a chemical composition that is largely UO_2, but also contains UO_3 and oxides of lead, thorium and rare earth elements.

19. Which one of the following is not a synthetic detergent?
(a) $CH_3(CH_2)_{10}CH_2OSO_3^-Na^+$
(b) $[CH_3(CH_2)_{15}—N—(CH_3)_3]^+Br^-$
(c) $CH_3(CH_2)_{16}COO^-Na^+$
(d) $CH_3(CH_2)_{16}COO(CH_2CH_2O)_n \cdot CH_2CH_2OH$

(***d***) Synthetic detergents are chemically alkyl sulphate or sulphonate or ammonium salt of long chain fatty acids. Thus, $CH_3(CH_2)_{16}COO(CH_2CH_2O)_n \cdot CH_2CH_2OH$, option (d) is not a synthetic detergent, while the remaining options contain synthetic detergents.

20. Which one of the following is an example of a clean fuel?
(a) Coke (b) Propane
(c) Petrol (d) Wax

(***b***) Clean fuels are fuels that have a lower carbon intensity than the standard for the fuel it replaces. Examples of clean fuels include ethanol, biodiesel, natural gas, biogas, propane and hydrogen.

21. Which one of the following metals does not react with cold water?
(a) Calcium (Ca) (b) Potassium (K)
(c) Magnesium (Mg) (d) Sodium (Na)

(c) *(**c**)* Magnesium (Mg) does not react with cold water, but reacts with hot water,
$Mg + 2H_2O \longrightarrow Mg(OH)_2 + H_2\uparrow$
$Mg + H_2O \longrightarrow MgO + H_2\uparrow$
On the other hand, calcium (Ca), potassium (K) and sodium (Na) are react with cold water.
Thus, option (c) is correct.

22. In which of the following pairs are the isoelectronic ions?
(a) Mg^{2+}, Ar (b) Na^+, O^{2-}
(c) Al^{3+}, Cl^- (d) K^+, Ne

(b) *(**b**)* According to question,

Ion	Atomic Number	Number of Electrons
Mg^{2+}	12	12 – 2 = 10
Ar	18	18
Na^+	11	11 – 1 = 10
O^{2-}	8	8 + 2 = 10
Al^{3+}	13	13 – 3 = 10
Cl^-	17	17 + 1 = 18
K^+	19	19 – 1 = 18
Ne	10	10

The species containing same number of electrons are isoelectronic. Thus, Na^+ and O^{2-} are isoelectronic species.
Hence, option (b) is correct.

23. Which one of the following is used as a binder in paints?
(a) Titanium dioxide (b) Novolac
(c) Phthalocyanine (d) Silicones

(d) *(**d**)* Binder refers to the substances that hold the particles of pigment together in paints.
Silicones are used as a binder in paints. They are synthetic organo-silicon polymers containing repeated R_2SiO units.

24. Basic scientific principle behind a nuclear reactor is
(a) nuclear fusion
(b) controlled nuclear fusion
(c) uncontrolled nuclear fission
(d) controlled nuclear fission

(d) *(**d**)* Basic scientific principle behind a nuclear reactor is controlled nuclear fission, in which a heavy atomic nucleus splits into two smaller nuclei and a large amount of energy is released in this process.

25. Which one of the following could be the melting point of iron?
(a) 25°C (b) 37°C
(c) 500°C (d) 1500°C

(d) *(**d**)* Iron (Fe) is a chemical element with atomic number 26. It is a metal that belongs to the first transition series and group 8 of the periodic table.
Its melting point is 1538°C (~1500°C) or 1811 K.

26. Which one of the following statements is not correct for the given reaction?
$Fe(s) + CuSO_4(aq) \longrightarrow FeSO_4(aq) + Cu(s)$
(a) Iron is the reducing agent
(b) The solution turns green in colour after the reaction
(c) Copper is a more reactive metal than iron
(d) The reaction is an example of a redox reaction

(c) *(**c**)* A more reactive metal displaces a less reactive metal from its solution.
$Fe(s) + CuSO_4(aq) \longrightarrow FeSO_4(aq) + Cu(s)$
In the above given reaction, iron (Fe) displaces copper (Cu) from its solution ($CuSO_4$), thus copper is less reactive than iron.
Therefore, option (c) is incorrect, while the remaining options are correct.

27. Which one of the following is an organic acid?
(a) Hydrochloric acid
(b) Nitric acid
(c) Acetic acid
(d) Sulphuric acid

(c) *(**c**)* An organic acid is an organic compound with acidic properties. The most common organic acids are the carboxylic acids. Among the given options, acetic acid (CH_3COOH) is an organic acid. The remaining options contains inorganic acids.

28. Dinitrogen (N_2) and dioxygen (O_2) are the main constituents of air, but they do not react with each other to form oxides of nitrogen because,
(a) the reaction requires initiation by a catalyst
(b) oxides of nitrogen are unstable
(c) the reaction is endothermic and requires very high temperature
(d) the stoichiometry of N_2 and O_2 in air is not ideal for the reaction to take place

(c) *(**c**)* The reaction of dinitrogen (N_2) and dioxygen (O_2) is endothermic and requires very high temperature.
$N_2 + O_2 \longrightarrow 2NO$
It is because, the nitrogen-nitrogen triple bond (N ≡ N) is very strong.
The energy cost to break that bond as well as the O = O is not compensated by the formation of nitrogen-oxygen (N = O) double bond.
Thus, they do not react with each other.

29. The equivalent weight of oxalic acid in $C_2H_2O_4 \cdot 2H_2O$ is
(a) 45 (b) 63
(c) 90 (d) 126

(b) *(**b**)* Equivalent weight of an acid
$= \frac{\text{Molecular weight}}{\text{Basicity}}$
Molecular weight of $C_2H_2O_4 \cdot 2H_2O$
$= 2\times12 + 2\times1 + 4\times16 + 2\times18 = 126$
$\therefore$ Equivalent weight of $C_2H_2O_4 \cdot 2H_2O$
$= \frac{126}{2} = 63$

30. On exposure to moist air, copper gains a green coat on its surface due to formation of which one of the following compounds?
(a) Copper carbonate
(b) Copper oxide
(c) Copper sulphate
(d) Copper nitrate

(a) *(**a**)* On exposure to moist air, copper gains a green coat on its surface due to formation of copper carbonate.
$\underset{\text{Copper}}{2Cu\,(s)} + \underbrace{CO_2(g) + O_2(g) + 2H_2O(l)}_{\text{from moist air}} \longrightarrow \underset{\text{Basic copper carbonate (green coating)}}{2CuCO_3{\cdot}Cu(OH)_2}$

31. Which one of the following will not produce carbon dioxide on reacting with an aqueous solution of hydrochloric acid?
(a) Limestone (b) Quicklime
(c) Chalk (d) Marble

(b) *(**b**)* Calcium carbonate occurs in nature in several form like limestone,chalk, marble, etc. It reacts with aqueous solution of hydrochloric acid (HCl) to liberate carbon dioxide.
$CaCO_3 + 2HCl \longrightarrow CaCl_2 + H_2O + CO_2\uparrow$
Quicklime on reaction with HCl does not liberate CO_2.
$\underset{\text{Quicklime}}{CaO} + 2HCl \longrightarrow CaCl_2 + H_2O$

32. Which one of the following substances is not a mixture?
(a) Ice (b) Ice-cream
(c) Air (d) Honey

(a) *(**a**)* Mixtures are obtained by mixing two or more substances in any proportions. Thus, among the given options, ice is not a mixture as it contains only water (liquid).
The other substances, i.e. ice-cream, air and honey are mixtures.

33. Tooth enamel is made up of which one of the following calcium compounds?
(a) Calcium carbonate
(b) Calcium sulphate
(c) Calcium hydroxide
(d) Calcium phosphate

(d) Enamel is the hardest substance in the human body. It contains the highest percentage of minerals, 96% with water and organic material composing the rest. The primary mineral present in enamel is hydroxyapatite which is a crystalline calcium phosphate.

2018 (II)

34. The atomic number of an element is 8. How many electrons will it gain to form a compound with sodium?
(a) One (b) Two (c) Three (d) Four

(b) The atomic number of an element = 8

Electronic configuration = $\begin{matrix} K & L \\ 2 & 6 \end{matrix}$

$\therefore$ Valency = 8 – 6 = 2
Sodium (atomic number 11) has electronic configuration

$= \begin{matrix} K & L & M \\ 2 & 8 & 1 \end{matrix}$

$\therefore$ Valency = 1
Thus, the given element will gain 2 electrons from two sodium atoms to complete its octet.

Formula of the compound,

= Na O
1 2

$= Na_2O$

35. A sample of oxygen contains two isotopes of oxygen with masses 16u and 18u respectively. The proportion of these isotopes in the sample is 3 : 1. What will be the average atomic mass of oxygen in this sample?
(a) 17.5 u (b) 17 u (c) 16 u (d) 16.5 u

(d) Average atomic mass of an element

$$= \frac{\text{Atomic mass of isotope I} \times \text{\% age of isotope I}}{100} + \frac{\text{Atomic mass of isotope II} \times \text{\%age of isotope II}}{100}$$

As it is given that, the proportion of two isotopes of oxygen with masses 16 u and 18 u is 3 : 1 respectively.

So, they have percentage composition 75% and 25% respectively.

$\therefore$ Average atomic mass of oxygen in the sample $= 16 \times \frac{75}{100} + 18 \times \frac{25}{100} = 16.5$ u

36. Which one of the following is a heterogeneous mixture?
(a) Hydrochloric acid
(b) Vinegar
(c) Milk
(d) Soda water

(c) A mixture that does not have uniform composition is called heterogeneous mixture.

Out of the given compounds, milk is a heterogeneous mixture of fats, carbohydrates, proteins etc. dispersed in water.

37. What is the formula mass of anhydrous sodium carbonate? (Given, that the atomic masses of sodium, carbon and oxygen are 23u, 12u and 16u respectively).
(a) 286 u (b) 106 u
(c) 83 u (d) 53 u

(b) Chemical formula of (anhydrous) sodium carbonate = Na_2CO_3

$\therefore$ Formula unit mass of Na_2CO_3
$= (2 \times 23 + 12 + 16 \times 3)$ u = 106 u

38. Which one of the following is called '*syn* gas'?
(a) $C(s) + H_2O(g)$
(b) $CO(g) + H_2O(g)$
(c) $CO(g) + H_2(g)$
(d) $NO_2(g) + H_2(g)$

(c) *Syn* gas or synthesis gas is a mixture of carbon monoxide (CO) gas and hydrogen (H_2) gas. It may also consist of carbon dioxide (CO_2) gas.

39. Which one of the following reactions will give NO (nitric oxide) gas as one of the products?
(a) $3Cu + 8HNO_3$ (dil.) $\longrightarrow$
(b) $Cu + 4HNO_3$ (conc.) $\longrightarrow$
(c) $4Zn + 10HNO_3$ (dil.) $\longrightarrow$
(d) $Zn + 4HNO_3$ (conc.) $\longrightarrow$

(a) (a) $\underset{\text{(dil.)}}{3Cu + 8HNO_3} \longrightarrow 3Cu(NO_3)_2 + 2NO\uparrow + 4H_2O$

(b) $\underset{\text{(conc.)}}{Cu + 4HNO_3} \longrightarrow Cu(NO_3)_2 + 2NO_2\uparrow + 2H_2O$

(c) $\underset{\text{(conc.)}}{Zn + 2HNO_3} \longrightarrow Zn(NO_3)_2 + H_2\uparrow$

(d) $\underset{\text{(dil.)}}{4Zn + 10HNO_3} \longrightarrow 4Zn(NO_3)_2 + N_2O\uparrow + 5H_2O$

Thus, in option (a) contains the reaction having NO (nitric oxide) gas as one of the products.

40. Which one of the following is a tribasic acid?
(a) Hydrochloric acid (b) Nitric acid
(c) Sulphuric acid
(d) Phosphoric acid

(d) H_3PO_4, phosphoric acid can be written as $O{=}P(OH)_3$. It has three acidic hydrogens (attached to oxygen) and is, therefore tribasic acid.
On the other hand, hydrochloric acid, nitric acid are monobasic, while sulphuric acid is dibasic.

41. Which one of the following statements is not correct?
(a) All carbons in diamond are linked by carbon-carbon single bond
(b) Graphite is layered structure in which layers are held together by weak van der Waals' forces
(c) Graphite layers are formed by hexagonal rings of carbon atoms
(d) Graphite layers are held together by carbon-carbon single bond

(d) Graphite layers are held together by van der Waals' forces and not by carbon-carbon single bond.
Hence, statement given in option (d) is incorrect.

42. Which one of the following solution will have pH less than 7?
(a) NaOH (b) KCl (c) $FeCl_3$ (d) NaCl

(c) $FeCl_3$ is a salt of strong acid (HCl) and weak base $Fe(OH)_3$. As a result, $FeCl_3$ is an acidic salt and hence in solution form its pH lies below 7.

43. Which one of the following is an oxidation-reduction reaction?
(a) $NaOH + HCl \longrightarrow NaCl + H_2O$
(b) $CaO + H_2O \longrightarrow Ca(OH)_2$
(c) $2Mg + O_2 \longrightarrow 2MgO$
(d) $Na_2SO_4 + BaCl_2 \longrightarrow BaSO_4 + 2NaCl$

(c) (a) $\overset{+1}{Na}\overset{-1}{OH} + \overset{+1\,-1}{HCl} \longrightarrow \overset{+1}{Na}\overset{-1}{Cl} + \overset{+1\,-2}{H_2O}$
(No change in oxidation states)
(b) $\overset{+2\,-2}{CaO} + \overset{+1\,-2}{H_2O} \longrightarrow \overset{+2\,+2\,-1}{Ca\,(OH)_2}$
(No change in oxidation states)
(c) $\overset{0}{2Mg} + \overset{0}{O_2} \longrightarrow 2\,\overset{+2\,-2}{MgO}$ (Mg: Oxidation; O: Reduction)
(d) $\overset{+1}{Na_2}\overset{-2}{SO_4} + \overset{+2\,-1}{BaCl_2} \longrightarrow \overset{+2}{Ba}\overset{-2}{SO_4} + 2\overset{+1}{Na}\overset{-1}{Cl}$

(No change in oxidation states)
$\therefore$ It is clear from the above equations, that in option (c), the reaction involves both oxidation and reduction.
Thus, it is a redox reaction.

44. Which one of the following is not used as fertilizer?
(a) Ammonium nitrate
(b) Ammonium sulphide
(c) Ammonium phosphate
(d) Ammonium sulphate

(b) Ammonium sulphide is not used as fertilizer. Fertilizer are commercially produced plants nutrients. It increase the fertility of soil by supplying nitrogen, phosphorus, potassium and other nutrients. These are used to ensure good vegetative growth giving rise to healthy plants.
There are following types of fertilizers– Nitrogenous fertilizers, phosphatic fertilizers and NP fertilizers.

45. Which one of the following is the chemical formula of gypsum?
(a) $CaSO_4.2H_2O$ (b) Ca_2SiO_4
(c) $2CaSO_4.H_2O$ (d) $CaSO_4$

(a) Gypsum has the chemical formula ($CaSO_4 \cdot 2H_2O$). It is a white crystalline solid which is associated with two molecules of water of crystallisation.
It is used in the production of plaster of Paris and in ammonium sulphate $(NH_4)_2SO_4$ fertilizer.

46. Which one of the following statements about the law of conservation of mass is correct?
(a) A given compound always contains exactly same proportion of elements.
(b) When gases combine in a reaction, they do so in a simple ratio by volume, provided all gases are at room temperature.
(c) Matter can neither be created nor be destroyed.
(d) Equal volumes of all gases at same temperature and pressure contain equal number of molecules.

(c) Law of conservation of mass states that, matter can neither be created nor be destroyed.
In a chemical reaction,
Total mass of reactants = total mass of products.

2018 (I)

47. Which of the following properties is true for a tooth paste?
(a) It is acidic
(b) It is neutral
(c) It is basic
(d) It is made up of calcium phosphate, the material of tooth enamel

(c) Tooth pastes are derived from different variety of components namely abrasives, fluoride and detergents. Abrasives includes particles of calcium hydrogen phosphates, aluminium hydroxide, etc. Tooth pastes are basic in nature and prevents tooth decay by neutralising the excess of acid.
They can neither be acidic nor normal.

48. Which one of the following gives the highest amount of hydrogen ions (H^+) ?
(a) Sodium hydroxide solution
(b) Milk of magnesia
(c) Lemon juice (d) Gastric juice

(c) Acidity is a measure of the concentration of hydrogen ions (H^+) present when a substance is dissolved in a liquid. Among the given options, lemon juice is the most acidic substance.
Thus, it has highest amount of hydrogen ions.

49. Brine is an aqueous solution of
(a) NaCl (b) NaOH
(c) $NaHCO_3$ (d) Na_2CO_3

(a) Brine is an aqueous solution of sodium chloride (NaCl). Electrolysis of brine produces sodium hydroxide (NaOH), chlorine (Cl_2) and hydrogen (H_2). This forms the basis of the chlor-alkali industries.

50. Which one of the following is the chemical formula of washing soda ?
(a) $NaHCO_3$ (b) $Na_2CO_3 \cdot 10H_2O$
(c) $Na_2CO_3 \cdot 5H_2O$ (d) NaOH

(b) Washing soda is sodium carbonate or salt of carbonic acid with chemical formula of $Na_2CO_3 \cdot 10H_2O$. Its anhydrous form, i.e. Na_2CO_3 is called soda ash.
It is a basic salt because when it dissolved in water, it gives a strong base NaOH.

51. Which one of the following is not true for bleaching powder?
(a) It is used as a reducing agent in chemical industries
(b) It is used for bleaching wood pulp in paper factories
(c) It is used for disinfecting drinking water
(d) It is used for bleaching linen in textile industry

(a) Bleaching powder ($CaOCl_2$) is produced by the action of chlorine on dry slaked lime [$Ca(OH)_2$]. The following are the uses of bleaching powder:
(i) It is used for bleaching purposes in textile industries, paper industry and in laundry.
(ii) It is also used as a disinfectant for water to make it free of germs.
(iii) It is used as an oxidising agent in many chemical industries.
Hence, statement given in option (a) is not true for bleaching powder.

52. Which one of the following is the number of water molecules that share with two formula unit of $CaSO_4$ in plaster of Paris?
(a) One (b) Two
(c) Five (d) Ten

(a) Plaster of Paris is chemically known as calcium sulphate hemihydrate $\left(CaSO_4 \cdot \frac{1}{2} H_2O\right)$.
Thus, with two formula unit of $CaSO_4$, one molecule of water is shared.
It is prepared by heating gypsum at 393 K.

$$\underset{\text{Gypsum}}{2(CaSO_4 \cdot 2H_2O)} \xrightarrow{393\ K} CaSO_4 \cdot \frac{1}{2}H_2O$$
Plaster of Paris

53. How is carbon black obtained?
(a) By heating wood at high temperature in absence of air
(b) By heating coal at high temperature in absence of air
(c) By burning hydrocarbons in a limited supply of air
(d) By heating coal at high temperature in presence of air

(c) Carbon black is obtained by burning hydrocarbons in a limited supply of air.

$$CH_4 + \underset{\text{Limited supply}}{O_2} \longrightarrow \underset{\text{Carbon black}}{C} + 2H_2O$$

54. Which one of the following properties is not true for graphite?
(a) Hybridisation of each carbon atom is sp^3
(b) Hybridisation of each carbon atom is sp^2
(c) Electrons are delocalised over the whole sheet of atoms
(d) Each layer is composed of hexagonal rings

(a) A graphite crystal consists of layers of carbon atoms or sheets of carbon atoms. Each carbon atom in a graphite layer is joined to other three carbon atoms by strong covalent bonds to form flat hexagonal rings. In this case, only 3 of the 4 valence electrons of C participate in bonding.
Thus, the hybridisation is sp^2 having covalent bonds with other 3C-atoms in same plane.
The electrons are delocalised over the whole sheet of atoms which makes it good conductor of electricity.
Thus, among the given options, option (a) is incorrect.

55. Which one of the following is the purest form of carbon?
(a) Charcoal (b) Coke
(c) Fullerene (d) Carbon black

(d) **Carbon black** (lamp black) is the most purest form of carbon. It contains about 98-99% of carbon. It is used for making printer ink, black paint, varnishes and carbon papers.

Charcoal is the impure form of carbon because of its porous structure and large surface area.

Coke contains 80-85% carbon. It is used as a fuel and as a reducing agent in metallurgy.

Fullerene is an allotrope of carbon whose molecules consists of carbon atoms connected by single and double bonds.

56. The poisonous nature of carbon monoxide (CO) is due to its
(a) insolubility in water
(b) ability to form a complex with haemoglobin
(c) ability to reduce some metal oxides
(d) property of having one σ-bond

(b) Carbon monoxide (CO) is highly poisonous to living beings because it has an ability to form more stable carboxyhaemoglobin. It is 300 times more stable than oxygen-haemoglobin complex.

In blood, when the concentration of carboxyhaemoglobin reaches about 3-4 percent. The oxygen carrying capacity of blood is greatly reduced. The oxygen deficiency results headache, weak eyesight, nervousness, etc.

57. Which one of the following alkali metals has lowest melting point?
(a) Sodium (b) Potassium
(c) Rubidium (d) Caesium

(d) The melting point of alkali metals decreases on moving down the group due to decrease in the extent of metallic bonding. Thus, the sequence of melting point among the given options is

$$Na > K > Rb > Cs$$

Therefore, caesium (Cs) has the least melting point.

58. Which one of the following metals is alloyed with sodium to transfer heat in a nuclear reactor?
(a) Potassium (b) Calcium
(c) Magnesium (d) Strontium

(a) Sodium-potassium alloy (NaK) is an alloy of two alkali metals-sodium and potassium. It is used as a heat-transfer. Coolant in fast-breeder nuclear reactors and experimentally in gas-turbine power plants.

2017 (II)

59. Which one of the following is a cause of acid rains?
(a) Ozone
(b) Ammonia
(c) Sulphur dioxide
(d) Carbon monoxide

(c) Acid rain is caused by a variety of human activities that emit the oxides of sulphur and nitrogen in the atmosphere. These substances can rise very high into the atmosphere, where they mix and react with water, oxygen and other chemicals to form more acidic pollutants, known as acid rain.

60. The desirable range of pH for drinking water is
(a) 6.5 to 8.5 (b) 5.0 to 6.5
(c) 6.5 to 7.0 (d) 7.0 to 8.5

(c) The pH of pure water is 7. In general, water with a pH lower than 7 is considered acidic and with a pH greater than 7 is considered basic. The normal range for pH in surface water systems is 6.5 to 8.5 and the pH range for groundwater systems is between 6 to 8.5.

61. Consider the following reaction,

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O.$$

Which of the following about the reaction given above is/are correct?
1. Carbon is oxidised
2. Hydrogen is oxidised
3. Hydrogen is reduced
4. Carbon is reduced

Select the correct answer using the code given below :
(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) 2 and 4

(a) According to the following reaction,

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

it is a combustion reaction of methane. In this kind of reaction, carbon dioxide and water are always the products.

Here, methane gets oxidised and oxygen gas gets reduced.

Hence, the reaction is oxidation-reduction type of reaction in which methane is reducing agent and oxygen gas get reduced.

62. Water boils at a lower temperature at high altitudes, because
(a) the air pressure is less
(b) outside temperature is less
(c) latent heat is less
(d) None of the above

(a) The boiling point of water is lower at higher altitudes due to the decreased atmospheric pressure. This may cause an increase in cooking time.

63. How much CO_2 is produced on heating of 1 kg of carbon ?
(a) $\frac{11}{3}$ kg (b) $\frac{3}{11}$ kg (c) $\frac{4}{3}$ kg (d) $\frac{3}{4}$ kg

(a) $C + O_2 \longrightarrow CO_2$

12 g of C on heating produces = 44 g of CO_2

$\frac{12}{1000}$ kg of C produces = $\frac{44}{1000}$ kg of CO_2

∴ 1 kg of C produces

$$= \frac{44}{1000} \times \frac{1000}{12} \text{ kg of } CO_2$$

$$= \frac{11}{3} \text{ kg}$$

64. Zinc is used to protect iron from corrosion because zinc is
(a) more electropositive than iron
(b) cheaper than iron
(c) a bluish white metal
(d) a good conductor of heat and electricity

(a) Zinc metal is used to protect iron from corrosion because zinc is more electropositive than iron.

The process of coating iron land steel objects with a thin layer of zinc is called galvanisation.

Zinc metal present on the surface of iron forms a thin protective layer of basic zinc carbonate due to the reaction between zinc, oxygen, CO_2 and moisture in air.

Since, zinc is more electropositive than iron.

65. Which one of the following gases is placed second in respect of abundance in the Earth's atmosphere?
(a) Oxygen (b) Hydrogen
(c) Nitrogen (d) Carbon dioxide

(a) The permanent gases whose percentages do not change from day to day are nitrogen, oxygen and argon.

Nitrogen accounts for 78% of the atmosphere, oxygen 21% and argon 0.9%: Gases like carbon dioxide, nitrous oxides, methane and ozone are trace gases that account for about a tenth of one percent of the atmosphere.

66. Which one of the following is a chemical change?
(a) Cutting of hair
(b) Graying of hair naturally
(c) Swelling of resin in water
(d) Cutting of fruit

› *(**b**)* A chemical change (chemical reaction) is a change of materials into other, new materials with different properties, and one or more new substances are formed.
These process are, in general are not reversible except by further chemical reaction. Hence, graying of hair is the correct answer as that cannot be reversed.

67. Which one among the following chemicals is used as washing soda?
(a) Calcium carbonate
(b) Calcium bicarbonate
(c) Sodium carbonate
(d) Sodium bicarbonate

› *(**c**)* Sodium carbonate (Na_2CO_3) is used as washing and cleansing purpose in houses. That's why, it is called as washing soda.

68. Why is potassium permanganate used for purifying drinking water?
(a) It kills germs
(b) It dissolves the impurities
(c) It is a reducting agent
(d) It is an oxidising agent

› *(**d**)* Potassium permanganate ($KMnO_4$) is a strong oxidising agent and is used for purifying water and treat stinking wells.

69. The principal use of hydrofluoric acid is
(a) in etching glass
(b) as a bleaching agent
(c) as an extremely strong oxidising agent
(d) in the preparation of strong organic fluorine compounds

› *(**a**)* The principal use of hydrofluoric acid [HF] is in etching glass. This technique is used for creating art on the glass surfaces.

70. The species that has the same number of electrons as, $^{35}_{17}Cl$ is
(a) $^{32}_{16}S$ (b) $^{34}_{16}S^+$ (c) $^{40}_{18}Ar^+$ (d) $^{35}_{16}S^{2-}$

› *(**c**)* Number of electrons = Number of protons = Atomic number.

$^{A}_{Z}X$ ← Mass number, Symbol; Z ↑ Atomic number

$^{35}_{17}Cl \rightarrow$ Number of electrons = 17.

$^{32}_{16}S \rightarrow$ Number of electrons = 16.

$^{34}_{16}S^+ \rightarrow$ Number of electrons
$= 16 - 1 = 15$

$^{40}_{18}Ar^+ \rightarrow$ Number of electrons
$= 18 - 1 = 17$

$^{35}_{16}S^{2-} \rightarrow$ Number of electrons
$= 16 + 2 = 18$

$\therefore$ $^{35}_{17}Cl$ and $^{40}_{18}Ar^+$ has the same number of electrons.

71. The compound $C_6H_{12}O_4$ contains
(a) 22 atoms per mole
(b) twice the mass percent of H as compared to the mass percent of C
(c) six times the mass percent of C as compared to the mass percent of H
(d) thrice the mass percent of H as compared to the mass percent of O

› *(**c**)* Molar mass of C in $C_6H_{12}O_4$
$= 6 \times 12 = 72$
Molar mass of H in $C_6H_{12}O_4$
$= 1 \times 12 = 12$
$\therefore$ Mass percent of C is six times as compared to the mass percent of H.

72. The ionisation energy of hydrogen atom in the ground state is
(a) 13.6 MeV (b) 13.6 eV
(c) 13.6 Joule (d) Zero

› *(**b**)* For a hydrogen atom, composed of an orbiting electron bound to a nucleus of one proton, an ionisation energy of 2.18×10^{-18}J (13.6 eV) is required to force the electron from its lowest energy level entirely out of the atom.

73. When pure water boils vigorously, the bubbles that rise to the surface are composed primarily of
(a) air
(b) hydrogen
(c) hydrogen and oxygen
(d) water vapour

› *(**d**)* When pure water boils vigorously, the bubble that rise to the surface are composed primarily of water vapour.

74. Which compound, when dissolved in water, conducts electricity and forms a basic solution?
(a) HCl
(b) CH_3COOH
(c) CH_3OH
(d) NaOH

› *(**d**)* NaOH, when dissolved in water, conducts electricity and forms a basic solution. Strong electrolyte conducts electricity in solution.

2017 (I)

75. Molecules of which of the following has cage like structure?
1. Diamond 2. Graphite
3. Fullerenes
Select the correct answer using the code given below.
(a) All of these (b) 2 and 3
(c) Only 2 (d) Only 3

› *(**d**)* Fullerenes C_{60} has cage like structure. It is made by heating graphite in an electric arc in the presence of inert gases such as helium or argon.
It is the only pure form of carbon.

76. Temporary hardness in water is due to which one of the following of calcium and magnesium?
(a) Hydrogen carbonates
(b) Carbonates
(c) Chlorides
(d) Sulphates

› *(**b**)* Temporary hardness or carbonate hardness is due to the presence of soluble calcium hydrogen carbonate $Ca(HCO_3)_2$ and magnesium hydrogen carbonate $Mg(HCO_3)_2$.
It can be removed by boiling and Clark's method.

77. Which one of the following elements is least reactive with water?
(a) Lithium (b) Sodium
(c) Potassium (d) Cesium

› *(**a**)* Among given elements, lithium is least reactive with water. Lithium density is only about half of water.

78. Rutherford's alpha-particle scattering experiment was responsible for the discovery of
(a) electron (b) proton
(c) nucleus (d) helium

› *(**c**)* Rutherford's alpha-particle scattering experiment was responsible for the discovery of atomic nucleus.
He proposed that, there is a positively charged spherical centre in an atom, called the nucleus. Nearly, all the mass of an atom resides in the nucleus.

79. Glass is a
(a) liquid (b) colloid
(c) non-crystalline amorphous solid
(d) crystalline solid

› *(**c**)* Glass is a homogeneous mixture of silicates of various alkaline metals of non-crystalline amorphous solid. The ordinary glass has approximately the composition of $Na_2O \cdot CaO \cdot 6SiO_2$.

80. Which one of the following statements is not correct?
(a) In the conduction mode of transference of heat, the molecules of solid pass heat from one molecule to another without moving from their positions.
(b) The amount of heat required to raise the temperature of a substance is called its specific heat capacity.
(c) The process of heat transfer in liquids and gases is through convection mode.
(d) The process of heat transfer from a body at higher temperature to a body at lower temperature without heating the space between them is known as radiation.

*(**b**)* The amount of heat required to raise the temperature of a substance is called its specific heat capacity is not correct, because specific heat capacity represents the amount of heat required to raise the temperature of 1 kg of a body by 1° C. Hence, statement given in option (b) is not correct.

81. The amount of heat required to change a liquid to gaseous state without any change in temperature is known as
(a) specific heat capacity
(b) mechanical equivalent of heat
(c) latent heat of vaporisation
(d) quenching

*(**c**)* The amount of heat required to change a liquid to gaseous state without any change in temperature is known as latent heat of vaporisation.

82. A homogeneous mixture contains two liquids. How are they separated?
(a) By filtration (b) By evaporation
(c) By distillation (d) By condensation

*(**c**)* Homogeneous mixture containing two liquids can be separated by distillation method. It is based on the difference in the boiling points of the liquids in the given mixture.

83. Which one of the following elements forms highest number of compounds?
(a) Oxygen (b) Hydrogen
(c) Chlorine (d) Carbon

*(**d**)* Among the given elements, carbon has maximum number of compounds. The estimated number of carbon compounds known today is about three millions.

84. Which one of the following elements corrodes rapidly?
(a) Aluminium (b) Iron
(c) Zinc (d) Silver

*(**a**)* Among the given elements, iron corrodes rapidly. The corrosion of aluminium starts rapidly first but a passive layer is formed after sometime due to which rate of corrosion decreases. Hence, the rate of corrosion is highest in iron.

85. 20 g of common salt is dissolved in 180 g of water. What is the mass percentage of the salt in the solution?
(a) 5% (b) 9% (c) 10% (d) 15%

*(**c**)* Mass of solute (salt) = 20 g
Mass of solvent (water) = 180 g
We know that,
Mass of solution =
Mass of solute + Mass of solvent
= 20 g + 180 g = 200 g
Mass percentage of salt in the solution

$$= \frac{\text{Mass of solute}}{\text{Mass of solution}} \times 100$$

$$= \frac{20}{200} \times 100 = 10\%$$

86. The valency of an element depends upon the
(a) total number of protons in an atom
(b) mass number of an atom
(c) total number of neutrons in an atom
(d) total number of electrons in the outermost shell of an atom

*(**d**)* Valency is the combining capacity of an atom. It depends upon the total number of electrons in the outermost shell of an atom. An outermost shell which has eight electrons is said to posses an octet.

87. Match List I with List II and select the correct answer using the code given below the Lists.

List I (Noble gas)	**List II** (Use)
A. Argon	1. In lights for advertising display
B. Neon	2. Airport landing lights and in light houses
C. Krypton	3. Light in photographer's flash gun
D. Xenon	4. In tungsten filament to last longer

Codes

	A	B	C	D
(a)	3	1	2	4
(b)	3	2	1	4
(c)	4	2	1	3
(d)	4	1	2	3

*(**d**)* The correct matching is A-4, B-1, C-2, D-3.
- **Argon** is used for filling in tungsten filament to last longer.
- **Neon** is used in fluorescent bulbs for advertisement display purposes.
- **Krypton** is used as airport landing lights and in light houses.
- **Xenon** is used in high intensity photographic flash tubes.

88. Why is it difficult to measure the coefficient of expansion of a liquid than solid?
(a) Liquids tend to evaporate at all temperatures
(b) Liquids conduct more heat
(c) Liquids expand too much when heated
(d) Their containers also expand when heated

*(**d**)* Liquids can be heated only by putting them in a container. So, when heat is provided to the liquid, the container also expands.

89. Radon is
(a) an inert gas
(b) an artificial fibre
(c) an explosive
(d) a metal

*(**a**)* Radon is an inert gas. It is placed in group 18 of the periodic table. It is called inert gas because of it is chemically inert in nature.

90. Which one of the following elements is used in pencil-lead?
(a) Zinc (b) Lead
(c) Carbon (Graphite) (d) Tin

*(**c**)* The allotrope of carbon, i.e. graphite is used for marking on the paper in the form of lead pencil. It is also known as the black lead.

2016 (II)

91. Match List I with List II and select the correct answer using the codes given below the Lists.

List I (Process)	**List II** (Type of change)
A. Heating camphor	1. Chemical
B. Cooling of water vapour upto room temperature	2. Evaporation
C. Cooking an egg	3. Condensation
D. Formation of water vapour at room temperature	4. Sublimation

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	1	3	4
(d)	2	3	1	4

(a) The correct matching is A-4, B-3, C-1, D-2.

- **Cooking an egg** is a chemical change because a substance with new chemical composition is formed, which is not converted into its initial state by process.
- **Evaporation** is the process of conversion of substance from liquid state to vapour state at any temperature below its boiling point is called evaporation.
- **Condensation** is the process of change of state of a substance from its gaseous state to liquid state at a particular temperature, is called condensation.
- **Sublimation** is the process of conversion of a substance from solid state to vapour state without going through liquid state is called sublimation. Heating camphor follows sublimation.

92. Identify the element having zero valency.

(a) Sulphur (b) Phosphorus
(c) Lead (d) Radon

(d) Valency of inert gases is zero because they have completely filled valence shell and do not combine with other elements to form compounds, i.e. their octet is completely filled.

Radon is an example of zero valency among given options.

93. There are six electrons, six protons and six neutrons in an atom of an element. What is the atomic number of the element?

(a) 6 (b) 12 (c) 18 (d) 24

(a) The number of protons or electrons of an atom is called atomic number. Here, number of protons and number of electrons are six each.

Hence, atomic number of element is 6.

94. Which one of the following has different number of molecules? (All are kept at normal temperature and pressure)

(a) 3 g of hydrogen (b) 48 g of oxygen
(c) 42 g of nitrogen (d) 2 g of carbon

(d) The mass of 1 mole of a substance is equal to its relative atomic or molecular mass in grams.

1 mole = 6.022×10^{23}

1g hydrogen = 6.022×10^{23} atoms

3g hydrogen = $3 \times 6.022 \times 10^{23}$ atoms

$= \frac{3}{2} \times 6.022 \times 10^{23}$ molecules

(Since, hydrogen is diatomic molecule.)

16 g oxygen = 6.022×10^{23} atoms

48 g oxygen = $3 \times 6.022 \times 10^{23}$ atoms

$= \frac{3}{2} \times 6.022 \times 10^{23}$ molecules

(As oxygen is diatomic molecule.)

12 g carbon = 6.022×10^{23} atoms

2 g carbon = $\frac{1}{6} \times 6.022 \times 10^{23}$ atoms

$= \frac{1}{6} \times 6.022 \times 10^{23}$ molecules

(As carbon is monoatomic molecules.)

Hence, 2 g of carbon contains different number of molecules.

95. Which one of the following statements is correct?

(a) Covalent bonds are directional
(b) Ionic bonds are directional
(c) Both covalent and ionic bonds are directional
(d) Both covalent and ionic bonds are non-directional

(a) Covalent bond is formed by sharing of electrons between atoms by overlapping of the atomic orbitals of participant atoms and usually only few patterns of overlap are possible.

Consequently, only few spatial arrangement of atom are possible. These limitations determine the direction of bonds.

96. When one strikes a safety match, the first step is

(a) burning of sulphur
(b) decomposition of potassium chlorate into potassium chloride and oxygen
(c) conversion of a small amount of red phosphorus into white phosphorus
(d) burning of glue and starch

(c) Head of matchstick of safety match is made up of antimony trisulphide and potassium chlorate, while frictionable surface is made of red phosphorus. When matchstick is rubbed against the striking surfaces, the friction generates enough heat to convert a trace of the red phosphorus into white phosphorus.

97. The LPG cooking gas contains propane and butane as the constituents. A sulphur containing compound is added to the LPG, because

(a) it lowers the cost of production
(b) it enhances the efficiency of LPG
(c) it facilitates easy detection of leakage of the gas
(d) it assists in liquefying hydrocarbons

(c) LPG gas is basically propane and butane, and it is odourless in its natural state. LPG is very dangerous, because if it leaks, it can cause terrible fires and explosions. To avoid this, ethyl mercaptan (CH_3CH_2SH) or ethanethiol ($CH_3\ CH_2\ SH$) is added to the gas, which possesses that strong odour of cabbages. The smell helps us to detect, when there is a leak, which actually makes a lot of sense from a safety and security perspective.

98. A plant having yellow leaves with dead spots has the deficiency of

(a) potassium
(b) magnesium
(c) nitrate
(d) phosphate

(a) A plant having yellow leaves with dead spots, has the deficiency of potassium.

Magnesium is the part of chlorophyll in all green plants, so lack of Mg^{2+} causes chlorosis and tissue damaged, while lack of **phosphate** causes leaves dull and causes curled disease.

Lack of **nitrate** causes green or white purple and red spots of anthocyanin, chlorosis with necrosis, first in older leaves or premature abscission, delayed flowering and fruiting.

99. Fluoride toothpaste is sometimes recommended, because fluoride

(a) prevents plaque formation
(b) hardens the enamel of the tooth
(c) kills pathogenic bacteria
(d) prevents toothache

(a) Fluoride toothpaste is sometimes recommended because fluoride prevents plaque formation and it promotes new bone formation. It is used for strengthening of bone. Fluoride is added to public drinking water to prevent tooth decay.

100. Combination of one volume of nitrogen with three volumes of hydrogen produces

(a) one volume of ammonia
(b) two volumes of ammonia
(c) three volumes of ammonia
(d) one and a half volumes of ammonia

(b) The law of combining volume (for gases) states that, when gases react with each other, they follow, Gay-Lussac's law.

The volume of reactants and products in a large number of chemical reactions are related to each other by small integers, provided the volumes are measured at the same temperature and pressure.

As per balance equation,

$$\underset{(1\ \text{volume})}{N_2} + \underset{(3\ \text{volume})}{3H_2} \xrightarrow[(200\ \text{atm})]{\text{Fe/Mo}\ (450\text{-}500^\circ C)} \underset{(2\ \text{volume})}{2NH_3} + \text{Heat}$$

101. The setting time of cement is lowered by adding
(a) oxides of aluminium
(b) gypsum
(c) oxides of magnesium
(d) silica

(*b*) Gypsum is often added to Portland cement to prevent early hardening or ''flash setting, allowing a longer working time.'' It shows down the setting of cement, so that cement is adequately hardened.

102. Emulsion is known as a
(a) colloidal solution of substances having different physical states
(b) true solution
(c) distillation mixture for making alcohols
(d) colloidal solution of two liquids

(*d*) Liquid-liquid colloidal systems in which finely divided droplets of a liquid are dispersed into other liquid, are called emulsions. These are formed by shaking the two immiscible liquids with each other.

103. If one mixes up ashes with animal fat, the substance received in the crude form is called
(a) pheromone (b) soap
(c) cement (d) concrete

(*b*) Metal salts (e.g. Na^+, K^+, Ca^{2+}) are one of the constituents of ash. When ashes are mixed up with animal fat, the salt of fatty acid is obtained, which is called **soap**.

104. Which one of the following gases give acidic solution on dissolving in water ?
(a) Hydrogen (b) Carbon dioxide
(c) Nitrogen (d) Oxygen

(*b*) Carbonic acid is a weak acid that causes a slight drop in pH (makes the solution more acidic). When CO_2 dissolves in water, it forms carbonic acid (H_2CO_3). This compound has a higher K_a value $\sim 10^{-3}$ than water. This means that, the H_2CO_3 is more likely to dissociate and produce H^+ ions in solution.

105. How many moles of hydrogen atom are present in one mole of aluminium hydroxide?
(a) One mole (b) Two moles
(c) Three moles (d) Four moles

(*c*) As molar formula of aluminium hydroxide is $Al(OH)_3$. Therefore, each mole of $Al(OH)_3$ contains three moles of hydrogen atom.

106. Which one of the following species is not capable of showing disproportionation reaction?
(a) ClO^- (b) ClO_2^- (c) ClO_3^- (d) ClO_4^-

(*d*) ClO_4^- is the perchlorate ion, where chlorine avails at its maximum oxidation stage i.e. +7, therefore it cannot be reoxidise. Disproportionation is a specific type of redox reaction in which an element from a reaction undergoes both oxidation and reduction to form different products.

2016 (I)

107. Which one of the following oxides dissolve in water ?
(a) CuO (b) Al_2O_3 (c) Fe_2O_3 (d) Na_2O

(*d*) Sodium oxide (Na_2O) is the base anhydride of sodium hydroxide, when water is added to it, NaOH is produced.

$$Na_2O + H_2O \longrightarrow 2NaOH$$

It is used in ceramics and glasses, though not in a raw form and rest of all are not dissolved in water.
CuO, Al_2O_3,Fe_2O_3 are water insoluble.

108. Matter around us can exist in three different states namely, solid, liquid and gas. Correct order of their compressibility is
(a) liquid < gas < solid
(b) solid < liquid < gas
(c) gas < liquid < solid
(d) solid < gas < liquid

(*b*) The correct order of compressibility is solid < liquid < gas.
Gases are highly compressible as compared to solids and liquids. In gas, the molecules have enough kinetic energy so that the effect of intermolecular forces is small and the typical distance between neighbouring molecules is much greater than the molecular size.

109. The temperature at which a solid melts to become a liquid at the atmospheric pressure is called its melting point. The melting point of a solid is an indication of
(a) strength of the intermolecular forces of attraction
(b) strength of the intermolecular forces of repulsion
(c) molecular mass
(d) molecular size

(*a*) The melting point of a solid is an indication of strength of intermolecular force of attraction. On increasing the temperature of solids, the kinetic energy of the particles increases. Due to the increase in kinetic energy, the particles start vibrating with greater speed.
The energy supplied by the heat overcomes the forces of attraction between the particles. A stage is reached when solid melts and is converted into a liquid.

110. Which one of the following elements will not react with dilute HCl to produce H_2?
(a) Hg (b) Al (c) Mg (d) Fe

(*a*) Hg will not react with dilute HCl to produce H_2 because in reactivity series, Hg is less reactive than the given, i.e. the hydrogen which has a stronger attraction to the Cl than the mercury.

111. When a solid is heated, it turns directly into a gas. This process is called
(a) condensation (b) evaporation
(c) sublimation (d) diffusion

(*c*) Sublimation is the change of state from solid state directly to gaseous state without going through liquid state and *vice-versa.*

112. Which one of the following is water gas?
(a) Mixture of carbon monoxide and hydrogen
(b) Mixture of carbon monoxide and nitrogen
(c) Mixture of carbon dioxide and water vapour
(d) Mixture of carbon monoxide and water vapour

(*a*) Water gas is a mixture of carbon monoxide (CO) and hydrogen (H_2) in which nitrogen and water vapour is in the form of impurity. The gas is made by passing steam over a red hot carbon fuel such as coke.

113. A glass vessel is filled with water up to the brin and a lid is fixed to it tightly. Then it is kept inside a freezer for hours. What is expected to happen?
(a) The water freezes to ice and the level of ice comes down
(b) The water in the glass vessel summer freezes to ice
(c) The glass vessel breaks due to expansion as water freezes to ice
(d) The water does not freeze at all

(*a*) Water shows anomalous behaviour in the expansion during temperature range of 4°C to 0°C. When the temperature falls below 4°C, water expands. As lid is fixed tightly, the stress will develop in the bottle due to this anomalous expansion which breaks the glass when water is converted into ice i.e., volume is maximum.

114. Suppose you have four test tubes labelled *A, B, C* and *D*. *A* contains water, *B* contains solution of an alkali, *C* contains solution of an acid, and *D* contains solution of sodium

chloride. Which one of these solutions will turn phenolphthalein solution pink ?
(a) Solution *A* (b) Solution *B*
(c) Solution *C* (d) Solution *D*

› ***(b)*** Solution *B*, i.e. solution of an alkali will turn phenolphthalein solution pink because phenolphthalein is a weak organic acid. It turns colourless in acidic solution and pink in basic solution. It is a synthetic indicator used in acid-base titrations.

115. An atom of carbon has 6 protons. Its mass number is 12. How many neutrons are present in an atom of carbon?
(a) 12 (b) 6
(c) 10 (d) 14

› ***(b)*** We know that,
Mass number (A) = number of proton (p) + number of neutron (n)
$12 = 6 + n$
⇒ Hence, number of neutrons present in atom of carbon is $n = 6$

116. Which one of the following is a reduction reaction?
(a) $2Mg(s) + O_2(g) \longrightarrow 2MgO(s)$
(b) $S(s) + O_2(g) \longrightarrow SO_2(g)$
(c) $2HgO(s) \xrightarrow{Heat} 2Hg(l) + O_2(g)$
(d) $Mg(s) + S(s) \longrightarrow MgS(s)$

› ***(c)*** $2HgO(s) \xrightarrow{Heat} 2Hg(l) + O_2(g)$
Above reaction is a reduction reaction. In this reaction, HgO decomposes on exposure to light or on heating above 500°C. Heating produces highly toxic mercury fumes and oxygen.

117. Which one of the following oxides of nitrogen is known as 'anhydride' of nitric acid?
(a) N_2O (b) N_2O_3
(c) NO_2 (d) N_2O_5

› ***(d)*** The anhydride of HNO_3 is N_2O_5, nitrogen pentaoxide and N_2O_3 is the anhydride of HNO_2, nitrous acid.
The anhydride of nitric acid is the acidic oxide which on hydration forms the acid. N_2O_5 dissolves in water to give nitric acid.
$N_2O_5 + H_2O \longrightarrow 2HNO_3$
Conversely, HNO_3 on dehydration with P_2O_5 yields its anhydride, N_2O_5,
$2HNO_3 + P_2O_5 \longrightarrow N_2O_5 + 2HPO_3$

118. Which one of the following is the chemical name for baking soda?
(a) Sodium bicarbonate (sodium hydrogen carbonate)
(b) Sodium carbonate
(c) Potassium bicarbonate (potassium hydrogen carbonate)
(d) Potassium carbonate

› ***(a)*** The chemical name of baking soda is sodium bicarbonate (sodium hydrogen carbonate, ($NaHCO_3$). It is produced using sodium chloride as one of the raw materials.
$NaCl + H_2O + CO_2 + NH_3 \longrightarrow$
$\underset{\text{(Sodium hydrogen carbonate) Baking soda}}{NaHCO_3} + \underset{\text{(Ammonium chloride)}}{NH_4Cl}$

Directions (Q. No. 119) *The following question consist of two statements,* ***Statement*** I *and* ***Statement*** II. *You have to examine these two statements carefully and select the answers to the question using the codes given below.*
Codes
(a) Both the statements are individually true and Statement II is the correct explanation of Statement I
(b) Both the statements are individually true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

119. Statements
I. Petroleum is a mixture of many different hydrocarbons of different densities.
II. The grade of petroleum depends mainly on the relative proportion of the different hydrocarbons.

› ***(b)*** Petroleum is a mixture of a very large number of different hydrocarbons; the most common are alkanes, cycloalkanes, aromatic hydrocarbons etc. The relative percentage of hydrocarbons varies and is responsible for different grades of petroleum.

120. Density of water is
(a) maximum at 0°C
(b) minimum at 0°C
(c) maximum at 4°C
(d) minimum at – 4°C

› ***(c)*** Water shows anomalous behaviour between temperatures 0°C and 4°C expands it temperature goes below 4°C.
Therefore, density decreases below 4°C. Also, it expands when temperature goes above 4°C. Therefore, density of water is maximum at 4°C because volume is minimum at 4°C.

121. The phosphorus used in the manufacture of safety matches is
(a) red phosphorus
(b) black phosphorus
(c) white phosphorus
(d) scarlet phosphorus

› ***(a)*** Red phosphorus is used in the manufacture of safety matches. Red phosphorus exists as an amorphous network. Red phosphorus does not ignite in air at temperature below 240°C.

122. Which one of the following is not a chemical change?
(a) Ripening of fruits
(b) Curdling of milk
(c) Freezing of water
(d) Digestion of food

› ***(c)*** Freezing of water is a physical change, i.e. not a chemical change because it is the interconversion of states, the physical property changes but the chemical composition remains the same.
Remaining all are examples of chemical change.

123. Soap is the sodium or potassium salt of
(a) stearic acid
(b) oleic acid
(c) palmitic acid
(d) All of the above

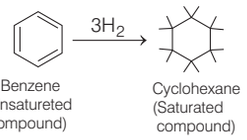
› ***(d)*** The mineral salts of higher fatty acids such as oleic acid ($C_{17}H_{33}COOH$), stearic acid ($C_{17}H_{35}COOH$) and palmitic acid ($C_{15}H_{31}COOH$) etc. are called soaps. Out of these, only sodium and potassium salts of fatty acids being water soluble, are widely used for cleaning purposes.

124. Which of the following carbon compounds will not give a sooty flame?
(a) Benzene
(b) Hexane
(c) Naphthalene
(d) Anthracene

› ***(b)*** Hexane (C_6H_{14}) will not give a sooty flame. Hexane is a significant constituent of gasoline. The darker or sootier the smoke, the more unsaturated the compound. On the other hand, if the flame is burning relatively clearer and clearer then the compound is saturated. Hexane burns with a clear and luminous flame.

125. What is the number of mole(s) of $H_2(g)$ required to saturate one mole of benzene ?
(a) 1 (b) 2
(c) 3 (d) 4

(c)



3 moles of $H_2(g)$ is required to saturate one mole of benzene.

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126. The symbol of the element 'tungsten' is
(a) Ta (b) W
(c) Tl (d) Tc

(b) W is the symbol for element 'tungsten'.
Ta is the symbol for element 'tantalum'.
Tl is the symbol for element 'thallium'.
Tc is the symbol for element 'technetium'.

127. Which one of the following statements is correct?
(a) Rutherford's α-particle scattering experiment led to the discovery of electron
(b) J.J. Thomson suggested that the nucleus of an atom contains protons
(c) The atomic number of an element is the same as the number of protons in the nucleus of its atom
(d) The mass number of an atom is equal to the number of electrons in its shells

(c) Atomic number of an element is equal to the number of protons in the nucleus of its atom. Hence, it is the correct statement.
In other options,
In 1911, Ernest Rutherford's α-scattering experiment led to the discovery of nucleus in an atom.
J.J. Thomson's cathode ray tube experiment led to the discovery of electrons.
Mass number of an atom is equal to the sum of total number of neutrons and protons in the nucleus.

128. The alkali metals have relatively low melting point. Which one of the following alkali metals is expected to have the highest melting point?
(a) Li (b) Na
(c) K (d) Rb

(a) On moving from Li to Cs, size of atom increases, so interatomic interaction becomes weak resulting in low melting point. Among alkali metals, Li has the highest melting point.

129. Consider the following reaction,
$CH_4(g) + H_2O(g) \xrightarrow{1270\ K} CO(g) + 3H_2(g)$
In the reaction given above, the mixture of CO and H_2 is
(a) natural gas (b) water gas
(c) producer gas (d) industrial gas

(b) According to question,
$CH_4(g) + H_2O(g) \longrightarrow CO(g) + 3H_2(g)$
The mixture of CO and H_2 is called **water gas**. This mixture of CO and H_2 is used for the synthesis of methanol and a number of hydrocarbons.
Natural gas is a naturally occurring hydrocarbon gas mixture consists of methane and varying amount of higher alkenes.
Producer gas, also called suction gas, is a fuel gas made from coke, anthracite or other carbonaceous fuel.
Industrial gases are a group of gases that are specifically manufactured for use in industries.

130. Addition of gypsum to cement
(a) reduces setting time of cement
(b) produces very light colour of cement
(c) increases setting time of cement
(d) shining surface is formed

(c) Gypsum is a mineral and is hydrated calcium sulphate in chemical form. It plays an important role in controlling the rate of hardening of cement. It is added to control the 'setting of cement'. If not added, the cement will set immediately after mixing of water leaving no time for concrete placing.

131. To weld metals together, high temperature is required. Such a high temperature is obtained by burning.
(a) acetylene in oxygen
(b) LPG in oxygen
(c) methane in oxygen
(d) acetylene in nitrogen

(a) To weld metals together, high temperature is obtained by burning acetylene in oxygen.
Pure oxygen is used to increase the flame temperature to allow localised melting of the work piece material.
An acetylene/oxygen flame burns at about 3,773 K.

132. Ammonia (NH_3) obtained from different sources always has same proportion of nitrogen and hydrogen. It proves the validity the law of
(a) reciprocal proportion
(b) constant proportion
(c) multiple proportions
(d) None of the above

(b) The law of definite or constant proportion was given by Joseph Proust. It states that, irrespective of source, a given compound always contains exactly the same elements in the same proportion by weight.
Hence, it follows law of constant proportion, not multiple and reciprocal proportion laws.

133. Boric acid is an acid because its molecule
(a) accepts OH^- from water releasing proton
(b) combines with proton from water molecule
(c) contains replaceable H^+ ion
(d) gives up a proton

(a) Boric acid is a weak monobasic acid. It is not a protonic acid, but acts as a Lewis acid by abstracting OH^- from water.
$B(OH)_3 + 2H_2O \rightleftharpoons B(OH)_4^- + H_3O^+$

134. The main constituent of vinegar is
(a) acetic acid
(b) ascorbic acid
(c) citric acid
(d) tartaric acid

(a) The main constituent of vinegar is acetic acid (CH_3COOH). Commercially, it is produced by fast or slow fermentation process which involves yeast (fungus).
Citric acid is present in lemons or oranges. Tartaric acid is present in tamarind. Ascorbic acid is present in all citrus fruits.

135. White phosphorus glows in the dark due to
(a) amorphous character
(b) slow oxidation
(c) high ignition temperature
(d) good conducting property of electricity

(b) The slow oxidation is actually a reaction of white phosphorus with oxygen (at some partial pressure) forming short lined molecules such as HPO and P_2O_2 that on stabilising emits visible light.
White phosphorus (P_4) + Oxygen (O_2) ⟶ [Short-lined intermediates] ⟶ [Final product] (HPO, P_2O_2, etc.) Light-energy (Glow)

136. Which of the following are the properties of an electron?
1. Electron is a constituent of cathode ray.
2. Electron is a negatively charged particle.

3. The mass of the electron is equal to the mass of the proton.
4. Electron is deflected by the electric field but not by magnetic field.

Select the correct answer using the codes given below.

(a) 1 and 2 (b) 1, 2 and 3
(c) 3 and 4 (d) 1 and 4

› *(**a**)* J.J. Thomson discovered electron in cathode ray tube experiment so, it is considered as the constituent of cathode ray.

It is a negatively charged particle and is deflected by both electric and magnetic field.

137. Which of the following statements regarding heavy water are correct?

1. It is extensively used as a moderator in nuclear reactors.
2. It cannot be used in exchange reaction to study reaction mechanism.
3. Viscosity of heavy water is relatively smaller than that of ordinary water.
4. The dielectric constant of heavy water is smaller than that of ordinary water.

Select the correct answer using the codes given below.

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 4

› *(**d**)* Heavy water is used as a moderator in nuclear reactors and its dielectric constant is smaller than the ordinary water.
[Deuterium is heavier than hydrogen]

It is extensively used in exchange reactions for the study of reaction mechanism.

Its density is higher than water. Hence, its viscosity is also higher than ordinary water.

138. Match List I with List II and select the correct answer using the codes given below the Lists.

List I (Element)		List II (Use)
A.	Li	1. Time keeper in atomic clocks
B.	Na	2. Batteries
C.	K	3. Transfer of nerve impulses
D.	Cs	4. Control of the water content in the blood

Codes

	A	B	C	D
(a)	2	3	4	1
(b)	1	2	3	4
(c)	2	4	3	1
(d)	1	3	2	4

› *(**a**)* The correct matching is A-2, B-3, C-4, D-1.

- **Lithium (Li)** is used in batteries and is widely known as lithium batteries. These are disposable and can produce 1.5-3.7 V.
- **Sodium (Na)** is used in transfer of nerve impulse. Sodium ions move inside the membrane when a stimulus reaches a resting neuron and at complete depolarisation, action potential is created.
- **Potassium (K)** is available in fruits and vegetables; and sodium (Na) is available in salts. Eating salt raises amount of sodium which reduces the ability of kidney to remove water, whereas potassium helps kidney to work efficiently in retaining water.
- **Caesium (Cs)** atomic clocks are the most accurate time keepers. These are the devices that contain a 'pendulum' of atoms that are excited into resonance by microwave radiation. Definition of 'SI' unit of time is based on these clocks.

So, option (a) is the correct answer.

139. Graphite is a much better conductor of heat and electricity than diamond. This is due to the fact that each carbon atom in graphite

(a) undergoes sp^2 hybridisation and forms three sigma bonds with three neighbouring carbon atoms
(b) undergoes sp^3 hybridisation
(c) is tetrahedrally bonded
(d) is free from van der Waals' force

› *(**a**)* Graphite is a good conductor of heat and electricity because of the presence of one free electron on each carbon atom as each carbon have sp^2 hybridisation (forms three sigma bonds with three neighbouring carbon atoms).

2015 (I)

Directions (Q. Nos. 146-148) *The following three items consist of two statements, Statement* I *and Statement* II. *You have to examine these two statements carefully and select the answers to these items using the codes given below.*

Codes

(a) Both the Statements are individually true and Statement II is the correct explanation of Statement I
(b) Both the Statements are individually true but Statement II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

140. **Statement I** The granules of modern gunpowder (also called black powder) are typically coated with graphite.
Statement II Graphite prevents the build-up of electrostatic charge.

› *(**b**)* The granules of modern gunpowder (black powder) are coated with graphite as graphite dramatically reduced the moisture absorbing capacity of the powder. Graphite prevents the build-up of electrostatic charge. So, option (b) is the correct answer.

141. **Statement I** Colour of nitrogen dioxide changes to colourless at low temperature.
Statement II At low temperature, nitrogen tetraoxide (N_2O_4) is formed, which is colourless.

› *(**a**)* It is obvious from above statements that with fall in temperature, nitrogen dioxide (NO_2) molecules associate to give formation of colourless nitrogen tetraoxide (N_2O_4).

$$2NO_2 \rightleftharpoons N_2O_4$$

NO_2 is favoured at higher temperatures, while at lower temperatures, N_2O_4 predominates. So, option (a) is the correct answer.

142. **Statement I** Oxygen gas is easily produced at a faster rate by heating a mixture of potassium chlorate and manganese dioxide than heating potassium chlorate alone.
Statement II Manganese dioxide acts as a negative catalyst.

› *(**c**)* When a mixture of potassium chlorate and manganese dioxide is heated, more amount of oxygen (O_2) is produced because here manganese dioxide (MnO_2) acts as a positive catalyst and, thus increases the rate of reaction.

So, option (c) is the correct answer.

143. Match List I with List II and select the correct answer using the codes given below the Lists.

List I (Compound)	List II (Nature)
A. Sodium hydroxide	1. Strong acid
B. Calcium oxide	2. Alkali
C. Acetic acid	3. Weak acid
D. Hydrochloric acid	4. Base

Codes

	A	B	C	D		A	B	C	D
(a)	2	3	4	1	(b)	2	4	3	1
(c)	1	4	3	2	(d)	1	3	4	2

(b) The correct matching is A-2, B-4, C-3, D-1

- **Sodium hydroxide** (NaOH) is an inorganic compound. It is highly caustic metallic base and alkali salt of sodium.
- **Calcium oxide** (CaO) or quicklime is caustic alkaline solid at room temperature. A chemical derivative of calcium hydroxide of which quicklime is the base anhydride.
- **Acetic acid** (CH_3COOH) is a weak monoprotic acid.
- **Hydrochloric acid** (HCl) is a highly corrosive, strong mineral acid with many industrial uses.

So, option (b) is the correct answer.

144. All life forms contain 'molecules of life'. These are

(a) carbohydrates (b) proteins
(c) nucleic acids (d) All of the above

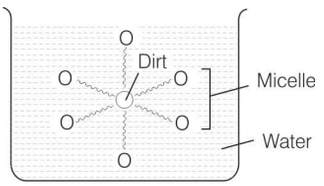
(d) All life forms are made up of carbon, hydrogen, oxygen, nitrogen, phosphorus and sulphur. Out of them, 97% composition consist of C, H, O.

A living cell contains inorganic materials (minerals and water) and organic materials like carbohydrates, lipids, proteins and nucleic acids.

145. The cleansing action of soap and detergent in water is due to the formation of

(a) micelle (b) salt
(c) base (d) acid

(a) Soap and detergent both have a water soluble polar end and fat soluble non-polar end. When it applied to a dirty cloth, their non-polar parts surround the dirt (fat or oil) and the polar part remains with water as shown below.



The structure is called micelle. When washed with plenty of water, it goes with water, leaving behind a clean cloth. Whereas, a salt formation occurs only when acid and base react with each other.

146. The chemical used as a fixer/developer in photography is

(a) sodium sulphate
(b) sodium sulphide
(c) sodium thiosulphate
(d) sodium sulphite

(c) Sodium thiosulphate ($Na_2S_2O_3$) has the capacity to dissolve insoluble silver halides, so it is used in photography for fixing, i.e. to remove unreacted silver bromide as shown below

$$AgBr + \underset{\text{Sodium thiosulphate}}{2Na_2S_2O_3} \longrightarrow Na_3[Ag(s_2O_3)_2] + NaBr$$

Sodium argento thiosulphate

147. Why is the graphite used in electrolytic cells?

(a) Graphite is soft and can be easily moulded into electrodes
(b) Graphite is made up of layers of carbon atoms which can be slide
(c) Graphite is inert to most of the chemicals and remains intact in electrolytic cells
(d) Graphite is a good conductor of electricity

(d) In graphite, each carbon atom is bonded to three other carbon atoms in the same plane giving a hexagonal array.

Atoms in the plane are bonded covalently, with only three of the four potential bonding sites satisfied.

The fourth electron is free to migrate in the plane, that makes graphite electrically conductive.
That's why, graphite is a good conductor of electricity.
Hence, it is used in electrolytic cells.

148. Washing soda is the common name for

(a) calcium carbonate
(b) magnesium carbonate
(c) sodium carbonate
(d) potassium carbonate

(c) Sodium carbonate ($Na_2CO_3 \cdot 10H_2O$) decahydrate, because of its cleansing property, also known by the name washing soda.

Common name for calcium carbonate ($CaCO_3$) is limestone, common name for magnesium carbonate ($MgCO_3$) is magnesia alba.

149. Which one among the following does not wet the walls of the glass vessel in which it is kept?

(a) Water (b) Alcohol
(c) Mercury (d) Phenol

(c) Mercury (Hg) is a heavy, silvery white metal. The cohesive force is stronger than the adhesive force. Hence, it does not stick to glass and as a result, it does not wet. Mercury is the only metallic element that is liquid at standard conditions of temperature and pressure.

150. Permanent hardness of water is due to the presence of

(a) sulphates of sodium and potassium
(b) sulphates of magnesium and calcium
(c) carbonates of sodium and magnesium
(d) bicarbonates of magnesium and calcium

(b) Chlorides and sulphates of magnesium (Mg) and calcium (Ca) are responsible for the permanent hardness of water, as hardness due to them cannot be removed by just boiling.

Note *Bicarbonates of Mg and Ca lead to temporary hardness which can be removed by boiling.*

151. In the reaction, $ZnO + C \longrightarrow Zn + CO$, 'C' acts as a/an

(a) acid (b) base
(c) oxidising agent (d) reducing agent

(d) Removal of oxygen (O) is reduction and its addition is oxidation. In the reaction, ZnO is reduced to Zn by C (i.e. C causes its reduction) so, C acts as a reducing agent.

152. Which one of the following are the characteristics of organic farming?

1. Use of chemical fertilizers to improve soil fertility.
2. Frequent decomposing and following.
3. Use of herbs to control pests.
4. Higher productivity per hectare.

Select the correct answer using the codes given below.

(a) 1 and 4 (b) 1, 3 and 4
(c) 2 and 3 (d) 2, 3 and 4

(d) Organic farming is a method of farming which primarily aimed at cultivating the land and raising crops in such a way, as to keep the soil alive and in good health by use of organic wastes (crop, animal and farm wastes, aquatic wastes) and other biological materials alongwith beneficial microbes (bio-fertilizers) to release nutrients to crops for increased sustainable production in an ecofriendly, pollution free environment.

Hence, statement I is incorrect.

153. Which one among the following is used in making gunpowder?

(a) Magnesium sulphate

(b) Potassium nitrate
(c) Sodium stearate
(d) Calcium sulphate

(b) Gun-powder is an intimate mixture of nitrate, i.e. potassium nitrate (6 parts), charcoal (1 part) and sulphur (1 part). Here, potassium nitrate (KNO_3) behaves as a good oxidising agent. Sulphur and charcoal burn vigorously in the molten state of KNO_3.

154. We use CFL to save electrical energy and to provide sufficient light. The full form of CFL is
(a) Condensed Filament Light
(b) Compact Filament Lamp
(c) Condensed Fluorescent Lamp
(d) Compact Fluorescent Lamp

(d) The full form of 'CFL' is Compact Fluorescent Lamp. It is a fluorescent lamp designed to replace an incandescent lamp.

155. When heat rays are reflected from the earth, gases like carbon dioxide, nitrous oxide do not allow them to escape back to the space causing our planet too heat up. These gases are known as
(a) noble gas (b) greenhouse gas
(c) hot gas (d) blue gas

(b) Greenhouse gases such as CO_2, methane, nitrous oxide trap the heat radiated from earth. This leads to an increase in earth's temperature. This heating up of earth and its objects due to the trapping of infrared radiation by greenhouse gases in the atmosphere. Greenhouse gases absorb and emit infrared radiation in the wavelength range emitted by earth.

2014 (II)

156. A sample of carbon dioxide that undergoes a transformation from solid to liquid and then to gas would undergo
(a) a change in mass
(b) a change in density
(c) a change in composition
(d) no change in physical properties

(b) A sample of carbon dioxide that undergoes a transformation from solid to liquid and then to gas would undergo a change in density.

$$D = \frac{M}{V}$$

During interconversion of the states, volume changes. Hence, density also changes.

157. Which of the following element combinations will form ionic compounds?
1. Ca ($Z = 20$) and Ti ($Z = 22$)
2. Si ($Z = 14$) and Br ($Z = 35$)
3. Mg ($Z = 12$) and Cl ($Z = 17$)
Select the correct answer using the codes given below.
(a) Only 2 (b) Only 3
(c) 2 and 3 (d) All of these

(b) An ionic bond is formed when one of the atom can donate electron to achieve the inert gas electronic configuration and the other atom needs electrons to achieve the inert gas electronic configuration.
Ionic bond is formed between metals and non-metals, i.e. Mg (metal) and Cl (non-metal) will form ionic compound but Ca (metal) and Ti (metal), Si(non-metal) and Br (non-metal) will not form ionic compound.

158. A thermodynamic process where no heat is exchanged with the surroundings is
(a) isothermal (b) adiabatic
(c) isobaric (d) isochoric

(b) In adiabatic process, there is no exchange of heat between the system and the surroundings. Whereas, in isothermal process, temperature remains constant. In isobaric process, change of state is brought at constant pressure and in isochoric process, volume of the system remains constant.

159. A compound X_2O_3 contains 31.58% oxygen by weight. The atomic mass of X is
(a) 34.66 g mol^{-1}
(b) 45.01 g mol^{-1}
(c) 52.00 g mol^{-1}
(d) 104.00 g mol^{-1}

(c) In compound X_2O_3,
Percentage of oxygen by weight = 31.58
Percentage of X by weight = 68.42
Let the atomic mass of $X = x$

$$\frac{2x}{2x + 48} \times 100 = 68.42$$

$$\therefore \quad x = 52$$

So, atomic mass of X is 52 g mol^{-1}.

160. Which one among the following contains the most neutrons?
(a) $^{59}_{26}Fe$ (b) $^{61}_{29}Cu$
(c) $^{61}_{30}Zn$ (d) $^{60}_{30}Zn^{2+}$

(a) Number of neutrons in an atom
= Mass number – atomic number
Number of neutrons in $^{59}_{26}Fe$ = 33
Number of neutrons in $^{61}_{29}Cu$ = 32
Number of neutrons in $^{61}_{30}Zn$ = 31
Number of neutrons in $^{60}_{30}Zn^{2+}$ = 30
So, $^{59}_{26}Fe$ contains the most neutrons.

161. Turpentine oil in paints is used as a
(a) pigment
(b) film-forming material
(c) thinner
(d) drier

(c) Turpentine oil in paints is used as a thinner. It act as a solvent. It can be used to thin oil paint to affect the flow of paint on to the canvas.

162. The burning sensation of a bee sting can be stopped by rubbing the affected area with soap. This is because
(a) a bee sting is acidic and soap, an alkali, neutralises it
(b) a bee sting is alkaline and soap, an acid, neutralises it
(c) soap cleans the affected area and removes the sting
(d) soap acts as an anaesthetic and dulls the sensation

(a) Bee sting contains formic acid which leads to high burning sensation when bites. By placing large amount of alkali near the sting site is unlike to produce a perfectly neutral pH to stop the burning sensation.

163. What is the oxidising agent in the following equation?

$$HAsO_2(aq) + Sn^{2+}(aq) + H^+(aq) \longrightarrow As(s) + Sn^{4+}(aq) + H_2O(l)$$

(a) $HAsO_2(aq)$
(b) $Sn^{2+}(aq)$
(c) $H^+(aq)$
(d) $Sn^{4+}(aq)$

(a) In the given reaction, $HAsO_2(aq)$ is the oxidising agent that helps in the oxidation of Sn^{2+} to Sn^{4+} and itself gets reduced from +3 to 0.

164. Which one of the following substances is most likely to be used as soap?
(a) $CH_3(CH_2)_{12}COOCH_3$
(b) $CH_3(CH_2)_5O(CH_2)_5CH_3$
(c) $CH_3(CH_2)_{12}COONa$
(d) $CH_3(CH_2)_{12}CHCl_2$

(c) $CH_3(CH_2)_{12}CO\overset{-}{O}\overset{+}{N}a$ is most likely to be used as soap. A soap is the sodium salt (or potassium salt) of a long chain carboxylic acid (or fatty acid).

165. A sample of gas is to be identified by means of its behaviour in the presence of a glowing splint. Which of the following gases will neither itself burn nor cause the splint to burn?
(a) Oxygen
(b) Nitrogen
(c) Hydrogen
(d) Methane

(b) Nitrogen is an inert gas neither itself burn nor cause the splint to burn. Oxygen does not burn itself but act as a supporter of combustion.
Hydrogen helps in producing flames, whereas methane helps in combustion and readily produces carbon dioxide.

166. Heat given to a body which raises its temperature by 1°C is known as
(a) water equivalent
(b) thermal capacity
(c) specific heat
(d) temperature gradient

(b) The heat capacity of a body is the quantity of heat required by the body to raise its temperature by 1°C. It is also termed as thermal capacity.

167. Consider the following reaction,
$x\mathrm{As_2S_3} + y\mathrm{O_2} \longrightarrow z\mathrm{As_2O_3} + w\mathrm{SO_2}$
What is y (the coefficient for $\mathrm{O_2}$) when this equation is balanced using whole number coefficients?
(a) 5 (b) 7
(c) 9 (d) 11

(c) Balanced chemical equation is
$2\,\mathrm{As_2S_3} + 9\mathrm{O_2} \longrightarrow 2\mathrm{As_2O_3} + 6\mathrm{SO_2}$
So, y is 9.

168. How many grams of $\mathrm{MgCO_3}$ contain 24.00 g of oxygen? (The molar mass of $\mathrm{MgCO_3}$ is 84.30 g $\mathrm{mol^{-1}}$)
(a) 42.15 g (b) 84.30 g
(c) 126.00 g (d) 154.00 g

(a) In $\mathrm{MgCO_3}$, there are 3 oxygen atoms in each mole.
Atomic mass of oxygen is 16 g/mol. So, there are
$(16 \times 3) = 48$ grams of oxygen in one mole of $\mathrm{MgCO_3}$
Molar mass of $\mathrm{MgCO_3} = 84.30$ g $\mathrm{mol^{-1}}$

$$\frac{48 \text{ g of O}}{84.3 \text{ MgCO}_3} = \frac{24 \text{ g O}}{x \text{ g MgCO}_3}$$

$$x = 42.15 \text{ g}$$

169. If the reaction of 1.0 mole $\mathrm{NH_3}(g)$ and 1.0 mole $\mathrm{O_2}(g)$
$4\mathrm{NH_3}(g) + 5\mathrm{O_2}(g) \longrightarrow 4\mathrm{NO}(g) + 6\mathrm{H_2O}(l)$ is carried to completion, then
(a) all the $\mathrm{O_2}(g)$ is consumed
(b) 4.0 moles of $\mathrm{NO}(g)$ is produced
(c) 1.5 moles of $\mathrm{H_2O}(l)$ is produced
(d) all the $\mathrm{NH_3}(g)$ is consumed

(a) $\underset{4 \text{ mol}}{4\mathrm{NH_3}(g)} + \underset{5 \text{ mol}}{5\mathrm{O_2}(g)} \longrightarrow 4\mathrm{NO}(g) + 6\mathrm{H_2O}(l)$
1 mole of $\mathrm{NH_3}$ requires $\frac{5}{4}$ moles of $\mathrm{O_2}$
$= 1.25$ mol
Since, for 1 mole of $\mathrm{NH_3}$, 1.25 moles of $\mathrm{O_2}$ are required, therefore, $\mathrm{O_2}$ is the limiting factor (here $\mathrm{O_2}$ is only 1 mole). Hence, all the $\mathrm{O_2}$ will be consumed in reacting with 1 mole of $\mathrm{NH_3}$.

170. The high heat of vaporisation of water is mainly a result of
(a) van der Waals' forces
(b) covalent bonds
(c) interionic attraction
(d) hydrogen bonding

(d) The heat of vaporisation is often quoted for the normal boiling temperature of the substance. In the molecule of liquid, water are held together by relatively strong hydrogen bonding. High value of heat of vaporisation shows the presence of stronger forces of attraction.

171. A monoatomic species that has 18 electrons and a net charge of 2– has
(a) the same number of electrons as a neutral argon atom
(b) more protons than electrons
(c) 2 unpaired electrons
(d) 20 protons

(a) A monoatomic species has the same number of electrons as a neutral argon atom.
The monoatomic species must be S^{2-}.

Monoatomic species	Number of electrons
Sulphur (S)	16
Sulphur ion (S^{2-})	16 + 2 = 18
Argon (Ar)	18

Hence, from above table, it is clear that S^{2-} have same number of electrons as neutral argon atom. So, a monospecies that has 18 electrons and net charge of 2– has the same number of electrons as a neutral argon atom.

2014 (I)

172. Which of the following pairs represents isoelectronic ions?
(a) Na^+, K^+ (b) K^+, Mg^{2+}
(c) Mg^{2+}, Ca^{2+} (d) Ca^{2+}, S^{2-}

(d) Isoelectronic ions are the ions that contain the same number of electrons.
Number of electrons in Na^+
$= 11 - 1 = 10$
Number of electrons in $K^+ = 19 - 1 = 18$
(Na^+, K^+) are not isoelectronic ions.
Number of electrons in Mg^{2+}
$= 12 - 2 = 10$
(K^+, Mg^{2+}) are not isoelectronic ions.
Number of electrons in Ca^{2+}
$= 20 - 2 = 18$
(Mg^{2+}, Ca^{2+}) are not isoelectronic ions.
Number of electrons in S^{2-}
$= 16 + 2 = 18$
(Ca^{2+}, S^{2-}) are isoelectronic ions.

173. Dihydrogen can be prepared on a commercial scale by the action of steam on hydrocarbons, when a mixture of CO and $\mathrm{H_2}$ gas is formed. It is known as
(a) water gas (b) producer gas
(c) industrial gas (d) fuel gas

(a) Water gas is a synthesis gas, containing CO and $\mathrm{H_2}$.
$\mathrm{H_2O} + \mathrm{C} \longrightarrow \mathrm{H_2} + \mathrm{CO}$
Producer gas is a fuel gas, i.e. manufactured from material such as coal.
$2\mathrm{C} + \mathrm{O_2} + 3.73\mathrm{N_2} \longrightarrow 2\mathrm{CO} + 3.73\,\mathrm{N_2}$

174. Which one of the following statements about hydrogen is/are correct?
1. Hydrogen has three isotopes of which protium is the most common.
2. Hydrogen ion (H^+) exists freely in the solution.
3. Dihydrogen (H_2) acts as a reducing agent.

Select the correct answer using the codes given below :
(a) Only 1 (b) 1 and 3
(c) Only 3 (d) All of these

(b) 1. Hydrogen has three isotopes; protium ($^1_1$H), deuterium ($^2_1$H or D) and tritium ($^3_1$H or T).
2. Hydrogen ion (H^+) does not exist freely in the solution. It combines with anion present in solution.

3. Dihydrogen (H_2) acts as a reducing agent and helps in the reduction.

Hence, statements 1 and 2 are correct and statement 3 is incorrect.

175. Which one of the following statements is correct?

(a) Fullerenes have only six-membered carbon rings
(b) Fullerenes are cage-like molecules
(c) Diamond is thermodynamically the most stable allotrope of carbon
(d) Graphite is slippery and hard and is therefore, used as a dry lubricant in machines

(b) *(b)* Fullerenes consist of 20 hexagonal and 12 pentagonal rings. They are the cage-like molecules.

Diamond is kinetically most stable allotrope of carbon.

Graphite is soft, smooth and slippery and is therefore, used as a dry lubricant in machines.

So, option (b) is the correct answer.

176. The number of valence electrons in the O^{2-} ion is

(a) 4 (b) 6 (c) 8 (d) 10

(c) *(c)* The number of valence electrons in the O^{2-} ion is 8.

O(atomic number) = 8 = $1s^2\ 2s^2\ 2p^4$

O^{2-} ion = $1s^2\ 2s^2\ 2p^6$

Outermost shell has valence electrons.

177. Which one of the following is the correct electronic configuration of chlorine?

(a) 2, 7, 8 (b) 2, 8, 7
(c) 2, 8, 8 (d) 7, 8, 9

(b) *(b)* We know that,
Atomic number of chlorine = 17
So, its electronic configuration = 2, 8, 7.

178. The number of aluminium ions present in 54 g of aluminium (atomic weight = 27) is

(a) 2 (b) 18
(c) 1.1×10^{24} (d) 1.2×10^{24}

(d) (*d*) Number of aluminium ions present in 54 g of aluminium

$$= \frac{6.023 \times 10^{23} \times 54}{27}$$

$$= 1.2 \times 10^{24}$$

179. The most stable form of carbon is

(a) diamond (b) graphite
(c) fullerene (d) coal

(a) *(a)* The most stable form of carbon is diamond because in its structure, directional covalent bonds are present throughout the lattice.

180. The latest discovered state of matter is

(a) solid
(b) Bose-Einstein condensate
(c) plasma
(d) liquid

(c) *(c)* There are four states of matter, which are given solid, liquid, gas and plasma. Plasma is the latest discovered state of matter. This state of matter exists at very high temperature.

181. The mass number of an atom is determined by

(a) adding the number of neutrons and number of electrons
(b) adding the number of protons and number of electrons
(c) the number of protons only
(d) adding the number of neutrons and number of protons

(d) *(d)* Mass number is defined as the sum of the number of protons and neutrons in an atom of the element. It is denoted by *A*.

$A = p + n$

where, p = number of protons
n = number of neutrons

182. Consider the following balanced chemical equation,

$$2CO + O_2 \rightleftharpoons 2CO_2$$

Which one of the following statements is significant in relation to the above chemical equation?

(a) One can add to a vessel only 2 moles of CO for each moles of O_2 is added
(b) No matter how much of these two reagents are added to a vessel, 1 mole of O_2 is consumed
(c) When they react, CO reacts with O_2 in a 2 : 1 mole ratio
(d) When 2 moles of CO and 1 mole of O_2 are placed in a vessel, they will react to give 1 mole of CO_2

(c) *(c)* According to the given reaction, 2 moles of carbon dioxide can form only when 2 moles of carbon monoxide reacts with 1 mole of oxygen. The formation of product always depends on the availability of reactant.

183. Which of the following acids is a mineral acid?

(a) Citric acid
(b) Hydrochloric acid
(c) Ascorbic acid
(d) Tartaric acid

(b) *(b)* A mineral acid (or inorganic acid) is an acid derived from one or more inorganic compounds. All mineral acids form hydrogen ions and the conjugate base ions when dissolved in water.

Commonly used mineral acids are sulphuric acid, hydrochloric acid and nitric acid. Citric acid, ascorbic acid and tartaric acid are organic acids.

184. Which one of the following is correct regarding the reaction of fluorine with water?

$$2F_2(g) + 2H_2O(l) \longrightarrow 4H^+(aq) + 4F^-(aq) + O_2(g)$$

(a) Fluorine is oxidised to F^-
(b) Water is oxidised to O_2
(c) Water is reduced to H^+
(d) Oxidation state of fluorine does not change

(b) *(b)* In this reaction, water is oxidised to O_2. Its oxidation state changes from –2 to 0.

185. Which one of the following is/are amphoteric?

(a) $Al(OH)_3(s)$ and $Fe(OH)_3(s)$
(b) $Al(OH)_3(s)$ and $HCO_3^-(aq)$
(c) $Ba(OH)_2(s)$ and $NaOH(aq)$
(d) Only $Al(OH)_3(s)$

(d) *(d)* Aluminium hydroxide [$Al(OH)_3$] is an amphoteric substance. It can acts as either a Bronsted-Lowry base or a Lewis acid.

$$Al(OH)_3 + 3H^+ \longrightarrow Al^{3+} + 3H_2O$$
$$Al(OH)_3 + OH^- \longrightarrow Al(OH)_4^-$$

186. The most of the mass of an atom can be found in

(a) electrons (b) charges
(c) nucleus (d) electron cloud

(c) *(c)* The atomic mass of an atom can be found in nucleus. For atoms, the protons and neutrons of the nucleus account for almost all of the mass. Atomic mass figures refer to an individual particle species.

187. A fertilizer contains 20% nitrogen by mass. To provide a fruit tree with an equivalent of 1 kg of nitrogen, the quantity of fertilizer required is

(a) 20 kg (b) 0.20 kg
(c) 0.05 kg (d) 5 kg

(d) (*d*) Mass per cent

$$= \frac{\text{Mass of nitrogen}}{\text{Mass of fertilizer}} \times 100$$

$$\Rightarrow \quad 20 = \frac{1}{x} \times 100$$

$$\Rightarrow \quad x = \frac{100}{20} = 5 \text{ kg}$$

Required quantity of fertilizer = 5 kg

43
BIOLOGY

2019 (II)

1. Blood is a type of

(a) epithelial tissue (b) muscular tissue
(c) nervous tissue (d) connective tissue

(d) Blood is a type of connective tissue, which forms 30-32% of total extracellular fluid. The connective tissues have a special function of linking and supporting other tissues/organs of the body.

Blood is a fluid, which helps in the transportation of various substances in the body. It is red because it contains a red pigment called haemoglobin in its red blood cells.

2. Mitochondria are able to produce their own

(a) nucleus (b) proteins
(c) chloroplasts (d) digestive enzymes

(b) Mitochondria are able to produce their own protein. It acts as the cell's power producer and also called as powerhouse of the cell.

It possesses single circular DNA molecules, a few RNA molecules, ribosomes and the components required for the synthesis of proteins. It provides energy for energy requiring processes of the cell like muscle contraction, nerve impulse, conduction, etc. It may store and release calcium when required.

3. Which one of the following statements is correct?

(a) Xylem consists of tracheids, vessels, xylem parenchyma and xylem fibres
(b) Flexibility in plants is due to sclerenchyma
(c) Parenchyma have no intercellular spaces
(d) Xylem consists of sieve plate, sieve tube and companion cells

(a) The statement given in option (a) is correct. Rest of the statements are incorrect and can be corrected as

(b) Flexibility in plants is due to the permanent tissue collenchyma. Sclerenchyma provides mechanical support to organs.

(c) Parenchyma cells are thin walled with small intercellular spaces.

(d) Phloem consists of sieve plates, sieve tube and companion cells.

4. Who among the following popularised the use of embryological characters in taxonomy?

(a) Carl Linnaeus
(b) Panchanan Maheshwari
(c) Birbal Sahni
(d) Bentham and Hooker

(b) Panchanan Maheshwari popularised the use of embryological characters in taxonomy. He is also known as 'The Father of Indian Plant Embryology'.

5. Which one of the following is the major constituent of biogas?

(a) Carbon dioxide (b) Nitrous oxide
(c) Methane (d) Oxygen

(c) Biogas usually contains 45-70% methane and 30-45% carbon dioxide.

It also contains small amounts of nitrogen, hydrogen sulphide, halogenated compound and organic silicon compounds. Thus, among the given options, methane is the major constituent of biogas.

6. Thyroid gland produces a hormone called 'thyroxine' which

(a) controls blood glucose levels
(b) controls ovulation
(c) controls metabolic rate
(d) maintains pregnancy

(c) Thyroid gland produces a hormone called thyroxine, which controls the metabolic rate of the body. It acts to increase the basal metabolic rate and affects protein, fat and carbohydrate metabolism affecting how human cells use energetic compounds.

7. Which one of the following is a bacterium that causes disease in the human body?

(a) *Varicella zoster*
(b) *Trypanosoma gambiense*
(c) *Salmonella typhi*
(d) *Plasmodium falciparum*

(c) *Salmonella typhi* is a bacterium which causes typhoid fever in the human body. This fever is contracted by the ingestion of contaminated food and water.

Varicella zoster is a virus causing chickenpox.

Trypanosoma gambiense and *Plasmodium falciparum* are protozoans which cause sleeping-sickness and malaria, respectively in the human body.

8. Which one of the following statements regarding viruses is not true?

(a) Viruses need living cells to reproduce
(b) All viruses are parasites
(c) Viruses can synthesise their food through photosynthesis
(d) Viruses are similar to chemical substances outside the host

(c) The statement given in option (c) is not true and can be corrected as Viruses do not perform photosynthesis to synthesise their food. They are inert, crystalline structure outside the living cell. Once they infect a cell, they takeover the machinery of the host cell to replicate themselves, killing the host.

Rest of the statements are correct.

9. Which one of the following diseases may be caused by the deficiency of vitamin-C?

(a) Rickets (b) Rabies
(c) Hepatitis (d) Scurvy

(d) The deficiency of vitamin-C (ascorbic acid) causes scurvy. This condition is characterised by general weakness, anaemia, gingivitis and skin haemorrhage .

2019 (I)

10. Which one of the following cell organelles does not possess nucleic acid?
(a) Nucleolus (b) Chloroplast
(c) Ribosome (d) Plasma membrane

› *(**d**)* The cell organelle given in option (d), i.e. **plasma membrane** does not contain nucleic acids. It is made up of lipids and proteins.
Chloroplast contains deoxyribonucleic acid whereas ribosomes are made up of ribonucleic acids and proteins.
Nucleolus is made up of proteins and ribonucleic acids (RNA) and the site for the synthesis of ribosomal RNA.

11. Which one of the following cell organelles does not possess its own genetic material encoding proteins?
(a) Ribosome (b) Nucleus
(c) Mitochondria (d) Chloroplast

› *(**a**)* Ribosome is the cell organelles, which does not possess its own genetic material (DNA) encoding proteins. Nucleus, mitochondria and chloroplast contain DNA which code for proteins needed by these cell organelles.

12. Which one of the following is not a component of conducting tissue in plants?
(a) Fibres (b) Tracheids
(c) Pericycle (d) Sieve tubes

› *(**c**)* Pericycle is not the component of conducting tissue. It is a part of the innermost layer of cortex.
Xylem and phloem both contain fibres. Tracheids are the part of xylem whereas sieve tubes are found in phloem.

13. Which one of the following organisms has vascular tissues?
(a) *Cladophora* (b) *Penicillium*
(c) *Marsilea* (d) *Anabaena*

› *(**c**) Marsilea* is pteridophytic plant, which has primitive vascular tissues. *Cladophora* is an algae, *Penicillium* is a fungi. *Anabaena* is Blue-Green Algae (BGA).

14. Which one of the following organisms represents the primary consumer category in an ecosystem?
(a) Caterpillar (b) Crabapple tree
(c) Frog (d) Sparrowhawk

› *(**a**)* Caterpillar is primary consumer in an ecosystem. It is because it feeds on leaves and is a herbivore.
Crabapple tree is a producer. Frog is a secondary consumer. Sparrowhawk is a top consumer.

15. As per the code of the nomenclature, which one of the following is the correct way of writing a biological name?
(a) *Amoeba Proteus*
(b) *Amoeba proteus*
(c) *amoeba proteus*
(d) *Amoeba Proteus*

› *(**b**)* Name written in option (b), i.e. *Amoeba proteus* is the correct way of writing biological name of *Amoeba*.
The first name represents genus and its first letter is capital. The second name represents species and its first letter is small. Both names are underlined separately when hand written and written in Italics when typed.

16. Which one of the following statements regarding electrocardiogram is correct?
(a) Electrocardiogram is graphical representation of electrical activity of cornea
(b) Electrocardiogram is graphical representation of activity of kidney
(c) Electrocardiogram is graphical representation of activity of brain
(d) Electrocardiogram is graphical representation of electrical activity of heart

› *(**d**)* Statement (d) is correct regarding electrocardiogram. Electrocardiogram is graphical representation of electrical activity of heart.
Electrodes are placed on patient's limbs and on the surface of the chest and connected to a machine called electrocardiograph. It detects any abnormality in the functioning of the heart.

17. Which one of the following statements regarding penicillin is correct?
(a) Penicillin resistant bacteria can store this antibiotic in vacuole
(b) Penicillin resistant bacteria can degrade this antibiotic by an enzyme called β-lactamase
(c) Penicillin resistant bacteria can degrade this antibiotic by an enzyme called lactic acid dehydrogenase
(d) Penicillin is not absorbed by bacteria, so most bacteria are resistant

› *(**b**)* Statement (b) regarding penicillin is correct. The bacteria which produce enzyme β-lactamase can degrade penicillin. Most of the bacteria are sensitive for penicillin as it interferes cell wall synthesis.

18. Which one of the following organelles of mammalian cell is rich in hydrolytic enzymes?
(a) Mitochondria (b) Ribosomes
(c) Lysosome (d) Nucleus

› *(**c**)* Cell organelle **lysosome** is rich in hydrolytic enzymes. These enzymes degrade the cell debris and recycle its components within the cells.
Mitochondria is powerhouse of the cell.
Ribosomes are called protein factory.
Nucleus is referred as head of the cell which controls all cellular activities.

19. Which one of the following statements regarding cholera is correct?
(a) Cholera is a disease that causes loss of memory
(b) Cholera is a disease of muscles due to consumption of alcohol
(c) Cholera is a disease due to the consumption of contaminated food or water
(d) Cholera is a genetic disease

› *(**c**)* Statement (c) regarding Cholera is correct. It is a waterborne disease and also spread through contaminated food. It is caused by bacterium *Vibrio cholerae*.
Cholera does not cause loss of memory. The consumption of alcohol does not cause cholera and it does not affect the functioning of muscles. This is also not a hereditary disease.
Hence, statement (c) is correct, and rest of the statements are incorrect.

2018 (II)

20. **Statement I** Phytoplanktons produce most of the organic carbon in the ocean.
Statement II Algae are produced in the cold water biome.
(a) Both the statements are true and statement II is the correct explanation of statement I.
(b) Both the statements are true, but Statement II is not the correct explanation of statement I.
(c) Statement I is true, but statement II is false.
(d) Statement I is false, but statement II is true.

› *(**c**)* Phytoplanktons are unicellular plants which carryout photosynthesis and are producers they fix up maximum CO_2 and form first trophic level.
Algae are a diverse group of aquatic organisms that have the ability to conduct photosynthesis in every kind of aquatic biomes, e.g. seaweeds.
So, statement I is true, but Statement II is false.

21. Which one of the following groups of cellular organelles contains DNA?
(a) Mitochondria, nucleus, chloroplast
(b) Mitochondria, Golgi bodies, nucleus
(c) Mitochondria, plasma membrane, nucleus
(d) Chloroplast, nucleus, ribosomes

(a) *(**a**)* The mitochondria, nucleus and chloroplast contains DNA. Mitochondria and chloroplasts are known as semiautonomous cell organelles because they can replicate themselves.

22. One of the additional function of Smooth Endoplasmic Reticulum (SER) is
(a) protein synthesis
(b) lipid synthesis
(c) storage of biomolecules
(d) detoxification of toxic substances

(d) *(**d**)* Detoxification of toxic substances is the additional function of Smooth Endoplasmic Reticulum (SER). Protein synthesis takes place on Rough Endoplasmic Reticulum (RER).

23. Damage to the apical meristem of a growing young plant will affect the
(a) length of the plant
(b) colour of the flower
(c) colour of the leaves
(d) taste of the fruits

(a) *(**a**)* Damage to the apical meristem of a growing young plant will affect the length of the plant. It is responsible for apical growth.

24. Which of the following kingdom(s) has/have only unicellular organisms?
(a) Monera
(b) Protista
(c) Monera and Protista
(d) Protista and Fungi

(c) *(**c**)* Monera and Protista, both have unicellular organisms. Monera includes all prokaryotic organisms, e.g. bacteria, mycoplasma. Protista includes unicellular eukaryotic organisms.

25. Which one of the following is a waterborne disease?
(a) Jaundice (b) Tuberculosis
(c) Rabies (d) Arthritis

(a) *(**a**)* Jaundice is caused by microbial infection present in water and food items. If left untreated, it damages liver.

26. The acidic, semidigested food coming out of the stomach is neutralised by
(a) pancreatic juice
(b) duodenal secretion
(c) large intestine secretion
(d) bile juice

(d) *(**d**)* The acidic, semidigested food coming out of the stomach is neutralised by sodium bicarbonate present in the bile juice secreted by liver.

Here, it makes the medium of food alkaline.

27. The oxygenated blood from the lungs is received by
(a) left auricle (b) left ventricle
(c) right auricle (d) right ventricle

(a) *(**a**)* The oxygenated blood from the lungs is received by left auricle through pulmonary vein from where it goes to left ventricle and then it is pumped into other body parts.

28. The oxygen evolved during photosynthesis comes from splitting of
(a) water (b) carbon dioxide
(c) oxygen (d) light

(a) *(**a**)* The oxygen evolved during photosynthesis comes from splitting of water.

Photosynthesis is the process that plants undergo in order to transform solar energy into chemical energy, which is stored in the bonds of sugar molecules. Besides O_2, other products of the light-dependent reactions are ATP and NADPH, oxygen production is simply by product of the splitting of water by sunlight.

The overall process of photosynthesis is typically represented by the following equation

$$6\,CO_2 + 6\,H_2O + \xrightarrow[\text{Chlorophyll}]{\text{Sunlight}} C_6H_{12}O_6 + 6O_2$$

29. Which one of the following depicts the correct circuit of a reflex arc?
(a) Effector→sensory neuron→spinal cord→motor neuron→receptor
(b) Receptor→sensory neuron→spinal cord→neuron→effector
(c) Receptor→sensory neuron→brain→motor neuron→effector
(d) Sensory neuron→receptor→brain effector→motor neuron

(b) *(**b**)* The correct reflex arc is depicted by option (b).

It is receptor → sensory neuron → spinal cord → neuron → effector.

30. If one set of chromosomes for a given plant is represented as *n*; in case of double fertilisation, the zygote and the endosperm nucleus of a diploid plant would have how many sets of chromosomes, respectively?
(a) *n* and 2*n* (b) 2*n* and 2*n*
(c) *n* and 3*n* (d) 2*n* and 3*n*

(d) *(**d**)* The zygote will have 2*n* chromosomes, while endosperm nucleus will have 3*n* set of chromosomes after double fertilisation in an angiospermic plant.

2018 (I)

31. AIDS is caused by a virus whose genetic material is
(a) single-stranded circular DNA
(b) double-stranded DNA
(c) single-stranded RNA
(d) double-stranded RNA

(c) *(**c**)* AIDS is caused by Human Immunodeficiency Virus (HIV). It belongs to family–Retroviridae and genus – *Lentivirus*.

HIV genome consists of two identical copies of the *ss*RNA genome, reverse transcriptase enzyme and some proteins. This machinary with the help of host genome can code for viral proteins that are essential during its replication. All these components are: enclosed within the core of viral particle.

32. Which one of the following organelles is not found in prokaryotic cells?
(a) Cell wall
(b) Mitochondria
(c) Plasma membrane
(d) Ribosome

(b) *(**b**)* Prokaryotic cells are simple and primitive type of cells, e.g. bacteria, methanogens, blue-green algae, etc. These cells do not contain membrane bound nucleus and other cellular organelles, i.e. mitochondria.

Prokaryotes lack all membrane bound cellular organelles.

Mitochondria is a double membrane bound cell organelle, which is responsible for the production of ATP molecules. It is found in only eukaryotic cells.

33. Which one of the following parts of body does not take part in the process of breathing?
(a) Bronchi
(b) Bowman's capsule
(c) Diaphragm
(d) Trachea

(b) *(**b**)* Bowman's capsule does not take part in process of breathing. It is a part of human excretory system. It is closed cup-shaped and double-walled structure of Malpighian body of a kidney.

The human respiratory system includes nose, larynx, trachea, bronchi and bronchioles, alveoli and lungs.

Diaphragm is a dome-shaped sheet that separate the thoracic cavity from abdominal cavity. It also serves major role during breathing. Trachea or Wind pipe conduct air to lungs and bronchi are the bronches of wind pipe inside lungs.

34. Which one of the following statements about classification of plants is correct?

(a) Thallophytes have well-differentiated body design
(b) *Funaria* is a fungus
(c) All pteridophytes are phanerogams
(d) Vascular system is not found among bryophytes

(d) *(**d**)* Bryophytes are non-vascular, very small, photosynthetic and first terrestrial plants.

Thallophyta is a group of plants in which body is not differentiated into true roots, stems and leaves, e.g. algae.

Funaria (Moss) is a bryophyte. All pteridophytes are cryptogams, while all gymnosperms and angiosperms are phanerogams.

35. Which one of the following is the correct sequence of levels of hierarchy of classification of organisms from higher to lower?

(a) Phylum → Class → Order → Family → Genus
(b) Phylum → Class → Family → Order → Genus
(c) Family → Order → Class → Species → Genus
(d) Class → Family → Order → Species → Genus

(a) *(**a**)* The correct level of hierarchy is Kingdom → Phylum (in animals)/Division (in plants) → Class → Order → Family → Genus → Species.

A hierarchical system is used for taxonomic classification of organisms. Each level in this classification includes, organisms with similar characteristics.

According to the Linnaen hierarchy, proposed by Carolus Linnaeus (Father of Systematic Botany), the hierarchy involves seven obligate categories.

36. Which one of the following statements about meristematic tissues in plants is correct ?

(a) These are dead tissues and form wood
(b) They provide flexibility to plant due to their thickened walls
(c) These are present in the bark of a tree only
(d) Growth occurs in plants due to division of cells of these tissues

(d) *(**d**)* The statement (d) about meristematic tissues in plants is correct. Growth occurs in plants due to the division of meristematic cells.

Meristematic tissues are a group of undifferentiated immature cells that are either preparing to divide or are in continuous state of division.

These tissues contribute towards the growth of the plants as they are present in the actively growing regions of plants.

Due to meristematic tissues, a plant body retains the capacity of unlimited growth throughout their life.

37. Which one of the following elements is needed in the human body to transfer electrical signals by nerve cells?

(a) Lithium (b) Sodium
(c) Rubidium (d) Caesium

(b) *(**b**)* Sodium in the form of ions are found in large proportions in human body fluids. It plays an important role in the transmission of nerve signals. It also regulates the flow of water across cell membranes and in transport of sugars and amino acids into the cells.

38. Who among the following first discovered cell ?

(a) Robert Brown (b) Robert Hooke
(c) Leeuwenhoek (d) Rudolf Virchow

(b) *(**b**)* **Robert Hooke** was the first who discovered and studied cell from a thin section of dead cork. He observed small honeycomb-like structures and named them as cells.

Robert Brown discovered nucleus. **Leeuwenhoek** was the first person to observe living cells under microscope. **Rudolf Virchow** proposed that all cells arise from pre-existing cells.

39. Which one of the following group of organisms forms a food chain?

(a) Grass, human and fish
(b) Grass, goat and human
(c) Tree, tree cutter and tiger
(d) Goat, cow and human

(b) *(**b**)* Grass, goat and human form a food chain. The series of organisms in which one organism is being eaten by other is called food chain. The organisms given in the option (b) form the following food chain:

Grass (Producers) → Goat (Primary Consumer) → Human (Secondary Consumer).

40. Which one of the following types of tissues will have contractile proteins?

(a) Nervous tissue
(b) Muscle tissue
(c) Bone tissue
(d) Blood tissue

(b) *(**b**)* Muscle tissues have contractile proteins. These proteins are actin and myosin and they are, the main components of thin and thick filaments, respectively.

Muscle contraction occurs when these filaments slide over one another in a series of repetitive events.

41. If by an unknown accident the acid secreting cells of the stomach wall of an individual are damaged. Digestion of which one of the following biomolecule will be affected to a greater extent?

(a) Protein only
(b) Lipid
(c) Carbohydrate only
(d) Protein and carbohydrate

(a) *(**a**)* If by any accident of individual the acid secreting cells of stomach wall are damaged then proteins will not be digested. Pepsinogen is an inactive form of pepsin which is secreted by oxyntic cells. It is converted into active form called pepsin in the presence of HCl. Pepsin converts protein molecules into proteases, peptones and ultimately into amino acids.

42. The accidental touch of nettle leaves creates a burning sensation, which is due to inject of

(a) hydrochloric acid
(b) methanoic acid
(c) citric acid
(d) sulphuric acid

(b) *(**b**)* The accidental touch of nettle leaves create a burning sensation.

This is due to the methanoic acid (HCOOH) secreted by them. It is a herbaceous plant that grows in the wild. The leaf of the dock plant, which often grows beside the nettle, is the traditional remedy for it.

2017 (II)

43. Which one of the following vitamins has a role in blood clotting?

(a) Vitamin-A (b) Vitamin-D
(c) Vitamin-E (d) Vitamin-K

(d) *(**d**)* Vitamin-K plays an essential role in clotting of blood. It acts as a cofactor for enzyme to bind with calcium ions, which are required for activation of coagulation factors such as prothrombin.

44. The term 'Probiotic' is applied to
(a) organic food
(b) antacid
(c) antibiotic
(d) live microbial food supplement

(d) *(**d**)* The term 'Probiotic' is used to live microbial food supplement. These are the live microorganism which provide health benefits when consumed.

45. Which one of the following microbes causes acidification and curdling of milk?
(a) Lactic acid bacteria
(b) *Clostridium botulinum*
(c) *Vibrio cholerae*
(d) *Saccharomyces cerevisiae*

(a) *(**a**)* Lactic Acid Bacteria (LAB) causes acidification and curdling of milk. It ferments lactose to lactic acid and other end products.

46. Who among the following shared the Nobel Prize in 1962 along with Francis Crick and James Watson for their discoveries concerning the molecular structure of nucleic acids?
(a) Erwin Chargaff
(b) Maurice Hugh Frederick Wilkins
(c) Rosalind Franklin
(d) Phoebus Levene

(b) *(**b**)* The Nobel Prize in Physiology or Medicine 1962 was awarded jointly to Francis Harry Compton Crick, James Dewey Watson and Maurice Hugh Frederick Wilkins. They were awarded prize for their discoveries associated with the molecular structure of nucleic acids and its significance for information transfer in living beings.

47. Tendons through which muscles are connected to bones are tightly compacted bundles of which one of the following long fibrous protein?
(a) Fibrin (b) Collagen
(c) Elastin (d) Cellulose

(b) *(**b**)* Tendons are made up of long fibrous protein called collagen. These are inelastic fibres of great tensile strength.

48. Which one of the following is the scientific name of the causal organism of elephantiasis?
(a) *Ascaris lumbricoides*
(b) *Culex pipiens*
(c) *Wuchereria bancrofti*
(d) *Fasciola hepatica*

(c) *(**c**) Wuchereria bancrofti* is a human parasitic roundworm which causes elephantiasis. It is carried from person-to-person by mosquitoes.

49. Melanin is the natural pigment that gives colour to human skin, hair and the iris provides protection against
(a) ultraviolet radiation
(b) infrared radiation
(c) X-ray radiation
(d) short wave radio radiation

(a) *(**a**)* Melanin provides protection against ultraviolet radiations. It acts as effective absorbent of light and dissipate over 99.9% of absorbed radiation.

50. Intake of which one of the following food components should be minimised by patients having gouty arthritis due to elevated serum uric acid level?
(a) Food fibres (b) Nucleic acids
(c) Lipids (d) Carbohydrates

(a) *(**a**)* Gouty arthritis is a disease due to defect in purine (a component of nucleic acid) metabolism that causes an excess of uric acid and its salts (urates).

The uric acid level is raised in the blood. When crystals of uric acid salts (e.g. sodium urate) accumulate in the joint, it causes gouty arthritis. Intake of nucleic acids rich food will promote the disease.

So, the patients of gouty arthritis are recommended to take such food that are low in nucleic acids components. Food fibres, lipids and carbohydrates rich food are taken is normal balanced diet.

51. Which one of the following statements about microbes is not correct?
(a) They are used in sewage treatment plants
(b) They are used in industrial fermenters for the production of beverages
(c) No antibiotic has been obtained from any microbes
(d) They are used to get many bioactive molecules for the treatment of diseases

(c) *(**c**)* Microbes are used for production of antibiotics. Penicillin antibiotics were first to be effective against many bacterial infections.

52. Golden rice is a genetically -modified crop plant where the incorporated gene is meant for biosynthesis of
(a) omega-3 fatty acids
(b) vitamin-A
(c) vitamin-B
(d) vitamin-C

(b) *(**b**)* The incorporated gene leads to synthesis of β-carotene which act as precursor of vitamin-A. It is to be grown and consumed in areas with shortage of vitamin-A.

2017 (I)

53. Cell wall of any fungus is different from plants in having
(a) cellulose (b) chitin
(c) cholesterol (d) glycogen

(b) *(**b**)* The cell wall in fungi is made up of chitin which is a heteropolysaccharides, while the cell wall of plant is made up of cellulose (polymer of glucose). Plant cell wall does not contain chitin.

54. Sleeping sickness is a parasitic disease of humans and other animals. It is caused by
(a) *Histomonas*
(b) *Trypanosoma*
(c) *Angomonae*
(d) *Naegleria*

(b) *(**b**)* Sleeping sickness is disease caused by protozoan and transmitted by insect. The name of causal organism is *Trypanosoma* which is transmitted *via* tse-tse fly (vector of sleeping sickness).

55. Dengue virus causes high fever, rashes and reduces the number of particular type of blood cells. Those blood cells are
(a) monocytes
(b) platelets
(c) eosinophils
(d) neutrophils

(b) *(**b**)* Dengue virus causes high fever, rashes and reduces the number of particular cells named platelets. This virus also causes bone marrow suppression leading to reduction in number of blood platelets.

56. Kidney secretes an enzyme, which changes plasma protein angiotensinogen into angiotensin. The enzyme is
(a) renin
(b) nitrogenase
(c) hydrolase
(d) mono-oxygenase

(a) Renin is an enzyme secreted by juxtaglomerular cells of kidney, when there is a fall in glomerular blood flow. It changes plasma protein angiotensinogen to angiotensin. Hence, it plays an important role in regulation of kidney function by Renin-Angiotensin mechanism.

57. Red Blood Cells (RBCs) have
(a) no nucleus, no mitochondria and no endoplasmic reticulum
(b) nucleus, mitochondria and endoplasmic reticulum
(c) nucleus, mitochondria, but no endoplasmic reticulum
(d) no mitochondria, but endoplasmic reticulum is present

(a) Red Blood Cells (RBCs) or Erythrocytes are the most abundant cells in blood. They do not contain nucleus and most cell organelles like mitochondria, endoplasmic reticulum, etc. This feature of RBCs is an adaptation that allows maximum binding of oxygen with haemoglobin.

58. Which one of the following statements is not correct?
(a) All proteins are enzymes
(b) Mostly enzymes are proteins
(c) All fats are energy rich compounds
(d) Glucose is a common carbohydrate

(a) In the given statements only statement (a) is not correct.
Almost all enzymes are protein, but all proteins are not enzyme because proteins can also be hormones, membrane receptors, etc. (Ribozyme is an exception which is a nucleic acid).

59. Sugarcane is one of the important cash crops in India. It is grown to obtain
(a) starch (b) glucose
(c) fructose (d) sucrose

(d) Sugarcane is the world's largest crop by production quantity. Sucrose is extracted from cane juice and purified in specialised mills. Ethanol is also produced by the sugarcane industry as a byproduct.

60. The colourful part of the sunflower or marigold plant is
(a) flower (b) inflorescence
(c) fruit (d) seed

(b) Flowers are specialised organs meant only for reproduction and have nothing to do with the maintenance of the plant itself. This is further evident from that the fact that in many plants flowers come as the fore-runners of death.
Flowers may arise singly, as in China rose or more commonly, in groups at the axil of special leaves or at the terminal position. A branch or a branch system bearing a clusters of flowers is called an **inflorescence**.
Hence, the colourful part of the sunflower or marigold plant is inflorescence.

2016 (II)

61. The genetic material of bacteria is found in
(a) nucleus (b) cytoplasm
(c) cell membrane (d) ribosome

(b) DNA is the genetic material of bacteria, which is scattered in cytoplasm. Histone proteins are absent in bacterial DNA. Bacterial cell is prokaryotic and found incipient nucleus, but lack of nuclear membrane. In eukaryotic cells, genetic material (DNA) found in nucleus.

62. A horse and a donkey can breed to produce mule, which is an infertile animal. The infertility is because horse and donkey belong to different
(a) class (b) order
(c) species (d) genus

(c) A horse *(Equus ferus caballus)* and a donkey *(Equus africanus asinus)* can breed to produce mule, which is an infertile animal. The infertility is because horse and donkey belong to different species. Mule are medium weight animal, which are more hardworker and long-live than horse and donkey.

63. A plant having yellow leaves with dead spots has the deficiency of
(a) potassium (b) magnesium
(c) nitrate (d) phosphate

(a) A plant having yellow leaves with dead spots, has the deficiency of potassium. Magnesium is the part of chlorophyll in all green plants, so lack of Mg^{2+} causes chlorosis and tissue damaged.
Lack of phosphate causes leaves dull and causes curled disease. Nitrate deficiency causes green or with purple and red spots of anthocyanin, chlorosis with necrosis, first in older leaves or premature abscission, delayed flowering and fruiting.

64. Consider the following statements about cactus.
1. The leaves are reduced to spines.
2. The stem does the photosynthesis.
Which of the statements given above is/are correct ?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Both statements are correct. Cactus is a xerophytic plant. Due to the environmental adaptation, leaves are reduced to spine and stem does the photosynthesis. This nature prevents the loss of water (transpiration).

65. In which one of the following methanogens, i.e. methane producing bacteria are not present?
(a) Rumen of cattle
(b) Aerobic environment
(c) Anaerobic sludge
(d) Excreta of cattle

(b) Methanogens, i.e. methane producing bacteria are not present in aerobic environment because they are archaebacteria, which are present in anaerobic condition and cannot function under aerobic conditions. They are present in rumen of cattle, anaerobic sludge, wet lands, sewage treatment plants, etc.

66. Malarial parasite is a
(a) bacteria (b) protozoan
(c) virus (d) fungus

(b) Malaria is a mosquito-borne infectious disease affecting human, caused by parasitic protozoans (a group of single-celled microorganisms) belonging to the *Plasmodium*.
Plasmodium parasite have two hosts, one is female *Anopheles* mosquito, another is human.

67. It is difficult to kill viruses because they
(a) consist of tough protein coat
(b) are very small in size
(c) lack cellular structure
(d) spend a lot of time inside the host's cells

(c) It is difficult to kill viruses because they lack the cellular structure. They are not hard to kill, once they are outside the body. Virus do not have self-protein synthesis machinery. They use host's protein synthesis machinery for protein synthesis. Virus also mutate the surface protein very rapidly.

2016 (I)

68. To prevent heart problems, blood of a normal healthy person should have
1. low cholesterol level
2. high HDL level
3. high VLDL level
4. high LDL level

Select the correct answer using the codes given below

(a) 1 and 2 (b) 1, 2 and 4
(c) 3 and 4 (d) 1, 2 and 3

(a) *(**a**)* Low cholesterol level and High Density Lipids (HDL) high level are helpful in preventing heart problems.

High Density Lipids are directly correlated with low level of VLDL. HDL is also called 'good cholesterol' and LDL is called 'bad cholesterol'. These cholesterol are harmful to our heart. So, high VLDL level and high LDL levels are not helpful in preventing heart problems.

69. Spruce and cedar are tree varieties of

(a) equatorial forest
(b) temperate coniferous forest
(c) monsoon forest
(d) temperate deciduous forest

(b) *(**b**)* Temperate coniferous forest are made up of mainly cone-bearing trees such as pine, fir, spruce, hemlock, etc., and other varieties include juniper, cedar, redwood.

70. 'Sal' tree is a

(a) tropical evergreen tree
(b) tropical semi-evergreen tree
(c) dry deciduous tree
(d) moist deciduous tree

(d) *(**d**)* 'Sal' tree is moist deciduous tree. The monsoonal forests are found in areas where rainfall lies between 70-100 cm. Sal is the most significant tree found in these forests. Teak, neem, peepal are other common varieties.

71. Which of the following pairs of vector and disease is/are correctly matched?

Vectors	Diseases
1. *Anopheles*	Malaria
2. *Aedes aegypti*	Chikungunya
3. Tse-tse fly	Filariasis
4. Bed bugs	Sleeping sickness

Select the correct answer using the codes given below

(a) 1, 2 and 3 (b) 1 and 2
(c) 1 and 4 (d) Only 2

(b) *(**b**)* The vector for malaria disease is female *Anopheles* mosquito. This disease is caused by a protozoan called *Plasmodium*. It is a epidemic disease. Chikungunya spread by a mosquito *Aedes aegypti*. This disease is caused by alpha virus. Monkey and human being act as a reservoir.

This disease spread by female *Culex*. It is also known as 'elephantiasis'. The causative agent for this disease is *Wuchereria bancrofti*. This worm affects the lymphatic system by blocking lymph vessels. The lymph fluid accumulate in different parts of body, leading to swelling of organs especially lower limbs.

72. Consider the following statements.

1. All echinoderms are not marine.
2. Sponges are exclusively marine.
3. Insects are found in all kinds of habitats.
4. Many primates are arboreal.

Which of the statements given above is/are correct?

(a) 1, 3 and 4 (b) 3 and 4
(c) 2 and 4 (d) Only 3

(b) *(**b**)* Insects constitute the largest group of animal kingdom and it includes more than 700000 species present in class–Insecta.

This is a group of the most successfully surviving animals in invertebrates, present in all kinds of habitats. Many primates such as monkey, lemurs, etc., are arboreal but human is not. Well-developed brain, binocular vision, cerebral hemisphere are present in primates.

All echinoderms are exclusively marine, with spiny skin and ambulacral system. But sponges are present in fresh and marine both kinds of aquatic habitats. Sponges are animals with pores, static and water canal system.

Hence, statements 3 and 4 are correct and statements 1 and 2 are incorrect.

73. Which of the following statements about DNA is/are correct?

1. DNA is the hereditary material of all living organisms.
2. All segments of DNA code for synthesis of proteins.
3. Nuclear DNA is doubled helical with two nucleotide chains which run anti-parallel.
4. DNA is also found in mitochondria.

Select the correct answer using the codes given below

(a) 1, 2 and 3 (b) 3 and 4
(c) 1, 3 and 4 (d) Only 4

(c) *(**c**)* DNA is made up of exon and intron regions. Exons are coding region and introns are non-coding region. Only exon part codes for protein synthesis. DNA is the genetic material of all living organisms. Except in few viruses where, RNA acts as a genetic material such as retrovirus (e.g. HIV virus). Here, RNA redirects DNA synthesis by 'reverse transcription' process with the help of 'reverse transcriptase' enzyme.

Nuclear DNA is a double helix with anti-parallel chains. DNA also present in mitochondria as well as in chloroplast. Circular DNA present in mitochondria is similar to bacteria which synthesise proteins related to its own function.

That is why these organelles are known as 'semi-autonomous organelles'.

Hence, statements 1, 3 and 4 are correct and statement 2 is incorrect.

74. Which of the following statements are correct for honeybee?

1. Males are haploid.
2. Workers are sterile and diploid.
3. Queen develops from diploid larvae that fed on Royal jelly.
4. Honey is collected/made by males.

Select the correct answer using the codes given below

(a) 1, 2, 3 and 4 (b) 2 and 3
(c) 1, 2 and 3 (d) 1 and 4

(c) *(**c**)* Honey is made by honeybee (*Apis indica*). In these flies 'Haplo-diploidy' is present. In this type of genetic constitution males are haploids and females are diploid.

Females are developed into queen and workers. Only one diploid larvae which larvae feed upon 'Royal jelly', developed into queen, rest larvae feed upon 'Bee bread' and developed into sterile worker. Honey is collected and made by workers.

75. **Statements I**. Plantation farming has mostly been practised in humid tropics. **Statements II**. The soil of humid tropics is highly fertile.

(a) Both the statements are true and Statement II is the correct explanation of Statement I.
(b) Both the statements are true, but Statement II is not the correct explanation of Statement I.
(c) Statement I is true, but Statement II is false.
(d) Statement I is false, but Statement II is true.

(c) *(**c**)* Humid tropics are most favoured location for plantation crops including rubber, tea, coffee, coconut, etc. However, the soil of this region is not very fertile due to the leaching of minerals because of heavy rainfall.

2015 (II)

76. In Egypt, ancient mummies can be found to have their arteries intact due to well-preserved
(a) mineralised blood
(b) fibroblast fibres
(c) elastic fibres
(d) brown fat

› *(**c**)* Elastic fibres are formed of elastin protein, which is probably the most resistant of all body proteins to chemical changes. Thousands of years old 'mummies' have their arteries intact due to the well-preserved elastin fibres. Elastin fibres are not present in mineralised blood, fibroblast fibres and brown fat, so they are not cause of well-preserved arteries in mummies.

77. Which one of the following statements is not correct?
(a) Application of lime makes the soil acidic
(b) High acidity in soil is typical of humid climate
(c) Increasing soil acidity results in declining soil fertility
(d) Arid climate is characterised by alkaline soil

› *(**a**)* The nature of lime is basic. So, when it is added to the soil, it makes it basic. If it is added to the acidic soil, then its acidity is neutralised by adding lime water. In humid climates due to the presence of calcium, magnesium and sodium in soil, it is acidic. Excess of acid in soil increases toxic level of soil, declining its fertility. In arid climate, soil is basic.

78. Which one of the following is useful in paper manufacturing industry?
(a) Fibrous plants
(b) Orchids
(c) Non-flowering plants
(d) Plants growing in high altitude

› *(**a**)* Fibrous plants are traditionally used to make paper in the paper manufacturing industry. These are also used to make cloth, rope, etc. Plant fibres include seed hairs, stem fibres, flax, leaf fibres and husk fibres.

79. Which one of the following is not a greenhouse gas?
(a) Water vapour
(b) Methane
(c) Ozone
(d) Carbon monoxide

› *(**d**)* Greenhouse gases are those gases which allow direct sunlight to reach the earth surface unimpeded. Many greenhouse gases occur naturally in the atmosphere such as CO_2, methane, water vapour, ozone and nitrous oxide. Carbon monoxide is not considered as a greenhouse gas. It is a very poisonous gas. It can kill people if inhaled for long time.

80. Muscle fatigue is due to the accumulation of
(a) cholesterol (b) lactic acid
(c) lipoic acid (d) triglycerides

› *(**b**)* Muscle fatigue is due to the accumulation of lactic acid. It is formed and accumulated in the muscle under conditions of high energy demand, rapid fluctuations of the energy requirement and insufficient supply of oxygen.

81. Living things are grouped into subgroups like plant kingdom/animal kingdom. Which one of the following is not correct for animal kingdom?
(a) Cannot make their own food
(b) Body contains cellulose
(c) Do not have chlorophyll
(d) Migrate from one place to another

› *(**b**)* Living beings of animal kingdom lack cellulose in their cells of the body, while plants have rigid cell wall that is composed of cellulose. Except *Euglena*, chlorophyll is not present in animals, so they cannot make their own food and ability for movement is present in them.

82. In blood, platelets are required for
(a) transporting oxygen
(b) transporting carbon dioxide
(c) initiating blood clotting
(d) initiating degradation of urea

› *(**c**)* In blood, platelets are required for blood coagulation (clotting and prevention of bleeding) at bleeding site in the body of animals. The normal platelet count ranges between 150000 and 450000 per microlitre (one millionth of a litre).

83. Which one of the following is not a place of action in human body for the malarial parasite *Plasmodium*?
(a) Liver (b) Kidney
(c) Red blood cells (d) Brain

› *(**b**)* Malarial parasite *Plasmodium* affects several body parts. In early stages, **liver** and **red blood cells** get affected.
Later on, malaria parasite affects liver and spleen causing hepatomegaly and splenomegaly, respectively. Also in advance cases, brain gets affected causing cerebral malaria. This parasite does not affect kidney.

84. Who among the following discovered antibiotic producing fungus from *Penicillium* genus?
(a) Louis Pasteur
(b) Sir Alexander Fleming
(c) Stanley Prusiner
(d) Robert Hooke

› *(**b**)* Sir **Alexander Fleming** was a Scottish biologist. His best known discovery is the antibiotic penicillin obtained from the mold *Penicillium notatum*.
Louis Pasteur was a French scientist well known for his discovery of various vaccination, microbial fermentation and pasteurisation.
Stanley Prusiner was an American, well-known for his discovery of prions. **Robert Hooke** first time discovered the cell.

85. Which one of the following is not an example of eukaryotic organism?
(a) Yeast (b) Bacteria
(c) Plant (d) Human being

› *(**b**)* Out of all, bacteria is not an example of eukaryotic organism. These are prokaryotes with a primitive nucleus without nuclear membrane. Yeast, plant and human being belong to eukaryotic organisms because they have well-developed nuclear membrane separating nucleus from cytoplasm.

86. Which one of the following vitamins is synthesised in our own skin?
(a) Vitamin-A (b) Vitamin-B
(c) Vitamin-C (d) Vitamin-D

› *(**d**)* Vitamin-D also known as calciferol refers to a group of fat-soluble vitamin responsible for enhancing intestinal absorption of calcium, iron, magnesium, phosphate and zinc. It is synthesised in our skin when we get exposed to sunlight. Vitamin-A, B and C are not synthesise in our body.

87. Carbon and energy requirements of autotrophic organisms are fulfilled by
(a) photosynthesis (b) gluconeogenesis
(c) glycogenesis (d) external sources

› *(**a**)* The energy and carbon requirements of autotrophic organisms like plants are fulfilled by the process known as photosynthesis. In this process, water and carbon dioxide are taken from atmosphere and in the presence of chlorophyll and sunlight, these raw materials are converted into carbohydrates which provides energy.
Excess of carbohydrates are stored in the form of starch. Gluconeogenesis is

the process of formation of glucose from non-carbohydrate sources. Glycogenesis is the process of formation of glycogen from glucose. External sources do not fulfil carbon and energy requirements of autotrophic organisms.

2015 (I)

88. **Statement I** Growth of plants is smooth with a complete fertiliser.
Statement II A complete fertiliser always contains N, P and K.
(a) Both the statements are individually true and Statement I is the correct explanation of Statement II
(b) Both the statements are individually true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

› *(**b**)* Plants grow smoothly with complete essential nutrients which are supplied in the form of fertilisers and there are 16 essential nutrients for proper growth of a plant. N, P and K are the parts of essential nutrients.
Hence, option (b) is the correct answer.

89. Energy is required for maintenance of life. It is obtained by a process called
(a) metabolism (b) phagocytosis
(c) photorespiration (d) decomposition

› *(**a**)* Energy required for maintenance of life is obtained by one of the most important life process, i.e. metabolism. This process includes anabolism (constructive) and catabolism (destructive) process.
Respiration is a catabolic process, which supplies energy for the maintenance of life. Process of intake of solids by the cell is known as phagocytosis.
Photorespiration occurs in plants. In this process, O_2 acts as an inhibitor of photosynthesis. This process occurs in chloroplast, mitochondria and peroxysomes.

90. Which one of the following is the smallest unit showing the properties of life such as capacity for metabolism, response to the environment, growth and reproduction?
(a) Gene (b) Chromosome
(c) Nucleus (d) Cell

› *(**d**)* Cell is the smallest unit which shows properties of life like reproduction and growth, metabolism, response to environment, etc., whereas genes, chromosomes and nucleus do not show properties of life.

91. In plants, which one of the following gases is released during photosynthesis?
(a) Carbon dioxide (b) Oxygen
(c) Hydrogen (d) Methane

› *(**b**)* Oxygen is released during process of photosynthesis by green parts of plant. The material of this process is CO_2, H_2O, chlorophyll (present in green parts of plant) and solar radiation. Carbon dioxide is consumed in this process, but nitrogen and methane are not released during this process.

92. Gametophytes of sexually reproducing flowering plants are
(a) haploid (b) diploid
(c) tetraploid (d) polyploid

› *(**a**)* Gametophytes of sexually reproducing plants are haploid. During sexual reproduction, haploid gametes are produced. They fuse together to form a diploid zygote in flowering plants.

93. The seeds of flowering plants are made up of
(a) ovary and ovary wall
(b) embryo, food reserves and seed coat
(c) cotyledons
(d) zygotes

› *(**b**)* Seeds of flowering plants are made up of embryo, food reserves, cotyledons and seed coat.
After fertilisation, ovule is changed into seed, while integument forms seed coat. In non-endospermic seed (dicot), food is reserved in cotyledons, while in endospermic seed (monocot), food is reserved in the form of endosperm.

94. A farmer in a semi-arid area claims that his farming practice is very environment friendly. Which of the following practices on his farm can justify his claim?
1. Planting a tree belt.
2. Practicing crop rotation.
3. Carrying out a large scale irrigation system.
4. Using organic fertilisers.
Select the correct answer using the codes given below
(a) 3 and 4
(b) 1, 2 and 4
(c) 1, 3 and 4
(d) 1 and 2

› *(**b**)* Semi-arid area is characterised by fertile soil with scarcity of water. For the very environment friendly farming practice–planting a tree belt, practicing crop rotation and organic fertilisers are essential.

95. Which endocrine gland requires iodine to synthesise a particular hormone whose deficiency may cause goitre disease?
(a) Hypothalamus (b) Pancreas
(c) Thymus (d) Thyroid gland

› *(**d**)* **Thyroid gland** requires iodine to synthesise thyroxine hormone which is essential for metabolism. Deficiency of this hormone may cause goitre. This disease is common in people of hilly areas.
Hypothalamus controls functions of pituitary gland. **Pancreas** is a mixed gland, its endocrine part secretes insulin, glucagon which play important role in carbohydrate metabolism.
Thymus gland present in neck region active in young, but degenerates in adults secreting thymosin hormone, plays an important role in immunity.

96. Which one of the following is not a part of nerves?
(a) Axons (b) Connective tissues
(c) Schwann cells (d) Smooth muscles

› *(**d**)* Smooth muscles are not parts of nerves. Smooth muscles or non-striated muscles are under control of central nervous system. These are involuntary in action. Axons, connective tissues and Schwann cells are parts of nerves.

97. Precursor of which one of the following vitamins comes from β-carotene?
(a) Vitamin-A (b) Vitamin-C
(c) Vitamin-D (d) Vitamin-K

› *(**a**)* Precursor of vitamin-A comes from β-carotene. In human body, β-carotene gets converted into retinol, which is called vitamin-A. Thus, β-carotene is a precursor (inactive form) of vitamin-A and also called provitamin-A carotenoid. β-carotene does not play any role in synthesis of vitamin-C, D and K.

98. 'Lubb-Dupp' sound is produced due to action of
(a) large intestine (b) lungs
(c) heart (d) oesophagus

› (*c*) Lubb-Dupp sound is produced by heart during closing of heart valves. Large intestine is related to absorption of water.
Lungs are meant for respiration and oesophagus is a conducting tube for food.

99. Perspiration cools the body because
(a) presence of water on the skin is cooling
(b) evaporation requires latent heat
(c) water has a high specific heat
(d) water is a poor conductor of heat

(b) For evaporation of sweat, the energy required (which is equal to latent heat of vaporisation) is taken from the body, due to which the body feels cool.

2014 (II)

100. Which one among the following is a plant hormone?
(a) Insulin
(b) Thyroxine
(c) Gibberellin
(d) Oestrogen

(c) Among the given options, 'Gibberellin' is a plant hormone. Others are the hormones found in animals.

Gibberellins regulate growth and influence various developmental processes, including stem elongation, germination, dormancy, flowering, leaf and fruit senescence.

Insuline is produced by beta cells in the pancreas. It regulates the metabolism of carbohydrates and fats by promoting the absorption of glucose from the blood to skeletal muscles and fat tissues.

Thyroxin is produced by the thyroid gland. It plays an important role in regulating body's metabolic functions.

Oestrogen primarily influence the development of female sexual characteristics and reproduction.

101. Which of the following groups of plants can be used as indicators of SO_2 pollution of air?
(a) Ferns (b) Mentha
(c) Lichens (d) Hornworts

(c) Lichens can be used as indicators of SO_2 pollution of air as they are sensitive to sulphur dioxide. If the air is badly polluted with sulphur dioxide (SO_2), there may be no lichens present.

Ferns, mentha and **hornworts** are not used as indicator of sulphur dioxide. The most tolerant lichens belong to those group that are crusty in appearance, whereas the leafy lichens are not very pollutant tolerant. Those lichen species that can tolerate quite high levels of pollution can often be found on pavements and walls in urban areas.

102. Which one of the following cell organelles is absent in animal cell?
(a) Cell membrane
(b) Endoplasmic reticulum
(c) Cell wall
(d) Mitochondria

(c) **Cell wall** is not found in animal cell. It is the characteristic feature of plant cell. In plants, it is made up of cellulose. It provides rigidity and mechanical strength to the plants. While cell membrane, endoplasmic reticulum and mitochondria are present in animal cells as well as plant cells.
Cell membrane is made up of lipoprotein arranged in a mosaic pattern. It is the outermost living covering of the animals and next to cell wall in plants.

Endoplasmic reticulum is a network of membrane bound cavities from nucleus to plasma membrane within the cell.
Mitochondria are cell organelles which are involved in respiration process and act as powerhouse of the cell.

103. Consider the following statements.
1. Carbohydrates are the only source of energy to humans.
2. Fats give maximum energy on oxidation as compared to other foods.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) Humans obtain energy from the following classes of biomolecules—carbohydrates, lipids and proteins.

Out of these, the maximum energy can be obtained from the oxidation of fats. Proteins and carbohydrates both provide 4 kilo calories per gram (approx.), while fats provide approximately 9 calories per gram.

104. Which one of the following is not a function of liver?
(a) Conversion of glucose into glycogen
(b) Production of urea
(c) Destruction of dead and worn out red blood cells
(d) Absorption of food and excess water from the undigested food

(d) Absorption of food is the function of small intestine, while absorption of excess water from the undigested food is the function of large intestine and is not the function of liver.
Liver is the largest gland present in humans. It is involved in conversion of glucose into glycogen (glycogenesis), production of urea through ornithine cycle, destruction of dead and worn out red blood cells by phagocytosis process of Kupffer cells. Other functions of liver are– Gluconeogenesis, Cori-cycle, production of bile juice and synthesis of heparin etc.

105. Which one of the following is not a part of female reproductive system?
(a) Fallopian tube (b) Cervix
(c) Urethra (d) Vagina

(c) Urethra is an accessory duct of male reproductive system that originates from urinary bladder. It acts as the common passage for both urine and semen.

The female reproductive system mainly comprises of ovaries, Fallopian tubes, uterus, cervix, vagina, etc.

106. Which one of the following animals is cold-blooded?
(a) Dolphin (b) Shark
(c) Whale (d) Tortoise

(b) Most sharks are cold-blooded. Some, like the mako and the white shark, are partially warm-blooded (they are endotherms). These sharks can raise their temperatures above the temperature of the water.

Dolphin and whale are warm-blooded. These animals keep their body temperature higher than their environment and have stable temperature, e.g. mammals and birds.

107. Leprosy is caused by
(a) virus (b) bacteria
(c) protozoan (d) retrovirus

(b) Leprosy is caused by the bacterium *Mycobacterium le*prae, which has a long incubation period. This microbe mainly affects the skin and peripheral nerves. Leprosy is not highly infectious. It is transmitted through droplets from nose and mouth.

Most effective drug is ofloxacin and thalidomide. It is diagnosed by lepromin skin test. Virus, protozoans and retrovirus play no role in causing this disease.

108. **Statement I** *Amoeba* is a unicellular organism and the single cell performs all functions of a living organism.
Statement II Cell is the fundamental unit of living organism.
(a) Both the statements are individually true and Statement II is the correct explanation of Statement I
(b) Both the statements are individually true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

(*a*) *Amoeba* is a unicellular organism and the single cell performs all functions like digestion, excretion, respiration, growth and reproduction.

A cell is the smallest unit of a living organism where the metabolic processes occur to keep the organism alive.

Hence, the cell is considered as the fundamental unit of living organism.

Hence option (a) is the correct answer.

109. Growth and repair of damaged tissue involves
(a) mitotic cell division only
(b) Both mitotic and meiotic cell divisions
(c) meiotic cell division only
(d) amitotic cell division only

(*a*) Mitosis is a type of cell division by which a single cell divides into two genetically identical daughter cells. This is the method by which the body produces new cells for both growth and repair of ageing or damaged tissues throughout the body.

Meiosis only provides cells for forming the zygote during fertilisation.

Amitosis is the direct cell division that takes place by simple cleavage of the nucleus without chromosome and spindle formation.

2014 (I)

110. After diagnosis of disease in a person, the doctor advises the patient iron and folic acid tablets. The person is suffering from
(a) osteoporosis
(b) anaemia
(c) goitre
(d) protein-energy malnutrition

(*b*) Anaemia is a condition in which the body lacks enough healthy red blood cells or haemoglobin. It is mainly caused due to the deficiency of minerals and vitamins in the body. So, in this case, doctor prescribes iron and folic acid tablets to the anaemic person (Folic acid is a form of vitamin-B).

In osteoporosis, bones become fragile and can easily break. **Goitre** is caused by hyposecretion of thyroxine hormone. **Marasmus** and **kwashiorkor** are caused due to protein energy malnutrition.

111. Dropsy is a disease caused due to adulteration in
(a) ghee (b) arhar dal
(c) mustard oil
(d) turmeric powder

(*c*) Dropsy is an abnormal collection of fluid in any part of the body. It is usually caused due to the adulteration of mustard oil with Argemone oil. Ghee, arhar dal and turmeric powder do not cause dropsy when they get adulterated.

112. Leaves of which of the following plants are not used for the rearing of silkworms?
(a) Mulberry (b) Castor
(c) Oak (d) Teak

(*d*) Teak plant is not used for rearing silkworms. Culture of silkworm is known as sericulture. Silk is obtained from cocoon of silkworm which represent pupa stage. The scientific name of silkworm is *Bombyx mori*.

Silk is first produced in China. Silk is made up of fibrin protein. Silkworm is most commonly reared on mulberry, oak and castor plants.

113. Which one among the following groups is the most abundant in terms of number of species identified?
(a) Fungi (b) Green plants
(c) Bacteria (d) Insects

(*d*) Insects are largest group in terms of number of species identified. Insects belong to phylum–Arthropoda.

114. Within an animal cell, the most abundant inorganic constituent of protoplasm is
(a) sodium and potassium salt
(b) water
(c) iron
(d) phosphate

(*b*) Inorganic constituents of protoplasm is comprised of water, various salts, gases, etc. Of these, water forms the main medium in which various metabolic processes are carried out. It is comprised of about 90% of the protoplasm.

115. In honey, which one among the following sugars predominates?
(a) Sucrose (b) Fructose
(c) Galactose (d) Maltose

(*b*) Fructose is the predominant sugar nearly 38.5% in the honey. Beside it, other contents are glucose, enzymes, proteins, vitamins and minerals.

116. What is 'breakbone fever' most commonly known as?
(a) Typhoid (b) Rhinitis
(c) Yellow fever (d) Dengue

(*d*) Dengue fever, also known as breakbone fever, is a mosquito-borne tropical disease caused by the dengue virus. Its symptoms include fever, headache, muscle and joint pains and a characteristic skin rash that is similar to measles.

Yellow fever is a viral disease. Its symptoms include fever, chills, loss of appetite, nausea, etc.

Rhinitis is an allergic response due to the viral infection of throat and nose.

Typhoid is caused by *Salmonella typhi* bacteria. It is diagnosed by widal test. Its symptoms are headache, fever and loss of appetite.

117. Which one among the following is not a sexually transmitted disease?
(a) Syphilis (b) Gonorrhea
(c) Scurvy (d) Hepatitis-B

(*c*) Scurvy is not a sexually transmitted disease. It is a deficiency disease that occurs due to lack of vitamin-C. Its symptoms include bleeding gums, weakness, anaemia, etc.

Sexually transmitted diseases are infections that spread primarily through person to preson by sexual contact, e.g. syphilis, hepatitis-B, gonorrhea, etc.

118. The site of cellular respiration in animal cell is
(a) ribosome
(b) mitochondria
(c) endoplasmic reticulum
(d) lysosome

(*b*) Mitochondria is the site of cellular respiration in animal cell. They carry out oxidation of food and transfer the energy to ATP (used to bring about energy requiring activities of the cell). The mitochondria are thus often described as the powerhouse or ATP mills of the cell.

119. Which of the following statements about vitamins are correct?
1. Vitamin-C is essential to make connective tissue in body.
2. Vitamin-D is needed for synthesis of eye pigment.
3. Vitamin-B_{12} helps in maturation of RBCs.
4. Vitamin-C is required to make use of calcium absorbed from the intestine.

Select the correct answer using the codes given below
(a) 1 and 2 (b) 1 and 3
(c) 2 and 4 (d) 1 and 4

(*b*) Vitamin-A is needed for synthesis of eye pigment and vitamin-D is required to make use of calcium absorbed from the intestine.

Hence statements 2 and 3 are correct and statements 2 and 4 are incorrect.

GENERAL STUDIES

44

HISTORY

2019 (II)

1. Which among the following struggles, based on Gandhi's philosophy of Satyagraha, involved the industrial working class?

(a) Champaran (b) Kheda
(c) Ahmedabad (d) Bardoli

(c) Ahmedabad Mill Strike, 1918 was based on Gandhi's philosophy of Satyagraha, that involved the industrial working class. The industrial dispute for wage hike was between the owners and workers of a cotton mill in Ahmedabad. The result of peaceful strike led by Gandhiji was successful and workers got a 35% wage increase.

2. Consider the following statements related to the Virashaiva tradition of Karnataka.

1. The Virashaiva Movement was led by a Brahmana named Basavanna, a minister in the court of the Kalachuri ruler in the twelfth century.
2. The Virashaivas encouraged practices, approved by the Dharmashastras, like post-puberty marriage and remarriage of widows.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) The twelfth century witnessed the emergence of a new movement called Virashaiva Movement in Karnataka. The movement was led by a Brahmana named Basavanna (1106-68) who was initially a Jaina and a minister in the court of a Kalachuri ruler (Chalukya, King of Gujarat). His followers were known as Virashaivas (heroes of Shiva) or Lingayats (wearers of the linga).
The Chalukyas were contested for the suzerainty by the kalachuris of Southern India in that period.
Hence, statement (1) is correct.

Also, the Dharmashashtras did not permit widow remarriages and post-puberty marriages. But the Virashaivas undertook them as a sign of refusal of the sacred Brahmanical scriptures. Hence, statement (2) is incorrect.

3. Consider the following statements related to Wellesley's administration.

1. In the six years of Wellesley's administration, the army accounted for 42.5% of the Company's total expenditure.
2. Wellesley's administrative measures were restricted to the affairs to the Company and had nothing to do with the commercial and military affairs of the Indian ruling families.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) During Lord Wellesley's tenure, army was an expensive machinery of the British East India Company. He had waged war on many Princely States and native rulers like Mysore, Marathas, Nizams etc. Hence, to sustain such a larger army, he spent huge amounts (42.5% of the company's total expenditure) on army.He also devised the infamous Subsidiary Alliance System which required to maintain British army in native ruler's courts.

Wellesley was an excellent administrator. He used his Subsidiary Alliance System to interfere with the commercial and military affairs to Indian rulers by making them mandatorily consult him on important decisions.
Hence, statements (1) & (2) are correct.

4. Consider the following statements concerning the Russian Revolution.

1. The Bolsheviks believed that in a repressive society like Tsarist Russia, the party should be disciplined and should control the number of its members.
2. The Mensheviks believed that the party should be opened to all (as in Germany).

Selects the correct answer using the codes given below.

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) The Bolsheviks and Mensheviks were the two factions of Russian Social Democratic Labour Party formed under leaderships of Vladimir Lenin and Julius Martov respectively.

Bolsheviks believed in limiting the membership of party to a core professional member body as it was easier to manage them which was necessary in a repressive society like Tsarist Russia.

On the other hand Mensheviks believed in power of masses and advocated liberal thoughts like entry of common masses in the party.
Hence, statements (1) & (2) are correct.

5. Which of the following was/were among the decisions taken by the First Continental Congress held in Philadelphia in September, 1774?

1. It rejected a plan for a colonial union under the British authority.
2. It drew up a statement of grievances.
3. It agreed upon a scheme to stop trade between the Colonies and England.

Which of the statement(s) given above is/are correct?

(a) 1 and 3 (b) Only 2
(c) 1 and 2 (d) 1, 2 and 3

(d) The First Continental Congress held in Philadelphia in September, 1774 is a major landmark in the history of USA. It was attended by representatives of 12 out of 13 British Colonies in USA, at the Carpenter's Hall. It was held in response to British impasses over the historic Boston Tea Party of December, 1773.

The major outcomes of this Congress were

- Rejection of Union of Colonies under British Crown.
- Scheme for boycott of trade between the Colonies and the Britishers.
- Mechanism of grievance discussion.

Hence, all the statements are correct.

6. Which among the following statements is not correct about the Reign of Terror?

(a) The Jacobins dashed out the Girondists with the help of Sans-Culottes.
(b) The Jacobins were in league with the French clergy.
(c) The arrest for anti-revolutionary activities was by law restricted to the political class.
(d) The Reign of Terror came to an end with the execution of Robespierre by guillotine in July, 1794.

(c) Reign of Terror was the period in French revolution in 1793 to 1794, marked by widespread violence and executions by the Jacobins, in association with the Sans-Culottes. The Jacobins were in league with the French clergy.

Jacobins dashed out Girondists with help of Sans–Culottes and were actively conspiring against Aristocracy. The laws for arresting were not limited to political class instead it was vaguely defined and anyone could be arrested.
Hence, option (c) is not correct about the Reign of Terror.

The reign came to an end on 27th July, 1794 with the execution of Robespierre by guillotine.

7. Who among the following was the first Indian to qualify for the Indian Civil Service?

(a) Satyendranath Tagore
(b) Surendranath Banerjee
(c) RC Dutt
(d) Subhas Chandra Bose

(a) Satyendranath Tagore was the first Indian to join the Indian Civil Service. He was selected in 1863 and posted in Bombay Presidency as a Civil Servant.

8. Which among the following was/were among the factors for England to experience the Industrial Revolution first?

1. The scientific inventions had paved the way for Industrial Revolution.
2. It had favourable social and political structures in place.
3. Navigable rivers had made inland transport easier.
4. It had seen the ascendancy of capitalist practices.

Select the correct answer using the codes given below.

(a) 1 and 3 (b) 1, 3 and 4
(c) Only 4 (d) All of these

(d) Industrial Revolution was the technological implementation in mass production during the second half of 18th century in England. It was a period of transition with new machines technologies and steam power driving the production capacity.
The causes/factors for England to experience the Industrial Revolution first were

1. Scientific innovations.
2. Favourable Social and Political Structure
3. Navigable rivers
4. Higher Capital in hands for setting up enterprise.

Hence, all the statements are correct.

9. Who was the Education Minister in the first Cabinet of Independent India?

(a) Sarvepalli Radhakrishnan
(b) Sardar Vallabhbhai Patel
(c) Maulana Abul kalam Azad
(d) Acharya Narendra Dev

(c) Maulana Abul Kalam Azad was the Education Minister in the first Cabinet of Independent India.

National Education Day is celebrated on 11th November every year to commemorate the birth anniversary of Maulana Abul Kalam Azad.

10. Who among the following died in exile?

(a) Rani Laxmibai
(b) Bahadur Shah Zafar
(c) Tantia Tope
(d) Nana Saheb

(b) **Bahadur Shah Zafar** → Died in Rangoon while in exile in 1862.

Rani Laxmibai → Died on the battlefield in Gwalior on 18th June, 1858.

Tantia Tope → Executed by Britishers in 1859 at Shivpuri.

Nana Saheb → Escaped to Nepal and is thoughts to have died in 1859.

11. Consider the following statements related to Dayanand's idea of social reform

1. Dayanand's reform envisaged a social order, where different castes performed functions suitable to their status determined by merit.
2. Dayanand's 'robust Vedic counterpart' challenged the masculine West that had enslaved the Aryavarta.

Which of the statement(s) given above is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Both the statements are correct as Dayanand envisioned a homogenous society where different castes performed all the functions suitable to their status determined by merit. Also, Dayanand's 'robust Vedic counterpart' challenged the masculine West that had enslaved the Aryavarta.

12. Which one of the following statements is not true about Ijarah system?

(a) It was a system of revenue farming.
(b) It encouraged the role of middlemen.
(c) Under this system, land revenue was fixed for the peasant.
(d) The Ijaradar paid a fixed amount to the state treasury.

(c) The Ijarah system was developed during sultanate period and gained prominence in Mughals after Akbar.
It was a system of revenue farming.
In this system, the revenue was collected by a middlemen called Ijaradars who had to pay a certain sum to ruler and keep rest with him. This led to ruthless extration of revenue from poor peasants. The revenue was not fixed and varied upon the production.
Hence, statement (c) is incorrect.

13. During the Swadeshi movement, a National College was started in Calcutta under the principalship of

(a) Rabindranath Tagore
(b) Aurobindo Ghosh
(c) Rajani Kant Sen
(d) Syed Abu Mohammad

*(**b**)* Aurobindo Ghosh founded the Bengal National College at Calcutta on 15th August, 1906 and became its first Principal. But following his trial in the cases like Bande Mataram (Sedition case), 1907 and Alipore Bomb Conspiracy Case, 1908, he resigned from the post.

14. The ruins of Vijayanagar (Hampi) were brought to public light in the year 1800 by the following British surveyor and engineer

(a) James Prinsep (b) Colin Mackenzie
(c) James Rennell (d) Charles Metcalfe

*(**b**)* **Colin Mackenzie** an engineer and antiquarian in East Indian Company, found the ruins of Vijayanagar (Hampi) in 1800. He prepared the first survey map of Hampi.

James Prinsep, deciphered Kharoshti and Brahmi scripts.

James Rennell, produce first accurate maps of Bengal and the outlines of India.

Charles Metcalfe, appointed as acting Governor-General of India in 1835.

15. Which country among the following was not a part of the Third Coalition against Napoleon?

(a) Russia (b) Prussia
(c) Sweden (d) France

(d) The Third Coalition was an alliance of Russia, Britain, Prussia, Austria and Sweden against the Napoleon I of France. Hence, France was a party to war, but not a coalition member.

16. Which one of the following pairs of travellers and their nationality is not correctly matched?

(a) Ibn Battuta — Moroccan
(b) Duarte Barbosa — Portuguese
(c) Jean Baptiste Tavernier — Dutch
(d) Niccolao Manucci — Venetian

*(**c**)* Option (c) is not correctly matched because Jean Baptiste Tavernier was a French gem merchant and traveller. He made 6 voyages to Persia and India in his lifetime from 1630 to 1668 and has left valuable accounts of Mughal empire, Nizams of Hyderabad, etc.

17. In the Manusmriti which form of marriage results from the 'Voluntary union of a maiden and her lover'?

(a) Eighth form
(b) Fifth form
(c) Seventh form
(d) Sixth form

*(**d**)* In the Manusmriti, sixth (Gandharva form of marriage) results from the 'Voluntary union of a maiden and her love'. In Gandharva Marriage, a groom and his bride could get married without their parents knowledge or sanction. Hence, option (d) is the correct answer.

18. Consider the following statements concerning the American War of Independence.

1. The Americans raised the slogan, 'No taxation without representation'.
2. The American merchants resorted to the boycott of the British products.

Select the correct answer using the codes given below.

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

*(**c**)* During the American War of Independence, in 1774, the first Continental Congress was held in Philadelphia which actively advocated three agendas

1. Rejection of Union of Colonies under British authority.
2. Scheme of boycott of trade with Britain.
3. No taxation without representation and redressal of grievances.

Hence, option (c) is correct.

2019 (I)

19. Who was the author of the book 'History of British India'?

(a) Charles Grant (b) John Stuart Mill
(c) James Mill (d) William Jones

*(**c**)* James Mill (1773-1836) was the author of the book 'History of British India'. This book is a history of company rule in India. Hence, option (c) is correct.

20. Which Viceroy had made the observation, 'It's a beautiful world if it wasn't for Gandhi ...'?

(a) Lord Irwin
(b) Lord Wavell
(c) Lord Mountbatten
(d) Lord Willingdon

*(**d**)* Lord Willingdon was 22nd Viceroy of India from 1931 to 1936. He mentioned Mahatma Gandhi in a letter written in 1933 by saying, ''It's a beautiful world if it was not for Gandhi who is really a perfect nuisance.'' Hence, option (d) is correct. The important events of his period include Second and Third Round Table Conferences, relaunching of Civil Disobedience Movement, Communal Award by Ramsay MacDonald (1932), Poona Pact (1932), the Government of India Act, 1935 and Separation of Burma from India.

21. Which Indian businessman favoured 'healthy capitalism' in helping Gandhiji to work towards a 'common object'?

(a) Ghanshyam Das Birla
(b) Ambalal Sarabhai
(c) Sir Biren Mukherjee
(d) TT Krishnamachari

*(**a**)* Ghanshyam Das Birla, an Indian businessman favoured 'healthy capitalism' in helping Gandhiji to work towards a 'common object.' Birla was a close associate and a steady supported of Mahatma Gandhi, whom he met for the first time in 1916. Gandhiji stayed at Birla's home in New Delhi during the last four months of his life.

In 1957, GD Birla was awarded India's second highest civilian honour, the Padma Vibhushan by the Government of India.

22. The art piece 'In Memoriam' was a creation of which one of the following European painters?

(a) Thomas Jones Barker
(b) Joseph Noel Paton
(c) Thomas Daniell
(d) Charles D'Oyly

*(**b**)* The art piece 'In Memoriam' was a creation of the European painter, Joseph Noel Paton. He was a Scottish artist, illustrator and sculptor. In 1844, Paton's first painting, **Ruth Gleaning**, was exhibited at the Royal Scottish Academy.

23. Who among the following historians have described the Quit India Movement as a 'spontaneous revolution'?

(a) Gordon Johnson
(b) David Arnold
(c) FG Hutchins
(d) Peter Robb

*(**c**)* FG Hutchins described the Quit India Movement as a 'spontaneous revolution'. Quit India Movement, 1942 was an important event of the Indian freedom struggle.

Directions (Q. Nos. 24-26) *The following questions items consist of two statements, statement I and statement II. Examine these two statements carefully and select the correct answer using the codes given below.*

Codes

(a) Both the statements are individually true and statement II is the correct explanation of statement I.

(b) Both the statements are individually true and statement II is not the correct explanation of statement I.

(c) Statement I is true, but statement II is false.

(d) Statement I is false, but statement II is true.

24. **Statement I** Abul Fazl shaped, represented and articulated the ideas associated with the reign of Akbar.

Statement II The qualities of Abul Fazl impressed Akbar who found the former suitable as an adviser and spokesperson for his policies.

› *(**a**)* Abul Fazl shaped, represented and articulated the ideas associated with the reign of Akbar because the qualities of Abul Fazl impressed Akbar, who found the former suitable as an adviser and spokesperson for his policies.

Hence, both the given statements are individually true and statement II is the correct explanation of statement I.

25. **Statement I** The Kisan Manifesto adopted by the All India Kisan Sabha in August, 1936 contained radical demands.

Statement II The All India Kisan Sabha was a part of the Congress and maintained close relationship with the Provincial Congress Committees.

› *(**d**)* The Kisan Manifesto released in August 1936, demanded abolition of the zamindari system and cancellation of rural debts.

All India Kisan Sabha was separate entity. It was founded at Lucknow in 1936 with Swami Sahajanand as its first President and NG Ranga as its General Secretary.

So, statement I is false, but statement II is true. Hence, option (d) is correct.

26. **Statement I** The British ruled India through a modern bureaucracy headed by the Indian Civil Service, whose members were recruited through merit based on open competition.

Statement II The Indian Civil Service was based on the whole hearted participation of Indians.

› *(**c**)* The British ruled India through a modern bureaucracy headed by the Indian Civil Service, whose members were recruited through merit based on open competition.

The Indian Civil Services was not based on the whole hearted participation of Indians.

Hence, the statement I is true and statement II is not true. Thus, option(c) is correct here.

27. The Karachi Resolution of Congress in 1931 advocated which one of the following issues?

(a) State shall not own or control key industries and services.

(b) State shall handover the key industries and services to the Indian business groups.

(c) State should allow the Indian business group to invest 50% of the capital.

(d) State shall own or control key industries and services.

› *(**d**)* The Karachi Resolution of Congress in 1931 advocated that, 'State shall own or control key industries and services'.

The Karachi Resolution was passed by the INC at its Karchi Session in 1931.

The session was conducted in the shadow of three major events—

First, Mahatma Gandhi had just been released from prison following his Salt Satyagraha.

Second, the Gandhi-Irwin pact had just been concluded which had brought the Civil Disobedience Movement to an end.

And third, the British Government had, a week before the session, executed Bhagat Singh and two of his associates in connection with the Kakori Conspiracy Case.

28. The treaty of Schonbrunn (1809) was signed after which one of the following battles?

(a) Battle of Austerlitz
(b) Battle of Tilsit
(c) Battle of Wagram
(d) Battle of Lisbon

› *(**c**)* The treaty of Schonbrunn (1809) or treaty of Vienna was signed after the **Battle of Wagram**. This treaty was signed between Austria and France.

Battle of Austerlitz (2nd Dec, 1805), the first engagement of the war of the third coalition and one of Napoleon's greatest victories.

Battle of Tilsit fought between French and Russia on 14th June, 1807.

The battle of Lisbon (1st July to 25th Oct, 1147) was the military action that brought the city of Lisbon under definitive Portuguese control and expelled its Moorish overloads.

29. Which of the following statements about the New Model Unions is/are correct?

1. The New Model Unions were formed in the 1850s.
2. The New Model Unions were formed in the 1880s.
3. The New Model Unions comprised a Labour Party idea.
4. The New Model Unions excluded women in the 1920s.

Select the correct answer using the codes given below.

(a) Only 1 (b) Only 2
(c) 3 and 4 (d) Only 3

› *(**a**)* New Model Union or New Model Trade Union (NMTU) were a variety of Trade Union prominent in the United Kingdom. These were formed in the 1850s. These were generally tended to be restricated to individual trades. These union were not comprised by a Labour Party idea and it also did not excluded women.

Hence, statement (1) is correct.

30. The Truman Doctrine of 1947 was announced to achieve which one of the following?

(a) Containment of the USSR
(b) Increasing agricultural production in the USA
(c) Offering friendship to Europe
(d) Strengthening the UNO

› *(**a**)* The Truman Doctrine of 1947 was announced to achieve containment of the USSR.

It was announced by Harry S Truman on 12th March, 1947 and further developed on 12th March, 1948.

31. Which of the following statements is/are correct?

1. In 1948, Burma was admitted to the United Nations and immediately supported the USA in the Cold War.
2. In 1948, Burma joined the United Nations, but refused to denounce China as the aggressor in the Korean War.

Select the correct answer using the codes, given below.
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(b) On 4th Jan, 1948 Burma became an independent republic, under the terms of the Burma Independence Act, 1947. The new country was named the Union of Burma.

Apart from this, in 1948, Burma rejoined the United Nations but refused to denounce China as the aggressor in the Korean war.
Hence, statement (2) is correct.

32. Who among the following presented 'The April Thesis' to the Russian people in 1917?
(a) Stalin
(b) Trotsky
(c) Bukharin
(d) Lenin

(d) Vladimir Lenin presented 'The April Thesis' to the Russian people in 1917. It was the series of ten directives issued by Lenin. It was aimed at fellow Bolsheviks in Russia and returning to Russia from exile. The April Thesis influenced the October Revolution.

33. The treaty of Yandabo was signed in
(a) 1826
(b) 1825
(c) 1824
(d) 1823

(a) The treaty of Yandabo was signed on 24th February, 1826 between the Britishers and Burmese. It brought peace between East India Company and the king of Ava that ended the first Anglo-Burmese War. This treaty started the British Rule in Assam.

34. The Azamgarh Proclamation of 25th August, 1857 stressed on which one of the following issues?
(a) Hindu-Muslim divide
(b) Support to the English government
(c) The return of the Badshahi
(d) The imposition of heavy Jumas (revenue demand)

(a) The Azamgarh Proclamation of 25th August, 1857 stressed on Hindu-Muslim divide. It was published in Delhi Gazette in the midst of the Sepoy Mutiny (1857). The author was most probably Firoz Shah (grandson of Bahadur Shah Zafar), who fought against the British in Lucknow and Awadh (Oudh). The original document was published in Urdu.

2018 (II)

Directions (Q.Nos. 35 and 36) *The following questions items consist of two statements, statement I and Statement II. Examine these two statements carefully and select the correct answer using the codes given below.*

Codes
(a) Both the statements are individually true and statement II is the correct explanation of statement I.
(b) Both the statements are individually true and statement II is not the correct explanation of statement I.
(c) Statement I is true, but statement II is false.
(d) Statement I is false, but statement II is true.

35. **Statement I** The Government of India Act, 1935 introduced Dyarchy at the centre.

Statement II The provincial autonomy was granted to the provinces.

(b) The Government of India Act, 1935 marked a milestone towards a completely responsible government in India. It was a lengthy and detailed document having 321 Sections and 10 Schedules.

The Act provided for the adoption of Dyarchy at the Centre. Consequently, the federal subjects were divided into reserved subjects and transferred subjects. And abolished dyarchy in the provinces and introduced 'provincial autonomy' in its place.

Both the statements are individually true and statement II is not the correct explanation of statement I.

36. **Statement I** Mughal Painting reached its climax during the reign of Jahangir.

Statement II Aurangzeb's Court was adorned by some of the best known artists of the Mughal School of Paintings.

(c) During the reign of Jahangir (1605–1627) the Mughal painting reached its climax. He favoured paintings of events from his own life rather than illustrated fiction. He encouraged portraiture and scientific studies of birds, flowers and animals, which were collected in albums. Mansur and Manohar were among his famous painters.
But during the reign of Aurangzeb (1659–1707) the Mughal painting started to decline, although his ornate Pearl Mosque (1662) at Delhi is worthy of mention. Many artists then joined Rajput courts, where their influence on Hindu painting is clearly evident.

Hence, statement (I) is correct and statement (II) is incorrect.

37. Consider the following statements about merchant guilds of South India
1. Ayyavole merchant guild was originally established in Aihole.
2. Manigraman merchant guild was sub-ordinated to the Anjuvannam merchant guild in the 13th century.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Southern Indian trade guilds were formed by merchants in order to organise and expand their trading activities. The two most important merchant guilds of South India were known as the Ayyavole and the Manigraman.

Ayyavole were a merchant guild from Aihole that provided trade links between trading communities in Tamil Nadu, Karnataka and Andhra Pradesh.

Manigraman guild flourished in Tamil Nadu in the Pallava and Chola periods and was active in South-East Asia. The Manigraman merchants enjoyed a special position as compared to the other merchants.

Anjuvannam was another body of merchants in South India, which almost certainly represented an association of foreign merchants who traded across the Arabian sea. It interacted both with local merchants as well as the Ayyavole and Manigramam organisations.
Hence, statement (1) is correct and statement (2) is incorrect.

38. Which one of the following pairs is correctly matched?

Bhakti Saint		Philosophy
(a) Shankara	:	Avadhuta
(b) Ramananda	:	Kevaladvaita
(c) Ramanuja	:	Vishishtadvaita
(d) Chaitanya	:	Advaita

(c) Philosophy of Vishishtadvaita was pioneered by Ramanujacharya during the 11th century. It literally means the Unique Advaita, i.e. Advaita with some amendments. Hence, option (c) is correct.

Kevaladvaita is another name for the philosophy of advaita which was expounded by Adi Shankara.

The meaning of **Advaita** is non-dualism or belief in one reality.

Avadhuta is a Sanskrit term used to refer to a person who has reached a stage in their spiritual development in which they are beyond worldly concerns.

39. When did the Stamp Act Congress consisting of delegates from 9 of the 13 colonies of America meet in New York City?

(a) 1763 (b) 1764 (c) 1765 (d) 1766

*(**c**)* The Stamp Act Congress or first Congress of the American colonies, was a meeting held between 7th and 25th October, 1765, in New York City. It consists of representatives from some of the British colonies in North America. At this meeting, the delegates discussed and united against the stamp act, which required the use of specially stamped paper for legal documents, playing cards, calendars, newspapers and dice for virtually all business in the colonies.

40. Who among the following travellers was from Italy and visited Vijayanagar Kingdom in the fifteenth century?

(a) Nikitin (b) Fa-Hien
(c) Bernier (d) Nicolo Conti

*(**d**) Important Foreign Travellers who visited India*

Nicolo Conti (AD 1420-1421), an Italian traveller, visited Vijayanagar empire during the reign of Devaraya-I (the Tuluva dynasty ruler).

Nikitin (AD 1470–1474) was a Russian merchant,who describes the condition of the Bahmani kingdom under Muhammad III (1463–82).

Fa-Hien was a Chinese pilgrim who visited India during the reign of Chandra Gupta II. His primary aim was to visit the Buddhist religious places and to take with him the copies of the Buddhist religious texts.

François Bernier (25th September, 1620–22nd September, 1688) was a French physician and traveller.

He was briefly personal physician to Mughal prince Dara Shikoh (son of the Mughal emperor Shah Jahan), and was attached to the court of the emperor Aurangzeb.

41. Where did the French East India Company first establish its factory in India?

(a) Calicut (b) Surat
(c) Pondicherry (d) Masulipatnam

*(**b**)* Among the Dutch, Danish, Portuguese and French, the French East India Company was the last to be formed. It was founded by Jean-Baptiste Colbert in 1664. Under Francis Caron, the company established it's first factory at Surat in 1667 and second factory was established at Masulipattanam a year later.

In 1741, Joseph Francois Dupleix began to cherish the ambition of a French Empire in India. But the problem began to emerge when the conflict of the British and French started. The hopes of the French company were finally ruined with the arrival of Robert Clive (a British General) in India in 1744.

42. The Central Vigilance Commission was established on the recommendation of which one of the following Committees?

(a) Santhanam Committee
(b) Dinesh Goswami Committee
(c) Tarkunde Committee
(d) Narasimham Committee

*(**a**)* The Central Vigilance Commission (CVC) was set up by the government in February,1964 on the recommendations of the Santhanam Committee on Prevention of Corruption, headed by K. Santhanam, to advise and guide Central Government agencies in the field of vigilance.

CVC is conceived to be the apex vigilance institution, free of control from any executive authority, monitoring all vigilance activity under the Central Government and advising various authorities in Central Government organisations in planning, executing, reviewing and reforming their vigilance work. It consists of a Central Vigilance Commissioner as a chairperson; and not more than two Vigilance Commissioners as its members.

Hence, option (a) is correct.

43. Match List I with List II and select the correct answer using the codes given below the lists

List I (Author)	**List II** (Book)
A. Bal Gangadhar Tilak	1. The Arctic Home in the Vedas
B. Dadabhai Naoroji	2. Hind Swaraj
C. Mahatma Gandhi	3. The Discovery of India
D. Jawaharlal Nehru	4. Poverty and Un-British Rule in India

Codes

	A	B	C	D
(a)	3	4	2	1
(b)	3	2	4	1
(c)	1	4	2	3
(d)	1	2	4	3

*(**c**)* The correct matching is A-1, B-4, C-2, D-3. **The Arctic Home in the Vedas** is a history book on the origin of Aryanic People by Bal Gangadhar Tilak. It propounded the theory that the North Pole was the original home of Aryans during the pre-glacial period which they had to leave due to the ice deluge around 8000 BC.

Hind Swaraj or **Indian Home Rule** is a book written by Mahatma Gandhi in 1909. In it, he expresses his views on Swaraj, modern civilisation, mechanisation etc.

The Discovery of India was written by India's first Prime Minister Pt. Jawaharlal Nehru during his imprisonment in 1942–46 at Ahmednagar fort in Maharashtra. It is an honour paid to the rich cultural heritage of India, its history and its philosophy .

Poverty and Un-British Rule in India was authored by Dadabhai Naoroji. It brought attention to the draining of India's wealth into Britain.

44. Who among the following leaders started the Indian Home Rule League?

(a) Gopal Krishna Gokhale
(b) Mahatma Gandhi
(c) Bal Gangadhar Tilak
(d) JB Kripalani

*(**c**)* Between the years 1916 and 1918, the Indian independence movement witnessed the growth and spread of the Home Rule League started by leaders like Bal Gangadhar Tilak and Annie Besant.

The aim of the Home Rule Movement was the attainment of home rule or a dominion status for India under the British Empire. This movement was carried out through the two home rule leagues – Indian Home Rule League and Home Rule League.

Tilak launched the Indian Home Rule League in April, 1916 at Belgaum, while Annie Besant launched the Home Rule League in September, 1916 at Madras. The two leagues worked closely with one another. However, they did not merge to avoid friction between both the leaders.

45. The palace of English East India Company settlement in Madras was known as

(a) Fort William
(b) Fort St. George
(c) Elphinstone Circle
(d) Marble Palace

*(**b**)* The English East India Company was originally known as 'The Company of Merchants of London trading into the

East Indies.' It was founded by a Royal Charter, signed by Queen Elizabeth-I, on 31st December, 1600. Sir Thomas Smith was the Company's first Governor.

Soon, English open their trading centres at several parts of India. In 1613, Mughal emperor Jahangir issued a farman (permission letter) to English to establish their trade centre in India. Hence, British established their 1st trading factory at Surat in 1613.

In 1639 AD, the East India Company obtained the lease of the city of Madras from the local king where it built Fort St. George to protect its factory. So, the palace of English East India Company settlement in Madras was known as Fort St. George.

46. The Theosophical Society was led by
(a) AO Hume
(b) Arthur Griffith
(c) Annie Besant
(d) Lord Dufferin

*(**c**)* The Theosophical Society was founded by Madame Blavatsky and Col. Olcott in 1875 in New York. It was only in 1879, that this ideology gained its roots in the India. It was crystallised in the Madras Presidency with its headquarters in Adayar.

The Theosophical Society was led by Annie Besant in India. It was based on the principles of Universal brotherhood and the study of comparative religion and philosophy. The chief aim of the Theosophical Society in India was to establish the lost glory and honour of Hindu religion. Mrs. Annie Besant became its President and worked for the upliftment of Indian mass.

47. Which one of the following statements about Bipin Chandra Pal is correct?
(a) He was a member of the moderate group of Congress.
(b) He was a member of the extremist group of Congress.
(c) He was the Minister of Defence in the first government of independent India.
(d) He was the Chief Minister of West Bengal.

*(**b**)* Bipin Chandra Pal was a member of the extremist group of Congress. In the beginning of the 20th century, a new class of national leaders emerged in India which was different from the moderate group. They were known as extremists.

The Important extremist leaders were Lala Lajpat Rai, Bal Gangadhar Tilak, Bipin Chandra Pal (Lal-Bal-Pal) leading the extremist cause in Punjab, Bombay and Bengal respectively. Their goal was 'Swaraj' and did not stick to constitutional methods of protest and demand rather they resorted to boycotts, strikes, etc. Hence, option (b) is correct.

48. Which of the following statements about Brahmo Samaj is/are correct?
1. The Brahmo Marriage Act of 1872 allowed inter-caste and widow re-marriage only if the contracting parties declared themselves to be non-Hindus.
2. Keshub Chandra Sen arranged the marriage of his minor daughter with the Maharaja of Burdwan.
3. Keshub Chandra Sen's followers broke away to form the Naba Brahmo Samaj.
4. The Brahmo Samaj grew from a small elite group to a mass movement in the 19th century.

Select the correct answer using the codes given below.
(a) Only 1
(b) 1 and 2
(c) 2 and 3
(d) All of the above

*(**a**)* The Brahmo Marriage Act was passed in 1872. It allowed inter-caste and widow remarriage if the contracting parties declared themselves to be non-Hindus.
- Keshub Chandra Sen arranged marriage of his minor daughter with Maharaja of Cooch Behar.
- His followers formed Sadhaman Brahmo Samaj of became a member.
- Brahmo Samaj in 1856 but founded his own breakaway "Brahmo Samaj of India" in 1866.

Hence, statement (1) is correct and statement (2, 3 & 4) is correct.

2018 (I)

49. Who among the following first used the term 'Industrial Revolution' in English to describe, the changes that occurred in British industrial development between 1760 and 1820?
(a) Karl Marx
(b) Georges Michelet
(c) Arnold Toynbee
(d) Friedrich Engels

*(**c**)* The term Industrial Revolution was first popularised by the English economic historian Arnold Toynbee to describe Britain's economic development from 1760 to 1820.

50. Which one of the following statements about the Olympe de Gouges (1748- 1793) is correct?
(a) She declared that although citizens should have equal rights, they are not entitled to the same honours by the state.
(b) She was a supporter of the Jacobin government.
(c) She was jailed for treason by the National Assembly.
(d) She declared that the nation is the union of woman and man.

*(**a**)* Only the first statement (a) is correct about the Olympe de Gouges. She declared that although citizens should have equal rights they are not entitled to the same honours by the state.

Olympe de Gouges was a French Playwright and political activist.

51. Which one of the following statements about Renaissance Humanist culture is NOT true?
(a) It slackened the control of religion over human life.
(b) It believed that human nature was many-sided.
(c) It was concerned with good manners.
(d) It criticised material wealth, power and glory.

*(**c**)* The term Renaissance Humanist was derived from the program of studies called the 'Studia Humanitatis'. However, the idea of calling this 'Humanism' only really arose in the nineteenth century. It majorly advocated that human outlook should be based on a modern plan and not blindly follow a religious plan. It never spoke about good manners. Hence, option (c) is not correct about Renaissance Humanist culture.

52. Who gifted the *Badshahnama* to King George in 1799?
(a) Abul Fazl
(b) Abdul Hamid Lahori
(c) Nawab of Awadh
(d) William Jones

(c) In 1799, the Nawab (Provincial Governor) of Awadh in Northern India sent the Badshahnama, to King George III of England. Badshahnama is a genre of works written as the official visual history of Mughal emperor, Shah Jahan's reign.

53. What is the name of the award given to meritorious men in the Mughal Court in the form of a robe of honour that was once worn by the Emperor?
(a) Sarapa (b) Patka
(c) Padma Murassa (d) Khilat

(d) The granting of title to men of merit was an important feature of Mughal polity. The title, Asaf Khan was for one of the highest minister. The title 'Mirza Raja' was accorded by Aurangzeb to his two high-ranking officials Jai Singh and Jaswant Singh.

Sarapa, consisted of tunic a turban and a sash (Patka). Jwelled ornaments were often given as gifts by the emperor. The lotus blossom set with jewels (Padma Murassa) was only given in exceptional circumstances.

The correct option is (d).

54. Who among the following social reformer started a society for the encouragement of widow remarriage in 1866 in Maharashtra ?

(a) Bal Gangadhar Tilak
(b) Jyotirao Phule
(c) Vishnushastri Pandit
(d) Pandita Ramabai

(c) Vishnushastri Pandit encouraged the widow marriage in 1866 in Maharashtra. He was inspired by Phule and also founded an institution named as Punar Vivahtojak Mandal (remarriage association) for remarriage of widows in 1866.

55. Name the first major voluntary association representing primarily Indian landlord interests that was set up in Calcutta in 1851 ?

(a) British Indian Association
(b) Landholder's Society
(c) Madras Native Association
(d) Bombay Association

(a) British Indian Association was created after amalgamating the "Landholders Society" and "British India Society" on 31st October, 1851 in Calcutta. This was the first political organisation that brought the Indian landlords together. The President of the first committee of this organisation was Raja Radhakanta Deb.

56. Who among the following introduced the Permanent Settlement of Bengal in 1793 ?

(a) Lord Cornwallis (b) Lord Ripon
(c) Robert Clive (d) John Adam

(a) The Permanent Settlement was introduced by Lord Cornwallis in 1793. It was an agreement between the British East India Company and the Landlords of Bengal to settle the Land Revenue to be raised. In this agreement it was agreed that the landlords would have perpetual and hereditary rights over the land, so long as they pay the fixed revenue to the British Government.

57. Name the rebel who fought against the British in the battle of Chinhat in the course of the 1857 Revolt?

(a) Ahmadullah Shah
(b) Shah Mal
(c) Mangal Pandey
(d) Kunwar Singh

(a) Ahmadullah Shah fought against the British in the Battle of Chinhat in the course of the 1857 Revolt.

The Battle of Chinhat between the British forces and Indian rebels, at Ismailganj, near Chinhat. The British were led by The Chief Commissioner of Oude, Sir Henry Lawrence. The insurgent force, which consisted of mutineers from the East India Company's army and retainers of local landowners was led by Barkat Ahmad, a mutineer officer of the Company's army, under Ahmadullah's command.

58. Which of the following is/are NOT historical biography/ biographies?

1. Dipavamsa
2. Harshacharita
3. Vikramankadevacharita
4. Prithvirajavijaya

Select the correct answer using the codes given below.

(a) Only 1 (b) 2 and 3
(c) 2, 3 and 4 (d) All of these

(a) Dipavamsa is one of the most important works in Pali Literature. It details the tooth relic and Bodhi Tree's arrival in Sri Lanka. It also deals with the arrival of Buddha's teaching and preachers in Sri Lanka. It mentions that Buddha visited Kelaniya and Dighavapi in Sri Lanka.

All the other options are biographies about kings.

Hence, option (a) is the correct answer.

59. Which of the following pairs are correctly matched ?

	List I (Traveller)	List II (Country from)
1.	Marco Polo	Italy
2.	Ibn Battuta	Morocco
3.	Nikitin	Russia
4.	Seydi Ali Reis	Turkey

Codes

(a) 1, 2 and 3 (b) 2 and 3
(c) 1 and 4 (d) All of these

(d) **Marco Polo** was an Italian merchant, explorer and writer, born in the Republic of Venice.

Ibn Battuta was a Moroccan scholar who widely travelled the medieval world.

Afanasy Nikitin was a Russian merchant of Tver and one of the first Europeans to travel and to document his visit to India.

Seydi Ali Reis was an Ottoman admiral and navigator from Turkey. He commanded the left wing of the Ottoman fleet at the naval Battle of Preveza in 1538.

So, all the given options are correct. Hence, option (d) is correct.

60. Which of the following clans are included in the Agnikula Rajputs ?

1. Pratiharas 2. Chalukyas
3. Paramaras 4. Chahamanas

Select the correct answer using the codes given below.

(a) 1 and 3 (b) 1, 3 and 4
(c) 1, 2, 3 and 4 (d) 2 and 4

(c) The four Rajput clans from Agnikula were Chauhans, Chalukyas, Parmaras and Pratiharas. This theory comes from the Prithvirajraso of Chandarbardai. The Rajputs descended from Agni and this theory is based upon the Agnikula Legend of Bhavishyapurana.

61. Who among the following was the author of Humayun Nama?

(a) Roshanara Begum
(b) Ruquaiya Sultan Begum
(c) Gulbadan Begum
(d) Gauhara Begum

(c) Gulbadan Begum is best known as the author of Humayun Nama, the account of the life of her half-brother, Emperor Humayun, which she wrote on the request of her nephew, Emperor Akbar.

2017 (II)

62. Ashoka's connection with Buddhism is evident from which one of the following edicts?

(a) Major Rock Edict 13
(b) Rock Edict 6
(c) Minor Rock Edict 1
(d) Pillar Edict 4

(a) Ashoka famously converted to Buddhism and renounced violence soon after being victorious in a Kalinga war. The Major Rock Edict 13 mentions the victory over Kalinga which later dramatically transformed Ashoka and he became Buddhist.

63. In the Mesopotamian records, which one of the following terms was used for the Indus Valley (Harappans)?

(a) Dilmun (b) Meluha
(c) Magan (d) Failaka

(b) The Mesopotamian records from about 2350 BC onwards refer to trade relations with Meluha, which was the ancient name given to the Indus region, and they also speak of two intermediate stations called 'Dilmun' (identified with Bahrain on the Persian Gulf) and Makan (Makran Coast).

64. Who among the following is the author of the book 'The Indian Struggle, 1920-42'?

(a) Maulana Abul Kalam
(b) Jayprakash Narayan
(c) Subhash Chandra Bose
(d) Manabendra Nath Roy

(c) 'The Indian Struggle, 1920–42' is a two-part book by the Indian nationalist leader Netaji Subhash Chandra Bose that covers the 1920–42 history of the Indian independence movement to end British imperial rule over India.
The Indian Struggle was published in the country in 1948 after India became independent. The book analyses a period of the Indian independence struggle from the Non-cooperation and Khilafat movements of the early 1920s to the Quit India and Azad Hind movements of the early 1940s.

65. Which one of the following about the Swadeshi Campaign in 1896 is not correct?

(a) Its centre was Maharashtra.
(b) Its main participants were students.
(c) It opposed the levy of tariff on imports.
(d) It publicly burnt foreign clothes.

(d) Swadeshi Campaign in 1896 was aimed to boycott products made in Britain and use product made in India. It publicly didn't burnt foreign clothes. Hence, option (d) is not correct.

66. Which one of the following associations was founded in London by Dadabhai Naoroji in 1866?

(a) The Bengal British India Society
(b) The East India Association
(c) The British Indian Association
(d) The Madras Native Association

(b) In 1867, Dadabhai Naoroji helped to establish The East India Association. It superseded the London Indian Society and was a platform for discussing matters and ideas about India. The association's first president was Lord Lyveden.

67. Consider the following movements :

1.Moplah Rebellion
2.Bardoli Satyagraha
3.Champaran Satyagraha
4.Salt Satyagraha

Which one of the following is the correct chronological order of the above ascending order?

(a) 1-3-4-2 (b) 3-1-2-4
(c) 2-3-1-4 (d) 4-2-1-3

(b) The **Champaran Satyagraha** of 1916, in the Champaran district of Bihar, (India) during the period of the British Raj, was the first Satyagraha movement inspired by Mohandas Gandhi and a major revolt in the Indian Independence movement.

The **Moplah Rebellion** or the Malabar Rebellion was an extended version of the Khilafat Movement in Kerala in 1921.

The **Bardoli Satyagraha** of 1928, in the State of Gujarat, India during the period of the British Raj, was a major episode of Civil Disobedience and revolt in the Indian Independence movement.

The **Salt Satyagraha** of 1930, also known as the Dandi March and the Dandi Satyagraha, was an act of Non-violent Civil Disobedience in Colonial India initiated by Mohandas Karamchand Gandhi to produce salt from the seawater in the coastal village of Dandi.

Hence, option (b) is correct.

68. Which one of the following travelogues has given an insight on the reign of Muhammad-bin-Tughluq?

(a) Ibn Battuta's Rihla
(b) Francois Bernier's Travels in the Mogul Empire
(c) Niccolao Manucci's Storia do Mogor
(d) Tavernier's Travels in India

(a) Ibn Battuta gave an insight on the reign of Muhammad-bin-Tughluq. Ibn Battuta has discussed his travels and incursions in contemporary Islamic world and documented them in *Rihla*.

He was appointed as Qazi by Muhammad-bin-Tughluq and was also appointed ambassador to China. Ibn Battuta gives details about the later part of his rule. Portrait of Muhammad-bin-Tughluq in Batutta's words, "his gateway is never free from a beggar whom he has relived and never free from a corpse, he has slain".

69. Which one of the following was not a Chishti Sufi saint?

(a) Khwaja Moinuddin
(b) Baba Fariduddin Gani-i-Shakar
(c) Nizamuddin Auliya
(d) Shaikh Bahauddin Zakariya

(d) Shaikh Bahauddin Zakariya (1170-1262) was not a Chishti Sufi saint. He was sufi of Suhrawardiyya order. He was sunni muslim scholar, Sufi saint and poet who established the Suhrawardiyya order of Baghdad in medieval South Asia.

70. Which one of the following statements about the Ilbert Bill is correct?

(a) It proposed that the Indian magistrates would try Europeans in criminal cases.
(b) It allowed Indians to file criminal cases against Europeans.
(c) It authorised Indian ICS officers to try Europeans in courts.
(d) It was an agitation led by Ilbert in support of the nationalists.

(a) Ilbert Bill was passed during the Viceroyship of Lord Ripon in 1883-84. As per this bill, Indian magistrate could preside over the trial of Englishmen, European and Anglo-Indians in criminal cases. Introduction of the bill led to intense opposition in Britain and this growing bitterness led to formation of Indian National Congress.

Hence, option (a) is correct.

71. Who among the following was believed to be a leader of the Sanyasis and Fakirs conspiring against the British in 1857?

(a) Mangal Pandey
(b) Bahadur Shah II
(c) Queen Zeenat Mahal
(d) Nana Sahib

(b) The Sanyasis and Fakirs revolted against British from AD 1763 to 1800 due to the prohibition imposed on their pilgrimage by the East India Company's officials.

The central leadership comprised of rebel council headed by emperor Bahadur Shah Jafar II. Though Bahadur Shah Jafar's wife Jeenat Mahal tried to conspire with English, but she was not the part of central leadership. Hence, option (b) is correct answer.

72. Who among the following was the founder of the Awadh Kingdom in the 18th century?

(a) Murshid Quli Khan
(b) Saadat Khan
(c) Alivardi Khan
(d) Sarfaraz Khan

(b) Saadat Khan was the founder of Awadh Kingdom in the 18th century. The Nawab of Awadh was the title of the rulers who governed the State of Awadh (anglicised as Oudh) in North India during the 18th and 19th centuries. The Nawabs of Awadh belonged to a dynasty of Persian origin from Nishapur (Iran).

73. Who among the following was the founder of the Young Bengal Movement?

(a) Henry Vivian Derozio
(b) David Hare
(c) Dwarkanath Tagore
(d) Prasanna Kumar Tagore

(a) Young Bengal Movement was launched by Henry Louis Vivian Derozio (1809-31), who had come to Calcutta in 1826 and was appointed in the Hindu College as a teacher of English literature and History.

Besides this, he edited Hesperus and Calcutta Literary Gazette. He was connected with India Gazette as well. Hence, option (a) is correct.

74. Which one of the following statements about the Quit India Movement is not correct?

(a) It broke out in August, 1942.
(b) Ahmedabad Textile Mills went on strike for more than three months.
(c) Muslim League and Hindu Mahasabha actively participated in the movement.
(d) Communist Party did not support the movement.

(c) Hindu nationalist parties like the Hindu Mahasabha openly opposed the call for the Quit India Movement and boycotted it officially. The Muslim League opposed the Quit India Movement as it was of the view that if the British left India in its current state, Muslims as a minority would be oppressed by the Hindu majority.

Hence, statement (c) is incorrect.

2017 (I)

75. Which one of the following statements regarding King Krishnadeva Ray is not correct?

(a) He was a great scholar of Telugu and Sanskrit.
(b) Foreign travellers Paes and Nuniz visited his court.
(c) Barbosa praised him for the great justice and equity prevailing in his empire.
(d) He wrote his magnum opus Amuktamalyada in Sanskrit.

(d) Amuktamalyada is an epic poem in Telugu composed by Krishnadeva Ray. Krishnadeva Ray was an emperor of the Vijayanagar empire who reigned from 1509-29. He was the ruler of the Tuluva dynasty. Hence, statement (d) is incorrect.

76. Which one of the following was not a cause of the Revolt of 1857?

(a) The rumour that the British had mixed the bone dust of cows and pigs into the flour being sold in the market.
(b) The prophecy that British rule would come to an end on the centenary of the Battle of Plassey on 23rd June, 1857.
(c) Popular discontent with British rule.
(d) The prophecy that the end of British rule would lead to the end of the Kaliyuga and the return of Ram Rajya.

(d) The several prophecies and rumours behind the Revolt 1857 are

1. Mixing of bone dust of cows and pigs into wheat flour.
2. British rule would come to an end on the centenary of the Battle of Plassey on 23rd June, 1857.
3. Greased cartridges with cow and pig fat.
4. Conversion by Christian missionaries of Hindu folks into Christian religion.

Hence, the end of Kaliyuga and advent of Ram Rajya was not an rumour of the 1857 revolt.

77. Which one of the following was not a feature of the Subsidiary Alliance of Lord Wellesley?

(a) The British were responsible for protecting the ally from any external threats.
(b) All internal threats were to be handled by the ally alone, with no help from the British.
(c) The ally was to provide resources for maintaining a British armed contingent stationed in its territory.
(d) The ally could not enter into any agreement with other rulers without the permission of the British.

(b) The major principle of subsidiary alliance was princely rulers were not allowed to have an independent armed force. They were to be protected by the East India Company.

Hence, option (b) is not a feature of the subsidiary alliance of word wellesley.

78. Who among the following is one of the authors of the book 'Philosophy of the Bomb?'

(a) Bhagat Singh (b) Jawaharlal Nehru
(c) Surya Sen (d) Yashpal

(a) Though it is commonly believed that Bhagat Singh wrote the book 'Philosophy of the Bomb', infact Bhagwati Charan Verma, the important member of Hindustan Socialist Republican Association (HSRA) was behind Philosophy of Bomb, who edited the book. Hence, option (a) is correct.

79. Which of the following statements about the Chittagong group is not correct?

(a) Its membership included a large number of youth including Ganesh Ghosh, Lokenath Baul and Anant Singh.
(b) Its leader Surya Sen had been a lawyer in Dhaka before joining the group.
(c) Surya Sen and his group were closely associated with Congress work in Chittagong.
(d) This group had prepared an action plan to occupy the armouries in Chittagong.

(b) Surya Sen was a school teacher by profession and was popularly called as 'Master Da', who is noted for leading the 1930 Chittagong armoury raid.

Hence, option (b) is incorrect.

80. Which one of the following was a significant feature of the Quit India Movement?

(a) Women did not play an important role in the movement.
(b) Nasik in Maharashtra was an important regional base during the movement.
(c) It was marked by anti-zamindar violence.
(d) It was marked by the emergence of parallel governments in different parts of India.

(d) During Quit India Movement, 1942 there were four parallel government established in India—

1. Baliya (UP), 2. Tamluk (Bengal), 3. Satara (Maharashtra) and 4. Talcher (Odisha).

Among these, Satara had the longest running parallel government. Hence, option (d) is the correct answer.

81. Which one of the following characteristics does not describe the Khilafat Movement?

(a) Mahatma Gandhi sought to link it to the Non-Cooperation Movement.
(b) It was not supported by the Congress.
(c) It demanded that the Turkish Sultan must retain control over Muslim sacred spaces in the erstwhile Ottoman empire.
(d) It was led by Muhammad Ali and Shaukat Ali.

(b) The support of the Congress was essential for the Khilafat Movement to succeed and Gandhiji put his weight behind the Khilafat Movement alongwith Swaraj Movement for Hindu-Muslim unity.

82. Which one of the following was not a feature of railways in colonial India?

(a) The main purpose of the setting up of railways in India was to serve the interest of the empire.
(b) British capital investments were invited with 15% guaranteed interest to be paid if necessary from Indian revenues.
(c) The construction work disturbed ecology.
(d) The construction of the railways was planned in such a way that it connected the internal markets with the ports, but provided no interconnection between internal market cities.

(c) The introduction and expansion of Railways in India by British though completely economic advantages for the British but it also played an important role for regional connectivity.

Among the given options, (c) is not a feature of railway in colonial India as construction work does not disturbed the ecology.

83. Which colonial administrator made the following declaration about the partition of Bengal in 1904? ''Bengal united is a power. Bengal divided will pull in different ways''. That is perfectly true and one of the merits of the scheme.

(a) Lord Curzon
(b) HH. Risley
(c) Lord Minto
(d) Sir Lancelot Hare

(b) In the official note, HH. Risley, the Home Secretary to the Government of India made this statement, intended to curb Bengalis influence by not only placing Bengalis under two administrations, but by reducing them to a minority in Bengal itself.

84. Who launched the Bardoli Satyagraha on 4th February, 1928?

(a) Mahatma Gandhi
(b) Vallabhbhai Patel
(c) Rajendra Prasad
(d) Kalyanji Mehta

(b) Sardar Vallabhbhai Patel was launched the Bardoli Satyagraha on 4th, February, 1928. It was a major episode of civil disobedience and revolt in the Indian Independence Movement led by Vallabhbhai Patel and its success gave rise to Patel becoming major leader.

85. Which one of the following Princely States did not support the Congress during the course of Civil Disobedience Movement?

(a) Bhavnagar (b) Mysore
(c) Junagadh (d) Kathiawar

(c) During the Civil Disobedience Movement, Congress formally extended the call for launching a struggle to the people of the Princely States. Among the given options, Junagadh did not support the Congress during the course of Civil Disobedience Movement.

86. Which one of the following is a feature of thought and philosophy of the Kandukuri Viresalingam?

(a) He believed that science and morality were unconnected to truth.
(b) He believed in universal education.
(c) He believed that language had no role in inculcating morality in students.
(d) He did not attempt to build a national consciousness on a cultural base.

(b) Kandukuri Viresalingam was one of the early social reformers who encouraged women education. He believed in universal education. He also started a school in Dowlaiswaram in 1874.

Hence, option (b) is correct.

87. Name the calligrapher in Akbar's court who was honoured with the title 'Zarrin Kalam' or Golden Pen.

(a) Abul Fazl
(b) Tansen
(c) Muhammad Husayn
(d) Muhammad Kasim

(c) Muhammad Husayn was one of the most famous artists of calligraphy in Akbar's court, who came from Kashmir and taught the Mughal Prince. He was honoured with the title 'Zarrin Kalam' or 'Golden Pen'.

88. Who among the following was the author of the 'Badshahnama'?

(a) Abdul Hamid Lahori
(b) Abul Fazl
(c) Shah Jahan
(d) Sadullah Khan

(a) Abdul Hamid Lahori was the author of the 'Badshahnama'. He was a traveller and historian during the period of Mughal emperor Shah Jahan. 'Badshahnama' or 'Padshahnama' is a genre of work written as the official visual history of Mughal emperor Shah Jahan.

89. Which one of the following statements about Fatehpur Sikri is not correct?

(a) It was located on the direct road to Ajmer.
(b) Akbar commissioned the building of a marble tomb for Sheikh Salim Chisti next to the Friday mosque at Fatehpur Sikri.
(c) The arched gateway or Bulund Darwaza was meant to remind visitor of the Mughal victory in Gujarat.
(d) In 1585, the capital of the Mughal emperor shifted from Fatehpur Sikri to Delhi.

(d) In 1585, the capital of the Mughal empire shifted from Fatehpur Sikri to Lahore due to the exhaustion of the small supply of water to city. Again in the year 1598, the capital shifted to Agra.

Hence, option (d) is incorrect.

2016 (II)

90. Which of the following was/were the reasons for the defeat of British during the American War of Independence?

1. The remoteness of the American Continent and the lack of good roads.
2. The British authorities failed to rally the loyalist Americans.
3. The Americans benefited from the extraordinary military leadership of George Washington.
4. The Americans had access to superior arms and ammunition.

Select the correct answer using the codes given below.

(a) 1, 2 and 4
(b) Only 3
(c) 2 and 3
(d) 1, 2 and 3

(d) *The reasons for defeat of the Britishers during the American War of Independence were*

1. The remoteness of the American Continent and the lack of good roads.
2. The British authorities failed to rally the loyalist Americans.
3. The Americans benefited from the extraordinary military leadership of George Washington.

Hence, statement 1, 2, 3 are correct.

91. The Industrial Revolution in England had a profound impact on the lives of people. Which of the following statements are correct about that?

1. Women were the main workers in the silk, lace-making and knitting industries.
2. Factory managers were well aware of the health risks of industrial work on children.
3. Children were often employed in textile factories.
4. The novel 'Hard Times' by Charles Dickens was a severe critique of the horrors of industrialisation.

Select the correct answer using the codes given below.

(a) 1 and 3 (b) 1 and 2
(c) 1, 3 and 4 (d) 3 and 4

(d) The Industrial Revolution in England had a profound impact on the lives of people.

Child labour was, integral to the first factories, mines and mills in England.

In textile mills, as new powerlooms and spinning mules took the place of skilled workers, factory owners used cheap, unskilled labour to decrease the cost of production.

And, child labour was the cheapest labour of all. Dickens novel 'Hard Times' (1854) is a great moral fable that not only provides a damning critique of industrial England of the 19th century, but also an indictment of global laissez faire capitalism of 21st century.

So, Statements 3 and 4 are true. Hence, option (d) is correct.

92. Although used earlier by French and German writers, the term 'Industrial Revolution' in English was first popularised by

(a) Adam Smith (b) Arnold Toynbee
(c) James Mill (d) Bertrand Russell

(b) The term 'Industrial Revolution' was first popularised by the English economic historian **Arnold Toynbee** in 1882, to describe Britain's economic development from 1760 to 1840.

93. In the 19th century, the majority of the workers in Japan's modern industries were mainly

(a) Japanese men and children
(b) Japanese women and Chinese men
(c) women
(d) Japanese and Chinese men

(c) As Western style factories and the mechanisation of production spread in the late 19th century, tens of thousands of individuals, mainly were women from rural backgrounds, took up factory work.
At their peak in the 1920s, the industries employed more than half a million workers, over 80% of them female. The growth of a modern industrial sector in pre-war Japan thus, depended largely on the labour of women.

Directions (Q.Nos. 94 and 95) *The following questions items consist of two statements, statement I and statement II. Examine these two statements carefully and select the correct answer using the codes given below.*

Codes

(a) Both the statements are individually true and Statement II is the correct explanation of Statement I.
(b) Both the statements are individually true, but Statement II is not the correct explanation of Statement I.
(c) Statement I is true, but Statement II is false.
(d) Statement I is false, but Statement II is true.

94. **Statement I** The city of Rome revived in a spectacular way in the 15th century.

Statement II From the 15th century onwards, artists were known individually by name, not as member of a group or a guild, in Roman society.

(a) The city of Rome revived in a spectacular way in the 15th century.
The ruins in Rome were carefully excavated by archaeologists. Artists and sculptors were also employed to decorate buildings with paintings, sculptures and reliefs. Some artists were skilled equally as painters, sculptors and architects, e.g. Michelangelo Buonarroti (1475-1564), Filippo Brunelleschi (137-1446).
Another remarkable change was that, artists were known individually, i.e. by name, not as members of a group or a guild, in Roman society.

Hence, both the statements are individually true and statement II is the correct explanation of statement I.

95. **Statement I** Chartism was the political reform campaign for democratic rights, which swept over Britain between 1838 and 1848.

Statement II The Trade Union Movement declined by the 1830s as it failed to fulfil the aspirations of the working class.

(b) Chartism was a working-class movement for political reform in Britain, which existed from 1838 to 1848. The aim of the Chartists was to gain political rights and influence for the working classes. Chartism got its name from the formal petition or People's Charter of 1838.

The origins of the Trade Union Movement can be traced to the time of the Industrial Revolution, which transformed Britain in the 18th and 19th centuries from an agrarian and rural society to one, which was based on industrial production in factories, textile mills and mines.

The Trade Union Movement declined by the 1830s as it failed to fulfil the aspirations of the working class.

Both the statements are individually true, but statement II is not the correct explanation of statement I.

96. The cylindrical stone seals were used in which civilisation?

(a) Harappan
(b) Egyptian
(c) Roman
(d) Mesopotamian

(d) Cylinder seals are small cylinders, generally made of stone. Most of them have a hole drilled through their length, so that they could be worn on a string or pin. They are engraved with a design, so that when they were rolled out on clay, they would leave a back-to-front impression of the design in relief.

Cylinder seals were probably invented in Southern Mesopotamia about 3500 BC and were used until about 300 BC. They are found in all countries where cuneiform writing was used. Stamp seals were used even earlier, from about 6000 BC and continued in use long after cylinder seals went out of fashion.

97. Which of the following statements with regards to the Civil Disobedience Movement is/are correct?

1. The movement received massive response from business groups and peasantry.
2. The movement coincided with large scale labour upsurge in Maharashtra.
3. The movement was marked by the mass participation of lawyers and students.

Select the correct answer using the codes given below
(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) All of these

› ***(d)*** The Civil Disobedience Movement was launched by Mahatma Gandhi. It began with the famous Dandi March of Gandhi in the year 1930.

This movement coincided with large scale labour upsurge in Maharashtra. The movement was marked by the mass participation of lawyers and students. The movement also received massive response from business groups and peasantry.

Hence, all given statements are true.

98. Who among the following invented the powerloom that revolutionised the cotton textile industry?
(a) Edmund Cartwright
(b) Samuel Crompton
(c) Richard Arkwright
(d) James Hargreaves

› ***(a)*** A powerloom is a mechanised loom powered by a drive shaft. It was one of the key developments in the industrialisation of weaving during the early Industrial Revolution. The powerloom was designed in 1784 by Edmund Cartwright and built in 1785. It was refined over the next 47 years until a design by Kenworthy and Bullough made the operation completely automatic (the Lancashire Loom).

2016 (I)

99. Consider the following statements about Rashtrakuta kings.
1. They were ardent patrons of Shaivism and did not support other forms of religion.
2. They promoted only Sanskrit scholars and gave them large at grants.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(d)*** The Rashtrakuta kings were known for their religious tolerance, however, they were more inclined towards Jainism. Many of the scholars who flourished in the court of Rashtrakuta kings wrote in Sanskrit, Kannada, and Prakrit languages. The Rashtrakutas built well-known jain temples at locations such as Lokapura in Bagalkot district and their loyal feudatory, the Western Ganga dynasty, built jain monuments at Shravanabelagola and Kamadahali. Hence, neither 1 nor 2 statements is correct.

100. Subsidiary Alliance was a system devised by
(a) Lord Wellesley (b) Lord Dalhousie
(c) Lord Canning (d) Lord Ripon

› ***(a)*** The Doctrine of Subsidiary Alliance was introduced by Lord Wellesley, then British Governor-General in India from 1798 to 1805. It was the system of ruling a dominated nation. The idea behind the doctrine was influenced by two factors; *firstly*, the British Empire should be well-defended and should have no danger from any external power. *Secondly*, the Empire should expand as widely as possible, so that, in course of time, the whole of India might become a British possession.

101. Which of the following statements about the first act of the Revolt of 1857 is/are true?
1. It occurred in Meerut when two sepoys stole a superior officer's rifle.
2. It began when the Rani of Jhansi declared war on the British.
3. It began when Mangal Pandey fired at a European officer in Meerut.
4. It began when Mangal Pandey fired at a European officer in Barrackpore.

Select the correct answer using the codes given below.
(a) Only 2 (b) 1, 2 and 3
(c) Only 4 (d) 1 and 4

› ***(c)*** Mangal Pandey played a key role in events that preceded the outbreak of Revolt of 1857. On 29th March, 1857, Mangal Pandey fired at an European officer in Barrackpore. The rebellion began on 10th May, 1857 in the form of a mutiny of sepoys of the company's is army in the Garrison town of Meerut. Hence, option (c) is correct.

102. Which of the following statements above Gandhiji's *Hind Swaraj* written in 1909 is/are true?
1. *Hind Swaraj* offers a civilisational concept of the Indian nation.
2. *Hind Swaraj* states that Parliament democracy was necessary for the amelioration of the sufferings Indians.
3. *Hind Swaraj* argues that industrial capitalism was responsible for the immorality of society.

Select the correct answer using the codes given below.
(a) Only 3 (b) 1, 2 and 3
(c) 1 and 3 (d) Only 1

› ***(c)*** Hind Swaraj was written by Gandhiji in 1909 to express his views on Swaraj, modern civilisation and industrialisation etc. He was in favour for independence not just from British but also from modern civilisation. He thought that Parliament is simply a costly toy and was not in favour of parliamentary democracy.

He was also against industrial capitalism and argues that industrial capitation was responsible for the immorality of society. Hence, option (c) is correct.

103. Consider the following statements.
1. Warren Hastings set-up the Calcutta Madrassa for the study and teaching of Muslim law related subjects.
2. Jonathan Duncan started a Sanskrit College at Varanasi for the study of Hindu law and philosophy.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(c)*** Madrasah-i-Aliah or Calcutta Madrassa was the first educational institution set-up for the study and teaching of Muslim law related subjects in India in 1780 by Warren Hastings. In 1791, Jonathan Duncan started the Sanskrit College at Varanasi for the study of Hindu law and philosophy. Therefore both, the statements are true. Hence, option (c) is correct.

104. Consider the following statements about Sufism in India.
1. Pilgrimage called *ziyarat* to tombs of Sufi saints is an important feature.
2. The most influential group of Sufis in India were Chishtis.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(c)*** One of the most popular rituals in sufism is visiting of grave-tombs of Sufi saints. This ritual of visiting tombs is called Ziyarat. The Chishti order in India

was the most influential group of Sufis and this order emerged from Central Asia and Persia. Khwaja Muinuddin Chishti introduced the Chishti order in India. Hence, option (c) is correct.

105. Which of the following thinkers movements influenced the development Gandhiji's political ideas?

1. Henry David Thoreau
2. John Ruskin
3. John Milton
4. Jainism

Select the correct answer using the codes given below.

(a) Only 1 (b) 1, 2 and 3
(c) 1, 2 and 4 (d) 2 and 4

› ***(c)*** Gandhiji was greatly influenced by thinkers like John Ruskin, Leo Tolstoy, Henry David Thoreau. John Ruskin's book 'Unto his last' had a lasting impact on Gandhiji's psyche. He paraphrased the book as 'Sarvodaya'. Gandhi also was influenced by non-violence principle preached by Jainism. Hence, option (c) is correct.

106. Consider the following statements.

1. The *Arthashastra* is the first Indian treatise to define a state.
2. The main concerns of the *Arthashastra* are theoretical issues like the origin of the state.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(a)*** The *Arthashastra* written in Sanskrit was the first Indian work that formally defined state. It is a practical treatise on statecraft, economic policy and military strategy. Its main concern is not to deal with theoretical issues like origin of state. So, statement (1) is true and (2) is false.

2015 (II)

107. Which one of the following statement(s) about Subsidiary Alliance devised by Lord Wellesley in the year 1728 is not correct?

(a) The territories entering into a Subsidiary Alliance with the British were responsible for their own internal and external protection.
(b) In the territory of the ally, a British armed contingent would be stationed.
(c) The ally would have to provide the resources for maintaining the British contingent in the territory.
(d) The permission of the British was needed for the ally to enter into agreements with other rulers.

› ***(a)*** The Subsidiary Alliance system was used by Wellesley to bring Indian states within the orbit of British power. It was "Non-Intervention Policy".

The system served the double purpose of asserting British supremacy in India and at the same time of saving India from the menace of Napoleon.

In the territory of the ally, a British armed contingent would be stationed the ally would have to provide the resources for maintaining the British contingent in the territory. Hence, option (a) is correct answer.

108. Who among the following was associated with the Mughal Court as a physician to Prince Dara Shikoh?

(a) Hakim Afzal Khan
(b) Ibn Battuta
(c) Francois Bernier
(d) Duarte Barbosa

› ***(c)*** Francois Bernier was a French physician and traveller. He was born at Joue-Etiau in Anjou. He was briefly personal physician to Prince Dara Shikoh (28th October, 1615—30th August, 1659), the elder son of Shah Jahan.

109. Which one of the following statements about Khilafat Movement is not correct?

(a) The Khilafat Movement demanded that the Khalifa must retain control over Muslim holy places.
(b) The radical trend in the Khilafat Movement was represented by younger leaders like Muhammad Ali, Shaukat Ali and Maulana Azad.
(c) Indian Muslim leaders used Khilafat as a symbol that could unite the Indian Muslim community.
(d) The Delhi conference of the Central Khilafat Committee in 1920 decided to launch a massive Non-cooperation Movement.

› ***(d)*** The leaders of the Khilafat Movement joined hands with Indian National Congress for the upcoming Non-cooperation Movement.

Again 19th March, 1920 was observed as Khilafat Day and following that there was an all party conference in June 1920 at **Allahabad**. There agenda of the Non-cooperation Movement was finalised. Hence, option (d) is not correct.

110. The British Officer who was a representative of the Governor General and who lived in a State which was not under direct British rule was called

(a) Collector (b) Viceroy
(c) Resident (d) Agent

› ***(c)*** A Resident or in full Resident Minister, is British India official. He was british officer and the representative of the governor general. He officially has diplomatic functions which are often seen as form of indirect rule. The Residency system has its origins in the system of subsidiary alliances devised by the British after Battle of Plassey in 1757.

111. Which of the following features of the Permanent Settlement of 1793 is/are correct?

1. The Permanent Settlement vested land ownership rights in the peasants.
2. The Permanent Settlement vested land ownership rights in the Zamindars.
3. The Zamindars had to pay a fixed amount of rent by a particular date.
4. The Zamindars benefitted hugely from the Permanent Settlement while the peasants suffered.

Select the correct answer using the codes given below.

(a) Only 1 (b) 2 and 3
(c) Only 4 (d) 1, 2 and 3

› ***(b)*** Permanent settlement recognised the landlords as the proprietors of the land.

The landlords were given the right to transfer or sell their lands if they liked.

All the rights of the landlords depended on their payment of the fixed revenue on the fixed date at the treasury of the Government.

It fixed once for all total amount of revenue to be paid by each landlord for his zamindari to the Government. So, statement 2 and 3 are true. Hence, option (b) is correct.

112. The Vijayanagara empire received its death blow at the battle of

(a) Talikota in 1565 (b) Panipat in 1661
(c) Talikota in 1665 (d) Raichur in 1510

› ***(a)*** The Battle of Talikota was fought between Vijayanagar kingdom and Sultanates of Deccan on 26th January, 1565. The Vijayanagara kingdom was defeated in the battle, and after the death of Krishna Deva Raya the fall of Vijayanagara Empire began.

113. Consider the following statements about the travellers who visited India.

1. Abdur Razzaq Samarqandi from Herat visited Delhi and Daulatabad.
2. Ibn Battuta provides detailed accounts of both Delhi and Daulatabad.
3. According to Francois Bernier, there was no private property in land in Mughal India.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) 1, 2 and 3
(c) 1 and 3 (d) 2 and 3

› ***(d)*** Abdur Razzaq Samarqandi visited the Hampi during the reign of Deva Raya II. He was entrusted with an embassy from Persia. He arrived at Calicut in 1442, where he resided till the beginning of April 1443. Being there he was summoned to Vijayanagara, and stayed till the 5th December, 1443 in Hampi.

Ibn Battuta provided detail account of Delhi and Daulatabad in his book 'RIHLA'. Francois Bernier, the French traveller mentioned that Mughal India had no system of private property. Hence, option (d) is correct.

114. Match List I with List II and select the correct answer using the codes given below the lists.

List I (Dynasty)		**List II** (Architecture)	
A.	Chalukyas	1.	Sun Temple, Konark
B.	Hoysalas	2.	Pattadakal Temples
C.	Pandyas	3.	Keshava Temple, Somnathpur
D.	Eastern Gangas	4.	Eastern *Gopura* of Chidambaram Temple

Codes

	A	B	C	D		A	B	C	D
(a)	1	3	2	4	(b)	1	2	3	4
(c)	2	4	3	1	(d)	2	3	4	1

› ***(d)*** The correct matching is A-2, B-3, C-4, D-1. **Pattadakal** is a small town that is renowned for its ancient temples in Karnataka. It is built by **Chalukya Rulers** during 7th- 8th centuries. Owing to its incredible temples, Pattadakal was titled a World Heritage Site by UNESCO in 1987.

The **Keshava temple** in Somnathpur in Karnataka is believed to have been built around AD 1268, under Somnatha (a general in the army of Narasimha III) of the **Hoysala dynasty**.

The **Eastern Gopura of Chidambaram Temple** or **Thillai Natarajah Temple** is a Hindu temple dedicated to Lord Shiva located in the town of Chidambaram in Tamil Nadu. It is built by **Pandyas ruler.**

The **Sun Temple** of Konark marks the highest point of achievement of Kalinga architecture depicting the grace, the joy and the rhythm of life all its wondrous variety. This temple was constructed by Raja Narasinghs Deva-I of the **Ganga Dynasty** was dazzling supreme in the political firmament of India.

115. The Senia tradition is

(a) musical tradition that emerged from the colonial army (Sena).
(b) dance drama written by Keshab Chandra Sen.
(c) textile tradition of Eastern Uttar Pradesh.
(d) musical tradition that invoked the name of Tansen, the celebrated musician of Akbar's Court.

› ***(d)*** The word 'Senia' is related to Tansen, the father of Indian Classical Music. The word 'Gharana' implies a style of music. The followers of Tansen's school of music are widely known as the followers of 'Senia gharana' (i.e. 'Seniya' style/school of music). The followers of this 'gharana' may either be related with the family of Tansen or by the age old tradition of 'Guru-Shishya parampara' (teacher to student relation).

116. Who among the following founded the Rajahmundri Social Reform Association in 1878 in support of widow remarriage?

(a) Vishanshastri Pandit
(b) Ishwar Chandra Vidyasagar
(c) Pandita Ramabai
(d) Veerasalingam Pantulu

› ***(d)*** Kandukuri Veerasalingam Pantulu was a strong advocate of women's rights in the 19th century. He hailed and campaigned for his cause from the district of Rajahmundri in Andhra Pradesh. Veerasalingam Pantulu founded the Rajahmundri Social Reform Association in 1878 in support of widow remarriage. Hence, option (d) is correct.

117. The 19th century Faraizi Movement in Eastern Bengal developed under the leadership of

(a) Titu Rai
(b) Haji Shariatullah
(c) Shah Sayyid Ahmad
(d) Dudu Miyan

› ***(b)*** The 19th century Faraizi Movement in Eastern Bengal developed under the leadership. He also regarded British rule in Bengal as injurious to the religious life of the Muslims.

118. The Indian States Committee was formed in 1928 under

(a) the Raja of Junagadh
(b) Ian Copland
(c) Sir Harcourt Butler
(d) Motilal Nehru

› ***(c)*** The Indian State Committee was formed in 1928 under Sir Harcourt Butler. Sir Harcourt Butler had reported that the quality of administration in the Princely States was continually improving. The Committee had yet to consider the large amount of material which had been laid before it, and its report would be as important in its own sphere as that of the Simon Commission.

2018 (I)

119. Harappan crafts display an amazing degree of standardisation. According to Kenoyer, what was the possible reason for such an achievement?

(a) Availability of raw materials at local level
(b) Centralised markets for crafts
(c) Specialised training centres for craftsmen
(d) State control

› ***(a)*** Kenoyer's main focus has been on the Indus Valley Civilisations where he has conducted research for the last 23 years. He has done a great deal of work in trying to replicate processes used by ancient people in the production of jewellery and pottery. During his research, he reveal that it was possible due to availability of raw materials at local level.

120. Consider the following statements about Ashokan rock edicts.

1. Major Rock Edict XIII records Ashoka's remorse at the sufferings caused by his Kalinga campaign.
2. Major Rock Edict X records Ashoka's visit to Lumbini.
3. Major Rock Edict XII refers to *Dhamma Mahamattas* as a new category of officers instituted by Ashoka.
4. Major Rock Edict XII speaks about showing tolerance towards all sects.

Which of the statements given above are correct?

(a) 1 and 4 (b) 2 and 3
(c) 3 and 4 (d) 1, 3 and 4

(a) Major Rock Edict XIII is the largest inscription from the Edict. It mentions about Ashoka's victory over Kalinga. It also mentions victory of Ashoka's Dhamma over Greek kings, Antiochus, Ptolemy Antigonus, Magas, Alexander and South Kingdom of Cholas and Pandyas.

Major Rock Edict XII directed and determined request for tolerance among different religious seets. Major Rock Edict X condemns the desire for fame and glory. It stresses on popularity of Dhamma. Hence, option (a) is correct.

121. Which one of the following is the common element among Rajagriha, Vaishali and Pataliputra?

(a) Pali canon of the *Sthaviravadins* was compiled there.
(b) Ashokan Major Rock Edicts were found there.
(c) Places where Buddhist Councils were held.
(d) Places associated with the compilation of Buddhist Canons of the *Mahasanghikas*.

(c) The followers gathered at Rajagriha for the first general council. The second council was held in Vaishali 100 years after the death of Buddha. The third one is said to be held in Pataliputra in the time of the Mauryan king Ashoka. These are the places where Buddhist Councils were held. Hence, option (c) is correct.

122. Which one among the following sects was associated with makkhali?

(a) Vajrayana (b) Ajivikas
(c) Sthaviravadins (d) Mahasanghikas

(b) **Ajivikas** sects was associated with makkhali gosala was a system of ancient Indian philosophy and an ascetic movement of the Mahajanapada period in the Indian subcontinent. **Ajivika** is classified as a nastika or 'heterodox' system.

The Ajivikas may simply have been a more loosely-organised group of wandering ascetics (sramanas or sannyasins).

123. Consider the following statements about the early modern technology in India.

1. The Portuguese brought European movable metal types to Goa in 1550.
2. The first Indian script of which types were prepared was Tamil.
3. The English Company's first experiment with printing press was at Calcutta.
4. The English Company imported printing press to India in 1674-75 at the request of Bhimji Parak.

Which of the statements given above are correct?

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1, 2 and 4

(b) Movable type is the system of printing and typography that uses movable components to reproduce the elements of a documents. Gutenberg in 1439 was the first European to use the printing press and movable type in Europe. The english company's first experiment with printing press was at calcutta.

The tamil Brahmi script has been paleographically and stratigrasnically constitutes the earliest known writing system. Bhimji parak was first to introduce printing press in Bombay (1674-75). Hence, option (b) is correct.

124. Which of the following statement(s) is/are true for Olympe de Gouges?

1. She was one of the most important politically active woman in France revolutionary.
2. She was one of the members of the committee that drafted the Declaration of Rights of Men and Citizen.
3. She wrote the Declaration of the Rights of Women and Citizen.

Select the correct answer using the codes given below.

(a) Only 1 (b) Only 3
(c) 1 and 2 (d) 1 and 3

(a) Olympe de Gouges (7th May, 1748 – 3rd November, 1793), born Marie Gouze, was a French playwright and political activist. She is perhaps best known as an early feminist, who demanded that French women be given the same rights as French men.
In her Declaration of the Rights of Woman and the Female Citizen (1791), she challenged the practice of male authority and the notion of male–female inequality. She was executed by guillotine during the reign for attacking the regime of the Revolutionary government and for her close relation with the Girondists. Hence, option (a) is correct.

125. Arrange the following inventions in the field of cotton industry in chronological order *(starting with the earliest)*.

1. James Hargreaves's Spinning Jenny.
2. John Kay's Flying Shuttle.
3. Samuel Crompton's Mule.
4. Richard Arkwright's Water Frame.

Select the correct answer using the codes given below.

(a) 1, 3, 4, 2 (b) 2, 1, 4, 3
(c) 4, 1, 3, 2 (d) 2, 4, 1, 3

(b) John Kay's Flying Shuttle In 1733. Spinning Jenny was invented in 1764 by James Hargreaves.
Richard Arkwright's Water Frame- in 1768. Samuel Crompton's Mule- in 1779. Hence, option (b) is correct.

126. While opposing the Public Safety Bill, 1928, who among the following said that it was 'a direct attack on Indian nationalism, on the Indian National Congress' and as 'the Slavery of India, Bill No. 1'?

(a) Motilal Nehru
(b) Lala Lajpat Rai
(c) T Prakasam
(d) Diwan Chaman Lal

(a) Motilal Nehru described the Public Safety Bill, 1928 as ''a direct attack on Indian nationalism, on the Indian National Congress and as the Slavery of India, Bill No 1,'' because he supported Soviet Union concept of propaganda.

Motilal Nehru was an Indian lawyer, an activist of the Indian National Movement and an important leader of the Indian National Congress, who also served as the Congress President twice in 1919 and 1928 respectively. He was co-founder of Swaraj Party along with Chittranjan Das. He chaired the famous Nehru Commission in 1928, that was counter to the Simon Commission.

127. Soon after the formation of Indian National Congress, the British became suspicious of nationalists. Who among the following called Congress representing only the elite — 'a microscopic minority'?

(a) Lord Napier (b) Lord Dufferin
(c) Lord Ripon (d) Lord Lytton

(b) Lord Dufferin initially did not take Congress much seriously. Then, there was a blast and all of a sudden a

Pamphlet appeared titled 'The Rising Tide'. Another pamphlet appeared titled 'An Old Man's Home'.

These were against the British and Lord Dufferin took no time to react and express objection to these mischief. He initially called Congress as representative of "microscopic minority of India" but later in the fourth session of Allahabad, the Government servants were disallowed to take part in the proceedings of the Congress.

128. What is the common element among Ram Prasad Bismil, Roshan Singh and Rajendra Lahiri?

(a) They threw a bomb in the Central Legislative Assembly on 8th April, 1929.
(b) They were the founder members of Swaraj Party.
(c) They were the founders of Hindustan Republican Association.
(d) They were associated with Kakori Conspiracy Case.

› *(**d**)* Ram Prasad Bismil, Roshan Singh and Rajendra Lahiri were associated with Kakori Conspiracy Case. Kakori conspiracy was the idea of Ram Prasad Bismil. It was an armed robbery on 9th August, 1925, the idea was accepted and it followed a plan that a running train at Kakori on the Lucknow-Saharanpur section should be looted.

The result was arrested of some 30 people and all of them were trailed in the Kakori Conspiracy Case. Only 4 members were given sentence of death, they were : Ram Prasad Bismil, Ashfaqullah Khan, Rajendra Lahiri and Roshan Singh. Rest all were either put in jail for long-term or were deported for life.

129. Mahatma Gandhi's *Hind Swaraj* is essentially

(a) a critique of Western modernity.
(b) an outline of his philosophy of Satyagraha.
(c) an invocation to Indians to free themselves from the British.
(d) a blueprint of Gram Swaraj.

› *(**a**)* Hind Swaraj or Indian Home Rule is a small tract written by Gandhi ji in 1909. It is a critique of Western modernity. Gandhi ji had been living in South Africa for some years, had been to India on a visit and on the voyage back to South Africa from London.

Hind Swaraj occupies a seminal place in Gandhi's oeuvre and can even reasonably be described as one of the most critical documents of the 20th century, it is because in this work he initiated what he himself described as 'a severe condemnation of 'modern civilisation'.

130. Pochampally in Nalgonda district of Telangana became famous in April, 1951 because

(a) the Bhoodan Movement was launched there by Vinoba Bhave.
(b) the landless peasants rose up in arms against the landlords.
(c) it witnessed violent anti-Hindi agitation.
(d) silk weavers cooperative was formed for the first time in India.

› *(**a**)* Pochampally is a small village situated in the district of Nalgonda in the State of Telangana, India. 18th April, 1951 was an historic day of the very genesis of the Bhoodan Movement. Vinoba Bhave visited Pochampally mandal in Nalgonda district. Vinayak Narahari Bhave was known as Vinoba Bhave who was an advocate for non-violence and human rights and is well known for his Bhoodan Movement to help the landless farmers.

131. Who among the following was not associated with the Sarvodaya Movement?

(a) Jaya Prakash Narayan
(b) Acharya Vinoba Bhave
(c) Dhirendra Mazumda
(d) GV Apparao

› *(**d**)* The Sarvodaya Movement was based on the three fold programmes– Gramdan, Khadi and Shanti Sena. It was led in India in 1950s. The prominent leaders associated with the movement were Acharya Vinoba Bhave, Jaya Prakash Narayan, Dhirendra Majumdar etc.

GV Apparao was a Telugu poet and writer of Andhra Pradesh. He wrote the Telugu play 'Kanyasulkam' which is often considered the greatest play in the Telugu language.

2014 (II)

132. Who among the following was associated with the formulation of the basic ideas of the *Mahayana* Buddhism?

(a) Nagarjuna (b) Kashyapa Matanga
(c) Menander (d) Kanishka

› *(**a**)* Nagarjuna was associated with the formulation of the basic ideas of the Mahayana Buddhism.
Nagarjuna gave the 'Madhamika' philosophy or the philosophy of the middle way and taught that there is neither reality nor non-reality but only relativity.

133. Consider the following statements about Harappan culture.

1. The Harappan culture matured in Sind and Punjab.
2. It spread from there to Southwards and Eastwards.
3. The area, where it spread, was bigger than Egypt and Mesopotamia.

Which of the statement(s) given above is/are correct?

(a) 1 and 2
(b) 2 and 3
(c) Only 3
(d) 1, 2 and 3

› *(**d**)* The mature Harappan extends from 2600-1900 BC. It matured in Sind and Punjab. The early Harappan communities turned into large urban centres. Such urban centres include Harappa, Ganeriwala, Mohenjo-Daro in modern day Pakistan and Dholavira, Kalibangan, Rakhigarhi, Rupar and Lothal in India.

The area, where it spread, was bigger than Egypt and Mesopotamia. All the given statements are true. Hence, option (d) is correct.

134. Megasthenes was a

(a) Greek ambassador to the court of Chandragupta Maurya.
(b) Greek trader during Ashoka's time.
(c) Greek trader in the Gupta period.
(d) Chinese pilgrim during Harsha's time.

› *(**a**)* Megasthenes was a Greek ambassador to the court of Chandragupta Maurya. He came to India in the 4th century BC as a representative of Seleucus Nicator. Megasthenes wrote an account of India and also that of Chandragupta's reign in his book named 'Indica'.

135. By the late 19th century, India was one of the largest producers and exporters of

(a) cotton yarn and wheat
(b) sugar and rice
(c) sugar and alcohol
(d) iron and steel

› (***a***) By the late 19th century till 1920s, India was one of the largest producers and exporters of cotton yarn and wheat. India was the world's main producer of cotton textiles and had substantial export trade to Britain, *via* East India Company.

The export of Indian wheat developed after the Suez Canal opened in 1869. By 1890s about 17% of India's wheat was exported.

136. Which one of the following is the correct sequence of appearance of the poet-saints of the Bhakti-Sufi tradition?

(a) Basavanna— Appar—Mira Bai—Lal Ded

(b) Appar—Mira Bai—Lal Ded—Basavanna
(c) Appar—Basavanna—Lal Ded—Mira Bai
(d) Basavanna—Mira Bai—Lal Ded—Appar

(c) Bhakti (or devotion) movement was the binding force in India, which irrespective of caste, creed and gender provided spiritual guidance and solace to the masses.

Appar Nayanar was a 7th century Saiva Tamil poet-saint, a prominent among Nayanars.

Bhandari Basavanna (1134-96) was a Kannada Bhakti Saint and social reformer.

Lalleshwari or Lal Ded (1320-92) was a mystic of the Kashmiri Shaivite sect. She was also the creator of poetry called Vatsun.

Mira Bai was a 16th century saint and devotee of Lord Krishna. She was one of the most prominent figure of Vaishnava bhakti movement. Hence, option (c) is correct.

137. Who among the following did not bring trading ships to the port of Surat in pre-British times?
(a) Portuguese and English
(b) Russian and German
(c) English and Arab
(d) French and Arab

(b) Russian and German did not bring trading ships to the port of Surat in pre-British times. Portuguese, English, Arab and French brought ships to port of Surat.

138. The Factory Act of 1891 in India was enacted to
(a) improve the condition of labour in India.
(b) ensure greater governmental control over industry in India.
(c) provide a level playing field for English manufacturers.
(d) enable greater political control over Indian industry.

(a) The Factory Act of 1891 in India resulted in the limitation on the factory working hours, and provided the solution to the pathetic working conditions of workers in factories. Hence, option (a) is correct.

139. Who among the following was not a member of the 'Big Four' in the Congress of Vienna (1815)?
(a) Great Britain (b) Russia
(c) Austria (d) France

(d) The Congress of Vienna (1815) was organised following the exile of Napoleon Bonaparte to Elba by Victorious Allied Powers. The 'Big Four' members were
(i) Britain (ii) Austria
(iii) Prussia (iv) Russia
Later on, the royalist France was also invited to join.

140. The Stamp Act Congress consisting of delegates from nine of the thirteen colonies met in 1765 at
(a) Philadelphia (b) New York City
(c) Boston (d) Providence

(b) The Stamp Act Congress met in the Federal Hall building in New York City between 7th-25th October, 1765. It was the first colonial action against a British measure and was formed to protest the Stamp Act issued by British Parliament on March, 1765. The Stamp Act Congress was attended by 27 representatives of nine of the thirteen colonies.

141. Which of the following American colonies did not attend the first Continental Congress held in Philadelphia?
(a) Rhode Island (b) Connecticut
(c) Georgia (d) Maryland

(c) The First Continental Congress convened in Philadelphia's Carpenters Hall on 5th September, 1774. The Congress was attended by 56 delegates appointed by the legislatures of twelve of the thirteen colonies, the exception being the province of Georgia, which was hoping for British assistance with native American problems on its frontier.

142. The Bolshevik Revolution started in Russia during the reign of
(a) Czar Alexander I
(b) Czar Alexander II
(c) Czar Alexander III
(d) Czar Nicholas II

(d) The Bolshevik Revolution started in Russia during the reign of Czar Nicholas II. The Bolshevik Revolution was abdicated on 2nd March, 1917, as a result of the February, Revolution. In July, 1918, the Bolsheviks executed Nicholas along with his wife, Alexandra and their children.

143. Which of the following statements is/are correct about the working of the Permanent Settlement in Bengal?
1. The traditional zamindars lost their lands.
2. The reason for the zamindars inability to pay up land revenues was that the *Ryots* defaulted on payment of revenue.
3. A new group of farmers—the *jotedars*—became influential.
4. The Collector replaced the zamindars as the alternatives focus of authority.

Select the correct answer using the codes given below.
(a) Only 1
(b) 1 and 4
(c) 2 and 3
(d) All of the above

(d) The Permanent Settlement had come into operation in 1793. In introducing the Permanent Settlement, British officials hoped to resolve the problems they had been facing since the conquest of Bengal.

Permanent Settlement was made with the *rajas* and *taluqdars* of Bengal. They were now classified as zamindars, and they had to pay the revenue demand that was fixed in perpetuity.

During permanent settlement in Bengal last their lands. The season belind the zamindars inability to pay up land revenues was that the ruois defaulted on payment of revenue. Apart from this, a new group of farmers known as jotedars became influential. In this settement the Collecter replaced the zamindars as the alternative focus of authority. Hence, all the given statements are correct.

Directions (Q. Nos. 144-146) *The following three questions consist of two statements, Statement* I *and Statement* II. *You have to examine these two statements carefully and select the answers to these items using the codes given below.*

Codes
(a) Both the Statements are individually true and Statement II is the correct explanation of Statement I.
(b) Both the Statements are individually true, but Statement II is not the correct explanation of Statement I.
(c) Statement I is true, but Statement II is false.
(d) Statement I is false, but Statement II is true.

144. **Statement I** The first coins to bear the names and images of rulers were issued by the Kushanas.
Statement II The first gold coins were issued by the Kushanas.

(c) The Indo-Greeks were the first to issue gold coins in India and these increased in number under the Kushanas. The first coins to bear the names and images of rulers were issued by the Kushanas. The Kushan gold coins influenced subsequent issues, notably those of the Guptas. The mostly gold coins were issued by Guptas. So, 149 statement I is true, but statement II is false. Hence, option (c) is correct.

145. **Statement I** Jahandar Shah's reign came to an early end in January, 1713.
Statements II He was defeated at Agra by Farrukhsiyar, his nephew.

(a) Jahandar Shah's reign came to an early end in January, 1713 when he was defeated at Agra by Farrukhsiyar, his nephew.

Farrukhsiyar owed his victory to the Sayyid brothers, Abdullah Khan and Husain Ali Khan Barahow, who were therefore given the offices of Wazir and Mir Bakshi, respectively. Both the statements are individually true and statement II is correct explanation of statement I. Hence, option (a) is correct.

146. **Statement I** The defects of the Regulating Act and the exigencies of British politics necessitated the passing of the Pitt's India Act.
Statement II The Pitt's India Act gave the British Government supreme control over the company's affairs and its administration in India.

(b) The East India Company Act, 1784 also known as Pitt's India Act, was an act of the Parliament of Great Britain intended to address the shortcomings of the Regulating Act of 1773 by bringing the East India Company's rule in India under the control of the British Government. Both the statements are individually correct, but statement II is not the correct explanation of statement I. Hence, option (b) is correct.

2014 (I)

147. Who among the following was associated with the Hindustan Socialist Republican Association?
(a) Subhash Chandra Bose
(b) Mahatma Gandhi
(c) Bhagat Singh
(d) Laxmi Sehgal

(c) Hindustan Socialist Republican Association was established at Feroz Shah Kotla, New Delhi in 1928 by Chandrashekhar Azad, Bhagat Singh, Sukhdev Thapar and others. It was a revolutionary organisation, earlier known as Hindustan Republican Association.

148. The Indian National Army was organised to fight against
(a) Germany and Japan
(b) British in India
(c) Japan
(d) Russia

(b) The Indian National Army was formed by India nationalists in 1942 in South-East Asia during Second World War with aim to secure Indian independence with Japanese assistance. It was organised to fight against the British in India. It was revived by Subhash Chandra Bose in 1943.

149. The Non-cooperation Movement was withdrawn after the
(a) First World War
(b) Chauri-Chaura Incident
(c) Bardoli Satyagraha
(d) Gandhi-Irwin Pact

(b) The Non-cooperation Movement was withdrawn after the Chauri-Chaura incident. The Non-cooperation Movement was launched by Mahatma Gandhi with the aim of self-governance and obtaining full independence.

First World War originating in Europe that lasted from 28th July, 1914 to 11th November, 1918. The Bardoli Satyagraha, 1928 was a movement in the independence struggle led by Sardar Vallabhbhai Patel for the famous of Bardoli against the unjust raising of taxes. The Gandhi-Irwin Pact was a political agreement signed by Mahatma Gandhi and Lord Irwin on 5th March, 1931.

150. Gandhiji's 'Harijan Campaign' sought to
1. attack caste system as a whole.
2. open wells, roads, temples, etc., to Harijans.
3. encourage social work among Harijans.

Select the correct answer using the codes given below.
(a) 1 and 2
(b) 2 and 3
(c) 1 and 3
(d) Only 3

(b) Gandhiji never attacked caste system as a whole as he did not question Hinduism and caste as part of it. He encouraged social work among Harijans and also encouraged opening of wells, roads, temples, etc, to Harijans. Hence, option (b) is correct.

151. The Salt Satyagraha called by Mahatma Gandhi saw the
1. violation of salt laws along coastal Indian.
2. participation of women.
3. upward swing when it came to the working class joining in.

Select the correct answer using the codes given below.
(a) 1 and 3 (b) Only 2
(c) 1 and 2 (d) 1, 2 and 3

(d) The March on foot undertaken by Gandhiji and seventy-eight Congress volunteers was the most significant event in the history of the breach of salt law in India, is known as Salt Satyagraha (1930). It was an act of non-violent Civil Disobedience Movement in colonial India to stop the taxation on salt production. The Salt Satyagraha saw the violation of salt laws along coastal Indian. In Salt Satyagraha marked the first time women became mass participants in the struggle for freedom and upward swing when it came to the working. Hence, option (d) is correct.

152. During the Civil Disobedience Movement, tribals in Chota Nagpur
1. gave up opium consumption
2. wore Khadi
3. gave up drinking liquor and eating meat

Select the correct answer using the codes given below.
(a) 1 and 3 (b) Only 1
(c) 1, 2 and 3 (d) 2 and 3

(d) In Chota Nagpur, Birsa Munda fought for the tribal natural right. After him many adivasis more than 26,000 started resistance movement and eventually merged with Mahatma Gandhi's Satyagraha and Civil Disobedience Movement. The Adivasis wore Khadi and gave up drinking liquor and eating meat. Hence, option (d) is correct.

153. Which of the following was/were the new feature/features visible in the Quit India Movement (1942)?
1. Large-scale movements in some of the industrial areas.
2. Large-scale mass involvement in some of the Princely States.
3. Increase in the activities of the Kisan Sabha.

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) Only 1
(c) 1 and 3 (d) Only 3

› ***(a)*** The Quit India Movement also called the August Movement of India or the 'Bharat Chhodo Andolan' was a Civil Disobedience Movement that was launched in August 1942. The movement was a call by Mahatma Gandhi for the country's immediate independence. The movement saw a large-scale mass involvement in some of the princely states. There ware also large-scale movements in some of the industrial areas. The activities of the Kisan Sabha also increased during the movement. So, the statements are true. Hence, option (a) is correct.

154. The Swadeshi Movement evoked serious responses in

(a) Madras and Hyderabad
(b) Bengal and Maharashtra
(c) Bihar and Odisha
(d) The Princely States

› ***(b)*** The Swadeshi Movement was started in 1905 against the partition of Bengal by Lord Curzon in 1905. The movement evoked serious responses in Bengal and Maharashtra.

In Bengal, Boycott Movement was adopted wherein people boycotted foreign goods, schools and colleges etc. The prominent leaders were Aurobindo Ghosh, Ashwini Kumar Dutta, Rabindranath Tagore etc.
In Maharashtra, the movement was supported by leaders like Gokhale and Bal Gangadhar Tilak though both followed different methods. The slogans of Swaraj, the concepts of Swadeshi, Boycott and National Education were spread.

155. Who among the following was associated with the Gudem-Rampa Rebellion?

(a) Birsa Munda
(b) Kushal Konwar
(c) Alluri Sitarama Raju
(d) Mangal Pandey

› ***(c)*** The Gudem-Rampa Rebellion (1922-33) was started in Andhra Pradesh to oppressive Madras Forest Act of 1882. The forest laws imposed by the British has infringed the rights of the tribal from time to time and they had to fight their grievances on their own with little or no help from outside.

Most of the tribal uprising were armed and guerilla warfare uprising against the British. The rebellion was led by Alluri Sitarama Raju until his death in 1923.

156. The 'Doctrine of Lapse' was a policy that aimed to

(a) control the rebellions in the Princely States.
(b) extend the military strength of the British.
(c) regulate the landlords.
(d) extend the territorial boundaries of the English East India Company.

› ***(d)*** The Doctrine of Lapse was annexation policy-purportedly devised by Lord Dalhousie, who was the Governor-General for the East India Company in India between 1848 and 1856. Doctrine of Lapse was a policy that ained to extend the territional boundaries of the English East India Company. Under the Dalhousie's Doctrine of Lapse, he led to the annexation of states like Satara, Nagpur, Jhansi, Sambhalpur, Udaipur, Aitpur and Bhagat. This raised the waves of discontentment against the British rule.

157. The Permanent Settlement of 1793 introduced

(a) peasant rights.
(b) bourgeois property rights in land to zamindars.
(c) shipping rights for the English East India Company.
(d) rights for women to have property rights.

› ***(b)*** The Permanent Settlement was introduced by Governor-General Lord Cornwallis in 1793. It was an agreement between the British East India Company and the Landlords of Bengal to settle the Land Revenue to be raised.

The following are features of Permanent Settlement

(i) It recognised the landlords as the proprietors of the land.
(ii) The landlords (bourgeois) were given the right to transfer or sell their land if they liked.
(iii) All the rights were depended upon their payment to the treasury of government.
(iv) The landlords were required to give to the tenant the patta. Thus, the tenants got rights on their holding and knew of the revenue to be paid.

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GEOGRAPHY

2019 (II)

1. Which one of the following is a Rabi crop in the Northern States of India?

(a) Rice (b) Bajra
(c) Barley (d) Ragi

(c) *(**c**)* Rabi crops are winter crops which are sown in October-November and harvested in March. The Rabi crops in Northern states of India includes barley, wheat, oats, gram, linseed and mustard.

2. The rivers of North-West Europe are good examples of

(a) parallel pattern of drainage
(b) radial pattern of drainage
(c) barbed pattern of drainage
(d) trellis pattern of drainage.

(c) *(**c**)* The rivers of North-West Europe are good examples of barbed drainage pattern. **Barbed drainage pattern** is a rare kind of drainage pattern which is formed when the tributaries flow in opposite direction to their master streams.

A parallel pattern of drainage system is a pattern of river caused by steep slopes with some relief. Because of the steep slopes, the streams are swift and straight, with very few tributaries and all flow in the same direction.

Radial drainage patterns form when streams and rivers flow off a central peak or dome like a volcano.

In a **trellis pattern**, the river forms a net like system and the tributaries flow roughly parallel to each other.

3. Which one of the following ocean currents is a cold current?

(a) South Atlantic Drift
(b) Mozambique Current
(c) East Australian Current
(d) Caribbean Current

(c) *(**a**)* **South Atlantic Drift/Current** is an eastward flowing cold ocean current, fed by the Brazil Current. It is continuous with the northern edge of the Antarctic Circumpolar Current.

The **Mozambique current** is a warm ocean current flowing along the east coast of Africa in Indian ocean. It flows between Mozambique and Madagascar islands.

The **East Australian Current** (EAC) is a warm ocean current that flows along the eastern cost of Australia.

The **Caribbean Current** is a warm ocean current that flows northwestward through the Caribbean sea along the coast of South America and into the Gulf of Mexico.

4. Paradip Port is located on the delta of river

(a) Rihand (b) Ganga
(c) Mahanadi (d) Bitarani

(c) *(**c**)* It is a natural, deep-water port on the East coast of India in Jagatsinghpur district of Odisha. Paradip Port is situated at confluence of the Mahanadi river and the Bay of Bengal.

5. Which one of the following hypothesis/theory explains the origin of the Universe?

(a) Nebular Hypothesis
(b) Binary Theory
(c) Big Bang Theory
(d) Planetesimal Hypothesis

(c) *(**a**)* Nebular Hypothesis was given by Immanuel Kant explain about the origin of Universe. According to this theory, plants were formed out of a cloud (nebula) of hydrogen, helium & dust surrounding the Sun.

Binary theory explains about the existence of companion star of the Sun.

According to **Big Bang theory**, all matter or substance forming this universe existed at one place as a tiny ball. This tiny ball had infinite density and temperature. At the Big Bang Theory the tiny ball exploded and started a process of expansion of the universe which continues to a day.

According to Planetesimal Hypothesis, when a planetary system was forming, there was a proto planetary disc with materials form the nebula from which the system came.

6. Which one of the following land territories of Indonesia is not touched by the Equator?

(a) Sumatra
(b) Sulawesi
(c) Java
(d) Kalimantan

(c) *(**c**)* Java island is located to the South of equator at around 5°S latitude in a West to East orientation, so Java is not touched by the equator. However, equator passes through the Sumatra, Sulawesi and Kalimantan portion of Borneo island in Indonesia.

7. Which one of the following sedimentary rocks is organically formed?

(a) Shale (b) Chert
(c) Halite (d) Chalk

(c) *(**d**)* Organically formed sedimentary rocks are formed from the remains of living organisms such as corals & shell fish (calcareous rock); or from vegetative remains like forests & swamps (carbonaceous rocks). Some examples of organically formed sedimentary rocks are chalk, limestone, peat, lignite & coal. Shale is a mechanically formed sedimentary rock. Halite & chert are chemically formed sedimentary rocks.

8. According to the Koppen climatic classification, the letter code Cfa denotes.

(a) Tropical wet climate
(b) Humid subtropical climate
(c) Tundra Climate
(d) Tropical wet and dry climate

(b) Koeppen's climate classification is based on annual & monthy average temperatures and precipitation. According to this classification **Cfa** denotes **Humid subtropical** climate. This type of climate is characterised by warm summer and absence of dry season. Tropical wet climate is denoted by Af; Tundra climate is denoted by ET and Tropical wet & dry climate is denoted by Aw.

9. In which one of the following folds in the axial plane found to be virtually horizontal?
(a) Isoclinal (b) Anticlinal
(c) Recumbent (d) Monoclinal

(c) In **recumbent fold**, axial plane found to be horizontal. The axial plane of a fold is the plane or surface that divides the fold as symmetrically as possible.
In **Isoclinal fold**, the axial surface and limbs slope in the same direction and at approximately the same angle.
An **Anticlinal fold** is an arch like fold which is convex upward.
Monoclinal fold is a step-like fold consisting of a zone of steeper dip within or otherwise horizontal topography.

10. How many Ramsar convention sites have been enlisted from India so far?
(a) 12 (b) 16 (c) 27 (d) 62

(c) India has 27 Ramsar sites like Dal lake, Pong Dam, Harike Wetland, Chilika lake etc. Ramsar Convention on Wetlands is an international treaty on conservation and sustainable use of Wetlands. It was signed in Ramsar, a city in Iran in 1971. India Chilika lake was first to be designated as a Ramsar site in India.

11. The Munda language group belongs to which family?
(a) Austric (b) Dravidian
(c) Sino-Tibetan (d) Indo-European

(a) Munda language belongs to the **Austroasiatic or Austric** group of language. It is a group of language with more than 9 million user from central and eastern India.
Dravidian languages are Tamil, Telugu, Kannada and Malyalam. **Sino-Tibetan languages** includes Chinese and the Tibeto-Busman languages.
Indo-European languages are dialects in Asia and Europe.

12. Which one of the following statements about the Suez Canal is not correct?
(a) The Suez Canal was constructed in 1869.
(b) It links the Mediterranean Sea and the Read Sea.
(c) It has a six lock system and ships cross the different levels through these locks.
(d) It has given Europe a new gateway to the Indian Ocean.

(c) Option, (c) is not the correct statement about the Suez Canal because. It has no lock system as the water level in Mediterranean Sea and Red Sea through Gulf of Suez is nearly same. The Suez Canal in Egypt connects the Red Sea with Mediterranean Sea. It was constructed between 1859 to 1869 and officially opened in 1869. It gives Europe a new gateway to the Indian Ocean allowing huge trade and cargo movement between the Europe and Asian countries.

13. The normal lapse rate of temperature of Earth's atmosphere drops to 0°C at the
(a) upper part of ionosphere
(b) upper boundary of the tropopause
(c) lower part of mesosphere
(d) upper boundary of stratopause

(b) The normal lapse rate of temperature of Earth's atmosphere drops to 0°C at the upper boundary of the tropopause. The lapse rate is the rate at which temperature in Earth's atmosphere changes with altitude.

14. Loktak lake is situated in the State of
(a) Sikkim (b) Manipur
(c) Odisha (d) Mizoram

(b) Loktak Lake is located at Moirang in Manipur, India. It is the largest fresh water lake of North-East India & is famous for phumdis (floating vegetative mass). Keibul Lamjao National Park, one of the phumdis of the Lake, is the only floating national park in the world. It is also the last natural habitat of endagered Sangai deer.

15. Stalactite, stalagmite and pillars are the depositional landforms of
(a) running water
(b) wind
(c) glacier
(d) underground water

(d) Stalactite, stalagmite and pillars are the depositional landforms of underground water. Stalactite and stalagmite are formed when the calcium carbonate dissolved in groundwater.

16. A deep valley characterised by steep step like slope is known as
(a) U-shaped valley (b) Blind valley
(c) Gorge (d) Canyon

(d) Canyon is a deep valley characterised by steep step like slopes. A **U-shaped** valley is a glaciated landform having a characteristic U-shape with steep straight sides & a flat or rounded bottom.
A **blind valley** is a deep, narrow & flat bottomed valley with an abrupt ending.
A Gorge is a deep, narrow valley with steep sides.

17. Which one of the following is a major plate?
(a) Pacific Plate (b) Cocos Plate
(c) Arabian Plate (d) Philippine Plate

(a) Among the given options, Pacific plate is a major plate. The outer shell of Earth, Lithosphere is broken into tectonic plates. There are seven major tectonic plates which include Pacific Plate, North American Plate, Eurasian Plate, African Plate, Antarctic Plate, Indo-Australian Plate and South American Plate. There are about 20 minor plates including Cocos Plate, Nazca plate, Arabian Plate Philippine plate and Caroline Plate, etc.

18. Lithosphere consists of
(a) upper and lower mantle
(b) crust and core
(c) crust and uppermost solid mantle
(d) mantle and core

(c) Lithosphere is the solid outer part of the Earth. It consists of the crust and solid outermost layer of the upper mantle. Hence option (c) is correct answer.

2019 (I)

19. Spring tides refer to
(a) greatest difference in the sea level at high and low tides
(b) lowest difference in the sea level at high and low tides
(c) no difference in the sea level at high and low tides
(d) counteraction of gravitational pull of the Sun to that of Moon

(a) The periodic rise & fall of sea level, once or twice a day, is called a Tide. When the Sun, the Moon and the Earth are in a straight line, the height of tides will be higher than normal. These are called spring tides, which occur on new Moon or full Moon day. At these times, high tides are very high & low tides are very low. Thus, spring tides refer to the greatest difference in the sea level at high and low tides.

20. Which one of the following Union Territories of India is the smallest in terms of geographical area?
(a) Daman and Diu
(b) Chandigarh
(c) Dadra and Nagar Haveli
(d) Lakshadweep

(d) Lakshadweep is the smallest Union Territories of India in terms of geographical area. According to Census 2011, Lakshadweep has an area of 32 km^2. It is an archipelago consisting of 36 islands and having the capital at Kavaratti. Among others, UTs Daman and Diu has an area of 112 km^2 whereas Chandigarh and Dadra and Nagar Haveli has an area of 114 km^2 and 491 km^2 respectively.

21. 'Campos' and 'Llanos', Tropical Savanna grasslands are generally found in
(a) Australia (b) Central Africa
(c) South America (d) East Asia

(c) 'Campos' and 'Llanos' are the tropical savanna grasslands found in South America. Campos is located in Brazilian Highlands and Llanos in Orinoco basin. Tropical Grasslands are located near the Equator, between the tropic of Cancer and the tropic of Capricorn. They are also known as 'Savannas'. It is a transitional zone found between the equatorial rainforests and the hot deserts. They includes savannas of Africa and Campos and Llanos of South America.

22. 'Viticulture' is a common feature of which one of the following Australian cities?
(a) Adelaide
(b) Darwin
(c) Hobart
(d) Brisbane

(a) Viticulture is a common feature of Adelaide, which is the capital of South Australia. Viticulture is the cultivation and harvesting of grapes. The Adelaide Hills wine region is a famous economic region of country for production of wine from grapes.

23. 'Shamal' warm and dry wind is a Local wind found in
(a) East Asia
(b) West Coast of Africa
(c) Sahara of Africa
(d) Mesopotamia

(d) 'Shamal local' winds are found in Mesopotamia. Mesopotamia is a region covering Iraq, Iran and the Arabian Peninsula. Shamal is a hot, dusty and dry wind which blows from the North or North-West in Mesopotamia. Other important warm local winds are Chinook in North America, Foehn in the Alps, Khamsin in Egypt, Sirocco in North Africa and Harmattan in West Africa.

24. 'Inversion of Rainfall' is associated with
(a) Orographic rainfall
(b) Convectional rainfall
(c) Cyclonic rainfall (Tropical)
(d) Cyclonic rainfall (Temperate)

(a) 'Inversion of Rainfall' is associated with orographic rainfall. The orographic rainfall occurs due to the ascent of air forced by a mountain barrier. The mountain barrier should be across the wind direction, so that the moist air is forced in obstruction to move upward and get cooled. The amount of the rainfall increases with increasing height of the barrier, but this is up to certain limit. After that there is a marked decrease due to lesser moisture content of the air and this phenomenon is called 'Inversion of Rainfall'.

25. Which one of the following is not a west flowing river?
(a) Periyar
(b) Bharathapuzha
(c) Pamba
(d) Tamraparni

(d) Tamraparni is not a West flowing river while others are west flowing rivers. Tamraparni river originates from Pothigai hills of Western Ghats (Tamil Nadu). It flows through Tamil Nadu and falls into Gulf of Mannar. It has been historically Known as 'Podhigai'. It is mentioned in the ancient Sangam and Tamil texts.

26. Which one of the following rivers was earlier known as 'Vitasta'?
(a) Tista
(b) Jhelum
(c) Tungabhadra
(d) Bharathapuzha

(b) 'Jhelum river' was earlier known as 'Vitasta', while Tista was known as 'Trisrota', Tungabhadra as 'Pampa' and Bharathapuzha as 'Pratichi'.
Jhelum river rises from northern slopes of Pir Panjal ranges at Verinag Spring. It flows through Srinagar and Wular lake before entering Pakistan. It is a tributary of the Indus river.

27. River Sharda drains in the Northern Plains of Uttar Pradesh. Before entering in the Plains, Sharda is known as
(a) Saraswati
(b) Bhagirathi
(c) Kali
(d) Pindar

(c) The Sharda river originates from the greater Himalayas at Kalapaani in the Pithoragarh district of Uttarakhand. The river is named after the Goddess Kali whose temple is situated in Kalapaani. It drains the Northern plains of Uttar Pradesh and Uttarakhand. It is known as 'River Sharda', when it reaches the plains of Uttarakhand and Uttar Pradesh. It forms India's continous border between India and Nepal.

28. Which of the following is/are environmental effects of Rotation of the Earth?
1. Daily or diurnal rhythm in day-light and air temperature
2. Flow path of both air and water are turned consistently in a sideward direction.
3. The movement of the tides

Select the correct answer using the codes given below:
(a) 1 and 2
(b) 1 and 3
(c) 1, 2 and 3
(d) Only 3

(c) Statement 1, 2 and 3 all are correct. The rotation of the earth causes day and night. This results in the diurnal changes in day-light and air temperature. So, statement 1 is correct.
Earth's rotation results in the Coriolis effect. This deflects winds and streams of water to the right in Northern Hemisphere and to the left in Southern Hemisphere. So, statement 2 is correct.
Tides are caused by the gravitational attractive forces of the moon and the sun as well as the centrifugal force due to the Earth's rotation. So, statement 3 is also correct.

29. Which one of the following is the correct sequence of proved coal reserves in the Indian States in decreasing order?
(a) Jharkhand, Chhattisgarh, Odisha, West Bengal
(b) Jharkhand, Odisha, Chhattisgarh, West Bengal
(c) Odisha, West Bengal, Jharkhand, Chhattisgarh
(d) Odisha, Chhattisgarh, West Bengal, Jharkhand

(b) Coal is the main source of energy in India as it fulfils about 67% of the total commercial energy consumed in the country.
The decreasing order of proved coal reserve is given below

State	Proved Coal Reserve (million tonnes)
Jharkhand	45563
Odisha	37391
Chhattisgrah	20428
West Bengal	14156

Hence option (b) is correct answer.

30. Consider the following statements relating to Richter scale
1. It was devised in 1935 by Charles F Richter.
2. It describes the quantity of energy released by a single earthquake.
3. Richter scale has no upper limit.

Which of the statement(s) given above is/are correct?
(a) Only 1
(b) 1 and 2
(c) 2 and 3
(d) All of these

(***b***) Richter scale is a scale of number used to indicate the magnitude of earthquakes. Charles Francis Richter developed the Richter scale in 1935. This scale describes the quantity of energy released by earthquake. Numbers for the Richter scale range from 0 to 9. So, statements (1) and (2) are correct, but statement (3) is not correct.

31. Which one of the following ocean currents is not a cold ocean current?
(a) Canary current
(b) California current
(c) Kuroshio current
(d) Oyashio current

(***c***) Ocean current is a continuous movement of ocean water from one place to another. It is created by wind, water temperature, salt content and gravitational force of Moon. It is of two types; cold and warm ocean currents. Examples of warm ocean current are Kuroshio, Alaskan, El Nino, East Australian, Florida, Gulf stream, Brazilian and Agulhas ocean currents. Examples of cold ocean current are Humboldt or Peruvian, Kuril or Oyashio, California, Antarctica, Okhotsk, Labrador, Canary and Falkland ocean currents. Hence, option (c) is correct.

32. What is the time gap in occurrence of two successive tides at a given place on the ocean surface?
(a) 12 hr
(b) 12 hr 26 min
(c) 24 hr
(d) 24 hr 52 min

(***b***) A lunar day has a length of 24 hours and 52 minutes. Earth rotates through two tidal 'bulges' every lunar day. So, coastal areas experience two high and two low tides in every 24 hours and 52 minutes. There are two high tides occur in the interval of 12 hours and 26 minutes.
Hence, option (b) is correct answer.

33. The Sun is seen little before it rises and for a short while after it sets. This is because of
(a) total internal reflection
(b) atmospheric refraction
(c) apparent shift in the direction of Sun
(d) dispersion

(***b***) The Sun is seen little before it rises and for a short while after it sets due to atmospheric refraction. Atmosphere refraction is the phenomena of bending of light due to earth's atmosphere. When light enters from vacuum to Earth's atmosphere, it basically enters from rarer to denser medium and bends towards horizon. During sunrise, when the Sun is just below the horizon, our atmosphere causes the light rays to bend and we see the Sun early. Similarly, at sunset, the apparent position of the Sun is visible to us and not the actual position due to the same bending of light rays effect.

34. Which one of the following can be said to be essentially related to 'Polar Front Theory'?
(a) Anticyclone
(b) Tropical Cyclone
(c) Temperate Cyclone
(d) Inter Tropical Convergence

(***c***) The 'Polar Front Theory' of cyclogenesis was developed to explain the formation of mid-latitudes cyclone or Temperate cyclone. According to this theory, the warm-humid air masses from the topics meet the dry-cold air masses from poles, and thus a polar front is formed. The cold air mass pushes the warm air mass upwards, forming a low pressure cyclonic system. A mid-latitude cyclone is a large-scale low pressure system that is developed. Hence, option (b) is correct.

35. Brahmaputra and Indus rivers are antecedent rivers. Which one of the following may be the true definition of an antecedent drainage?
(a) Which follows the initial slope of the Himalaya
(b) Which existed before the Himalayan range came into existence
(c) Which followed the dip or rock beds of the Himalaya
(d) Which followed the strikes of rock beds of the Himalaya

(***b***) Option (b) is correct because **an antecedent drainage** is one whose path of flow neither a valley was established before the mountaineous structure was uplifted. Antecedent drainage existed before the Himalayan range came into existence. The Indus, Sutlej, Ganga, Kali, Arun, Tista and Brahmaputra are some of the important antecedent rivers originated from beyond the Greater Himalayas.

36. Kamarajar Port was commissioned in 2001 to handle thermal coal requirements. It is situated along the coast of which Indian State?
(a) Andhra Pradesh (b) Odisha
(c) Tamil Nadu (d) Karnataka

(**c**) Kamarajar (Ennore) Port is situated along the coast of Tamil Nadu. It is the 12th major port of India. This port was commissioned in 2001 to handle thermal coal requirement.

2018 (II)

37. Consider the following places of India :
1. Itanagar 2. Imphal
3. Agartala 4. Aizawl

Which one of the following is the correct chronological order of the above places in terms of sunrise time?
(a) 3-2-1-4 (b) 2-1-4-3
(c) 1-4-3-2 (d) 4-3-2-1

(***b***) *The correct sequence in order of Sunrise timing in the following places are as:*

At **Imphal**, the capital city of Manipur, the sunrise time is 4:59 AM.

At **Itanagar**, the capital city of Arunachal Pradesh, the sunrise time is 5:00 AM.

At **Aizawl**, the capital city of Mizoram, the sunrise time is 5:05 AM.

At **Agartala**, the capital city of Tripura, the sunrise time is 5:10 AM.
Hence 2-1-4-3 is correct answer.

38. Which one of the following is known as uplands of delta region?
(a) Bef (b) Bils (c) Peh (d) Chars

(***d***) The Deltaic plain is the extension of the Khadar land. It covers large area in the lower reaches of the Ganga river. In the delta region, the upland is called as 'Chars'.

39. Consider the following Wildlife Sanctuaries of India :
1. Shikari Devi
2. Bhadra
3. Simlipal
4. Pachmarhi

Which one of the following is the correct order of the above Wildlife Sanctuaries in terms of their location from South to North?
(a) 1-2-3-4 (b) 2-4-3-1
(c) 2-3-4-1 (d) 3-1-2-4

(***c***) The correct order of the given wildlife sanctuaries in terms of their location from South to North is

Bhadra Wildlife Sanctuary It is located in Chikkamagaluru town in Karnataka. It was established in the year 1951. It is a protected area and a tiger reserve as part of Project Tiger.

Simlipal Wildlife Sanctuary It is located in the Mayurbhanj district of Odisha. It is a National Park, an elephant reserve and a tiger reserve. It was established as a National Park in 1980.

Pachmarhi Wildlife Sanctuary It is a biosphere reserve in the Satpura Range of Madhya Pradesh. It was established in the year 1999.

Shikari Devi Wildlife Sanctuary It is located near the Shikari Devi Temple in Mandi district of Himachal Pradesh. It was established in the year 1974.

Hence, option (c) is correct.

40. Which one of the following statements about temperature is correct?

(a) Temperature decreases with height in the stratosphere.
(b) Temperature is constant at different heights in the stratosphere.
(c) Temperature increases with height in the troposphere at an average rate of 6.5°C per km.
(d) Temperature decreases with height in the troposphere at an average rate of 6.5°C per km.

(***d***) The temperature of the troposphere is highest near the surface of the Earth and decreases with height or altitude. On average, the temperature of the troposphere declines by 6.5°C per km on going above. However, in the stratosphere, temperature increases with altitude. The reason is that the direct heat source for the stratosphere is the Sun. Hence option (d) is correct answer.

41. Which one of the following is known as a zone of sharp salinity change in the vertical section of ocean?

(a) Thermocline
(b) Halocline
(c) Photic zone
(d) Pycnocline

(***b***) Halocline is the zone of sharp salinity change in the vertical section of ocean. A halocline is a strong, vertical salinity gradient within a body of water. As salinity affects the density of seawater, it can play an important role in its vertical stratification.

- Thermocline is a thin but distinct layer in a large body of fluid.
- Photic zone is the upper most layer of the ocean that receives sunlight enabling it to perform photosynthesis.
- Pycnocline is a layer in ocean or other body of water in which water density increases rapidly with depth.

42. Match List I with List II and select the correct answer using the codes given below the Lists :

List I (River Basin)	**List II** (Town)
A. Bhagirathi	1. Lansdowne
B. Alaknanda	2. Narendra Nagar
C. Nayar	3. Uttarkashi
D. Ganga	4. Pauri

Codes

	A	B	C	D		A	B	C	D
(a)	3	1	4	2	(b)	3	4	1	2
(c)	2	4	1	3	(d)	2	1	4	3

(***b***) The correct match is A-3, B-4, C-1, D-2. Uttarkashi lies in Bhagirathi basin. It is also called as 'Shivnagri'. Pauri lies in Alaknanda basin. It is most culturally rich place. Lansdowne lies in Nayar basin. It is popular since Britishers came to India. Narendra Nagar lies in Ganga basin and situated near Rishikesh.

43. Match List I with List II and select the correct answer using the codes given below the Lists :

List I (Local Wind)	**List II** (Place)
A. Yamo	1. Sudan
B. Black Roller	2. France
C. Bise	3. Japan
D. Haboob	4. North America

Codes

	A	B	C	D		A	B	C	D
(a)	1	4	2	3	(b)	1	2	4	3
(c)	3	4	2	1	(d)	3	2	4	1

(***c***) The correct match is A-3, B-4, C-2, D-1. Yamo is warm and dry wind that blows down the steep valleys in Japan. Black roller is a very strong and dust-laden wind blowing from the South–West or North-West in the Great Plains of North America.

Bise is a cold, dry wind in Switzerland and France.

A Haboob is a type of intense dust storm that blows in Sudan. Hence, option (c) is correct.

44. 'Majuli', the river island, is located in which one of the following rivers?

(a) Jamuna
(b) Padma
(c) Ganga
(d) Brahmaputra

(***d***) 'Majuli' is a river island, located in Brahmaputra river in Assam. It is formed by river Brahmaputra in the South and Subansiri in the North. In 2016, Majuli became the first island to be made a district in India. It is the world's largest river island.

45. Which one of the following types of cloud is characterised by continuous precipitation?

(a) Cirrocumulus (b) Cumulus
(c) Nimbostratus (d) Cumulonimbus

(***c***) Clouds are visible accumulations of tiny water droplets or ice crystals in the Earth's atmosphere. **Nimbostratus clouds** bring continuous precipitation that can last for many hours. These low-level clouds are full of moisture.

- **Cirrocumulus Clouds** are small rounded puffs that usually appear in long rows high in the sky.
- **Cumulus clouds** look like pieces of floating cotton.
- **Cumulonimbus clouds** are a type of cumulus cloud associated with thunderstorms and heavy precipitation.

46. Match List I with List II and select the correct answer using the codes given below the Lists :

List I (Major Port)	**List II** (Location)
A. Kolkata	1. Land-locked area
B. Mormugao	2. In the delta region
C. Visakhapatnam	3. On the riverside
D. Paradip	4. On the entrance of the estuary

Codes

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	4	1	3
(d)	2	1	4	3

(***b***) The correct matching is A-3, B-4, C-1, D-2.

The Port of Kolkata (renamed as Dr. Syama Prasad Mukherjee Port) is a riverine port in the city of Kolkata. It is the oldest operating port in India.

Mormugao is the main port of Goa. It is a natural port located on the entrance of an estuary.

Visakhapatnam Port is the deepest land-locked area of India. It is located in Andhra Pradesh. It is India's second largest port by volume of cargo handled.

Paradip Port is a natural, deep-water, and deltaic port located in Odisha. It is situated at confluence of the Mahanadi river and the Bay of Bengal.

47. Match List I with List II and select the correct answer using the codes given below the Lists :

List I (Koppen's Climatic Type)	List I (Letter Code)
A. Tropical wet	1. Af
B. Mid-latitude desert	2. Cs
C. Mediterranean	3. Df
D. Humid Continental	4. BWk

Codes

	A	B	C	D		A	B	C	D
(a)	1	4	2	3	(b)	1	2	4	3
(c)	3	2	4	1	(d)	3	4	2	1

(***a***) The correct match is A-1, B-4, C-2, D-3

Tropical Wet climate dented with letter code (Af) and also known as no dry season.

Mid-latitude desert is coded as BWk and also known as mid-latitude arid or dry climate.

Mediterranean climate is coded as Cs and also known as dry hot summer.

Humid continental is coded as Df and also known as no dry season or severe winter.

48. Match List I with List II and select the correct answer using the codes given below the lists :

List I (Railway Zone)	List II (Headquarters)
A. West Central	1. Jabalpur
B. South-East Central	2. Gorakhpur
C. East-Central	3. Bilaspur
D. North-Eastern	4. Hajipur

Codes

	A	B	C	D		A	B	C	D
(a)	2	3	4	1	(b)	1	4	3	2
(c)	2	4	3	1	(d)	1	3	4	2

(***d***) The correct match is A-1, B-3, C-4, D-2. Indian Railways is divided into 18 zones, which are further sub-divided into divisions. Each zonal railway is made up 73rd of a certain number of divisions, each having a divisional headquarter.

Railway Zone	Headquarters
West Central	Jabalpur
South-East Central	Bilaspur
East-Central	Hajipur
North-Eastern	Gorakhpur

49. Steppe (temperate continental) climate is not experienced in which one of the following places?

(a) Pretoria (b) Saskatchewan
(c) Perth (d) Buenos Aires

(***a***) Steppe climate is not experienced in Pretoria, a city located along the Apies river in South Africa. Pretoria has a humid sub-tropical climate with long, hot & rainy summers and short, cold & dry winters. Steppe climate or temperate continental climate is found in the interiors of the continents in the mid-latitudes. Steppe regions are practically treeless and the grasses here are shorter.

50. Which one of the following crops was introduced by the Portuguese in India?

(a) Opium (b) Coffee
(c) Betel leaf (d) Chili

(***d***) Chili is reported to be originated in South America and is widely scattered in all tropical and sub-tropical countries including India. It was first introduced in India by the Portuguese towards the end of the 15th century.

51. Sentosa island is located in

(a) Singapore (b) China
(c) Australia (d) Sri Lanka

(***a***) Sentosa, previously called Pulau Blakang Mati, is a resort island in Singapore. It was once a British military base and a Japanese prisoner of war camp. The island was renamed Sentosa and turned into a tourist destination in 1972. The island hosted the 2018 North Korea–United States summit between the United States President Donald Trump and Kim Jong-UN of North Korea on 12th June 2018, at the Capella Hotel.

2018 (I)

52. Consider the following statements about Roaring Forties

1. They are strong Westerly winds found in the oceans of Southern Hemisphere.
2. The strong east to west air currents are caused by the combination of air being displaced from the Equator towards the South Pole and the earth's rotation and abundance of landmasses to serve as wind breaks.

Which of the statements given above is/ are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) Regarding Roaring Forties, statement 1 is correct. The Roaring Forties are strong westerly winds found in the Southern Hemisphere, presents generally between the latitudes of 40 and 50 degrees. Hence, statement (1) is correct.The strong west-to-east air currents are caused by the combination of air being displaced from the Equator towards the South Pole and the Earth's rotation, and there are few landmasses to serve as windbreaks. Hence, statement (2) is correct.

53. Consider the following statements

1. The doldrums is a low pressure area around Equator where the prevailing winds are calm.
2. Chinook is a hot and dry wind that blows in winter and therefore raises the temperature in a short time.

Which of the statements given above is/ are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***c***) Statements (1) and (2) Both are correct. Doldrums, (also called equatorial calms) is the area where the Northeast and Southeast trade winds converge. It is a low pressure area around the equator where the prevailing winds are calm with no movement.

Chinook are local winds that blow over the Rocky mountains in USA. They are essentially hot and dry wind that blow in the winter season and raises the temperature in a short time thus removing the frost from the crops.

Hence, both the statements are correct.

54. Which one of the following is the driest desert of the world ?

(a) Atacama (b) Gobi
(c) Sahara (d) Kalahari

(***a***) Atacama desert located in the Andes mountain is the driest desert in the world. It is also the only true desert to receive less precipitation than the polar deserts. The desert owes its aridity to the fact that the desert lies on the leeward side of the Andes that blocks any winds from reaching here.

55. In which one of the following places, Headquarters of a Railway Zone is located?

(a) Kanpur (b) Lucknow
(c) Hajipur (d) New Jalpaiguri

(***c***) The East Central Railway (ECR) is one of the 18 Railway Zones in India. It is headquartered at Hajipur (Bihar) and comprises Sonpur, Samastipur, Danapur, Mughalsarai and Dhanbad divisions.

56. Which one of the following is not a tributary of Indus River?

(a) Beas (b) Ravi
(c) Chenab (d) Tawi

(***d***) Tawi is not one of the tributaries of Indus river. It is one of the left bank tributary of the Chenab River. The Tributaries of Indus are- Beas, Chenab, Gar, Gilgit, Gomal, Hunza, Jhelum, Kabul, Kunar, Kurram, Panjnad, Ravi, Shyok, Soan, Suru, Satluj, Swat, Zanskar, Zhob.

57. Which one among the following is the largest tiger reserve of India in terms of area of the core/critical tiger habitat?
(a) Manas
(b) Pakke
(c) Nagarjunasagar Srisailam
(d) Periyar

(***c***) Nagarjunsagar Srisailam Tiger Reserve is the largest tiger reserve in India in terms of area of core and critical habitat. The reserve spreads, over Andhra Pradesh and Telangana. It was made a tiger reserve in 1983. The total area of the tiger reserve is 3728 km^2.

58. Which one of the following is NOT a coastal depositional feature ?
(a) Tombolo
(b) Sandbar
(c) Stack
(d) Spit

(***c***) A stack or sea stack is a geological landform consisting of a steep and often vertical columns of rock in the sea near a coast formed by wave erosion. They are formed when part of a headland is eroded by hydraulic action. Tombolo, sandbar and spit are forms of coastal depositional landform.

59. Which of the following is/are coastal erosional feature(s) ?
1. Notch 2. Sea Arch
3. Cliff 4. Hook

Select the correct answer using the codes given below
(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 2 and 3
(d) Only 1

(***a***) Notch, sea arch and cliff are the examples of coastal erosional land forms.

A wave cut Notch is a small indent at the base of a cliff formed when a cliff is undercut by the sea.

A sea arch is a natural opening eroded out of a cliff face by marine processes.

A cliff is a steep vertical rock exposure, formed by the erosion of coastal rocks.

A Hook is a coastal depostional land form. It is a shorter spit with one end curved towards the land. Hence, option (a) is correct.

60. Chemical weathering of rocks is much greater in a place with
(a) cold and dry climate
(b) hot and humid climate
(c) hot and dry climate
(d) cold and humid climate

(***b***) Chemical weathering results from chemical reactions between minerals in rocks and external agents like air or water. Oxygen oxidizes minerals to alteration products whereas water can convert minerals to clays or dissolve minerals completely. It occurs majorly in areas with hot and humid climate.

61. Which of the following statements about Ring of Fire is/are correct?
1. It is a zone of convergence plate boundaries.
2. It is an active seismic and volcanic zone.
3. It is associated with deep trench.

Select the correct answer using the codes given below
(a) 1 and 2 (b) 2 and 3
(c) Only 1 (d) 1, 2 and 3

(***d***) The Ring of Fire is a ring of volcanoes around the Pacific Ocean that result from subduction of oceanic plates beneath lighter continental plates. This is also where the Earth's deep ocean trenches and where the deep earthquakes occur. The trenches form because the plate is bent downward as it subducts. It is the zone of convergence plate boundaries.

62. Which one of the following Himalayan vegetation species is found between the altitudes of 1800 to 2600 metres?
(a) Saal (b) Chir
(c) Spruce (d) Deodar

(***d***) Deodar tree is also known as Himalayan Cedar. These evergreen coniferous trees thrive in the Himalayas at altitudes of 1,500-3,200 meters above sea level.
Deodar grows to a height of 40–60 meters with a trunk diameter of 3 meters. The timber of Deodar tree is used as construction material due to its durability.
Other trees grow at altitude of
Saal – upto 1400mt.
Chir – 500-2000mt.
Spruce – 3000-4000mt.

63. Which one of the following rivers is NOT a tributary of river Cauvery?
(a) Hemavati (b) Arkavati
(c) Indravati (d) Amaravati

(***c***) The tributaries of Cauveri include Amaravati, Arkavati, Bhavani, Chinnar, Hemavati, Honnuhole, Kabini, Kannika, Kollidam, Lakshmana Tirtha, Lokapavani, Noyyal, Pambar, Shimsha, Sujyothi. Indravati is a tributary of the Godavari river. Hence, option (c) is correct answer.

64. Which of the following conditions is/are essential for tea cultivation?
1. Tropical and sub-tropical climate
2. Heavy rainfall ranging from 150 cm to 250 cm
3. Soil should contain good amount of lime.

Select the correct answer using the codes given below
(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) Only 1

(***b***) Tea is an evergreen plant. It grows both in tropical and sub-tropical regions. The tea plant grows well in hot and humid climate.
(i) The ideal temperature for growing tea is between 20 °C and 30 °C.
(ii) It needs heavy rainfall between 150 cm to 250 cm. It should be well-distributed throughout the year and should be in the form of frequent showers.

However, it does not require soil with good amount of lime for its cultivation. It requires well-drained fertile soil with acidic nature. Hence, option (b) is correct.

65. Which one of the following is a local wind that blows out from Siberia ?
(a) Bora (b) Purga
(c) Mistral (d) Blizzard

(***d***) Blizzards are the local winds of Siberia which blows out from the Siberian region towards south Asia. Blizzards are dangerous winter storms that are a combination of blowing snow and wind resulting in very low visibilities.

66. Which one of the following centres is NOT known for iron and steel industry ?
(a) Bhadravati
(b) Salem
(c) Visakhapatnam
(d) Renukoot

(***d***) Mysore Iron and Steel Company (MISCO) is located at Bhadravati in Shimoga district of Karnataka.
Salem Steel Plant is located in the Salem district of Tamil Nadu.
Visakhapatnam steel plant or vizag steel, is an integrated steel producer located in

Visakhapatnam, Andhra Pradesh. It is a Navratna company.
HINDALCO'S plant is located in Renukoot which is involved in the smelting of Aluminium. Hence, option (d) is the correct answer.

67. Which of the following are essential pre-requisites for establishment of a thermal power station?
1. Availability of fossil fuels
2. Proximity to a river, lake or sea
3. Good transport network
4. Proximity to an urban centre

Select the correct answer using the codes given below
(a) 1, 2 and 3 (b) 2 and 4
(c) 2 and 3 (d) 1 and 3

(***a***) The essential pre-requisites for establishment of a thermal power plant are that it must be in proximity to the source of fossil fuels and proximity to a water source like river, lake or sea. It should also have a good transport network for the transportation of raw materials. However, proximity to an urban centre is not one of the essential conditions for establishment of a power plant. Hence, option (a) is the correct answer.

2017 (II)

68. Sunrise in Eastern Arunachal Pradesh would be about how many hours before the sunrise in Western Gujarat?
(a) One hour (b) Two hours
(c) Three hours (d) Four hours

(***b***) The longitudinal extent of India's mainland is about 30°. Thus, from Gujarat to Arunachal Pradesh, there is a time lag of two hours. But the watches show the same time throughout the country because the time along the Standard Meridian of India (82°30′E) passing through Mirzapur (in Uttar Pradesh) is taken as the standard time for the whole country.

69. Consider the following States of India in terms of percentage of forest area in relation to the total area of the State.
1. Karnataka
2. Odisha
3. Kerala
4. Andhra Pradesh

Which one of the following is the correct descending order?
(a) 1-2-4-3 (b) 3-1-2-4
(c) 3-2-1-4 (d) 2-3-1-4

(***c***) According to the Indian State of Forest Report (ISFR), 2015, State which covers highest forest cover is Mizoram (88.93%), Kerala covers 49.50%, Odisha covers 32.34%, Karnataka covers 18.99% and Andhra Pradesh covers 15.25%. According to report 2018 forest cover in Kerela is 52.30% of its total geographical area. It is 32.98% for Odisha, 19.58% for Karnataka and 17.27% for Andhra Pradesh.

70. Which one of the following States in India has the longest coastline?
(a) Odisha (b) Tamil Nadu
(c) Karnataka (d) West Bengal

(***b***) Out of the given options, Tamil Nadu has the longest coastline. The total coastline of India measures about 7517 km, which is distributed among nine coastal states and four Union Territories. The nine coastal states are Gujarat, Maharashtra, Goa, Karnataka, Kerala, Tamil Nadu, Andhra Pradesh, Odisha and West Bengal.Gujarat is strategically located with largest share in India's coastline, followed by Andhra Pradesh and Tamil Nadu.

71. Which one of the following states in India has the largest area under forest cover?
(a) Maharashtra
(b) Chhattisgarh
(c) Madhya Pradesh
(d) Andhra Pradesh

(***c***) Madhya Pradesh has the largest area under forest cover (77414 sq km). It is followed by Arunachal Pradesh with forest cover of 67321 sq km. The maximum increase in forest cover has been observed in West Bengal (3810 sq km) followed by Odisha (1444 sq km) and Kerala (622 sq km).

72. Which one of the following is not an igneous rock?
(a) Gabbro (b) Granite
(c) Dolomite (d) Basalt

(***c***) Dolomite is not an igneous rock. It is a sedimentary rock.
Igneous rocks are formed from the solidification of molten rock material. There are two basic types of rocks namely Intrusive and Extrusive and Extrusive. Intrusive igneous rocks, which crystallise below Earth's surface and the slow cooling that occurs there allows large crystals to form. Examples of intrusive igneous rocks which are dolerite, gabbro, granite, pegmatite, and peridotite. Extrusive igneous rocks, which are formed on the surface, where they cool quickly to form small crystals. These rocks include andesite, basalt, obsidian, pumice, rhyolite, scoria and tuff.

73. The Coriolis effect is the result of
(a) Pressure gradient
(b) Earth's axis of inclination
(c) Earth's rotation
(d) Earth's revolution

(***c***) The Coriolis effect is the result of the Earth's rotation. The Earth rotates faster at the Equator than it does at the poles. This is because the Earth is wider at the Equator and thus a point on the Equator has **further to travel** in a day. The actual paths of winds and ocean currents, gets deflected as a result of the Coriolis effect.

74. Where is Mekong Delta located?
(a) Thailand (b) Cambodia
(c) Myanmar (d) Vietnam

(***d***) The Mekong is the region in South-Western Vietnam where the Mekong river approaches and empties into the sea through a network of distributaries. The Mekong delta region encompasses a large portion of South-Western Vietnam of over 40500 sq km.

75. Which one of the following pairs of rivers and tributaries is not correctly matched?
(a) Godavari : Indravati
(b) Ganga : Penganga
(c) Krishna : Bhima
(d) Luni : Sukri

(***b***) Penganga is a tributary of river Godavari, not Ganga. Major tributaries of Godavari are – **Indravati, Penganga,** Wainganga, Pranhita, Wardha, Manjia and Sabari.

Principal tributaries of Ganga are Gomati, Ghaghara, Gandak, Kosi, Yamuna, Son and Damodar.

Major tributaries of Krishna river are **Bhima**, Tungabhadra, Ghataprabha, Malaprabha and Musi.

Principal tributaries of Luni river are **Sokri**, Jowai and Jojari.

76. Consider the following characteristics of a tropical cyclone :
1. A warm sea temperature of > 26° C
2. High relative humidity of atmosphere at a height of > 700 m.
3. Atmospheric instability.

The above mentioned characteristics are associated with which one of the following cycles of its development?
(a) Formulation and initial stage
(b) Modification stage
(c) Full maturity
(d) Decay

(**a**) The given characteristics of a tropical cyclone are associated with formulation and initial stage. Hence option (a) is correct answer.

Following four atmospheric and oceanic conditions are necessary for development of a cyclonic storm during its formulation and inital stage.

(i) A warm sea temperature in excess of 26°C to a depth of 60 m, which provides abundant water vapour in the air by evaporation.

(ii) High relative humidity of the atmosphere to a height of above 700 m facilitates condensation of water vapour into water droplets and clouds, releases heat energy thereby inducing a drop in pressure.

(iii) Atmospheric instability encourages formation of massive vertical cumulus cloud convection with condensation of rising air over ocean.

(iv) A location of at least 4-5 latitude degree from the equator allows the influence of the forces due to the earth's rotation to take effect in reducing cyclonic wind circulation around low-pressure centres.

77. The shortest day length that occurs in the Northern hemisphere is on

(a) March 21 (b) September 23
(c) November 22 (d) December 22

(**d**) The shortest day and the longest night of the year in the Northern hemisphere occur during winter solstice which is observed on December 22. This is also the day of the longest day and the shortest night in the Southern hemisphere.

78. The Indian Railways have gone in for qualitative improvements since independence. Which of the following have taken place in recent years?

1. Gauge conversion
2. Track electrification
3. Automatic Signals

Select the correct answer using the codes given below

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) All of these

(**d**) Indian railways have an impressive record of qualitative improvements. The major areas of qualitative improvement during recent years are gauge conversion, rolling stock, track electrification, automatic signalling, introduction of fast trains and amenities and facilities for rail users. Hence, all statements are correct.

79. In India, maximum rainfall is received from

(a) Western Disturbances
(b) North-East Monsoon
(c) South-West Monsoon
(d) Retreating Monsoon

(**c**) In India, maximum amount of rainfall is received from South-West Monsoon. The South-West Monsoon occurs from June to September. The Northern and central Indian subcontinent heats up considerably during the hot summers. This causes a low pressure area over the Northern and central Indian subcontinent. To fill this void, the moisture-laden winds from the Indian Ocean (South-West) rush into the Subcontinent. On reaching the Southern most point of the Indian peninsula, the moisture-laden winds divide into two parts : the Arabian Sea Branch and the Bay of Bengal Branch, both of which causes considerable rainfall.

80. Which set of the following biosphere reserves in India is included in the World Network of Biosphere Reserves?

(a) Gulf of Mannar, Nokrek, Panchmarhi and Simlipal
(b) Gulf of Mannar, Kanchenjunga, Nokrek and Seshachalam
(c) Nilgiri, Nokrek, Panchmarhi and Panna
(d) Nilgiri, Nokrek, Panchmarhi and Seshachalam

(**a**) There are 18 Biosphere Reserves in India. Out of these 11 are included in the World Network of Biosphere Reserves. These are Gulf of Mannar, Nokrek, Pachmarhi, Simlipal, Nilgiri, Sundarbans, Nanda Devi, Great Nicobar, Achanakmar-Amarkantak, Agasthyamalai and Kanchenjunga. Other 7 Biosphere Reserves are Manas, Dihang-Dibang, Great Rann of Kutch, Cold Desert, Dibru-Saikhowa, Sesha Chalam Hills and Panna.

81. Which of the following statements about magnetite ore of iron is/are correct?

1. It is known as black ore.
2. It contains 60% to 70% of pure iron.
3. It possesses magnetic properties.

Select the correct answer using the codes given below:

(a) Only 1 (b) 2 and 3
(c) 1 and 3 (d) All of these

(**d**) Magnetite is known as black ore. It contains 60% to 70% of pure iron. Magnetite (Fe_3O_4) ore possesses magnetic properties. Hence, all the statments are correct.

82. Mariana Trench is located in the ocean floor of

(a) Southern Atlantic Ocean
(b) Western Pacific Ocean
(c) Eastern Pacific Ocean
(d) Northern Atlantic Ocean

(**b**) The Mariana Trench is the deepest part of the world's oceans. It is located in the Western Pacific Ocean to the East of the Mariana Islands. Its deepest point is known as the 'Challenger Deep' having a depth of 10,984 metres.

83. Taklamakan Desert is situated in

(a) Western Asia
(b) Southern fringe of Sahara in Africa
(c) South America
(d) Central Asia

(**d**) Taklamakan Desert is a desert of Central Asia and one of the largest sandy deserts in the world. The Taklamakan occupies the central part of the Tarim Basin in the Uygur Autonomous Region of Xinjiang (Western China). The desert area extends about 600 mi from West to East and it has a maximum width of 260 mi and a total area of approximately 123550 sq mi.

84. Rudraprayag is situated at the confluence of rivers Alakananda and

(a) Bhagirathi (b) Mandakini
(c) Nandakini (d) Dhauliganga

(**b**) Rudraprayag is situated at the confluence of rivers Alakananda and Mandakini. It is one of the Panch Prayag of Alaknanda and is located in Uttarakhand. The other four of Panch Prayags are Vishnuprayag (Alaknanda and Dhauliganga), Nandaprayag (Alaknanda and Nandakini), Karnaprayag (Alaknanda and Pindar) and Devprayag (Alaknanda and Bhagirathi).

85. Arrange the following Indian cities according to their locations from West to East :

1. Bilaspur 2. Jodhpur
3. Bhopal 4. Ranchi

Select the correct answer using the codes given below :

(a) 3-2-1-4 (b) 2-3-1-4
(c) 4-1-2-3 (d) 2-1-3-4

(**b**) Cities from West to East
Jodhpur (Rajasthan), Bhopal (Madhya Pradesh), Bilaspur (Chhattisgarh), Ranchi (Jharkhand).

86. The Kashmir region receives additional amount of precipitation during the winter brought by
(a) South-West Monsoon
(b) Western Disturbances
(c) Retreating Monsoon
(d) Temperate Cyclone

(***b***) The Kashmir region recieves additional amount of precipitation during the winter brought by Western disturbances. Western disturbances are low pressure depressions which originate as extratropical cyclonic storms over the Mediterranean sea and travel eastwards to enter the Indian Subcontinent. Hence option (b) is correct answer.

87. Which part of India has the Kalakot tertiary coal field?
(a) Brahmaputra river basin of Assam
(b) Damodar river basin of Jharkhand and West Bengal
(c) Himalayan mountain region
(d) Cardamom hills in Kerala

(***c***) Kalakot is a tertiary coal field located in Himalayan Mountain region. It is situated in Jammu & Kashmir near Pirpanjal Range. Other important tertiary coal fields of Jammu & Kashmir are Mohogala, Metka, Wadda and Saugar Marg.

88. Which one of the following statements about a satellite orbiting around the Earth is correct?
(a) Satellite is kept in orbit by remote control from ground station.
(b) Satellite is kept in orbit by retro-rocket and solar energy keeps it moving around the Earth.
(c) Satellite requires energy from solar panels and solid fuels for orbiting.
(d) Satellite does not require any energy for orbiting.

(***d***) Satellite does not requires any energy for orbiting. It requires energy to be transferred to its orbit only. Hence option (d) is correct.

89. Koradi Thermal Power Station is located in
(a) Nagpur
(b) Raipur
(c) Mumbai
(d) Secunderabad

(***a***) Koradi Thermal Power Station is located at Koradi near Nagpur, Maharashtra. The plant operates 8 units and has a total power generation capacity of 1700 mw.

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90. At which place Earth's magnetic field becomes horizontal?
(a) Magnetic meridian
(b) Magnetic equator
(c) Geographical pole
(d) Tropic of Cancer

(***b***) At magnetic equator of the Earth, the Earth's magnetic field only has horizontal component.

91. Match List I with List II and select the correct answer using the codes given below the lists :

List I (Industry)	**List II** (Place)
A. Petrochemical	1. Coimbatore
B. Aircraft	2. Pinjore
C. Machine tools	3. Bengaluru
D. Cotton textiles	4. Bongaigaon

Codes

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	4	2	3	1
(c)	1	2	3	4	(d)	1	3	2	4

(a) The correct match is A-4, B-3, C-2, D-1. Bongaigaon in Assam has Indian Oil refinery and petrochemical complex. Hindustan Aeronauticals Limited is situated at Bengaluru. Pinjore in Haryana is known for Hindustan Machine Tools Limited and textiles industries cluster is situated in Coimbatore.
Hence, option (a) is correct.

92. Consider the following statements pertaining to Coffee plantation in India :
1. Need warm and moist climate with a spell of dry weather during the ripening period.
2. Rolling fields having good drainage.
3. Strong sunshine over hilly slopes exceeding temperature 35°C.
4. Karnataka is the leading producer in India.

Which of the statements given above are correct?
(a) 1 and 4 (b) 1, 2 and 3
(c) 3 and 4 (d) 1, 2 and 4

(***d***) Karnataka is the leading producer of coffee in India. It is mostly due to the suitable climatic conditions for coffee production like warm and moist climate. Among other conditions are well drained soil and a dry weather during the ripening season. Direct sunlight is harmful for coffee plant, hence, it is planted under shade of taller trees, such as banana. It is grown on hill slopes (rolling fields), at elevations from 600 to 1600 metres with good drainage.

93. Sugarcane is one of the important cash crops in India. It is grown to obtain
(a) starch (b) glucose
(c) fructose (d) sucrose

(***d***) Sugarcane is the world's largest crop by production quantity. Sucrose is the primary product obtained from sugarcane. Other products derived from sugarcane include molasses, bagasse, falernum and ethanol.

94. Match List I with List II and select the correct answer using the codes given below the lists :

List I (Mineral)	**List II** (Leading State)
A. Manganese	1. Odisha
B. Gypsum	2. Rajasthan
C. Limestone	3. Karnataka
D. Magnesite	4. Uttarakhand

Codes

	A	B	C	D		A	B	C	D
(a)	1	3	2	4	(b)	1	2	3	4
(c)	4	2	3	1	(d)	4	3	2	1

(d) The Correct match is A-4, B-3, C-1, D-2. Uttarakhand is the second largest producer of magnesite (19%) after Tamil Nadu (74.1%). Karnataka has the largest limestone reserves (27%) in India. About 80% of Gypsum deposits are located in Rajasthan. Odisha has the largest deposits of Manganese (44%).

95. Which one among the following is the correct order of tiger reserves situated from North to South in India?
(a) Corbett—Simlipal—Sariska—Periyar
(b) Periyar—Sariska—Simlipal—Corbett
(c) Corbett—Sariska—Simlipal—Periyar
(d) Periyar—Simlipal—Sariska—Corbett

(***c***) Corbett National Park is in Uttarakhand, Sariska Tiger Reserve is in Alwar district of Rajasthan, Simlipal National Park is in Odisha while Periyar National Park is in Western Ghats of Kerala. Hence, option (c) is correct.

96. Which of the following are correct with regard to Indian Monsoonal Rainfall?
1. Largely governed by the topographical features.
2. Regional and seasonal variation in the distribution of rainfall.
3. Heavy downpour resulting in considerable runoff.
4. Beginning and end of rain is regular and on time.

Select the correct answer using the codes given below

(a) 1 and 2 (b) 1, 2 and 3
(c) 3 and 4 (d) 2, 3 and 4

(**b**) Statements 1, 2 and 3, are correct but 4 is incorrect. The beginning and end of monsoon is not necessarily regular and on time because it depends upon several factors including EL-Nino and westerly components. Other statements are correct with regard to Indian Monsoonal Rainfall.

97. Match List I with List II and select the correct answer using the codes given below the lists :

	List I (Type of Climate)		**List II** (State)
A.	Monsoon with Short Dry Season (Amw)	1.	Uttar Pradesh and Bihar
B.	Cold Humid Winter with Short Summer (Dfc)	2.	Tamil Nadu Coast
C.	Monsoon with Dry Winter (Cwg)	3.	N E India
D.	Monsoon with Dry Summer (As)	4.	Kerala and Karnataka Coast

Codes

	A	B	C	D		A	B	C	D
(a)	4	3	1	2	(b)	4	1	3	2
(c)	2	1	3	4	(d)	2	3	1	4

(**a**) The correct match is A-4, B-3, C-1, D-2. Based on Koeppen's method, Kerala and Karnataka coast comes under monsoon with short dry season (Amw). The North-East region except Assam falls under cold humid winter with short summer (Dfc). The entire Northern India, including U.P. and Bihar comes under monsoon with Dry winter (Cwg). Tamil Nadu and parts of Andhra Pradesh comes under Monsoon with dry summer (As).

98. Which one of the following Himalayan rivers does not originate from across the Himalayas?

(a) Indus (b) Sutlej
(c) Ganga (d) Brahmaputra

(c) Ganga river does not originate from across the Himalayas. It originates as Bhagirathi, from Gangotri glacier near Gomukh in Uttarkashi district of Uttarakhand. Indus, Sutlej and Brahmaputra are trans- Himalayan rivers which originate from across the Himalayas. Indus originates from a Glacier near Bokhar chu in Tibet; Sutlej originates near lake Rakshastal in Tibet and Brahmaputra rises from Chemayundung Glacier near Lake Mansarovar.

99. The headquarters of Eastern Railway Zone of Indian Railway is located at

(a) Bhubaneswar (b) Kolkata
(c) Hajipur (d) Guwahati

(b) The headquarters of Eastern. Railway Zone of Indian Railway is located at Kolkata. It comprises of 4 divisions- Howrah, Asansol, Malda and Sealdah. Bhubaneswar is the headquarter (HQ) of East Coast Railway Zone. Hajipur is the HQ of East Central Railway Zone. Guwahati is the HQ of North-East Frontier Railway Zone.

100. Match List I with List II and select the correct answer using the codes given below the lists :

	List I (Place)		**List II** (Mineral)
A.	Lakwa	1.	Copper
B.	Malanjkhand	2.	Petroleum
C.	Kalakot	3.	Zinc
D.	Zawar	4.	Coal

Codes

	A	B	C	D		A	B	C	D
(a)	3	4	1	2	(b)	3	1	4	2
(c)	2	1	4	3	(d)	2	4	1	3

(c) The correct matching is A-2, B-1, C-4, D-3. **Lakwa** in Assam is known for the petrochemical unit of BPCL and GAIL **Malanjkhand** in Madhya Pradesh has the biggest open copper mines in Asia. **Kalakot** at Rajauri,

(Jammu and Kashmir) is famous for coal. Zawar mines in Udaipur is famous for zinc, which is run by Hindustan Zinc Limited. Hence, option (c) is correct.

101. Which of the following are the major coral reef areas of India?

1. Gulf of Kutch
2. Gulf of Mannar
3. Lakshadweep
4. Andaman and Nicobar Islands

Select the correct answer using the codes given below

(a) 1 and 3 (b) 2 and 4
(c) 1, 2 and 3 (d) All of these

(**d**) Coral reefs are made of tiny marine organisms called polyps. Major coral reef areas of India include Gulf of Kutch, Gulf of Mannar, Lakshadweep and Andaman and Nicobar Islands.

102. Which of the following regions is/are not known for cotton-textile industry?

1. Mumbai—Pune region
2. Madurai—Coimbatore region
3. Dhanbad—Jamshedpur region
4. Indore—Ujjain region

Select the correct answer using the codes given below

(a) 1 and 3 (b) 2 and 3
(c) 1, 2 and 4 (d) Only 3

(**d**) Dhanbad-Jameshedpur region is famous for its coal mining and has some of the largest mines in India. Dhanbad is called the 'Coal Capital of India'.

103. Which one of the following states does not have the Headquarters of any Railway Zone?

(a) Jharkhand (b) Chhattisgarh
(c) Odisha (d) Bihar

(**a**) The East Central Railway, headquartered at Hajipur (Bihar) comprises Dhanbad (Jharkhand) division, so, there is no separate zone for Jharkhand.

Other states with headquarters of railway zones are

- Bilaspur (Chhattisgarh)–South-East Railway.
- Hajipur (Bihar)–North-East Central Railway.
- Bhubaneswar (Odisha)-East Coast Railway.

104. Which one of the following sections of the Golden Quadrilateral Highway in India is the longest in terms of route distance?

(a) Delhi-Kolkata
(b) Kolkata-Chennai
(c) Chennai-Mumbai
(d) Mumbai-Delhi

(**b**) The longest section of golden quadrilateral is a 1684 km stretch from Kolkata to Chennai. It consists of NH6 (Kolkata to Kharagpur), NH60 (Kharagpur to Balasore) and NH5 (Balasore to Chennai). Route Distance for given sections of Golden Quadrilateral are : Delhi-Kolkata 1454 km, Chennai-Mumbai-1290 km, Mumbai-Delhi 1419 km.

105. Match List I with List II and select the correct answer using the codes given below the lists :

	List I (Classification of Resources)		**List II** (Example)
A.	Basic inexhaustible resource	1.	Hydel power
B.	Conventional non-renewable resource	2.	Coal
C.	Non-conventional renewable resource	3.	Solar energy
D.	Non-conventional non-renewable resource	4.	Natural gas

Codes

	A	B	C	D		A	B	C	D
(a)	3	2	1	4	(b)	3	1	2	4
(c)	4	1	2	3	(d)	4	2	1	3

(*a*) The correct matching is A-3, B-2, C-1, D-4. Basic inexhaustible resources are present in unlimited quantity in nature and which cannot be exhausted by human activity. e.g. water, sand, solar energy, wind, etc.
Conventional non-renewable resources are conventional resources that cannot be replaced at a speed enough to keep up with consumption e.g. coal, petroleum, etc.
Non-conventional renewable resources include solar energy, wind energy, tidal energy, hydel power, biogas, etc.
Non-conventional non-renewable resources include nuclear energy, natural gas, etc.

106. Match List I with List II and select the correct answer using the codes given below the lists :

List I (Mineral Deposit)	List II (State)
A. Gypsum	1. Odisha
B. Graphite	2. Gujarat
C. Fluorspar	3. Arunachal Pradesh
D. Nickel	4. Rajasthan

Codes

	A	B	C	D		A	B	C	D
(a)	1	3	2	4	(b)	1	2	3	4
(c)	4	3	2	1	(d)	4	2	3	1

(*c*) The correct matching is A-4, B-3, C-2, D-1. The largest deposits of gypsum in India are located in Rajasthan. Bopi, Khetabari and Zaliha in Arunachal Pradesh, is known for graphite deposits. Fluorspar in Kadipani (Gujarat) is largest place of reserves. The largest deposits of Nickel is found in Odisha (over 90%).

107. Which one of the following pairs of Tribe and State is not correctly matched?
(a) Tharu : Madhya Pradesh
(b) Adi : Arunachal Pradesh
(c) Irula : Kerala
(d) Shaharia : Rajasthan

(*a*) The Tharu people are an ethnic group indigenous to the Southern foothills of the Himalayas. They live in Nepal Terai and Indian Terai of Uttarakhand, Uttar Pradesh and Bihar.

108. Consider the following Indian States :
1. Bihar
2. Rajasthan
3. Jammu and Kashmir
4. Haryana

Which one of the following is the correct **ascending order** of the above States on the basis of percentage of State's population to total population of India (based on census 2011)?
(a) 3-4-1-2 (b) 4-2-1-3
(c) 3-4-2-1 (d) 2-3-4-1

(*c*) As per 2011 Census of India, Bihar is the third most populated State of India after Uttar Pradesh and Maharashtra. The correct ascending order of given states on the basis of population (census 2011 is) as under:
1. Jammu & Kashmir (1.25 crore)
2. Haryana (2.53 crore)
3. Rajasthan (6.85 crore)
4. Bihar (10.40 crore)

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109. Match List I with List II and select the correct answer using the codes given below the lists

List I (Ultra mega power project at different stages of development)	List II (State)
A. Sasan	1. Gujarat
B. Mundra	2. Madhya Pradesh
C. Tilaiya	3. Andhra Pradesh
D Krishnapatnam	4. Jharkhand

Codes

	A	B	C	D		A	B	C	D
(a)	2	4	1	3	(b)	3	4	1	2
(c)	2	1	4	3	(d)	3	1	4	2

(*c*) The correct Matching is A-2, B-1, C-4, D-3. Ultra Mega Power Projects (UMPP) is a series of ambitious power stations planned by the Government of India. This would entail the creation of an additional capacity of atleast 100000 MW by 2022.
Sasan Ultra Mega Power Project is located in Sasan village in Singrauli district of Madhya Pradesh.
Mundra Ultra Mega Power Project is a sub-bituminous coal-fired power plant in Tunda village at Mundra, Kutch district, in Gujarat. Tilaiya Ultra Mega Power Project is a proposed coal-fired power plant in Jharkhand, India. Krishnapatnam Ultra Mega Power Project is the 4000 MW project which is located in Andhra Pradesh and is being developed by Reliance Power.

Directions (Q. Nos. 112-114) *The following items consist of two statements, Statement I and Statement II. Examine these two statements carefully and select the correct answer using the codes given below.*

Codes
(a) Both the statements are true and Statement II is the correct explanation of Statement I
(b) Both the statements are true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, but Statement II is false
(d) Statement I is false, but Statement II is true

110. **Statement I** Variability of annual rainfall is very high in western part of Rajasthan.
Statement II Average annual rainfall is extremely low in western Rajasthan.

(b) The variability of annual rainfall is very high in western part of Rajasthan, as almost entire annual rainfall occurs during the season of South-West Monsoon. Also Western cyclonic disturbances does not cause any winter rainfall in this region. This region lies in the rainshadow region of Aravallis and the Monsoon winds reaching here contains very little moisture. Thus average rainfall in western Rajasthan is very low (< 50 cm).
Hence, both the statements are true, but statement II is not the correct explanation of statement I.

111. **Statement I** Ozone is a tri-atomic molecule of oxygen.
Statement II Ozone is concentrated mainly in stratosphere.

(*b*) Ozone gas composed of three atoms of oxygen. Its chemical symbol is O_3. It is blue in colour and has a strong odour, while oxygen is colourless and odourless gas. Ozone gas (layer) is present in stratosphere, nearby 10-35 km above from the sea level, that absorbs most of the sun's ultraviolet (UV) radiation. These protect organism from many diseases.
Hence, both the statements are true, but statement II is not the correct explanation of statement I.

112. **Statement I** The laterite soils develop in areas with high temperature and high rainfall.
Statement II Laterite soils are the result of intense leaching process.

(*a*) The laterite soils develop in areas with high temperature and high rainfall. It is developed due to intense leaching of soil caused by heavy rainfall. This results in washing away of nutrients by surface water runoff. Thus, lime and silica are leached and a soil rich in oxides of iron & aluminium is obtained.

113. 'Sahel' region of Sahara desert is associated with
(a) core area of Sahara desert
(b) southern moving edge of Sahara desert
(c) northern fringe of Sahara desert bordering Mediterranean sea
(d) another name of Sahara desert

(**b**) The Sahel region is an eco-climate zone located on the southern moving edge of the Sahara desert. The region spans across Nigeria, Sudan, Senegal, Mauritania, Mali, Burkina Faso, Niger and Chad. The name, 'Sahel' comes from the Arabic word for 'border' or 'margin'. The region gained this name because it serves as the southern border to the Sahara.

114. Which one of the following pairs of a river and its tributary is not correctly matched?
(a) Godavari : Wainganga
(b) Cauvery : Bhavani
(c) Narmada : Amaravati
(d) Krishna : Bhima

(c) Amravati is a tributary of Cauvery river. Major tributaries of Cauvery are **Bhavani**, Hemavati, Kabini, Arkavati, Harangi and Noyyal etc.
Main tributaries of Godavari are **Wainganga**, Penganga, Pranhita, Indravati, Wardha, Manjra, Sabari, etc.
Main tributaries of Narmada are Tawa, Shakkar, Dudhi, Sher, etc.
Main tributaries of Krishna river are **Bhima**, Tungabhadra, Ghataprabha, Malaprabha and Musi.

115. In which one of the following countries, the Mediterranean type of biome is found?
(a) Chile (b) Kenya
(c) Argentina (d) Bolivia

(**a**) A biome is a large ecosystem, extending over a wide geographic region, characterised by certain dominant life-forms most notably, trees. Mediterranean biome is localised in the coastal areas surrounding the Mediterranean sea including parts of Europe, North Africa and Asia Minor. It is also located in the West coast of the USA and central Chile, in the Western part of South Africa and parts of southern Australia.

116. Match List I with List II and select the correct answer using the codes given below the lists

List I (River)		List II (Mouth of the river)
A. Danube	1.	North sea
B. Rhine	2.	Black sea
C. Rhone	3.	Bay of Biscay
D. Loire	4.	Mediterranean sea

Codes

	A	B	C	D		A	B	C	D
(a)	2	1	4	3	(b)	3	4	1	2
(c)	2	4	1	3	(d)	3	1	4	2

(**a**) The correct matching is A-2, B-1, C-4, D-3. **Danube** It is Europe's second longest river. It originates in Germany and empties into the Black sea.
Rhine It originates from Swiss alps in Swiss canton of Graubunden. It empties itself into the North sea at Cotterdam.
Rhone It is one of the major river of Europe and rises in Switzerland. It passes through Lake Geneva and empties itself in Mediterranean sea.
Loire It is longest river of France and rises in cevennes ranges. It empties itself into the Bay of Biscay (Atlantic Ocean at Saint Nazaire).

117. Which one of the following is the correct descending order of countries in terms of per capita availability of arable land?
(a) India, China, Brazil
(b) China, Brazil, India
(c) Brazil, China, India
(d) Brazil, India, China

(**d**) Per capita availability of arable land is as follows:

Country	Per capita availability of arable land
Brazil	0.37
India	0.12
China	0.08

118. Match List I with List II and select the correct answer using the codes given below the lists

List I (Geomorphic agent)		List II (Feature)
A. Ground water	1.	Plunge pool
B. Running water	2.	Horns
C. Glacier	3.	Playas
D. Wind	4.	Lapies

Codes

	A	B	C	D		A	B	C	D
(a)	4	1	2	3	(b)	3	2	1	4
(c)	3	1	2	4	(d)	4	2	1	3

(**a**) The correct Matching is A-4, B-1, C-2, D-3. **Plunge pools** are formed under the force of running water bodies such as waterfall. Plunge pools is an erosional features which occurs in the youthful stage of a river.
Horns is result of a glacier landform. It is made when glaciers erode three or more aretes, usually forming a sharp-edged peak.
Playas are wind erosion landforms formed in interior of desert basins. It is a flat-bottom depression surrounded by elevated topography and is periodically filled with water.
Lapies are formed by the ground water in Karst topography. It consists etched, fluted and pitted rock pinnacles separated by deep grooves.

119. Which of the following statement(s) concerning natural regions of the world is/are correct?
1. Equatorial climatic regions have less urbanisation.
2. Mediterranean climatic regions have high level of urbanisation.
3. Equatorial climatic regions have high concentration of human population.
4. Mediterranean climatic regions practice plantation farming.

Select the correct answer using the codes given below
(a) 1 and 2 (b) 1, 2 and 3
(c) 2 and 4 (d) Only 1

(**a**) Equatorial climate regions are located near equator within 5° North & South latitudes. The region has dense forests and scarce human population. These regions have less urbanisation & industrial development. So, statement (1) is correct and (3) is incorrect. Mediterranean climatic regions are located near the Mediterranean Sea. These are among the most developed regions of the world. They have high level of urbanisation and high level of industrial development. Plantation farming is not usually practised here. So, Statement (2) is correct and (4) is incorrect.

120. Consider the following statements
1. Periplus is a Greek word meaning sailing around.
2. Erythraean was the Greek name for the Mediterranean sea.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**a**) Periplus is the latinisation of the Greek word, which means sailing around. Hence, statement (1) is correct. The Erythraean sea is also a Greek word meaning Red sea. The Erythraean sea is the name in ancient cartography for a body of water located between the Horn of Africa and the Arabian Peninsula. Hence, statement (2) is incorrect.

121. Which of the following statements concerning Mediterranean climatic region are correct?

1. It is found in the latitude range 30° to 45° Northern and Southern hemisphere.
2. Mediterranean climate experience extreme temperature conditions.
3. In Mediterranean climate, rainfall occurs mainly during summer season.
4. In Mediterranean climate, rainfall is mainly due to Westerlies.

Select the correct answer using the codes given below.

(a) 1, 2 and 4 (b) 1 and 4
(c) 1, 2 and 3 (d) 3 and 4

(***b***) Mediterranean climate is found between the 30° and 45° degree North and South latitudes and often found on the western sides of continents. Hence, Statement (1) is correct. Mediterranean climate gets its name from the climate found around the Mediterranean sea. Mediterranean climate is mild (few extreme temperatures), so that there is seen generally two seasons i.e. summer and winter. Summers are longer than winter and the winter is very mild. Hence, Statement (2) is incorrect. Mediterranean climate is a fairly dry climate. Almost no rainfalls during the summer, so most of the rainfalls occurs during the cooler winter, which is strongly influenced by westerly air streams. Hence, statement (3) is in correct and statement (4) is correct.
Hence, option (b) is correct.

122. Arrange the following States in decreasing order of their total area under forest cover

1. Jharkhand
2. Chhattisgarh
3. Arunachal Pradesh
4. Madhya Pradesh

Codes

(a) 4, 3, 2, 1 (b) 4, 2, 3, 1
(c) 3, 4, 1, 2 (d) 2, 1, 3, 4

(***a***) According to Forest Report, 2017, the total area under forest cover is:

State	Area under forest cover
Madhya Pradesh	77414
Arunachal Pradesh	66964
Chhattisgarh	55547
Jharkhand	23553

Hence, option (a) is the correct answer.

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123. Spruce and Cedar are tree varieties of

(a) equatorial forest
(b) temperate coniferous forest
(c) monsoon forest
(d) temperate deciduous forest

(***b***) Temperate coniferous forests are made up of mainly cone-bearing trees such as pine, fir, spruce, hemlock etc and other varieties include juniper, cedar, redwood. The eastern himalayan subalpine conifer forests is a temperate coniferous forests which extends through Bhutan, India and Nepal.

124. 'Sal' tree is a

(a) tropical evergreen tree
(b) tropical semi-evergreen tree
(c) dry deciduous tree
(d) moist deciduous tree

(***c***) Dry deciduous forests are monsoonal forests found in areas where rainfall lies between 70 to 100 cm. Sal is the most significant tree found in these forests. Teak, neem, peepal are other common varieties.

125. Deserts, fertile plains and moderate mountains are the characteristics of which one of the following regions?

(a) South-Western border along sea
(b) Coromandel coast
(c) North-Eastern frontier
(d) North-Western India

(***d***) North-Western India has all the features described in the question. Area lying to the West of Aravallis is a desert. Punjab and Haryana have fertile plains suitable for wheat cultivation. Aravallis are moderately forested in rainier areas.

126. Which organisation prepares the topographical maps of India?

(a) Geological Survey of India
(b) Archaeological Survey of India
(c) Survey of India
(d) National Atlas and Thematic Mapping Organisation

(***c***) Survey of India, the national survey and mapping organisation of the country, is responsible for preparing topographical maps of India. These maps are printed on 1 : 250000, 1 : 50000 and 1 : 25000 scales.

127. Which one of the following is not an international boundary line?

(a) Dresden Line
(b) Durand Line
(c) Maginot Line
(d) 38th Parallel North

(***a***) **Dresden Line** is not an international boundary line. It is a railway line in Germany.
Durand Line forms the boundary between Pakistan and Afghanistan.
Maginot Line was drawn as a boundary line separating France from Switzerland, Germany and Luxembourg.
38th Parallel North refers to border between North and South Korea.

128. The brightness of a star depends on its

(a) size and temperature
(b) size and distance from the Earth
(c) size, temperature and mass
(d) size, temperature and distance from the Earth

(***a***) According to Stefan's law, rate of loss of heat energy in the form of radiation (brightness) of a star depends upon
(i) nature of surface of radiant object
(ii) surface area (size) of the star
(iii) temperature of the star

129. The 'Amarkantak Hills' is the source of which of the following rivers?

1. Narmada 2. Mahanadi
3. Tapti 4. Son

Select the correct answer using the codes given below

(a) 1 and 2 (b) Only 2
(c) 1, 3 and 4 (d) 1, 2 and 4

(*******) Narmada river originates from Narmada Kund in Amarkantak hills. Son river also rises from Amarkantak. Tapti river rises from Betul in Satpura range. Mahanadi originates from Sihawa mountain in Chhattisgarh. Hence, Statement (1) and (4) is correct.

130. Which of the following statements in the context of Northern hemisphere is/are correct?

1. Vernal equinox occurs on 21st March.
2. Summer solstice occurs on 22nd December.
3. Autumnal equinox occurs on 23rd September.
4. Winter solstice occurs on 21st June.

Select the correct answer using the codes given below

(a) Only 1 (b) 1 and 3
(c) 2 and 4 (d) 1, 2 and 3

(***b***) Equinox are days when days and nights are equal. In Northern Hemisphere, 21st March is Vernal equinox and 23rd September is

Autumnal equinox. Summer solstice falls on 21st June, when Sun is vertically overhead the tropic of cancer. Winter solstice is observed on 22nd December when Sun is vertically overhead at Tropic of Capricorn. Hence, option (b) is correct.

131. In India, glacial terraces known as 'Karewas' are found in
(a) Sapt Kosi Valley
(b) Jhelum Valley
(c) Alakananda Valley
(d) Teesta Valley

(**b**) Karewas are glacio-fluvial deposits found in Jhelum Valley of Kashmir. Karewas are important for the cultivation of zaffron, a local variety of saffron.

132. During solar eclipse,
(a) the Earth comes in between the Sun and the Moon
(b) the Moon comes in between the Sun and the Earth
(c) the Moon comes exactly halfway between the Earth and the Sun
(d) the Sun comes in between the Earth and the Moon

(**b**) A solar eclipse is a type of eclipse that occurs when the Moon passes between the Sun and Earth. The Moon's shadow fully or partially, casts itself on Earth blocking the Sun's light.

133. The Nagarjuna Sagar Project is located on which one of the following rivers?
(a) Godavari (b) Krishna
(c) Kavery (d) Mahanadi

(**b**) Nagarjuna Sagar project is the world's largest masonry dam built across Krishna river at Nagarjuna Sagar. It is one of the earliest irrigation and hydro-electric projects in India. It is situated in states of Andhra Pradesh and Telangana.

134. Consider the following statements.
1. Rajmahal highlands consist of lava flow deposits.
2. Bundelkhand gneiss belong to the oldest Archaean rocks of India.

Which of the statements given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(**c**) Rajmahal highlands were formed as a result of volcanic activity in the Jurassic period. It stretches from Sahibganj district to Dumker district in Jharkhand.
The Bundelkhand gneiss belong to the oldest rock system in India, known as Archaean rocks. It occurs in Bundelkhand, Baghelkhand, Maharashtra, Rajasthan, Andhra Pradesh and Tamil Nadu. Hence, both the statements are correct.

135. Which one of the following is a conventional energy source?
(a) Tidal energy
(b) Geothermal energy
(c) Solar energy
(d) Bio-mass-energy

(**d**) Conventional sources of energy are the natural energy sources which are being used for a long time and are usually present in a limited quantity. They include Biomass energy (wood, crop residue, dried vegetation etc), coal, petroleum and natural gas. Tidal energy, Geothermal energy and solar energy are non-conventional sources of energy.

136. After a ban on unscientific coal mining and transportation of coal in this region by the National Green Tribunal, many of those engaged in the activity have been switching to turmeric farming for their livelihood. The region referred above is
(a) Jaintia Hills, Meghalaya
(b) Koriya, Chhattisgarh
(c) Angul, Odisha
(d) Bardhaman, West Bengal

(**a**) The National Green Tribunal (NGT) ban on coal mining and transportation of coals in Jaintia Hills region has resulted in many of those enjoyed in the activity switching to turmeric farming.

Directions (Q. Nos. 140-142) *The following five items consist of two statements, Statement I and Statement II. Examine these two statements carefully and select the answers to these items using the codes given below.*

Codes
(a) Both the statements are individually true and Statement II is the correct explanation of Statement I.
(b) Both the statements are individually true, but Statement II is not the correct explanation of Statement I.
(c) Statement I is true, but Statement II is false.
(d) Statement I is false, but Statement II is true.

137. Statements
I. Volcanic eruption is accompanied by earthquakes.
II. Volcanoes erupt water vapours and dust particles in the atmosphere.

(b) Volcanic eruption at times might be accompanied by earthquake.
However, not all volcanoes result into earthquakes. Volcanoes erupt lava, water vapour, gases, dust particles etc. Hence, option (b) is correct.

138. Statements
I. Plantation farming has mostly been practised in humid tropics.
II. The soil of humid tropics is highly fertile.

(**c**) Humid tropics are most favoured location for plantation crops including rubber, tea, coffee, coconut etc. However, the soils of this region is not very fertile due to leaching of minerals because of heavy rainfall. Hence, option (c) is correct.

139. Statements
I. India has wide variation in population density.
II. Factors like agricultural productivity and history of settlements have greatly influenced the population density pattern in India.

(**a**) India has a wide variation in population density from 17 persons sq km in Arunachal Pradesh to 1106 persons /sq km in Bihar. Many factors contributes to this like agricultural productivity, history of settlements, relief, topography, water resources and historical reasons. So, Statement II is the correct explanation of statement I.

140. In Thar region, the shifting sand dunes are locally known as
(a) Dhrian (b) Daurs
(c) Dhoros (d) Dhaya

(**a**) The shifting sand dunes in 'Marusthali (Thar)' are locally known as Dhrian and the blow out depressions are known as 'Dhand'.
Dhoros are the long narrow depressions which are the remnants of the course of a dried out former river.
Dhaya are the heavily gullied blutts in Punjab and Haryana Plains.
Duars are the alluvial flood plains in the north-eastern India that lie South of the foothills of the Himalayas.

141. Which of the following pairs is/are correctly matched?

	List I (National Park)	List II (Famous for)
1.	Ranthambhore	Tiger
2.	Periyar	Elephant
3.	Manas	Lion
4.	Gir	Rhinoceros

Select the correct answer using the codes given below

(a) 1, 2 and 3 (b) 1 and 2
(c) 1 and 4 (d) Only 2

(*b*) Ranthambhore National Park is present in Sawai Madhopur in Rajasthan. It is famous for tiger.
Periyar National Park present in Idukki, Kerala. It is famous for elephants. Manas National Park is situated in Assam. It is famous as a project tiger reserve and an elephant reserve. Gir National Park is situated in Gujarat, famous for asiatic lions.

142. Jelep La pass is located in

(a) Punjab Himalaya
(b) Sikkim Himalaya
(c) Kumaon Himalaya
(d) Kashmir Himalaya

(*b*) Jelep La is a high altitude mountain pass located in Sikkim Himalayas. It links Lhasa in Tibet to Sikkim in India. This pass is an important trade route between India and Tibet.

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143. Which one of the following pairs of properties of typical air masses is correct?

	Air Mass	Source Region
(a)	Maritime Equatorial	Warm oceans in the equatorial zone
(b)	Maritime Tropical	Warm oceans in the tropical zone
(c)	Continental Tropical	Less warm oceans in the tropical zone
(d)	Continental Polar	Moist oceans in the polar zone

(*a*) Maritime Equatorial air mass originates from warm oceans in the equatorial zone.
Maritime Tropical originates from the warm water where heat and moisture are transformed to the overlying air from the water below. Continental tropical is a dry air mass formed over land in the area close to equator.
Continental polar is a cold, dry stable air mass originating over land surface of Canada and Alaska.

144. According to the Geo-scientists, the shape of the Earth is

1. round
2. spherical
3. close to that of a sphere
4. an oblate ellipsoid

Select the correct answer using the codes given below

(a) 2, 3 and 4 (b) 1, 2 and 3
(c) 1 and 2 (d) 3 and 4

(*d*) According to the Geo-scientists, the shape of the Earth can be considered as close to that of sphere and also as an oblate ellipsoid termed as 'Geoid'. The shape of Earth is not completely round and spherical because it is flattened at the poles and bulges at the Equator. Hence, option (d) is correct.

145. Match List I with List II and select the correct answer using the codes given below the lists

List I (Manufacturing Site)	List II (Industry)
A. Ludhiana	1. Auto parts
B. Kanpur	2. Woollen garments
C. Varanasi	3. Leather
D. Vijayawada	4. Handloom

Codes

	A	B	C	D
(a)	1	4	3	2
(b)	2	3	4	1
(c)	2	4	3	1
(d)	1	3	4	2

(*b*) The correct matching is A-2, B-3, C-4, D-1 **Ludhiana** is famous all over India for its woollen sweaters and cotton T-shirts. Most of the top Indian woollen apparel brands are based in Ludhiana.

Kanpur is renowned for its leather industries. The largest centre of the industry is in Jajmau suburb of Kanpur.
Varanasi is famous for handloom works and its Bari bazar is famous for Banarasi Silk Sarees.

Vijayawada is well known as the auto parts capital of India. The Jawaharlal Nehru Auto Nagar Industrial Estate, located here, is the largest of its kind in Asia.

146. Which one of the following is not a sea port?

(a) Paradeep
(b) Haldia
(c) Diamond Harbour
(d) Dhamra

(*c*) **Diamond Harbour** is in the Southern suburbs of Kolkata. It is a famous tourist centre situated on the eastern banks of Hooghly river, but it is not a port. Therefore, correct answer is option (c).
Paradeep Port is one of the major ports of India serving the Eastern and Central parts of the country. It is located in Odisha.
Haldia is a major riverport and industrial belt located in Kolkata near the mouth of the Hooghly river.

Dhamra Port is a newly developed port in Bhadrak district, Odisha, on the shore of the Bay of Bengal.

147. The Manas National Park is situated in the state of

(a) Madhya Pradesh
(b) Jharkhand
(c) Assam
(d) West Bengal

(*c*) The Manas National Park is situated in the state of Assam. It is UNESCO Natural World Heritage Site. It is mainly a project to harbour elephant, wild buffalo, tiger and rhino. It is one of the famous biosphere reserves in India.

148. Which one of the following statements is not correct?

(a) Temperatures decrease from the equator to poles.
(b) Temperatures in equatorial regions change substantially from January to July.
(c) Large land masses located in the Sub-arctic and Arctic zones develop centres of extremely low temperatures in winter.
(d) Highlands are always colder than surrounding lowlands.

(*b*) From January to July there is very small change in temperature. In equatorial region the changing angle of the Sun through the seasons has a proportionally smaller effect on the total amount of sunlight received. So, there is very little change in temperature.

149. An upfold in rock is

(a) graben (b) horse
(c) anticline (d) syncline

(*c*) An **anticline** is an fold that arches up as both sides of the rock are pushed inward.

A **syncline** is a fold that sinks down as both sides of the rock are pushedu pward.

A **Graben** is a fault which is produced when tensional stresses result in the subsidence of a block of rock. On a large scale, these features are known as Rift Valleys.

Horse is a latitude which is either of two belts or regions in the neighbourhood of 30° N and 30° S latitude characterised by high pressure and light variable winds.

150. Which one of the following gases is found in highest quantity in Exosphere?

(a) Hydrogen (b) Helium
(c) Nitrogen (d) Oxygen

(*a*) Exosphere, the outermost layer of atmosphere is extremely thin and here the atmosphere merges into outer space. It is mainly composed of very widely dispersed particles of hydrogen and some amount of helium.

Composition of Gases in Atmosphere

Gas	Percentage in Air
Nitrogen	78.08%
Oxygen	20.95%
Argon	0.93%
Carbon dioxide	0.033%
Other gases	0.02%

151. Identify the place that is not an oil field.

(a) Naharkatiya (b) Kalol
(c) Ledo (d) Ankleshwar

› *(**c**)* Ledo is not an oil field. It is a small town in Tinsukia district of Assam. It is the easternmost broad gauge railway station in India. In Assam Naharkatiya, Rudrasagar, Moran, Hugrijan, Lakoa and Galeki have been developed by Oil India Ltd. Gujarat's oil wells are at Ankleshwar (largest), Cambay, Kalol, Kosamba, Mehsana, Nowgam, Dholka, Lunej, Sananda, Wavel Bakal and Kathana.

152. Match List I with List II and select the correct answer using the codes given below the lists

	List I (Low-Latitude Climate)		**List II** (Characteristic)
A.	Wet Equatorial	1.	Uniform temperatures, mean near 27°C
B.	Monsoon and trade wind coastal	2.	Marked temperature cycle with very high temperature before the rainy season
C.	Wet-dry tropical	3.	Temperatures show an annual cycle with high temperature in the high-Sun season
D.	Dry tropical	4.	Strong temperature cycle, with intense temperature during high-Sun season

Codes

	A	B	C	D		A	B	C	D
(a)	2	3	4	1	(b)	1	2	3	4
(c)	2	4	3	1	(d)	1	3	2	4

› *(**b**)* The correct matching is A-1, B-2, C-3, D-4.
The wet equatorial climate is characterised by uniformity of temperature throughout the year. The average monthly temperature is about 26-28 degree Celsius.
Monsoon and trade wind coastal climates are characterised by distinct wet and dry seasons associated with marked temperature differences in summer and winter. Temperature ranges from 30-45°C in summer before onset of monsoon.
The wet-dry tropical climate is characterised by a warm climate but with a more marked temperature range. During the high Sun season, proximity to the Inter Tropical Convergence Zone brings heavy rains and during the cooler period, the subtropical high pressure cell produces very dry conditions.
The dry tropical climate is dominated by the subtropical high-pressure cell. It experiences very low precipitation and intense daytime heating under predominantly clear skies and includes many of the world's great deserts.

153. The IST meridian $82\frac{1}{2}^{\circ}$E passes through a number of states in India. Which one of the following sets of states is correct in this respect ?

(a) Uttarakhand, Uttar Pradesh, Chhattisgarh and Andhra Pradesh
(b) Uttar Pradesh, Jharkhand, Chhattisgarh and Odisha
(c) Uttarakhand, Uttar Pradesh, Madhya Pradesh and Chhattisgarh
(d) Uttar Pradesh, Odisha, Andhra Pradesh and Chhattisgarh

› *(**d**)* The standard meridian of India i.e. 82.5° E longitude passes through the states of Uttar Pradesh, Madhya Pradesh, Chhattisgarh, Andhra Pradesh and Odisha.
82.5°E meridian passes through Mirzapur district of UP is taken as Indian standard time. It is 05:30 hr ahead of Greenwich mean time.

154. Which one among the following places is not an iron ore mining area?

(a) Badampahar (b) Zawar
(c) Bailadila (d) Anantpur

› *(**b**)* Only six states, i.e. Jharkhand, (Badampahar) Odisha, Madhya Pradesh, Chhattisgarh, (Bailadila) Andhra Pradesh (Anantapur) and Goa account for over 95% of the total reserves of India. Zawar, in Udaipur, is a zinc mine.

155. Match List I with List II and select the correct answer using the codes given below the lists

	List I (Forest Conservation Type)		**List II** (Place)
A.	National Park	1.	Dudhwa
B.	Sanctuary	2.	Bhitarkanika
C.	Biosphere Reserve	3.	Chilika
D.	Tiger Reserve	4.	Nokrek

Codes

	A	B	C	D		A	B	C	D
(a)	2	3	4	1	(b)	1	4	3	2
(c)	2	4	3	1	(d)	1	3	4	2

› *(**a**)* The correct matching is A-2, B-3, C-4, D-1. **National Park** Bhitarkanika is in Odisha and the second largest mangrove ecosystem in India. Bhitarkanika mangroves is in the delta of river Baitarni and Brahmani.
Sanctuary Chilika, a brackish water lagoon in Odisha is a wildlife sanctuary.
Biosphere Reserve Nokrek, a hotspot biodiversity in Garo hills of Meghalaya. It is famous for Red Panda.
Tiger Reserve Dudhwa, a Tiger reserve in high diverse and productive region of Terai ecosystem of UP has tigers and swamp deer as major attractions.

156. Which of the following statements about hill stations of colonial India is/are correct?

1. The architecture of hill stations sought to recreate the European style.
2. Hill stations were developed as sanitariums where soldiers were treated for illness.
3. Shimla became the official residence of the Commander in Chief of the Indian Army.
4. In 1864, Shimla had to be evacuated because of a typhoid epidemic.

Select the correct answer using the codes given below

(a) Only 1 (b) 1, 2 and 3
(c) 3 and 4 (d) 2 and 3

› *(**b**)* Among the given options statements 1, 2 and 3 are correct while statement 4 is incorrect. In 1864, Shimla was declared as the summer capital of British India by the viceroy John Lawrence. It also became the official residence of Commander in chief of the Indian army. There is no evidence of 'Shimla had to be evacuated because of a typhoid epidemic.'

157. Match List I with List II and select the correct answer using the codes given below the lists

	List I (Place)		**List II** (Normal Vegetation Type)
A.	Western Ghats	1.	Tropical Moist Deciduous
B.	Himachal Pradesh	2.	Tropical Evergreen
C.	Haryana and Punjab	3.	Himalayan Moist
D.	Chhotanagpur Plateau	4.	Tropical Thorny

Codes

	A	B	C	D		A	B	C	D
(a)	2	3	4	1	(b)	1	4	3	2
(c)	2	4	3	1	(d)	1	3	4	2

(*a*) The correct matching is A-2, B-3, C-4, D-1. **Tropical Evergreen Forests** or Tropical Rainforests are usually found in areas receiving more than 200 cm of rainfall and having a temperature of 15°C to 30 °C and have annual humidity exceeding 77%. Evergreen forests are found on the Eastern and Western slopes of the Western Ghats in such states as Tamil Nadu, Karnataka, Kerala and Maharashtra.

The Himalayan subtropical pine forests or **Himalayan Moist Forests** are a large subtropical coniferous forest covering the Northern Indian states of Jammu and Kashmir, Himachal Pradesh, Uttarakhand and Sikkim.

The **Thorny Forests** and Scrubs are found in the semi-arid areas of Gujarat, Rajasthan, Madhya Pradesh, Chhattisgarh, Uttar Pradesh, Haryana and Punjab.

The **Tropical Moist Deciduous Forests** are found in areas where the rainfall is between 100 cm to 200 cm. They are found in the North Eastern states of Jharkhand, West Odisha and Chhattisgarh (Chhotanagpur Plateau). Moist Deciduous Forests are also found in the eastern slopes of the Western Ghats.

158. The term 'Regur' is used to mean
(a) laterite soil
(b) deltaic soil
(c) red soil
(d) black cotton soil

(*d*) **Black Soil** The Black soil of Deccan Plateau is also called 'Regur soil' or the 'Black cotton soil'. It is clayey, deep and impermeable, also called 'self ploughing' soil. The soil is most suitable for cotton cultivation.

159. Which one of the following combinations of stalactites and stalagmites occurrences is correct?
(a) Stalactites hang as icicles of different diameters and stalagmites hang from the floor of the caves.
(b) Stalactites hang as icicles of different diameters and stalagmites rise up from the floor of the caves.
(c) Stalactites rise up from the floor of the caves and stalagmites hang as icicles of different diameters.
(d) Stalactites hang as icicles of different diameters and stalagmites also hang as icicles of different diameters.

(*b*) Stalactites and stalagmites are depositional land forms formed by the action of ground water. These depositional landforms develop within limestone caves. Stalactites hang as icicles of different diameters. They are broad at their bases and taper towards the free ends. Stalagmites rise up from the floor of the caves. Hence, option (b) is correct.

160. Which one of the following is the correct sequence about various levels of organisation of Biosphere?
(a) Ecosystem — Biosphere — Community — Population
(b) Population — Organism — Ecosystem — Biosphere
(c) Organism — Community — Population — Biosphere
(d) Organism — Population — Ecosystem — Biosphere

(*d*) Organism is the smallest unit. A group of organisms together form population. A group of organism in their biotic and abiotic environment and the interaction with each other forms an Ecosystem. The combination of all ecosystems forms the biosphere. So, option (d) is the correct sequence about various levels of organisation of biosphere.

161. Which one of the following describes the Lithosphere?
(a) Upper and lower mantle
(b) Crust and upper mantle
(c) Crust and core
(d) Mantle and core

(*b*) Lithosphere is rigid, rocky outer layer of the Earth, consisting of the crust and the solid outermost layer of the upper mantle. It extends to a depth of about 70-100 km. It is broken into about a dozen separate rigid blocks or plates. Lithosphere is bounded by the atmosphere above and the asthenosphere below.
There are two types of lithosphere - Oceanic lithosphere and Continental lithosphere. The former is associated with oceanic crust and is slightly denser than the later one.

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162. Which one of the following statements is not correct?
(a) Kerala is the largest producer of natural rubber in India.
(b) Neyveli is an important thermal power generating area of Tamil Nadu.
(c) Ratnagiri bauxite mining area is located in Karnataka.
(d) Assam is the only largest tea producing state in India.

(*c*) Kerala contributes 90% of India's total production of natural rubber. Therefore, Kerala is the largest producer of natural rubber in India.

Major thermal power plants of Tamil Nadu are Mettur Thermal Power Station, Neyveli Thermal Power Station-1, Neyveli Thermal Power Station-2, Tuticorin Thermal Power Station and North Chennai Thermal Power Station.

India's major tea producing states are Assam, West Bengal, Tamil Nadu, Kerala and Karnataka. Assam is the largest tea producing state (52% of total tea production).

Ratnagiri bauxite mining area is located in Ratnagiri district of Maharashtra. Hence, option (c) is not a correct statement.

163. Match List I with List II and select the correct answer using the codes given below the lists

	List I (Paper Industry Centre)		**List II** (State)
A.	Kamptee	1.	Karnataka
B.	Rajahmundry	2.	Maharashtra
C.	Shahdol	3.	Andhra Pradesh
D.	Belagola	4.	Madhya Pradesh

Codes

	A	B	C	D		A	B	C	D
(a)	1	4	3	2	(b)	2	3	4	1
(c)	1	3	4	2	(d)	2	4	3	1

(*b*) The correct matching is A-2, B-3, C-4, D-1. Kamptee is a city in Nagpur district of Maharashtra.
Rajahmundry is a major city of Andhra Pradesh and located on the banks of the Godavari river, in East Godavari district of the state.
Shahdol city is a district of Madhya Pradesh.
Belagola is a village in Mandya district of Karnataka.

164. The newly formed state of Telangana is surrounded by
(a) 5 states (b) 4 states
(c) 6 states (d) 3 states

(*a*) Telangana touches boundaries of five states — Andhra Pradesh, Chhattisgarh, Karnataka, Odisha and Maharashtra. Telangana is the 29th state of India formed on 2nd June, 2014 with Hyderabad as its capital.

165. Rain shadow effect is associated with
(a) cyclonic rainfall
(b) orographic rainfall
(c) convectional rainfall
(d) frontal rainfall

› *(**b**)* When the saturated air mass come across a barrier. Such as mountain it is forced to ascend giving greater rainfall in wind ward slopes. Descending through leeward slopes it looses its moisture resulting into rainless area generally called rain-shadow area. This type of rainfall is called orographic rainfall.

166. Match List I with List II and select the correct answer using the codes given below the lists

	List I (Place)		**List II** (Industry)
A.	Jabalpur	1.	Petro-chemical industry
B.	Bengaluru	2.	IT industry
C.	Mathura	3.	Paper industry
D.	Ballarpur	4.	Automobile industry

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	3	2	1	4
(c)	4	2	1	3
(d)	3	1	2	4

› *(**c**)* The correct matching is A-4, B-2, C-1, D-3. Jabalpur is famous for automobile industry. Jabalpur has Vehicle Factory, Jabalpur, Grey Iron Foundry, Gun Carriage Factory Jabalpur and the Ordnance Factory Khamaria.

Bengaluru is famous for IT industry also known as 'Silicon Valley of India.'

Mathura is famous for petro-chemical industry. The Mathura Refinery, owned by Indian Oil Corporation, is located here. Ballarpur is located in Maharashtra and it is famous for paper industry. Ballarpur Industries Limited (BILT) is India's largest manufacturer of Writing and Printing (W&P) paper.

167. Which one of the following irrigation canals is the most important in terms of area coverage in Haryana?

(a) Bhakra Canal
(b) The Western Yamuna Canal
(c) Jawaharlal Nehru Canal
(d) Gurgaon Canal

› *(**b**)* **The Western Yamuna Canal** begins at the Hathnikund Barrage. The canals irrigate vast tracts of land in the region in Ambala, Karnal and Sonepat district becoming one of the most important canal in terms of area coverage in Haryana.

Bhakra Canal provides irrigation to Punjab, Haryana and Rajasthan.

Jawaharlal Nehru Canal supplies water to Haryana , but once a year due to drying up of its supply. It requires repair and proper availability or water.

Gurgaon Canal has been drawn at the Okhla Barrage from the Yamuna. With the waters of this canal, the districts of Gurgaon and Faridabad are irrigated.

168. Which one of the following Indian states has the highest proportion of area under forest cover ?

(a) Madhya Pradesh
(b) Sikkim
(c) Meghalaya
(d) Mizoram

› *(**d**)* As per State of Forest Report , 2017

State	**Forest Cover**
Mizoram	86.27%
Meghalaya	76.45%
Sikkim	77.13%
Madhya Pradesh	25.11%

169. Match List I with List II and select the correct answer using the codes given below the lists

	List I (National Park/Wildlife Sanctuary)		**List II** (State)
A.	Chandra Prabha	1.	Uttarakhand
B.	Silent Valley	2.	Chhattisgarh
C.	Valley of Flowers	3.	Uttar Pradesh
D.	Indravati	4.	Kerala

Codes

	A	B	C	D		A	B	C	D
(a)	2	1	4	3	(b)	3	4	1	2
(c)	2	4	1	3	(d)	3	1	4	2

› *(**b**)* The correct matching A-3, B-4, C-1, D-2. Chandra Prabha Wildlife Sanctuary is located in Chandauli district of Uttar Pradesh.

The Silent Valley National Park, also known locally as Sairandhrivanam, is located in the Nilgiri hills, in Palakkad district of Kerala.

Valley of Flowers National Park is located in West Himalaya in Uttarakhand.

Indravati National Park is the most famous wildlife sanctuary of Chhattisgarh.

170. To a perpendicular to the plane of ecliptic, the Earth's axis of rotation makes an angle of $23\frac{1^\circ}{2}$.

Had this angle been zero degree, which one among the following would result?

(a) There would have been no season.
(b) The length of day and night would have been the same throughout the year.
(c) The length of the day and night would have been the same all over the Earth.
(d) All of the above

› *(**d**)* The Earth axis is tilted at $23\frac{1^\circ}{2}$. This tilt of the Earth is responsible for yearly cycle of seasonal weather changes. If Earth's axis or rotation makes an angle 0° i.e. if it was not tilted, the plane of the Earth's poles would always be perpendicular to the Sun. Hence, there would be no variations in the duration of sunlight on any location on different days of the year and the length of day and night would have been the same all over the Earth.

171. 'Yakutsk' are the nomadic herders of

(a) Gobi
(b) Sahara
(c) Tundra
(d) Kalahari

› *(**c**)* The Yakutsks are a Siberian population that primarily lives in the tundra biome of Republic of Sakha (also known as Yakutia), an autonomous region within the Russian Federation. They are traditionally cattle and horse-breeders who speak a Turkic language, which differs from the subsistence patterns and languages of neighbouring populations.

172. The luxuriant growth of natural vegetation in tropical rainforest is due to

1. fertile soil.
2. hot and wet climate throughout the year.
3. intense sunlight for photosynthesis.
4. seasonal change to facilitate nutrient absorption.

Select the correct answer using the codes given below

(a) 1 and 4
(b) 2 and 3
(c) 1, 2 and 3
(d) All of these

› *(**b**)* Tropical rainforests are found in places with hot and wet equatorial climate. The profile of the soil in the rainforest is called a Latosol (reddish brown and red colour of soil) due to presence of oxides and aluminium and remain deficient in major nutrients such as calcium and potassium. As there is great competition for sunlight to do photo synthes is and water in the rainforests, the trees try to grow very tall.

173. Match List I with List II and select the correct answer using the codes given below the lists

List I (Region)	List II (Vegetation)
A. Selvas	1. Coniferous
B. Savannas	2. Mosses and lichens
C. Taiga	3. Epiphytes
D. Tundra	4. Grasses and trees

Codes

	A	B	C	D		A	B	C	D
(a)	4	1	2	3	(b)	3	2	1	4
(c)	3	4	1	2	(d)	4	2	1	3

› *(**c**)* The correct matching is A-3, B-4, C-1, D-2 **Selvas** is the largest tract of tropical rainforest in Mexico and contains the majority of terrestrial biodiversity in the country. Dense vegetation including epiphytes, ebony, Mahogany, cinchona, rosewood etc. are found here.

Savannas is rolling grassland scattered with shrubs and isolated trees, which can be found between tropical rainforest and desert biome.

Taiga is a biome characterised by coniferous forests consisting mostly of pines, spruces and larches.

In **Tundra,** the vegetations are composed of dwarfs shrubs, sedges and grasses, mosses and lichens. Hence, option (c) is correct.

174. Which of the following statements with regard to the Western coastal plain of India are correct?

1. It is a narrow belt.
2. It is an example of submerged coastal plain.
3. It provides natural conditions for development of ports.
4. It has well developed deltas.

Select the correct answer using the codes given below

(a) 1, 2 and 3
(b) 1 and 2
(c) 3 and 4
(d) All of the above

› *(**a**)* The western coastal plains are a narrow setup of coastal plains with an average width of 50 km between Arabian sea and Western Ghats. It is a narrow plain with an average width of 64 km. This plain is an example of submerged coastal plain. Because of this submergence, it is a narrow belt and provides natural conditions for the development of ports and harbours.

175. Which one of the following weather conditions indicates a sudden fall in barometer reading?

(a) Stormy weather
(b) Calm weather
(c) Cold and dry weather
(d) Hot and sunny weather

› *(**a**)* The word 'stormy' describes weather conditions like thunder, lightening, dark clouds, wind and pelting rain. This is the only option given in the question that indicates a sudden fall in barometer reading.

176. Match List I with List II and select the correct answer using the codes given below the lists

List I (Island)	List II (Location)
A. Continental island	1. Mauritius
B. Coral island	2. Madagascar
C. Volcanic island	3. Andaman and Nicobar islands
D. Mountain island	4. Maldives

Codes

	A	B	C	D		A	B	C	D
(a)	2	4	1	3	(b)	2	1	4	3
(c)	3	1	4	2	(d)	3	4	1	2

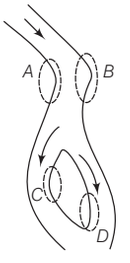
› *(**a**)* The correct matching is A-2, B-4, C-1, D-3. **Continental Island** An island that is near to and geographically related to a continent. e.g. Barneo, Java, Madagascar, New Zealand.

Coral Island An island formed from coral detritus and associated organic material in tropical and subtropical areas. e.g. - Maldives, Lakshadweep Island etc.

Volcanic Island An Island formed from volcanoes erupting from the ocean floor. e.g. Aleutian Island, Mariana Island, Mauritius Island.

Mountain Island An island formed by upraising of under ocean mountains. e.g. Andaman and Nicobar Islands.

177. Consider the following diagram



In which one among the following lettered areas of the diagram would erosion most likely change the shapes of the riverbed?

(a) *A* (b) *B*
(c) *C* (d) *D*

› *(**b**)* Area *A* have more angular surface and very small erosion than B, so it is unlikely to change the shapes of riverbed. *C* and *D* cannot be correct option because inside river stream will never changes the shapes of riverbed.

178. Match List I with List II and select the correct answer using the codes given below the lists

List I (Cloud)	List II (Characteristic)
A. Cirrus	1. Rain giving
B. Stratus	2. Feathery appearance
C. Nimbus	3. Vertically growing
D. Cumulus	4. Horizontally spreading

Codes

	A	B	C	D		A	B	C	D
(a)	3	1	4	2	(b)	3	4	1	2
(c)	2	4	1	3	(d)	2	1	4	3

› *(**c**)* The correct matching is A-2, B-4, C-1, D-3. Clouds are visible mass of condensed water vapour floating in the atmosphere high above the ground surface. Clouds are of following types

Cirrus Cloud are having fathery appearance thin, wispy clouds formed at high altitude (about 20,000 ft).

Stratus Cloud are low level cloud characterised by horizontal layering with a uniform base.

Nimbus Cloud are dark clouds formed at low altitudes and produces heavy rainfall/precipitation.

Cumulus are low level vertical cloud with ''Puffy appearance and having flat bases.''

2014 (II)

179. Which one of the following pairs of power projects is not correctly matched?

(a) Papanasam—Hydropower
(b) Neyveli—Hydropower
(c) Ukai—Thermal power
(d) Rana Pratap Sagar—Hydropower

› *(**b**)* Neyveli is a thermal power plant, hence this pair is not correct.
Major Hydropower projects in India are Lower Jhelum, Pong, Bhakra Nangal, Tehri, Rana Pratap Sagar, Hirakund, Ukai, Salal, Koyna, Bhadra, Iduki, Omkareshwar, Nagarjun Sagar, Kopili and Khardong, Chamera etc.

180. Which one of the following is the correct sequence of oil refineries in India in respect of their time of establishment (starting from the earliest)?

(a) Barauni—Haldia—Guwahati—Mathura

(b) Barauni—Mathura—Guwahati—Haldia
(c) Guwahati—Haldia—Mathura—Barauni
(d) Guwahati—Barauni—Haldia—Mathura

(d) **Guwahati Oil Refinery** is the first public sector oil refinery of India, started in January, 1962. It is operated by the Indian Oil Corporation.

Barauni Oil Refinery lies in the state of Bihar and was made with the collaboration of the USSR in July, 1964.

Haldia Oil Refinery is also operated by Indian Oil Corporation. It is located in West Bengal and was established in January, 1975.

Mathura Oil Refinery is also owned by the Indian Oil Corporation. It was started in Mathura (Uttar Pradesh) in October, 1983.

181. Movements of tides are mostly determined by
(a) albedo effect
(b) wind velocity
(c) rotation of the Earth
(d) revolution of the Earth

(c) Tides are periodic rise and fall of sea levels. They are caused by the gravitational forces of the Moon and the Sun. However, they are also influenced by the rotation of the Earth and the relative position of the Moon, which causes the level of tide to change in a given location.

182. Match List I with List II and select the correct answer using the codes given below the lists

List I (Textile Industry)		List II (Place)
A.	Woollen textile	1. Sualkuchi
B.	Cotton textile	2. Rishra
C.	Silk textile	3. Ludhiana
D.	Jute textile	4. Davangere

Codes

	A	B	C	D		A	B	C	D
(a)	3	4	1	2	(b)	2	1	4	3
(c)	2	4	1	3	(d)	3	1	4	2

(a) The correct matching is A-3, B-4, C-1, D-2. Sualkuchi is a town in Kamrup district of Assam. It is famous for Muga silk, Pat silk and Eri silk.

Rishra is a place in West Bengal, famous for jute textiles.

Ludhiana is one of the major towns in Punjab and is very famous for woollen textiles; while Davangere is a city in Karnataka, famous for cotton textiles.

183. Quartzite is metamorphosed from
(a) limestone
(b) plutonic rock
(c) sandstone
(d) shale

(c) Metamorphic rocks are formed due to complete alternation in the appearance and constitution of pre-existing rocks due to change in mineral composition and texture through temperature and pressure.

Metamorphic rocks formed through the sedimentary rocks

Sehist	→	from shale
Marble	→	from limestone
Quartzite	→	from sandstone and conglomerate
Coal	→	from graphite
Gneiss	→	from granite

184. The permanent wind that blows from the horse latitude to the equatorial region is known as
(a) westerly
(b) trade winds
(c) doldrum
(d) easterly

(b) **Trade winds** are the winds blowing from 30° North and South latitudes (Horse latitude) towards the Equator. They blow from North-East in the Northern hemisphere and South-East in the Southern hemisphere. These winds flow according to the distribution of pressure gradients and are steady and warm.

185. Which one of the following is a warm ocean current?
(a) Labrador current
(b) Kuroshio current
(c) Peru current
(d) Benguela current

(b) **The Kuroshio current** is a warm ocean current flowing Northwards on the west side of the North Pacific ocean.

Labrador current is a cold current in the North Atlantic ocean which flows from the Arctic ocean along the coast of Labrador.

Peru current also called as Humboldt current is a cold, low-salinity ocean current that flows North along the coast of Chile and Peru.

Benguela current is a cold ocean current that flows northward in Atlantic ocean along the west coast of southern Africa.

186. The place located at the confluence of Alaknanda and Bhagirathi rivers is
(a) Badrinath
(b) Rishikesh
(c) Rudraprayag
(d) Devprayag

(d) **Devprayag** is the place where the Alaknanda and Bhagirathi meet to form the Ganga river. Devprayag is located in Uttarakhand, and is one of the five Prayags. It is also a very famous Hindu religious town. In Sanskrit, the meaning of the term 'Devprayag' is 'Godly Confluence'.

Badrinath is one of India's Char Dham pilgrimage sites. It is located in Chamoli district of Uttarakhand.

Rishikesh is located in the foothills of Himalayas beside the river Ganga. It is located in Dehradun district of Uttarakhand.

Rudraprayag is one of the panch Prayag of Alaknanda river, the point of confluence of rivers Alaknanda and Mandakini. It is located in Rudraprayag district of Uttarakhand.

187. If a news is broadcast from London at 1 : 45 pm on Monday, at what time and on what day it will be heard at Dhaka (90° E)?
(a) 7 : 45 pm on Monday
(b) 7 : 45 am on Monday
(c) 7 : 45 pm on Tuesday
(d) 7 : 45 am on Sunday

(a) The International Time Zone of England (+ 00 : 00), that is, it falls at the International Time Zone itself. And Bangladesh follows the time zone (+ 06 : 00) of Greenwich Mean Time (GMT), which implies that when it is 1 : 45 pm (Monday) in London, it would be 7 : 45 pm (Monday) in Dhaka, Bangladesh, which is 6 hours ahead of the GMT.

188. Match List I with List II and select the correct answer using the codes given below the lists

List I (Geographical Feature)		List II (Type of Geographic Process)
A.	Cirque	1. Erosional feature of wind
B.	Yardang	2. Depositional feature of glacier
C.	Barkhan	3. Depositional feature of wind
D.	Drumlin	4. Erosional feature of glacier

Codes

	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	4	3	1	2
(c)	2	3	1	4	(d)	2	1	3	4

(b) The correct matching is A-4, B-3, C-1, D-2. A **Cirque** is an amphitheatre-like valley head, formed at the head of a glaciated valley due to

erosional feature of accumulated ice moving down the mountain tops.
A Yardang is a large wind eroded mass of soft or poorly consolidated rock in a desert region which lies parallel to the prevailing winds.

A **Barkhan** or **Barchan** is an erosional feature of wind where curved shapes of sand are formed.

A **Drumlin** is an depositional feature of a glacier where a topography of elongated hill in the shape of an inverted spoon or half buried egg are formed.

189. A topographical map with scale 1 : 50000 indicates 1 cm to
(a) 50 km (b) 500 m
(c) 50 m (d) 5 km

(b) A topographical map shows the topographical features in accordance with a scale, which is expressed in ratio (like 1 : 50000). Here, 1 unit at the map indicates 50000 units in real topography. Thus, 1 cm of the map indicates 50000 cm or 500 m.

190. **Statement I** Grand Banks are one of the major fishing grounds of the world due to the presence of a vast continental shelf.
Statement II Planktons grow in the shallow waters.
Codes
(a) Both the statements are individually true and Statement II is the correct explanation of Statement I.
(b) Both the statements are individually true, but Statement II is not the correct explanation of Statement I.
(c) Statement I is true, but Statement II is false.
(d) Statement I is false, but Statement II is true.

(a) Grand Banks are one of the major fishing grounds of the world, due to the presence of the North American continental shelf which lifts the nutrients to the surface. The planktons are found in shallow waters because there is not enough sunlight to sustain photosynthesis in deeper waters.

191. Which of the following statements is/are correct?
1. The tropical cyclones of China Sea are called 'typhoons'.
2. The tropical cyclones of the West Indies are called 'tornadoes'.
3. The tropical cyclones of Australia are called 'willy-willies'.
4. Formation of an anticyclone results in stormy weather condition.

Select the correct answer using the codes given below
(a) 1, 2, 3 and 4
(b) 1, 2 and 4
(c) 1 and 3
(d) Only 3

(c) The tropical cyclones are most frequent during the late summer season when the Doldrum belt has moved farthest from the Equator.

These, are called with following common names in the respective regions :

Name	Region
Typhoons	South China Sea
Hurricanes	West Indies and Gulf of Mexico
Willy-Willies	North Australia
Tornadoes	USA
Cyclones	Bay of Bengal

192. Rotterdam of Netherlands is largely famous for
(a) Textiles (b) dairying
(c) shipbuilding (d) paper industry

(c) Rotterdam of Netherlands is primarily famous for its well-established ship building industry.
Other options are-
Textile – Lanchashire (UK)
Dairying – Switzerland
Paper industry – Brussels

193. Which of the following sequences is correct for rainfall?
(a) Slow ascent of air → Slow condensation → Heavy downpour
(b) Rapid ascent of air → Large raindrops → Heavy downpour
(c) Pressure decreased → Air compressed → Heavy downpour
(d) Descent of air → Air warmed → Heavy downpour

(c) There is negative relationship between rainfall and pressure.
Air rises in a low pressure area. Rising air cools and this is likely to condense water vapour and form clouds that lead to heavy downpour.

2014 (I)

194. Which one among the following is the coral group of islands of India?
(a) Andaman (b) Nicobar
(c) Minicoy (d) Lakshadweep

(d) Lakshadweep is the coral group of islands of India that are built by corals and have fringing coral reefs close to their shores. Lakshadweep group of islands are located in the Arabian Sea.

195. Consider the following surface winds
1. Doldrums 2. Trade winds
3. Westerlies 4. Polar winds

Which one among the following is the idealised global pattern of these winds from the Equator to the Pole?
(a) 1 , 2 , 3 , 4 (b) 1 , 3 , 2 , 4
(c) 2 , 1 , 4 , 3 (d) 3 , 1 , 2 , 4

(a) **Doldrums** are a low pressure area around Equator. It is the region where northeast and Southeast trade winds converge. They are followed by Trade winds. Trade winds are permanent winds that blow from sub-tropical high pressure belt to equatorial low pressure belt. They prevail between 30° North to 30° South throughout the Earth.

They are followed by westerlies that blows from sub-tropical high pressure belt towards sub-polar law pressure belt between 30° to 60° latitudes.

Polar winds are dry, cold prevailing winds blowing in the polar region from polar high pressure belt to sub-polar low pressure belt.

196. Taungup Pass is a mountain corridor connecting India with
(a) Afghanistan (b) China
(c) Pakistan (d) Myanmar

(d) **Taungup Pass** is a mountain corridor that connects India with Myanmar.

Nathula Pass links the state of Sikkim in India with the Tibetan autonomous region in China. (Lipulekh La and Shipki La are other two passes between India and China).

Khyber Pass connects Afghanistan and Pakistan.

Karakoram Pass is trisection of the Indian, Chinese and Pakistani borders.

197. The summer and winter seasons in a year are caused by
(a) aphelion (farthest) and perihelion (nearest) positions of the Earth from the Sun during the annual revolution
(b) rotation of the Earth on its axis
(c) variation in solar insolation
(d) revolution of the Earth on its inclined axis

(d) The summer and winter seasons in a year are caused by revolution of the Earth on its inclined axis. Summer happens in the hemisphere tilted towards the Sun and winter happens in the hemisphere tilted away from the Sun.

198. Which of the following is/are correct relating to the North-Eastern Region Community Resource Management Project for upland areas?

1. It is a livelihood and rural development project aimed to transform the lives of the poor and marginalised tribal families in North-East India.
2. This project is initiated exclusively by the North-Eastern Council.

Select the correct answer using the codes given below

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(›) ***(c)*** North-Eastern Region Community Resource Management Project for upland areas is a project aimed to improve the livelihood of vulnerable groups in a sustainable manner through improved management of their natural resource base. It is implemented through the North-Eastern Council embracing the three State Governments and Regional and District societies. Hence, both the statements are correct.

Directions (Q. Nos. 202-206) *The following five* (5) *items consist of two statements, Statement* I *and Statement* II. *You have to examine these two statements carefully and select the answers to these items using the codes given below.*

Codes

(a) Both the statements are individually true and Statement II is the correct explanation of Statement I.
(b) Both the statements are individually true, but Statement II is not the correct explanation of Statement I.
(c) Statement I is true, but Statement II is false.
(d) Statement I is false, but Statement II is true.

199. **Statement I** Minerals are formed by slow cooling of the Magma.

Statement II Very small crystals are formed when lava cools quickly on the surface.

(›) ***(b)*** Magma cools slowly as it rises towards Earth's surface, it takes thousands to millions of years to become solid rocks. These rocks contains minerals. e.g. Granite. Hence, statement (1) is correct.
Lava on the Earth's surface cools quickly compared to Magma in Earth. As a result, rocks form quickly and mineral crystals are very small, e.g. Rhyolite. Hence, Statement (2) is correct.
Hence, option (b) is correct.

200. **Statement I** The Kullu Valley in Himachal Pradesh receives copious snowfall during winter.

Statement II The Kullu Valley receives moisture-bearing wind of the western disturbances during winter.

(›) ***(a)*** The period between October to February is winter season in Kullu. Heavy frost occurs during this period. Heavy (copious) Snowfall generally occurs during December and January western disturbances bring more rain and snow in Kullu, Manali, Shimla, Srinagar during winter time. Hence, option (a) is correct.

201. **Statement I** The hills with dense vegetation cover do not experience heavy soil erosion.

Statement II The vegetation cover helps infiltration of rainwater and binding of soils.

(›) ***(a)*** Due to dense vegetation cover, rain water is unable to take away the soil with it. The roots bind the soil tightly and also help infiltration of water deep inside. Hence, option (a) is correct.

202. **Statement I** The decrease of air temperature with increasing altitudes in the atmosphere is called the vertical temperature gradient.

Statement II In Troposphere, air temperature decreases with increasing altitude due to radiation from the Earth.

(›) ***(a)*** Vertical temperature gradient is variation in air temperature with increase in altitude and radiation of the Earth is one of the main factors that contribute to it. The temperature in troposphere decreases with height at the average rate of 6.5°C per km. Hence, option (a) is correct.

203. **Statement I** Chemical weathering processes are found more active in hot and humid environment.

Statement II High temperature and rainfall help in the process of decomposition of rocks.

(›) ***(a)*** High temperature and rainfall lead to decomposition of rocks which helps in the process of decomposition of rocks through chemical weathering. Hence, option (a) is correct.

204. Tuareg is a pastoral nomad living in the desert of

(a) Kalahari
(b) Sahara
(c) Arabia
(d) Patagonia

(›) ***(b)*** Tuareg are berber people who inhabit Sahara. They are found in Niger, Mali and Algeria. Being nomadic, they move constantly across national borders.

205. Match List I with List II and select the correct answer using the codes given below the lists

	List I (Region)		**List II** (Characteristic Vegetation)
A.	Selvas	1.	Tropophytes
B.	Savanna	2.	Mosses and lichens
C.	Tundra	3.	Epiphytes
D.	Monsoon land	4.	Grasses and trees

Codes

	A	B	C	D		A	B	C	D
(a)	3	2	4	1	(b)	1	4	2	3
(c)	1	2	4	3	(d)	3	4	2	1

(›) ***(d)*** The correct matching is A-3, B-4, C-2, D-1. Selvas are evergreen equatorial forest of Amazon. The type of vegetation occurs in this belt includes epiphytes, ebony, mahogany, rosewood etc.

Savanna is characterised by a rich growth of tall grass with scattered trees.

The Tundras or cold deserts are too cold for the growth of trees and the only vegetation is mosses and lichens.

The natural vegetation of monsoon lands is characterised with deciduous trees which lose their leaves during the hot season. Such trees are called tropophytes.

206. Which of the following is/are the most likely cause/causes of sheet-flood found in western part of Rajasthan?

1. Scanty rainfall
2. Sudden high intensity rain
3. Loose sandy soil with scanty vegetation

Select the correct answer using the codes given below

(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 3

(›) ***(c)*** Sheet-flood takes away a thin layer of soil with it. Scanty rainfall cannot be a reason for it. Sudden high intensity rain causes it and loose sandy soil along with scanty vegetation do not help much in protecting the soil from running away. Hence, option (c) is the correct answer.

207. Mulching, an agronomic measure of soil conservation, is very effective because it

1. protects soil from gully erosion
2. protects soil from sheetwash and wind erosion
3. helps soil to retain moisture and nutrients

Select the correct answer using the codes given below

(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 3

(*d*) Mulching is an activity in which crop residue is evenly spread on the soil. It protects soil from gully erosion, sheet-wash and wind erosion. It also helps soil to retain moisture and nutrients.

208. Why is hydrolysis an effective form of chemical decomposition of bedrock in humid tropics?

1. Humid tropics experience high temperature and humidity.
2. There is high diurnal range in temperature.

Select the correct answer using the codes given below

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(*c*) In tropic countries, where there is high rainfall and warm climate, the process of hydrolysis proceeds rapidly. The high diurnal range in temperature also support hydrolysis in region of tropics. Hydrolysis is the most important process in chemical weathering. It is due to the dissociation of water (H_2O) in H^+ and OH^- ions which chemically combine with minerals of rocks and causes decomposition.

209. Lapland is a cultural region largely within the Arctic Circle in the North of the Scandinavian Peninsula. Who inhabited the Lapland?

(a) Sami people
(b) Padaung people
(c) Hamar people
(d) Himba people

(*a*) The name Lapland refers to land inhabited by the Sami people, formerly called Lapp people.

Padaung people also known as Kayan, are Tibero - Burman ethnic minority of Myanmar.

Hamar people are an omotic community inhabiting South-Western Ethiopia. Himba people are indigenous ethnic group living in the kunene region of Namibia.

210. Considering the locations of mountains in India, which one among the following is in right sequence from South to North?

(a) Doddabetta, Kailash, Dhaulagiri, Vindhayachal
(b) Doddabetta, Vindhyachal, Dhaulagiri, Kailash
(c) Dhaulagiri, Kailash, Doddabetta, Vindhayachal
(d) Dhaulagiri, Vindhyachal, Doddabetta, Kailash

(*b*) Doddabetta is the highest mountain in the Nilgiri hills located primarily in state of Tamil Nadu.

Vindhyachal is a mountain range in West Central India located primarily in madhya Pradesh and Chhattisgarh.

Dhaulagiri is a mountain range in Nepal.

Kailash is a peak in the Kailash range, which also form part of the Trans-himalaya in Tibet. Hence, the correct sequence of mountain in India according to location (South to North) are

211. Which one among the following lakes is situated on the West coast of India?

(a) Chilika (b) Ashtamudi
(c) Pulicat (d) Kolleru

(*b*) **Ashtamudi lake** is situated in the Kollam district of the Kerala on the West Coast of India.

Chilika lake is a brackish water lagoon in Odisha on the East coast of India.

Pulicat lake is the second largest brackish water lake or lagoon in India, after Chilika and is on Coromandal coast.

Kolleru lake is one of the largest fresh water lakes in India located between Krishna and Godavari delta in Andhra Pradesh.

212. What is the correct sequence from the smallest to the largest grain of the following types of clastic rocks?

(a) Shale, sandstone, conglomerate, siltstone
(b) Shale, siltstone, sandstone, conglomerate
(c) Conglomerate, sandstone, shale, siltstone
(d) Sandstone, siltstone, conglomerate, shale

(*b*) **Shale** is a fine-grained, clastic sedimentary rock composed of mud that is a mix of flakes of clay minerals and tiny fragments of other minerals, especially quartz and calcite.

Siltstone is a sedimentary rock which has a grain size in the silt range, finer than sandstone and coarser than claystones.

Sandstone is a clastic sedimentary rock composed mainly of sand sized minerals or rock grains.

Conglomerate is a coarse grained sedimentary rock composed of rounded fragments embedded in a matrix of cementing material such as silica. Hence, option (b) is the correct sequence on the basis of grain size.

213. The Faroe is a group of islands lying in the Atlantic Ocean between Scotland and Iceland. This island group is also known as

(a) Island of Sheep
(b) Island of Goats
(c) Island of Cows
(d) Island of Buffaloes

(*a*) The Faroe Island, also known as 'Island of Sheep', is an archipelago and autonomous territory within the Kingdom of Denmark, situated between the Norwegian sea and the North Atlantic ocean.

46

POLITY

2019 (II)

1. Article-46 of the Constitution of India refers to the promotion of educational and economic interests of

(a) religious minorities
(b) Scheduled Castes, Scheduled Tribes and other weaker sections
(c) displaced persons from large irrigation projects
(d) the economically deprived

(*b*) Article 46 in part IV of the Constitution of India is related to the promotion of educational and economic interests of the Scheduled Castes, Scheduled Tribes and other weaker sections. Article 46 also provides that the state shall protect the weaker sections of the society from social in justice and all forms of exploitation.

2. The provisions of the Fifth Schedule of the Constitution of India shall not apply to the administration and control of the Scheduled Areas and Scheduled Tribes in

1. Chhattisgarh, Jharkhand and Odisha
2. Assam and Tripura
3. Meghalaya and Mizoram

Select the correct answer using the codes given below.

(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 3

(*c*) The provisions of the Fifth Schedule of the Constitution of India apply to the administration and control of the Scheduled Areas and Scheduled Tribes in 10 states namely: Andhra Pradesh, Chhattisgarh, Gujarat, Himachal Pradesh, Jharkhand, Madhya Pradesh, Maharashtra, Odisha, Rajasthan and Telangana. Hence, statement (2) and (3) are correct.

2019 (I)

3. An Election Commissioner can be removed from office on the recommendation of

(a) the Chief Justice of India
(b) the Chief Election Commissioner
(c) the President of India
(d) the Parliament

(*b*) An Election Commissioner can be removed from office on the recommendation of the Chief Election Commissioner (CEC) by President of India.

Article-324 (5) of the Constitution safeguards the CEC from arbitrary removal. CEC can be removed only by the President, just like Judge of the Supreme Court.

4. Which one of the following statements regarding the Rajya Sabha is not correct?

(a) Its members are elected by the elected members of the Legislative Assembly of a State.
(b) The election follows the system of proportional representation by means of a single transferable vote.
(c) 1/3rd of its members retire after every two years.
(d) It is a permanent body, but can be dissolved earlier by the President.

(*d*) Option (d) is incorrect because Rajya Sabha (Article-80) is a permanent body and not the subject to dissolution. Its maximum strength is 250. Its members are elected by the elected member of the Legislative Assembly of the State.

The election of Rajya Sabha members follows the system of proportional representation by means of a single transferable vote. One-third (1/3rd) of its members retire after every two years.

5. Fundamental Rights guaranteed under which one of the following Articles of the Constitution of India is available only to the citizens of India?

(a) Article-19 (b) Article-20
(c) Article-21 (d) Article-22

(*a*) Under **Article-19** of the Constitution of India, Fundamental Rights are guaranteed only to the citizens of India. There are five fundamental articles which are only to Indian citizens i.e. Articles-15, 16, 19, 29, 30. This article guarantees the six rights. *These are*

1. Right to freedom of speech and expression
2. Right to assemble peacefully and without arms.
3. Right to form associations or unions or co-operations.
4. Right to move freely throughout the territory of India.
5. Right to reside and settle in any part of the territory of India.
6. Right to practice any profession or to carry on any occupation, trade or business.

While **Article 20** deals with the protection of certain rights incase of conviction for offences. **Article 21** deals with the protection of life and personal liberty, and **Article 22** grants protection to persons who are arrested or detained.

6. Consider the following description of the Samadhi of a former Prime Minister of India.

Central Samadhi Platform comprises nine square black polished granite solid stone blocks, capped with a 'Diya' in the centre. The number nine holds significance and represents the Navarasas, Navaratras and Navagrahas. Then placement of

the Nine-Square Samadhi is in a circular lotus shaped pattern. The Nine-Square platform is accessed in four cardinal directions by pathways made in white composite tiles so that the floor does not get heated.

Identify the Samadhi.

(a) Shakti Sthal (b) Shantivan
(c) Sadaiv Atal (d) Veer Bhumi

(c) The Samadhi, which is mentioned in above question, is **Sadaiv Atal**. It is the crematorium (Samadhi) of former Prime Minister of India, Atal Bihari Vajpayee (1924-2018). He served three terms as the Prime Minister of India. He was the first Indian Prime Minister who was not a member of the Indian National Congress party to have served a full five year term in office.

Shakti Sthal is a memorial of former Prime Minister Indira Gandhi.

Shantivan is the Samadhi or Cremation spot of Prime Minister Jawaharlal Nehru and the **Veer Bhumi** is a memorial dedicated to the former Prime Minister Rajiv Gandhi.

2018 (II)

7. Which one among the following is not a Fundamental Right under the Constitution of India?

(a) Right to equality
(b) Right to freedom
(c) Right to citizenship
(d) Right against exploitation

(c) Fundamental Rights are the basic rights of the common people and inalienable rights of the people who enjoy it under the charter of rights contained in Part III (Article-12 to 35) of the Constitution of India. Right to Citizenship is not a Fundamental Right.

All the Fundamental Rights have been classified under the following six categories

1. Right to be Equality (Articles14-18)
2. Right to Freedom (Articles 19-22)
3. Right against Exploitation (Articles 23-24)
4. Right to Freedom of Religion (Articles 25-28)
5. Cultural and Educational Rights (Articles 29-30)
6. Right to Constitutional Remedies (Article 32)

The **Right to Property** is no longer a Fundamental Right.

8. Every Judge of the Supreme Court of India is appointed by

(a) the Supreme Court Collegium
(b) the Cabinet
(c) the President of India
(d) the Lok Sabha

(c) Every Judge of the Supreme Court shall be appointed by the President of India. Article-124 provides for the establishment and Constitution of the Supreme Court of India, which is the Apex Court of India. The sanctioned strength of the judges is 34, consisting of the Chief Justice and 33 other judges. Every Judge of Supreme Court shall attend the office till the age of 65 years.

9. Which one of the following is not a part of the Directive Principles of State Policy as enshrined in the Constitution of India?

(a) Equal justice and free legal aid.
(b) Protection of monuments and places and objects of national importance.
(c) Protection of personal law.
(d) Separation of Judiciary from executive.

(c) Part-IV of Indian Constitution deals with Directive Principles of State Policy (DPSP). Basic aim of DPSP is to set-up social and economic goals before the law makers. *Some of the important DPSPs are*

- Uniform civil code for the citizens.
- Provide free and compulsory education for children below 14 years.
- To promote international peace and amity.
- Protection of personal law does not come under Directive principles of state policy.
- Protection and improvement of environment and safeguarding of forests and wildlife.
- Organisation of Village Panchayats and to promote cottage industry.
- Equal pay for equal work.
- Equal justice and free legal aid.
- Protection of monuments and places and objects of national importance.
- Separation of judiciary from executive.

Hence, option (c) is not a part of Directive Principles of State Policy.

10. The word 'Socialist' was inserted into the Preamble to the Constitution of India through which one of the following Amendment Acts?

(a) 41st Amendment Act
(b) 42nd Amendment Act
(c) 43rd Amendment Act
(d) 44th Amendment Act

(b) The 42nd Amendment Act amended the Preamble and changed the description of India from 'Sovereign democratic republic' to a 'Sovereign, Socialist, Secular, Democratic Republic' and also changed the words 'Unity of the Nation' to 'Unity and Integrity of the Nation'.

The Preamble to the Constitution of India is a brief introductory statement that sets out the guiding purpose, principles and philosophy of the Constitution.

11. Who among the following is the ex-officio Chairman of the North-Eastern Council?

(a) The President of India
(b) The Prime Minister of India
(c) The Union Home Minister
(d) The Union Minister of State (Independent Charge), Ministry of Development of North-Eastern Region

(c) The North-Eastern Council is the nodal agency for the economic and social development of the North-Eastern Region which consists of the eight states: Arunachal Pradesh, Assam, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim and Tripura.

It was constituted in 1971 by an Act of Parliament. The Union Home Minister acts as ex-officio Chairman of North-Eastern Council (NEC). The secretariat (Headquarters) of the council is located at Shillong, Meghalaya. The President of India nominates the Chairman of the Council.

12. Article-352 of the Constitution of India contains provisions related to

(a) financial emergency.
(b) failure of constitutional machinery in states.
(c) suspension of the enforcement of rights conferred in Part III of the Constitution.
(d) general emergency.

(d) Article 352 of the Constitution of India contains provisions related to general or national emergency.

This Article states that if the President is satisfied that a grave emergency exists whereby the security of India or any part of the Territory of India is threatened by war or external aggression or armed rebellion, he may proclaim an general emergency.

2018 (I)

13. Who among the following are the two civil servants who assisted the Constituent Assembly in framing the Constitution of India?
(a) B. N. Rau and K.M. Munshi
(b) S. N. Mukherjee and Alladi Krishnaswamy Iyer
(c) B. N. Rau and S. N. Mukherjee
(d) K. M. Munshi and Alladi Krishnaswamy Iyer

(c) *(**c**)* Sir Benegal Narsing (B.N.) Rau was an Indian civil servant, jurist, diplomat and statesman known for his key role in drafting the Constitution of India. S.N Mukherjee was another civil servant who assisted in the framing of the Constitution of India. The Constitution of India was framed between December, 1946 and December, 1949. The Constitution of India came into effect on 26th January, 1950.
K.M. Munshi and Alladi Krishnaswamy Iyer were the members of Drafting Committee.

14. Which member of the Constituent Assembly proposed the resolution that the National Flag of India be a ''horizontal tricolour of saffron, white and dark green in equal proportion'', with a wheel in navy blue at the centre?
(a) Jawaharlal Nehru
(b) B. R. Ambedkar
(c) Rajendra Prasad
(d) Sardar Vallabhbhai Patel

(c) *(**c**)* In 1947, when India gained freedom from the British, a committee headed by Dr. Rajendra Prasad, decided to adopt the flag of the Congress as the National Flag of India with a few modifications. With this in mind, the flag of 1931 was adopted as the National Flag of India, but the Charkha in the middle was replaced with the Ashoka Chakra. In National Flag the saffron colour on top represents sacrifice, white represents peace and purity and green signifies the Law of Dharma (righteousness).

15. Which one of the following about the Parliament of India is NOT correct?
(a) The Parliament consists of the President, the Lok Sabha and the Rajya Sabha.
(b) There are no nominated members in the Lok Sabha.
(c) The Rajya Sabha cannot be dissolved.
(d) Some members of the Rajya Sabha are nominated by the President.

(b) *(**b**)* Option (b) is not correct because Anglo-Indians are the only community that has its own representatives nominated to the Lok Sabha (Lower House) in India's Parliament. It is done only if the President of India feels that the Anglo Indian community has not been adequately represented in the Lok Sabha.

16. Which one of the following statements with regard to the Comptroller and Auditor General (CAG) of India is NOT correct?
(a) He is appointed by the President of India.
(b) He can be removed from office in the same way as the judge of the Supreme Court of India.
(c) The CAG is eligible for further office under the Government of India after he has ceased to hold his office.
(d) The salary of the CAG is charged upon the Consolidated Fund of India.

(c) *(**c**)* Option (c) is not correct because the CAG is not eligible for further office either under the Government of India or under the Government of any State after he has ceased to hold his office. The CAG is an authority established by Article 148 of the Constitution of India. It was founded in 1858. Hence, option (c) is incorrect, while options (a), (b) and (d) are correct.

17. The superintendence, direction and control of elections in India is vested in
(a) the Supreme Court of India
(b) the Parliament of India
(c) the Election Commission of India
(d) the Chief Election Commissioner

(c) *(**c**)* According to Article 324 of the Constitution of India, the superintendence, direction and control of the preparation of the electoral rolls for, and the conduct of all elections shall be vested in the Election Commission of India. Therefore, it is the sole authority in India that takes charge of elections in India.

18. Which of the following provision(s) of the Constitution of India became effective from 26th November, 1949?
1. Elections
2. Citizenship
3. Emergency provisions
4. Appointment of the Judges
Select the correct answer using the codes given below.
(a) Only 1 (b) 1 and 2
(c) 1, 2 and 3 (d) 2 and 4

(b) *(**b**)* The Constitution of India was adopted on 26th November, 1949. Though, it came into force on 26th January, 1950 with some provisions related to Citizenship, Elections, Provisional Parliament etc.
The Constitution of India is the Supreme Law of India. The document lays down the framework demarcating fundamental political code, structure, procedures, powers and duties of government institutions. It also sets out the Fundamental Rights, Directive Principles and the Duties of Citizens.

2017 (II)

19. The creation of the institution of Lokpal was first recommended by
(a) Law Commission
(b) Santhanam Committee
(c) Shah Commission
(d) Administrative Reforms Commission

(d) *(**d**)* The Administrative Reforms Commission (ARC), sets up in 1966 recommended the Constitution of a two-tier machinery of a Lokpal at the Centre and Lokayukts in the states. While Law Commission of India is an executive body established by an order of the Government of India. Santhanam Committee was formed to guide Central Government agencies in the field of vigilance. Shah Commission was appointed by the Government of India to inquire into all the excesses committed in the Indian Emergency.

20. Who among the following is the Chairman of interdisciplinary committee constituted recently by the Government of India to examine framework for virtual currencies?
(a) Secretary, Department of Financial Services
(b) Special Secretary, Department of Revenue
(c) Special Secretary, Department of Economic Affairs
(d) Deputy Governor, Reserve Bank of India

(c) *(**c**)* Special Secretary of Department of Economic Affairs is the Chairman of interdisciplinary committee constituted to examine framework for virtual currencies.

21. Which one of the following is not a subject that has been devolved to the Panchayati Raj Institutions by the 11th Schedule of the Constitution of India?

(a) Non-conventional energy resources
(b) Roads
(c) Higher education
(d) Libraries

*(**c**)* 11th Schedule of the Constitution of India doesn't include **higher education.** This schedule includes 29 subjects including non-conventional energy resources, roads, libraries, etc.

22. Who among the following can attend the meetings of both Houses of Parliament while not being a member of either House?

(a) The Solicitor General of India
(b) The Vice-President of India
(c) The Comptroller and Auditor General of India
(d) The Attorney General of India

*(**d**)* The AGI (Attorney General of India) is the only person under the Indian system who can take part in the proceedings of the Parliament or any parliamentary committee, but cannot vote.

He has the right to speak and to take part in the proceedings of both the Houses of Parliament or their joint sittings and in any committee of the Parliament of which he may be named a member, but without the right to vote.

23. The Cabinet Mission Plan for India envisaged a

(a) Federation
(b) Confederation
(c) Unitary form of government
(d) Union of States

*(**d**)* The Cabinet Mission Plan for India envisaged a Union of States. The mission spent some three weeks to discuss with the leaders of various political parties, but could not arrive at any agreed solution. So, finally it announced its own recommendations on 16th May, 1946.

The Cabinet Mission Plan of 1946 proposed that there shall be a Union of India which was to be empowered to deal with the defense, foreign affairs and communications.

The Cabinet Mission recommended an undivided India and turned down the Muslim League's demand for a separate Pakistan. The Cabinet Mission restricted the Communal representation.

2017 (I)

24. Which one among the following states of India has recently proposed to frame the first Internal Security Act to deal with the challenges of terrorism, insurgency, communalism and caste violence?

(a) Maharashtra (b) Gujarat
(c) Uttar Pradesh (d) Chhattisgarh

*(**a**)* Maharashtra is the first state in the country to draft its own Internal Security Act. It has proposed setting up of Special Security Zones (SSZs), where the movement of arms, explosives and inflows of unaccounted funds will be prohibited.

25. By fulfilling which of the following conditions can a political party claim the status of a national party?

1. It secures at least six per cent (6%) of the valid votes polled in any four or more States, at a general election to the House of the People or to the State Legislative Assembly.
2. It wins at least four seats in the House of the People from any State or States or wins at least two per cent (2%) seats in the House of the People (i.e. 11 seats in the existing House having 543 members), and these members are elected from atleast three different states.
3. The party in question has got recognition as a State party in atleast two states.
4. It must have its headquarters in New Delhi.

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) 2 and 4
(c) 1 and 2 (d) 1, 3 and 4

*(**c**)* A registered political party is recognised as a national party only if it fulfills any one of the following three conditions.

- The party at a general election to Lok Sabha should poll 6% of votes in any four or more states won at least four seats in a Lok Sabha general election from any state or states.
- A party should win 2% of seats in the Lok Sabha from at least three different states.
- A party gets recognition as a state party in four states.
- It is not mandatory to have its headquarters in New Delhi.

So, only statements (1) and (2) are correct.

26. Match List I with List II.

List I (Amendments to the Constitution of India)	List II (Subjects)
A. 52nd Amendment Act, 1985	1. Reduction of voting age from 21 to 18
B. 73rd Amendment Act, 1992	2. Right to Education
C. 61st Amendment Act, 1988	3. Panchayati Raj
D. 86th Amendment Act, 2006	4. Disqualification on grounds of defection

Codes

	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	4	3	1	2
(c)	2	3	1	4	(d)	2	1	3	4

*(**b**)* The correct matching is A-4, B-3, C-1, D-2.

52nd Amendment Act, 1985 provided provisions related to anti-defection in India. It laid down the process by which legislators may be disqualified on the grounds of defection.

The 73rd Amendment Act, 1992 introduce the system of local self government or Pachayati Raj in India.

The 61st Amendment Act, 1988 reduce the age of voting from 21 to 18.

The 86th Constitutional Amendment provided Right to Education for all children upto age of 14 years.

27. Which one of the following is not an objective of the National Civil Aviation Policy, 2016?

(a) To establish an integrated ecosystem which will lead to significant growth of civil aviation sector.
(b) To promote tourism, increase employment and lead to a balanced regional growth.
(c) To ensure safety, security and sustain ability of all sectors through the use of technology.
(d) To enhance regional connectivity through fiscal support and infrastructure development.

*(**c**)* The most striking feature of National Civil Aviation is its Regional Connectivity Scheme (RCS) which seeks to encourages airlines to fly on regional routes. The new policy focuses upon competition, consumers, connectivity and investment. Hence, option (c) is not correct.

2016 (II)

28. Consider the following statement(s) about the Constitution of India.

1. A Member of Parliament enjoys freedom of speech in the Parliament as a Parliamentary privilege protected by the Constitution of India.
2. The Constitution has vested the power to amend the Constitution in the Parliament.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(c)*** Indian Constitution provides parliamentary privileges to the members of the Parliament. These are special rights, immunities, exceptions enjoyed by the members of the two houses of Parliament e.g. A member of Parliament enjoys freedom of speech in the Parliament and no member is liable to any proceedings in any given court for anything said by him in Parliament. Article 368 of the Indian Constitution gives power to the Parliament to amend the Constitution.
Hence, both Statements are correct.

29. Which one of the following statements about the Union Executive in India is correct?

(a) According to the Constitution of India, the total number of members is the Council of Ministers cannot exceed 20% of the total number of Members of the House of the People.
(b) The rank of the different Ministers is determined by the President.
(c) The Ministers are appointed by the President on the advice of the Prime Minister.
(d) Only a member of either House of Parliament can be appointed as a Minister.

› ***(c)*** Union executive consists of the President, the Vice-President and the Council of Ministers with the Prime Minister as the head to aid and advice the President. The Ministers (included Council of Minister) are appointed by the President on the advice of the Prime Minister. Hence, option (c) is correct.
The correct form of statement (a), (b) and (d) are –

- According to the Constitution of India, the total number of ministers in the Council of Ministers must not exceed 15% of the total number of members of Lok Sabha.
- The rank of different ministers in the Union Council of Ministers is determined by the Prime Minister.
- Ministers are appointed by legislature.

30. Which one of the following statements about Money Bill is correct?

(a) A Bill shall be deemed to be a Money Bill only if it provides for imposition of fines or penalties.
(b) A Money Bill shall be introduced in the Rajya Sabha.
(c) The Rajya Sabha can reject the Money Bill.
(d) The Speaker of the Lok Sabha finally decides if it is a Money Bill, should any dispute about it arise.

› ***(d)*** Under Article 110 (1) of the Constitution, a bill is deemed to be a Money Bill if it contains provisions dealing with following matters namely

- Imposing, abolishing and regulating a tax
- Regulating government borrowings
- The consolidated and contingency funds of India etc.

Money Bills can be introduced only in Lok Sabha. Rajya Sabha cannot make amendments in it. Money Bill passed by Lok Sabha and transmitted to it. Speaker of the Lok Sabha finally decides if it is a Money Bill, should any disputes about it arise.

Hence option (a), (b), (c) are incorrect and (d) is correct.

31. Which one of the following powers of the Prime Minister of India is codified in the Constitution of India itself?

(a) The power of advising the President as regards the appointment of other Ministers.
(b) The power to allocate business amongst the Ministers.
(c) The power to summon the meeting of the Cabinet.
(d) The power to transfer a Minister from one department to another.

› ***(a)*** *Powers of Prime Minister are as follow*

- He has the power of selecting other Ministers and also of advising the President to dismiss anyone of them individually or require anyone of them to resign. Virtually, the Ministers hold office at the pleasure of the Prime Minister.
- The Prime Minister takes decisions with regard to assignment of various ministries to individual ministers. He may transfer a Minister from one department to another.
- The Prime Minister stands between the President and the Cabinet. Communications relating to policy are made only through the Prime Minister.

Hence, option (a) is correct.

2016 (I)

32. Which of the following statements relating to the Comptroller and Auditor General (CAG) of India is/are correct?

1. The CAG can attend the sittings of the Committee on Public Accounts.
2. The CAG can attend the sittings of Lok Sabha and Rajya Sabha.
3. The jurisdiction of CAG is co-extensive with powers of the Union Government.

Select the correct answer using the codes given below.

(a) Only 1 (b) 2 and 3
(c) 1 and 2 (d) All of these

› ***(a)*** The CAG is an authority established by Constitution of India under Article-148 to audit all receipts and expenditure of Central and State Governments. CAG assists the Public Account Committee during the examination of audit report of CAG.
Further, CAG cannot attend the sittings of Lok Sabha and Rajya Sabha. The jurisdiction of CAG is not co-extensive with powers of the Union Government.
Hence, only statement (1) is correct.

33. Which of the following statements relating to election as the President of India is/are correct?

1. A person above the age of thirty-five years is eligible for election as the President of India.
2. The President of India is eligible for re-election for more than one term.
3. A person is not eligible for election as the President of India if the person holds an office of profit.

Select the correct answer using the codes given below.

(a) 1 and 2 (b) Only 2
(c) 1, 2 and 3 (d) Only 3

› ***(c)*** A person to be eligible for election as President should be a citizen of India, must have completed the age of 35 years and should not hold any office of profit. The President of India is eligible for re-election for more than one term. The manner of election of President is provided by 55th Article of the Constitution.

Therefore, all the statements are true. Hence, option (c) is correct.

34. The citizens of India do not have which of the following Fundamental Rights?

(a) Right to reside and settle in any part of India.
(b) Right to acquire, hold and dispose property.
(c) Right to practice any profession.
(d) Right to form cooperative societies.

› ***(b)*** Article-300A in Part-XII deals with Right to property and deems it to be a legal right rather than Fundamental Right.

Originally, the Right to property was one of the seven Fundamental Rights and dealt by Articles-19(1)(f) and Article-31. Article-19(1)(f) guaranteed to every citizen the right to acquire, hold and dispose property. The 44th Amendment Act abolished this right as Fundamental Right.

35. Which one among the following Acts for first time allowed Indians, at theoretically, entry to higher posts in British Indian administration?

(a) Charter Act, 1813
(b) Charter Act, 1833
(c) Charles Wood's Education Desp, 1854
(d) Indian Councils Act, 1861

› ***(b)*** Charter Act, 1833 for first time allowed Indians, at theoretically, entry to higher posts in British Indian Administration.

The **Charter Act of 1833** was the first act which made provision to freely admit the natives of India to share administration in the country. This act also provided that Haileybury College of London should make quota to admit future civil servants.

Charter Act of 1813 compelled the East India Company to accept responsibility for the education of the Indian people.

Charles Wood's Education Despatch, 1854 promoted the use of vernacular languages in the primary schools.

Indian Charter Act, 1861 was passed by British Parliament for executive and legislative purposes.

36. Which one of the following pairs of the Schedule in the Constitution of India and its content is not correctly matched?

Schedules	Content
(a) Eighth Schedule	Languages
(b) Second Schedule	The forms of oaths and affirmations
(c) Fourth Schedule	Allocation of seats in the Council of States
(d) Tenth Schedule	Provisions as to disqualification of the ground of defection

› ***(b)*** Option (b) is not matched correctly because the Second Schedule of the Indian Constitution deals with emoluments of President, Governors, Judges of High Court and Supreme Court and Comptroller and Attorney General of India. All other pairs are correctly matched.

37. Consider the following statements.

1. The Constitution of India empowers the Parliament to form new states and to alter the areas, boundaries or names of existing states by passing a resolution in simple majority.
2. Jammu and Kashmir has been given special status under Article-370 of the Constitution of India.

Which of the statement(s) given above is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

› ***(c)*** Both the statements are true. The Constitution of India empowers the Parliament to form new states and to alter the areas, boundaries or names of existing states by passing a resolution in simple majority. Hence, statement (1) is correct.

Article 370 along with Article 35A of the Indian Constitution gave special states to Jammu and Kashmir, the Government of India abrogated the Article 370 in August, 2019, has making all the provisions of the Indian Constitution applicable to Jammu and Kashmir was bifurcated and reorganised into Union Territory of Jammu and Kashmir and Union Territory of Ladakh. Hence, statement (2) is correct.

38. The Speaker of the Lok Sabha may resign his office by writing, addressed to the

(a) President of India
(b) Prime Minister of India
(c) Vice-President of India
(d) Deputy Speaker of the Lok Sabha

› ***(d)*** The Speaker of the Lok Sabha may resign from office by writing under his hand to the Deputy Speaker of the Lok Sabha. The Speaker of the Lok Sabha is the presiding officer of the Lok Sabha, the lower house of the Parliament of India.

39. The Election Commission recognises a political party as a national party if

1. it secures at least 6% of the total valid votes polled in four or more states in a general election to the Lok Sabha or to the State Legislative Assemblies.
2. it wins at least four seats in a general election to the Lok Sabha from any State or States.

Select the correct answer using the codes given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(c)*** Both statements are true. A political party is recognised as a national party if it secures at least 6% of valid votes polled in any four or more states. Further, it wins at least 4 seats in the house of people from any State or States or wins atleast 2% seats in house of the people.

40. Consider the following statements.

1. The Amendment procedure has been provided in Article-368 of the Constitution of India.
2. The consent of the States is mandatory for all Amendments to the Constitution of India.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› ***(a)*** Article-368 of the Indian Constitution lays down the procedure for Amendment of the Constitution. This article provides for two types of amendments.

(i) By a special majority of Parliament.
(ii) Through ratification of half of the states by a simple majority.

Besides, some other articles also provide for Constitutional Amendment by simple majority of Parliament and no consent of states are required.
Hence, statement (1) is correct and (2) is incorrect.

41. Which one of the following nations adopted first democratic Constitution in September 2015?
(a) Bhutan (b) Myanmar
(c) Nepal (d) Singapore

(c) On 20th September, 2015, Nepal adopted its first democratic Constitution replacing the interim Constitution of 2007. This new Constitution has restructured the nation into a federal republic. Nepal also adopted bi-cameral parliamentary system.

2015 (II)

42. The word 'Secular' was inserted into the Constitution of India by
(a) 44th Amendment Act
(b) 52nd Amendment Act
(c) 42nd Amendment Act
(d) 34th Amendment Act

*(**c**)* The word 'Secular' was inserted into the Preamble of the Constitution by the 42nd Amendment Act, 1976. Besides adding the word 'Secular', this Amendment Act also added two other words 'Socialist and Integrity', Hence, option (c) is correct.

44th Amendment Act was enacted to nullify some of the amendments made by 42nd Amendment Act.

43. The two provisions of the Constitution of India that most clearly express the power of judicial review are
(a) Article-21 and Article-446
(b) Article-32 and Article-226
(c) Article-44 and Article-152
(d) Article-17 and Article-143

*(**b**)* Though the phrase 'Judicial Review' has nowhere been mentioned in the Indian Constitution, but **Article-32** and **Article-226** explicitly confers the power of judicial review on the Supreme Court and High Court, respectively. Under Article 32, the Supreme Court has power to issue directions or writs for the enforcement of any of the Fundamental Rights.

Under Article-226, the Constitutional validity of a Legislative enactment or an executive order can be challenge in a High Court on the grounds that it infringes the Fundamental Rights or it is outside the competence of the authority which framed it.

44. An emergency under Article-352 of the Constitution of India can be declared only during
(a) war, external aggression or internal disturbance.
(b) war, external aggression or armed rebellion.
(c) failure of Constitutional machinery in the State.
(d) financial instability in the country.

*(**b**)* Under Article 352, the President can declare a national emergency when the security of India or a part of it is threatened by war or external aggression or armed rebellion.

A national emergency under this article can be declared even before the actual occurrence of war or external aggression or armed rebellion, if the President is satisfied that there is an imminent danger.

45. The 4th Schedule to the Constitution of India deals with
(a) provisions related to the administration of tribal areas.
(b) allocation of seats in the Council of States.
(c) the Union List, the State List and the Concurrent List.
(d) recognised languages of the Union of India.

*(**b**)* The 4th Schedule to the Constitution of India deals with allocation of seats in the Rajya Sabha to the states and the union territories.

The provisions related to administration of tribal areas is dealt in 6th schedule, while 7th schedule has provisions for division of powers between union territories and states. 8th schedule deals with recognised languages of the union of India.

46. The 'Basic Structure Doctrine' was enunciated by the Supreme Court during the
(a) Golak Nath case
(b) Maneka Gandhi case
(c) Keshavananda Bharati case
(d) SR Bommai case

*(**c**)* The 'Basic Structure Doctrine' is a judiciary-made doctrine enunciated by the Supreme Court on 24th April, 1973 in Keshavananda Bharati case to put limitation on the amending powers of the Parliament. The doctrine was propounded so that the 'basic structure of the basic law of the land' cannot be amended in exercise of its 'constituent power' under the Constitution.

47. The Panchayati Raj system under Part-IX of the Constitution of India does not apply to the states of
(a) Asom, Mizoram and Nagaland
(b) Nagaland, Meghalaya and Tripura
(c) Nagaland, Meghalaya and Mizoram
(d) Sikkim, Tripura and Meghalaya

*(**c**)* The Panchayati Raj system under Part-IX does not apply to the states of Nagaland, Meghalaya, Mizoram and certain other areas. These areas include hill area of Manipur for which a district council exists, Darjeeling district of West Bengal and scheduled and tribal areas in various states.

2015 (I)

48. Which of the following is/are not Fundamental Right(s) under the Constitution of India?
1. Right to Education
2. Right to Work
3. Right to Form Associations
4. Right to Practise any Profession

Select the correct answer using the codes given below.
(a) 1 and 2 (b) 2 and 4
(c) Only 2 (d) 1 and 3

*(**c**)* Right to work is not a Fundamental Right. It is a legal right and also a directive under Directive Principles of State Policy (DPSP). Article-41 direct the state to secure the Right to work, to education and to public assistance in cases of unemployment, old age, sickness and disablement.

Right to Education (Article-21A), Right to form association (Article-19(1)(c) and Right to practice any profession (Article-19(1)(g) are Fundamental Rights covered under Part-III of the Constitution. Hence, option (c) is not correct.

49. The 7th Schedule of the Constitution of India contains
(a) provisions regarding the administration of tribal areas.
(b) the Union List, the State List and the Concurrent List.
(c) a list of recognised languages.
(d) provisions about the disqualification of Members of Legislatures on grounds of defection.

*(**b**)* The 7th Schedule comes under Article-246 of the Constitution. It gives allocation of powers and functions between Union Territories and States. It contains 3 lists–

1. Union list (for Central Government)-97 subjects (now 100 subjects).
2. State list (powers of State Government) - 66 subjects (now 61 subjects).
3. Concurrent list (Both Union and State) - 47 subjects (now 52 subjects)

Hence, option (b) is correct.

50. Article-368 of the Constitution of India deals with
(a) the powers of the Parliament of India to amend the Constitution.
(b) financial emergency.
(c) reservation of seats for Scheduled Castes and Scheduled Tribes in the Lok Sabha.
(d) official language of the Union of India.

(a) Article 368 in part XX of Indian Constitution deals with the powers of the Parliament of India to amend the Constitution. It states that the Parliament may in exercise of its constituent power, add/remove an article, repeal any provisions of the Constitution. However, parliament has no power to amend the basic part of the Constitution.

51. Which among the following is not a basic feature of the Constitution of India?
(a) Fundamental Rights
(b) Independence of Judiciary
(c) Federalism
(d) The unquestioned right of the Parliament to amend any part of the Constitution.

(d) The unquestioned right of the Parliament to amend any part of the Constitution is not a basic feature of the Constitution of India. Fundamental Rights, Independence of Judiciary and Federalism are the basic features of Constitution of India.

52. Which one among the following is not a Fundamental Duty?
(a) To abide by the Constitution of India and respect the National Flag and National Anthem.
(b) To safeguard public property.
(c) To protect the sovereignty, unity and integrity of India.
(d) To render compulsory military service.

(d) Fundamental Duties are defined as the moral obligations of all citizens to help promote a spirit of patriotism and to uphold the unity of India. The Fundamental Duties of citizens were added to the Constitution by the 42nd Amendment in 1976, (Article-51 A, part IV A) upon the recommendations of the Swaran Singh Committee. They were originally ten in number. To render compulsory military service is not a Fundamental Duty.

53. Which of the following are the basic features of 73rd Amendment Act of the Constitution?
1. Provides for a three-tier structure of Panchayats in the village, intermediary and district levels.
2. Reservation of seats for Scheduled Castes and Scheduled Tribes and women in all the tiers of Panchayats.
3. Election to Panchayats under the supervision of the State Election Commissions.
4. Introduction of the 11th Schedule to the Constitution.

Select the correct answer using the codes given below.
(a) 1, 2 and 3 (b) 3 and 4
(c) 1, 2, 3 and 4 (d) 2 and 4

(c) 73rd Amendment Act, 1992 went into effect on 24th April, 1993. The salient features of the Act are
1. To provide 3-tier system of Panchayati Raj for all states having population of over 20 lakh.
2. To hold Panchayat elections regularly every 5 years under the supervision of state election commission.
3. To provide reservation of seats for Scheduled Castes, Scheduled Tribes and women (not less than 33%).
4. To appoint State Finance Commission to make recommendations as regards the financial powers of the Panchayats.
5. The 11th Schedule of India Constitution was added which contains 29 subjects. This schedule covers important topics such as Panchayat's powers, rural development poverty alleviation, drinking water etc.

All the statements are true. Hence, option (c) is correct.

2014 (II)

54. Which of the following statements is correct?
(a) The presiding officer of the Rajya Sabha is elected every year.
(b) The presiding officer of the Rajya Sabha is elected for a term of 5 years.
(c) The presiding officer of the Rajya Sabha is elected for a term of 6 years.
(d) The Vice-President of India is the ex-officio presiding officer of the Rajya Sabha.

(d) The presiding officer of Rajya Sabha is known as the Chairman. The Vice-President of India is the Ex-officio Chairman of the Upper House. He is not elected to chairman of the House and can only be removed from his office if he is removed from the office of the Vice-President. Therefore, only option (d) is correct.

55. Who among the following was not a member of the Drafting Committee of the Constitution of India?
(a) BR Ambedkar (b) KM Munshi
(c) Krishnaswamy Iyer (d) MK Gandhi

(d) MK Gandhi was not a member of the Drafting Committee of the constitution of India. The Drafting Committee for framing the Constitution was appointed on 29th August, 1947. The Committee comprised of a Chairman and 6 other members.

The members of the Committee were
- Dr. BR Ambedkar ® Chairman
- KM Munshi
- Alladi Krishnaswamy Iyer
- N. Gopalaswami Ayengar
- B.L. Mitter
- MD Saadullah
- DP Khaitan

The Drafting Committee was entrusted with the responsibility to prepare the Draft Constitution.

56. Which of the following is/are not related to Fundamental Duties?
1. To cherish and follow the noble ideals which inspired our national struggle for freedom.
2. To value and preserve the rich heritage of our composite culture.
3. To promote the educational and economic interests of the weaker sections of the people, especially the Scheduled Castes and Scheduled Tribes.
4. To protect all monuments of historic interest and national importance.

Select the correct answer using the codes given below.
(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) Only 4

(c) First two statements/duties are among the Fundamental Duties mentioned in the Article 51A.

To promote the educational and economic interests of the weaker sections of the people, especially the SCs and STs is a part of Directive Principles of State Policy under Article 46.

Further, Article 49 under the DPSP direct the state to protect all monuments of historic interest and national importance. Therefore, last two statements 3 and 4 are parts of Directive Principles of State Policy and not Fundamental Duties.

57. Joint sittings of the two Houses of Indian Parliament are held to

(a) elect the President of India.
(b) elect the Vice-President of India.
(c) adopt a Constitution Amendment Bill.
(d) consider and pass a Bill on which the two Houses disagree.

(d) Under the Article-108 of the Constitution of India, a joint sitting of both the Houses can be called if a Bill has been passed by one House and rejected by the other, if the two Houses have finally disagreed on the amendments to be made in the Bill or if more than six months have elapsed after a Bill is passed by one House but not passed by the other.

Till so far, the joint session of Indian Parliament has been called for only 3-bills i.e. the Dowry Prohibition Act, 1960, the Banking Service Commission Repeal Bill, 1977 and the Prevention of Terrorism Act, 2002. Hence, option (d) is correct.

58. The President of India can issue proclamation of Emergency

(a) on the advice of the Prime Minister.
(b) on the advice of the Council of Ministers.
(c) in his own discretion.
(d) when the decision of the Union Cabinet for the issue of such proclamation has been communicated to him in writing.

(d) Article-352 of the Indian Constitution talks about the national emergency. National emergency is imposed whereby there is a grave threat to the security of India or any of its territory due to war, external aggression or armed rebellion. Such emergency shall be imposed by the President on the basis of written request by the Council of Ministers headed by the Prime Minister.

2014 (I)

59. Which of the following statements are true for the Fundamental Right to Life and Personal Liberty as guaranteed under Article-21 of the Constitution of India?

1. The Right is available to citizens as well as aliens.
2. It covers protection against arbitrary executive and legislative action.
3. It includes the right to live with human dignity.
4. It can be taken away according to the procedure established by law.

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 2, 3 and 4 (d) 1 and 4

(c) Article 21, under Part III of the Constitution deals with Fundamental Right to protection of life and personal liberty. This article declares that no person shall be deprived of his life or personal liberty except according to procedure established by law.

The right is available to both citizens and aliens. The Supreme Court in Menaka Gandhi Case (1978) has given a wide interpretation to this Fundamental Right and affirmed that protection under Article 21 should be available not only against arbitrary executive action, but also against arbitrary legislative action.

The court also held that right to life is not merely confined to animal existence but it include within its sphere the Right to live with human dignity.

So, all the statements are true. Hence, option (c) is correct.

60. Which one among the following is not a characteristic of a federal system?

(a) There are two distinct levels of government.
(b) The responsibilities and powers of each level of government are clearly defined in a written Constitution.
(c) There is no separation of powers between the legislative and Executive branches of government.
(d) A Supreme Court is entrusted with the responsibility of interpreting these provisions and arbitrating in matters of dispute.

(c) A federal system of government deals with governance at two levels i.e. centre and state. It has nothing to do with separation of power between Legislative and Executive branches of government.

61. Which of the following is/are true relating to the Lokpal Bill, 2013, which was assented to by the President on 1st January, 2014?

1. It provides for creation of **antigraft** ombudsman to investigate corruption charges against public functionaries including the Prime Minister, Ministers and Members of the Parliament.
2. The Bill makes it incumbent upon states to make within a year their own law for setting up Lokayuktas on the lines of the Lokpal Bill.
3. States are free not to set-up Lokayuktas, if they do not so desire.

Select the correct answer using the codes given below.

(a) 1 and 2 (b) 2 and 3
(c) Only 1 (d) 1, 2 and 3

(a) The Lokpal Bill, was introduced in the Lok Sabha on 22nd December, 2011 and passed on 27th December, 2011 as the Lokpal and Lokayuktas Bill, 2011.

It received assent from President on 1st January, 2014 and came into force from 10th January. It is an Anti-Corruption Act of Indian Parliament which seek to provide for the establishment of the institution of Lokpal to inquire into allegations of corruption against certain public functionaries including Prime Minister, ministers and member of parliament and for matters connecting them. The bill also makes it incumbent upon states to make within a year their own law for setting up of Lokayuktas on th lines of the Lokpal Bill.

Hence, option (a) is correct.

62. The Constitution of India guarantees the Fundamental Right to Freedom of Religion to all its citizens. Which among the following is not true of this Right?

(a) It gives freedom of conscience and freedom to profess, practise and propogate any religion.

(b) It gives freedom to establish and maintain institution for religious and charitable purposes.
(c) The Right is subject to public order, morality and health.
(d) The state cannot make any law which abrogates this Right for citizens.

(d) The Constitution of India does guarantee Fundamental Right to Freedom of Religion to all its citizens, but the state can make laws which abrogate the right in case there is a threat to public order, morality and health. The provision for Right to Freedom of Religion is mentioned in Articles 25-28 of the Constitution. Hence, option (d) is not correct.

63. The Scheduled Castes and Scheduled Tribes (Prevention of Atrocities) Act, 1989 may be seen as giving effect to which of the following Fundamental Rights?

1. Equality before the Law
2. Right against Discrimination
3. Abolition of Untouchability
4. Right to Freedom of Religion

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) 2 and 3 ,
(c) 2, 3 and 4 (d) All of these

(a) The Scheduled Castes and Scheduled Tribes (Prevention of Atrocities) Act, 1989 may be seen as giving effect to Equality before Law, Right against Discrimination and Abolition of Untouchability. Right to Freedom of Religion is not covered as this act deals with intra-religious domain.

64. Under which of the following Acts is the National Population Register being created?

(a) The Citizenship Act of India, 1995 as amended in 2004.
(b) The Census Act, 1948
(c) The UID Act, 2010
(d) None of the above

(a) The National Population Register (NPR) is a Register of usual residents of the country. It is being prepared at the local (village/sub-town), sub-District, District, State and National level under provisions of the Citizenship Act, 1955 and the Citizenship (Registration of Citizens and issue of National Identity Cards) Rules, 2003.

It is mandatory for every usual resident of India to register in the NPR.

A usual resident is defined for the purposes of NPR as a person who has resided in a local area for the past 6 months or more or a person who intends to reside in that area for the next 6 months or more.

65. The Financial Sector Legislative Reforms Commission Report submitted recently, among others, contained which of the following proposals?

1. All regulators will have an empowered board, the role of each of the members has been defined.
2. The Commission envisages an important process of judicial review of the regulations.
3. The draft code does not mention about executive powers.

Select the correct answer using the codes given below.

(a) 1 and 2 (b) Only 2
(c) 1 and 3 (d) 1, 2 and 3

(a) The Financial Sector Legislative Reforms Commission (FSLRC) was constituted with a view to rewrite and clean up the financial sector laws to bring them in tune with the current requirements. FSLRC was set-up on 24th March, 2011 by the Government of India, Ministry of Finance. In its report, the commission recommended that regulators will have an empowered board with a precise selection-cum-search process appointed of members. It further recommended judicial review of regulations. Hence, option (a) is correct.

47

ECONOMY

2019 (II)

1. SAMARTH is a flagship skill development scheme of which one of the following Ministries?
(a) Ministry of Textiles
(b) Ministry of Agriculture and Farmers Welfare
(c) Ministry of Skill Development and Entrepreneurship
(d) Ministry of Human Resources Development

(***a***) SAMARTH is a flagship skill development scheme of Ministry of Textiles, Government of India. The objective of the scheme is to provide skill development to the youth for gainful and sustainable employment in the textile sector.

2. The principal component(s) of Nehru-Mahalanobis strategy of economic development was/were
1. Restructuring economic dependency on metropolitan capitalism into independent economic development.
2. Transition from semi-feudal agricultural to capitalist farming.

Select the correct answer using the codes given below.
(a) Only 1 (b) Only 2
(d) Both 1 and 2 (d) Neither 1 nor 2

(***c***) The Nehru-Mahalanobis strategy was developed by PC Mahalanobis, under the guidance of Jawaharlal Nehru in 1953. *The Principal components of this development strategy were*

(i) Restructuring economic dependency on metropolitan capitalism into independent economic development.
(ii) State capitalist and capitalist developments in the urban sector and
(iii) Transition from semi-feudal agricultural to capitalist farming.

Hence, Statements 1 and 2 both are correct.

2018 (II)

3. 'Seva Bhoj Yojana', a scheme of the Government of India that seeks to reimburse Central share of CGST and IGST on Food/Prasad/Langar/Bhandara offered by religious institutions, is introduced recently by which one of the following Ministries?
(a) The Ministry of Culture
(b) The Ministry of Home Affairs
(c) The Ministry of Consumer Affairs, Food and Public Distribution
(d) The Ministry of Social Justice and Empowerment

(***a***) The Ministry of Culture, Government of India has introduced a new scheme namely 'Seva Bhoj Yojana' to provide financial assistance on purchase of specific food items by Charitable Religious Institutions (CRIs) for free distribution among people.

Under this scheme, Centre's share of Central Goods and Services Tax (CGST) and Integrated Goods and Services Tax (IGST) charged on the raw food materials purchased by the religious institutions will be refunded.

4. Which one of the following Indian states has recorded negative growth of population as per Census 2011?
(a) Manipur (b) Mizoram
(c) Tripura (d) Nagaland

(***d***) As per Census 2011, Nagaland is the only state of the country which has negative growth rate of (0.58%). Growth rate of Indian population was 17.69% between 2001 and 2011.
In other options, the growth of population are-Manipur (18.6%), Tripura (14.8%), Mizoram (23.5%).

5. Which one of the following is the correct descending order of Indian states in terms of sex ratio as per Census 2011?
(a) Mizoram — Manipur — Tripura — Meghalaya
(b) Tripura — Manipur — Meghalaya — Mizoram
(c) Meghalaya — Manipur — Mizoram — Tripura
(d) Manipur — Meghalaya — Tripura — Mizoram

(**c**) Sex ratio is used to describe the number of females per 1000 of males. In the Population Census of 2011, it was revealed that the sex ratio is 943 females per 1000 of males.

As per census, sex ratio of following states is

State	**Sex Ratio** (2011)
Meghalaya	989
Manipur	987
Mizoram	976
Tripura	960

2018 (I)

6. Which one of the following Union Territories has the highest female literacy rate?
(a) Chandigarh
(b) Lakshadweep
(c) Andaman and Nicobar Islands
(d) Puducherry

(*b*) Lakshadweep has a literacy rate of 92.28%. The female literacy rate of Lakshadweep is 87.95% which is the highest among all the Union Territories in India. Daman and Diu comes next with 79.59% in female literacy.

2017 (II)

7. Which one of the following was set as a target of average growth of GDP of India over the plan period 2012-17 by the Approach Paper to the Twelfth Five Year Plan?

(a) 7% (b) 8%
(c) 9% (d) 10%

(*b*) The final growth target was set at 9% as per the approach paper of the 12th Five Year Plan which lasted from 2012 to 2017. However, 12th Five Year Plan 2012-17 as per the draft document released by the planning commission aimed at a growth rate of 8%. This was the revised rate when compared to the initial approach paper.

2017 (I)

8. According to the Census 2011, the density of population in which one among the following States is the lowest?

(a) Sikkim (b) Nagaland
(c) Manipur (d) Mizoram

(*d*) Mizoram is the second least populous state in the country after Arunachal Pradesh. Mizoram has population density of 52 persons per sq km.

The population density of Sikkim is 86 persons per sq km, Nagaland is 119 persons per sq km and Manipur is 128 persons per sq km.

9. In its emphasis on enhancing human capabilities, which one among the following does not figure in the Twelfth Five Year Plan?

(a) Life and longevity
(b) Education
(c) Delivery of public service
(d) Skill development

(*c*) The period of Twelfth Five Year Plan is 2012-17. In its emphasis on enhancing human capabilities, delivery of public service does not figure in the Twelfth Five Year Plan. The theme of the approach paper for Twelfth Five Year Plan was faster, sustainable and more inclusive growth.

The Twelfth Five Year Plan emphasised on enhancing capabilities in the field of life and longevity, education, skill development, nutrition, health, drinking water and sanitation.

10. Which one of the following is not a target of the Twelfth Five Year Plan?

(a) Real GDP Growth Rate of 8%
(b) Agriculture Growth Rate, not of 5%
(c) Manufacturing Growth Rate of 10%
(d) Increase in green cover by 1 million hectare every year during the Plan period

(*b*) The Twelfth Five Year Plan (2012-17) of the Government of India aimed 4% Agriculture Growth Rate, not 5%, but the sector grew by an average 1.6% annual growth due to lower production. Hence, option (b) is incorrect.

11. Which of the following are the features of the ideology of utilitarianism?

1. Utilitarians believed that all value derives from land.
2. The most celebrated spokesmen of utilitarianism were Jeremy Bentham and John Stuart Mill.
3. Utilitarians were advocates of the idea that India could be ruled through indigenous laws and customs.
4. Utilitarians were advocates of the idea of the 'greatest good for the greatest number of people'.

Select the correct answer using the codes given below.

(a) 1 and 4 (b) 2 and 4
(c) 1, 2 and 4 (d) 2 and 3

(*b*) Utilitarianism is propounded by Jeremy Bentham, later the concept was developed by John Stuart Mill. Utilitarians were advocates of the idea of 'greatest good for the greatest number of people'. Hence, Statements 2 and 4 are correct.

It is a theory in normative ethics holding that the best moral action is the one that maximises utility. Whereas, utility is defined in various ways, but is generally, it is related to the well-being of sentient entities. It is not connected with indigenous laws and customs.

Hence, Statements (1) and (3) are incorrect.

12. Surge pricing takes place when a service provider

(a) raises the price of its product or service as demand outstrips supply
(b) follows present prices immune to demand and supply dynamics
(c) fixes a minimum price for its services
(d) fixes an average price on the basis of transactions carried over a day

(*a*) Surge pricing also referred as dynamic pricing. It is a pricing strategy in which businesses set flexible prices for products or service based on current market demands. It is a common practice in hospitality, travel, retail etc. Hence, option (a) is the correct answer.

2016 (II)

13. Which of the following was/were the feature(s) of Lenin's New Economic Policy (NEP) for the Soviet Union?

1. Private retail trading was strictly forbidden.
2. Private enterprise was strictly forbidden.
3. Peasants were not allowed to sell their surplus.
4. To secure liquid capital, concessions were allowed to foreign capitalists, but the state retained the option of purchasing the products of such concerns.

Select the correct answer using the codes given below.

(a) Only 1
(b) 2 and 3
(c) 1, 2 and 4
(d) Only 4

(*b*) In March 1921, Lenin introduced the New Economic Policy in order to revive the economy. The new programme signified a return to a limited capitalist system. *The features of Lenin's New Economy Policy (NEP) were as follow*

- Forced requisition of grain was replaced by a specific tax in kind, peasants could retain excess produce and sell it for a profit.
- Smaller businesses were permitted to operate as private enterprises.
- Large industries remained under State control. They operated on the open market, but the State controlled the fixing of prices and the appointment of Board of Directors.
- Private trade and wages were restored, and compulsory labour service was abolished. Hence, option (b) is correct.

14. As per the RBI guidelines, which one of the following is the minimum tenure of Masala Bonds that an Indian company can issue offshore?

(a) Five years (b) Four years
(c) Three years (d) Two years

(c) To make Masala Bonds more attractive, the Reserve Bank of India (RBI) has reduced the minimum tenure of such bonds that an Indian company can issue offshore to three years from the previously stated five years. Masala bonds are Indian rupee denominated bonds issued in offshore capital markets. These are rupee-denominated bonds issued to offshore investors settled in dollars and therefore, the currency risk resides with investors.

15. Which of the following statement(s) about Krishi Kalyan Cess (KKC) is/are correct?

1. KKC is calculated in the same way as Service Tax is calculated.
2. The current rate of KKC is 0.50%.
3. KKC is similar to the Krishi Kalyan Surcharge (KKS).

Select the correct answer using the codes given below.

(a) Only 1 (b) 1 and 2
(c) 2 and 3 (d) All of these

(b) Both the statements (1) and (2) are correct. Krishi Kalyan Cess (KKC) is a cess, which will be implied and collected by the government as Service Tax on all taxable services at a rate of 0.50% of the worth of taxable services. It will be calculated in the same way as Service Tax is calculated. KKC comes into effect from 1st June, 2016.

Krishi Kalyan Cess is different from the Krishi Kalyan Surcharge, which is announced by the government in the same Union Budget, 2016-17. In order to provide a stable and predictable taxation regime and reduce black money, it was announced in the budget.

16. As per census 2011, which one of the following States has the highest number of inhabitated villages?

(a) Uttar Pradesh (b) Odisha
(c) Rajasthan (d) Maharashtra

(a) According to census 2011, there are a total of 638,596 villages in India. Out of those, 593,731 are inhabited villages and 44,865 are uninhabited (Ghost) villages. Among states, Uttar Pradesh has the highest number of uninhabited villages (9,510) followed by Bihar (6,083).

17. Which of the following statements concerning Scheduled Castes (SCs) population in India are correct?

1. The highest percentage of SC population out of the total population of the state is found in Punjab.
2. Bihar is the second largest state in respect of total SC population in India.
3. Uttar Pradesh has the highest number of SC population in India.
4. Among the Indian States, the lowest percentage of SC population out of the total population of the state is found in Goa.

Select the correct answer using the codes given below.

(a) 1, 2 and 3 (b) 1 and 3
(c) 2 and 4 (d) 1, 2 and 4

(b) According to census 2011 the total population of the Scheduled Castes in the country is 201378372, which constitute 16.6% of the total population

Uttar Pradesh (41357608) has the largest SC population followed by West Bengal (21463270).

Punjab has the highest percentage of SC to its population (31.9%).

Hence statements (1) and (3) are correct.

Himachal Pradesh comes second with 25.2% of its population being SC. Mizoram has lowest proportion of Scheduled Castes (0.1%).

Hence, statement (2) and (4) are incorrect.

18. Arrange the following States in order of their decreasing population size as per the census 2011.

1. West Bengal 2. Bihar
3. Maharashtra
4. Andhra Pradesh

Codes

(a) 2, 3, 1, 4 (b) 3, 2, 1, 4
(c) 1, 4, 3, 2 (d) 3, 1, 4, 2

(b) As per the census 2011, the States in order of their decreasing population size is

States	**Population** (In India)
1. Maharashtra	112372972 (2nd)
2. Bihar	103804637 (3rd)
3. West Bengal	91347736 (4th)
4. Andhra Pradesh	49386799 (10th)

Hence, option (b) is correct.

2016 (I)

19. Areas which are engines for economic growth supported by quality infrastructure and complemented by an attractive fiscal package are known as

(a) Export Processing Zones
(b) Duty Free Tariff Zones
(c) Special Economic Zones
(d) Technology Parks

(c) Special Economic Zone (SEZ) are those areas in a country that possess special economic regulations that differ from other areas. These areas has quality infrastructure and attractive fiscal packages to promote export.

20. Division of labour often involves

1. specialised economic activity.
2. highly distinct productive roles.
3. involving everyone in many of the same activities.
4. individuals engage in only a single activity and are dependent on others meet their various needs.

Select the correct answer using the codes given below.

(a) 1, 3 and 4 (b) 1, 2 and 4
(c) Only 3 (d) 2 and 4

(b) Division of labour is an specialised economic concept that states that dividing production process into different stages enables workers to forces on specific tasks. It is a highly specialised economic activity in which workers are given highly distinct productive roles. It does not involve many of the same activity. Hence, Statements (1), (2) and (4) are correct and 3 is incorrect.

21. Which of the following is/are the component/components of the Integrated Power Development Scheme launched by the Government of India recently?

1. Strengthening of sub-transmission and distribution networks in the urban areas.
2. Metering of distribution transformers/feeders/consumers in the rural areas.

Select the correct answer using the codes given below.

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Integrated Power Development Scheme (IPDS) was launched in 2014 with the objective of strengthening of sub-transmission and distribution network in urban areas and metering of distribution transformers/feeder/consumers in urban areas.

This scheme will help in reduction in AT&C losses along with establishment of IT enabled accounting system.

Hence, Statement (1) is correct and (2) is incorrect.

22. Which of the following statements about the aims of the United Nations is/are true?

1. To foster a mutual appreciation of each others culture and literature among nations.
2. To achieve international co-operation in solving problems of an economic, social, cultural or humanitarian character.
3. To foster relations between scholars and academics in different countries.
4. To organise international countries.

Select the correct answer using the codes given below.

(a) Only 1
(b) 1, 2 and 4
(c) Only 2
(d) 1, 2 and 3

(c) The aims of United Nations Organisation includes maintaining international peace and security, developing friendly relations among nations, achieving international co-operation in solving international problems of economic, social, cultural and to be a centre for harmonising actions of nations towards these common ends. UN was founded on 24th October, 1945. Its headquarter located at New York. Hence, option (c) is correct.

23. Which of the following best explains the stated broad vision and aspirations of the Twelfth Five Year Plan (2012-17)?

(a) Faster, sustainable and more inclusive growth
(b) Modernisation of industries and strengthening infrastructure
(c) Enhancing agricultural and rural incomes
(d) Checking inflation and strengthening non-economic variables like nutritional requirements, health and family planning

(a) The broad vision and aspirations that the Twelfth Plan seeks to fulfill are reflected in its subtitle 'faster, sustainable and more inclusive growth'. Twelfth Five Year Plan was India's last Five Year Plan.

24. Which of the following is/are not FDI policy change(s) after 2010?

1. Permission of100% FDI in automotive sector.
2. Permitting foreign airlines to make FDI up to 49%.
3. Permission of up to 51% FDI under the government approval route in multi-brand retailing, subject to specified conditions.
4. Amendment of policy on FDI in single-brand product retail trading for aligning with global practices.

Select the correct answer using the codes given below.

(a) Only 1 (b) 2 and 4
(c) 1 and 2 (d) 1, 2 and 3

(a) The government permitted 100% FDI in automatic sector in 2002. Foreign airlines were allowed to make FDI up to 49% in 2012. Multi-brand retail was opened for FDI (51%) in 2012. And in the same year, the amendment to the policy was done. Hence, option (a) is correct.

25. Which of the following are the features of Pradhan Mantri Jan-Dhan Yojana?

1. Ensuring financial inclusion of the poor.
2. Enhancing financial literacy.
3. Provision for accidental insurance to account holders.
4. Allowing bank accounts with zero balance.

Select the correct answer using the codes given below.

(a) 1 and 4 (b) 1, 2 and 3
(c) 2 and 4 (d) All of these

(d) Pradhan Mantri Jan-Dhan Yojana is a national mission on financial inclusion encompassing an integrated approach to bring about comprehensive financial inclusion of all household. It is applicable to 10 to 65 years age groups. The plan envisages one basic account with zero balance for every household, financial literacy, access with zero balance to credit, insurance and pension facility. It was launched in 2014. All the given statements are true. Hence, option (d) is correct.

26. Which one of the following is not a feature of the Nehru-Mahalanobis model of development strategy?

(a) Development of capital goods industries
(b) Major involvement of the state in the economy
(c) Industrial deregulation and disinvestment in the public sector
(d) Enhancing the scope and importance of the public sector

(c) The Nehru-Mahalanobis model was followed during Second Five Year Plan. The essence of this model was a shift in the pattern of industrial investment towards building up a domestic consumption goods sector. It was in favour of enhancing public investment and was against industrial deregulation and disinvestment. Hence, option (c) is correct.

2015 (II)

27. Which one of the following is not correct in the context of balance of payments of India during 2013-14?

(a) India's exports were less than its imports.
(b) Trade balance was negative.
(c) Net invisibles were positive.
(d) Capital account balance was negative.

(d) According to the Economic Survey 2013-14 the India's exports were less than imports. Trade balance is the calculation of a country's exports minus its imports.

Net invisibles refer to the services and products that do not result in the transfer of any physical object, it was positive in 2013-14. Capital account is the net result of public and private international investments following in and out of a country, which were positive in 2013-14. Hence, option (d) is not correct.

28. Which one of the following continents accounts for the maximum share in exports from India?

(a) Asia (b) Europe
(c) Africa (d) North America

(a) Share of export from India in 2013-14 was

Asia	—	49.6 %
Europe	—	18.6 %
America	—	17.3 %
Africa	—	9.9 %

As per 2019 report, top continent accounts for the maximum share-in exports from India is Asia (47.62%).

29. During the Eleventh Five Year Plan, agriculture sector in India witnessed a growth rate of 3.3% per annum which is higher than 2.4% per annum in the previous Five Year Plan. This is largely due to better performance of
(a) crops and livestock
(b) oilseeds and fibres
(c) fishing and oilseeds
(d) fibres and fishing

(*a*) The Eleventh Five Year Plan (2007-2012) had some success in reversing the declaration of agricultural growth witnessed during the 9th and 10th Plan. The foodgrain production registered record level of growth during Eleventh Five Year Plan. The livestock emerged as a important source of alternate income for small and marginal farmers.

30. Which of the following statement(s) about the livestock sector in India is/are correct?
1. Livestock contributed about 25% of gross value added in agriculture.
2. It provides self employment to a large segment of population.
3. Rapid growth of livestock sector can be egalitarian and inclusive.

Select the correct answer using the codes given below.
(a) Only 3 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 3

(*d*) India's livestock sector is one of the largest in the world. It provides food items such as milk, meat and eggs for human consumption. Livestock contributes to 25% of total agriculture GDP.

It provides alternate source of income to small and marginal farmers. Hence, livestock growth leads to egalitarian and inclusive growth.

31. Which one of the following is not correct in the current Indian scenario?
(a) Life expectancy is on the increase and is about 67 years as of now.
(b) Infant mortality rate is on the decline and has reached 47 per thousand.
(c) Maternal mortality rate is on the rise due to lack of medical facilities.
(d) Percentage of women giving birth in health institutions is on the rise.

(*c*) The Maternal Mortality Rate (MMR) is the annual number of female deaths per 1,00,000 live births from any cause related to aggravated by pregnancy or its management (excluding accidental or incidental causes). India's maternal rate reduced from 212 deaths per 1 lakh live births in 2007 to 174 deaths in 2015.

Under National Health Mission Central Government had launched several schemes such as the Janani Suraksha Yojana, Accredited Social Health Activist (ASHA) programme providing economic incentives to pregnant women which were important strategy in improving the maternal mortality ratio and institutional deliveries in India. Hence, option (c) is incorrect.

32. The First Five Year Plan (1951-56) was drafted by
(a) PC Mahalanobis
(b) KN Raj
(c) JC Kumarappa
(d) Jawaharlal Nehru

(*b*) The First Five Year Plan (1951-56) was drafted by KN Raj. It mainly focused in development of the primary sector. KN Raj also computed India's Balance of Payments for the first time for RBI.

2015 (I)

33. The concept of Demographic Transition provides a model for
(a) analysing government responses to changing pattern of population growth
(b) explaining major patterns of global population movement throughout the history
(c) analysing spatial variations in the distribution, growth and movement of population overtime
(d) examining the relationship between economic development and natural population change

(*d*) Demographic Transition (DT) refers to the transition from high birth and death rates to low birth and death rates as a country develops from a pre-industrial to an industrialised economic system. It examines the relationship between population change and economic development in particular regions of the world.

34. The Second Five Year Plan that called for the establishment of socialist pattern of society was commonly referred to as the
(a) Harrod-Domar Plan
(b) Mahalanobis Plan
(c) Nehru Plan
(d) People's Plan

(*b*) The Second Five Year Plan (1956-61) was to set India on the path of Industrialisation. PC Mahalanobis was the moving spirit behind the Second Five Year Plan. He gave the highest priority to strengthening the industrial base of the economy.

2014 (II)

35. Which one of the following is the correct sequence of economic sectors in terms of their contribution to the GDP of India in decreasing order?
(a) Service—Industry—Agriculture
(b) Agriculture—Industry—Service
(c) Industry—Service—Agriculture
(d) Agriculture—Service—Industry

(*a*) In GDP, service sector contributes about 64.8% (in 2013), while the Industrial sector contributes about 21.5% and Agriculture sector only 13.7%. However, Agriculture sector employs the largest portion of the Indian labour force.

As per 2018-19 India, Service sector contributes about 54.3% in GDP, while the Industrial sector contributes about 29.60% and agriculture sector only 14.4% in GDP of India.

36. Which one of the following is not an ASEAN member?
(a) Cambodia (b) Laos
(c) Myanmar (d) Taiwan

(*d*) Taiwan is not an ASEAN member. Taiwan is the political and economic organisation of 10 South-East Asian countries formed on 8th August, 1967 by Indonesia, Malaysia, the Philippines, Singapore and Thailand.

Its membership has expanded to include Brunei, Cambodia, Laos, Myanmar and Vietnam. Now, there are 10 members in ASEAN.

37. Which one among the following countries was the top contributor to India's FDI for the year 2013-14?
(a) Mauritius (b) Singapore
(c) USA (d) Japan

(*b*) Singapore overtook Mauritius last year as the leading source of FDI into India, according to data released by India's Department of Industrial Policy and Promotion (DIPP).

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GENERAL KNOWLEDGE

2019 (II)

1. The headquarters of National Research Laboratory for Conservation of Cultural Property (NRLC) is located at
(a) New Delhi (b) Lucknow
(c) Kolkata (d) Bhopal

(*b*) The National Research Laboratory for Conservation of Cultural Property (NRLC) is a scientific institution of the Ministry of Culture, Government of India. It is responsible for the Conservation of Cultural Heritage. The NRLC was established in 1976 with its headquarters located at Lucknow.

2. The first Utkrisht Double Decker Air-Conditioned Yatri (UDAY) Express train runs between
(a) Delhi and Kalka
(b) Bengaluru and Coimbatore
(c) Chennai and Visakhapatnam
(d) Chennai and Bengaluru

(*b*) The first Utkrisht Double Decker Air-Conditioned Yatri (UDAY) Express was a specially designed trains which started service between Coimbatore and Bengaluru in June, 2018. Second, such service was launched in September, 2019 that runs between Visakhapatnam and Vijayawada.

3. Who among the following were selected for the Rajiv Gandhi Khel Ratna Award, 2019?
(a) Vijay Kumar and Yogeshwar Dutt
(b) Sakshi Malik and Jitu Rai
(c) Virat Kohli and Mirabai Chanu
(d) Bajrang Punia and Deepa Malik

(*d*) Rajiv Gandhi Khel Ratna Award is the highest sporting honour of India. It is annually awarded to players by the Ministry of Youth Affairs and Sports. Bajrang Punia (Wrestling) and Deepa Malik (Parathlete) were awarded with this award in 2019.

4. Betel leaf of which one of the following states has been accorded a GI tag recently?
(a) Andhra Pradesh
(b) Madhya Pradesh
(c) Kerala
(d) Mizoram

(*c*) The Tirur Vettila (Betel leaf) from Kerala has got its GI tag. The leaf given its unique component i.e., Eugenol in leaves along with high protein and chlorophyll content has several medicinal properties.

Apart from this Arnamula Kannadi, Payyannur Pavithra Ring, Navara Rice, Alleppey Coir also gets GI tag in Kerala.

5. CORAS is a separate Commando Unit of
(a) National Security Guard
(b) Central Industrial Security Force
(c) Railway Protection Force
(d) Central Reserve Police Force

(*c*) The Ministry of Railways had launched the Commando for Railway Security (CORAS) of Indian railways in August, 2019. CORAS is a separate Commando Unit of Railway Protection Force and are raised keeping in mind the threat from disruptive forces. The first deployment of the unit will be in the naxalism affected Chhattisgarh state.

6. World Humanitarian Day is celebrated on
(a) 26th June (b) 17th July
(c) 19th August (d) 27th September

(*c*) World Humanitarian Day is an international day celebrated on 19th August every year to recognise the humanitarian personnel and those who laid down their lives for the causes of humanity. It was designated by the United Nation General Assembly for the first time in 2009 on 19th August.

7. Which one of the following is the name given to the lander of Chandrayaan-2?
(a) Vigyan
(b) Anusandhan
(c) Pragyan
(d) Vikram

(*d*) Chandrayaan-2 was the second lunar exploration mission of Indian Space Research Organisation (ISRO). It consisted of a lunar orbiter, lander named 'Vikram' and lunar rover called 'Pragyan'. The Mission's Lander was named after Vikram Sarabhai, who is widely regarded as the founder of Indian Space Programme.

8. Along with Pranab Mukherjee, how many more persons were conferred the Bharat Ratna Award in the year 2019?
(a) One (b) Two
(c) Three (d) Four

(*b*) In the year 2019, along with Pranab Mukherjee, two personalities were awarded Bharat Ratna, the highest civilian award of India. The awardees were:
1. Pranab Mukherjee → Former President of India.
2. Nanaji Deshmukh → Social Activist from Maharashtra.
3. Bhupen Hazarika → Assamese Composer, Musician and Lyricist.

9. Aishwarya Pissay excels in which one of the following sports?
(a) Badminton
(b) Boxing
(c) Motorsports
(d) Chess

(*c*) Aishwarya Pissay is an Indian circuit and off road motor cycle racer. She became the first ever Indian athlete to won a world title in motorsports.

10. Which one of the following films was adjudged as the Best Feature Film in the 66th National Film Awards?
(a) Bulbul can Sing (b) Hellaro
(c) Badhaai Ho (d) Uri

(*b*) The movie 'Hellaro' won the National Film Award for the Best Feature Film in the 66th National Film Awards. 'Hellaro' is a Gujarati language period drama, co-written and directed by Abhishek Shah.

2019 (I)

11. National Water Academy, a centre of excellence in training and capacity building in water resource, is located at
(a) New Delhi (b) Kolkata
(c) Pune (d) Chennai

(*c*) National Water Academy (NWA), a centre of excellence in training and capacity building in water resource, is located in Pune (Maharashtra). It was set-up under Central Water Commission by the Ministry of Water Resources in 1988.

The NWA is invisaged to function as 'Centre of Excellence' for in-service training of water resources engineering personnel. It is addressing the wider training needs of water resources engineers of State and Central Agencies in the field of planning, design, evaluation, construction, operation and monitoring of water resources projects.

12. 'Mission Indradhanush' is related to
(a) Bullet train project
(b) Agriculture development
(c) Women empowerment
(d) Full immunisation

(*d*) 'Mission Indradhanush' is related to full immunisation. This health mission was launched by the Ministry of Health and Family Welfare on 25th December, 2014. The scheme seeks to drive towards 90% full immunisation coverage of India and sustain same by year 2020. Vaccination is being provided against eight vaccine preventable diseases, i.e. Diphtheria, Pertussis, Tetanus, Polio, Measles, Tuberculosis, Hepatitis B, Meningitis, Rotavirus, Diarrhea and Japanese Encephalitis.

13. IMBEX is a joint exercise conducted regularly between Armies of India and
(a) Malaysia
(b) Maldives
(c) Mauritius
(d) Myanmar

(*d*) India-Myanmar Bilateral Army Exercise (IMBEX) is a joint exercise conducted regularly between Armies of India and Myanmar.

The first edition of this exercise was held in 2017 and 2nd edition was conducted on 14th January 2019 in Haryana.

This joint exercise is also the first of its kind military training exercise on United Nations Peacekeeping Operations between India and Myanmar.

14. Who among the following was awarded The Hindu Prize in Fiction category for the year 2018?
(a) Neelum Saran Gour
(b) N. Kalyan Raman
(c) Manoranjan Byapari
(d) Arunav Sinha

(*a*) Neelum Saran Gour was awarded The Hindu Prize in Fiction category for the year 2018. Neelum Gour got this award for her novel 'Requiem in Raga Janki'. She is the author of five novels, four collection of short stories and one work of literary non-fiction. She has also edited a pictorial volume on the history and culture of Allahabad.

15. Who among the following was appointed as Head of the seven Members committee to look into revenue shortfall being faced by the States after the GST rollout in India and suggest steps for augmenting collections?
(a) Himanta Biswa Sharma
(b) Thomas Isaac
(c) Sushil Modi
(d) Capt. Abhimanyu

(*c*) Sushil Modi was appointed as Head of the seven members committee to look into revenue shortfall being faced by the States after the GST rollout in India and suggest steps for augmenting collections by the States, since July, 2017.

As per the term of reference of the Group of Ministers, the Modi led committee will undertake data analysis using econometric and statistical tools and suggest suitable measures for course correction for revenue augmentation.

16. The 15th Pravasi Bharatiya Divas, 2019 was held in
(a) New Delhi (b) Gandhi Nagar
(c) Prayagraj (d) Varanasi

(*d*) The 15th Pravasi Bharatiya Divas, 2019 was held in Varanasi (Uttar Pradesh) from 21st to 23rd January, 2019. It was inaugurated by the Prime Minister Narendra Modi and Prime Minister of Mauritius, Pravind Jugnauth. The theme of this Pravasi Bharatiya Divas was 'Role of Indian Diaspora in building a new India'. On 9th January, 2020, 16th Pravasi Bhartiya Divas was celebrated.

17. Which one of the following cities was named by UNESCO as World Capital of Architecture for 2020?
(a) Tokyo (b) Johannesburg
(c) Rio-de-Janerio (d) New Delhi

(*c*) Rio-de-Janerio (Brazil) was named by UNESCO as World Capital of Architecture for 2020. Rio-de-Janerio will be hosting the World Congress of the International Union of Architects in July, 2020.

18. India's first private sector Howitzer gun-making unit is located at
(a) Jamshedpur (b) Kolkata
(c) Hazira (d) Gwalior

(*c*) India's first private sector Howitzer gun-making unit is located at Hazira in Surat (Gujarat). It was inaugurated by the Prime Minister Narendra Modi on 19th January, 2019 as India's first Armoured System Complex (ASC). It is built by a private company-Larsen and Toubro (L & T).

19. Which of the following pairs of old names and new names of islands in India is/are correctly matched?
1. Ross Island-Shaheed Dweep
2. Neil Island-Netaji Subhas Chandra Bose Dweep
3. Havelock Island-Swaraj Dweep

Select the correct answer using the codes given below.
(a) 1, 2 and 3 (b) 2 and 3
(c) 1 and 2 (d) Only 3

(*d*) Ross, Neil and Havelock Islands have been renamed on 75th anniversary of freedom fighter Subhash Chandra Bose.

The new names of these islands are as follows:

Ross Island — Netaji Subhash Chandra Bose Dweep

Neil island — Shaheed Dweep

Havelock Island — Swaraj Dweep

Hence option (d) is correct.

20. Who among the following was posthumously conferred with the Ashoka Chakra, India's highest peacetime gallantry award, in 2019?
(a) Jyoti Prakash Nirala
(b) Nazir Ahmad Wani
(c) Hangpan Dada
(d) Mohan Nath Goswami

(***b***) Lance Naik Nazir Ahmad Wani was posthumously conferred with the Ashoka Chakra, India's Highest Peacetime Gallantry Award, in 2019. He is the first recipient of the Ashoka Chakra award from Kashmir region of the Indian State Jammu and Kashmir. He was from Jammu and Kashmir Light Infantry of the Indian Army.

2018 (II)

21. Which one of the following is the Official Mascot of the FIFA World Cup, 2018?
(a) Fuleco (b) Zakumi
(c) Pille (d) Zabivaka

(***d***) The official mascot for the FIFA World Cup, 2018 tournament was Zabivaka—A wolf dressed in the colours of the Russian national team.
The 2018 FIFA World Cup was the 21st FIFA World Cup, an international football tournament contested by the men's national teams of the member associations of FIFA, once every four years. France became the champion of FIFA World Cup, 2018.
FIFA 2022 will be scheduled in Qatar.

22. The Headquarters of the proposed National Sports University (as per the National Sports University Ordinance, 2018) will be set-up in
(a) Chhattisgarh (b) Manipur
(c) Kerala (d) West Bengal

(***b***) National Sports University is a sports university proposed to be set-up with its headquarters at Manipur in India. It was proposed to be set-up by the National Sports University Ordinance, 2018. The Ordinance was designed on the lines of the National Sports University Bill, 2017.

23. India, in June, 2018, asserted that any mega connectivity project must respect sovereignty and territorial integrity of the countries. The project referred to above is
(a) North-South Corridor Project
(b) Belt and Road Initiative
(c) Chabahar Port
(d) Panama Canal Expansion

(***b***) The Belt and Road Initiative (BRI), also known as One Belt One Road (OBOR) initiative is a Chinese effort which focusses on improving connectivity and cooperation among Asian countries, Africa, China and Europe. With the exception of Bhutan, which has no diplomatic ties with China, almost every other South Asian country has signed into OBOR, but India has not signed it. The main reason behind India's opposition towards the policy is the China-Pakistan Economic Corridor (CPEC), which is a part of OBOR. Keeping this in mind, In 2018 India asserted that to be a part of any mega project, it is necessary that the project must respect sovereignty and territorial integrity of the countries.

24. Who among the following is not a member of G-7?
(a) France (b) Germany
(c) Russia (d) Japan

(***c***) Russia is not a member of G-7. The Group of Seven (G-7) is a group of seven largest advanced economies in the world, consisting of Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. The 44th edition of G-7 Summit, 2018 was hosted by Canadian Prime Minister Justin Trudeau in Quebec (Canada) on 8-9th June. The 45th G-7 Summit, 2019 was held at France.

25. Dr. Bindeshwari Pathak, who was awarded Nikkei Asia Prize, 2018 for Culture and Community, is the founder of
(a) Bachpan Bachao Andolan
(b) PRS Legislative Research
(c) Sulabh Sanitation and Social Reform Movement
(d) Smile Foundation

(***c***) Dr. Bindeshwari Pathak, a noted social reformer and founder of Sulabh Sanitation and Social Reform Movement, was honoured with Japan's prestigious 'Nikkei Asia Prize for Culture and Community' for his significant work in tackling poor hygiene and discrimination.
The award was launched in 1996, and awarded for significant contributions in one of the three areas–regional growth; science, technology and innovation; and culture and community. Earlier former Prime Minister Manmohan Singh and Infosys Chairman Narayan Murthy are among the few Indians who have won the prize.

2018 (I)

26. Which of the following statements about Indian Academy of Highway Engineers is/ are correct?
1. It is a registered society.
2. It is a collaborative body of both Central Government and State Governments.

Select the correct answer using the codes given below
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(***c***) Indian Academy of Highway Engineers (IAHE) is a registered society under the administrative control of Ministry of Road Transport and Highways, Government of India. IAHE is a collaborative body of the central and state governments. The primary objective of IAHE is to fulfill the long standing need for Training of Highway Engineers at the entry level and during the service. Hence both statements 1 and 2 are correct.

27. Bharatmala Project is related to
(a) improving road connectivity
(b) interlinking ports and railways
(c) interlinking of rivers
(d) interlinking major cities with gas pipelines

(***a***) Bharatmala Project is the second largest highways construction project in the country. Bharatmala will look to improve road connectivity particularly on economic corridors, border areas and far flung areas with an aim of quicker movement of cargo and boosting exports. The ambitious umbrella programme is to subsume all existing highway projects including the flagship National Highways Development Project (NHDP).

28. Which of the following statements about 'Aadi Mahotsav' held recently in New Delhi is/are correct?
1. The theme of the festival was 'A Celebration of the Spirit of Tribal Culture, Cuisine and Commerce'.
2. The festival was organised by the Ministry of Tourism, Government of India.

Select the correct answer using the codes given below.
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(***a***) The Aadi Mahotsav, was held at Dilli Haat, New Delhi, on 30th November, 2017. The theme of this festival was 'A Celebration of the Spirit of Tribal Culture, Cuisine and Commerce'. Hence statement (1) is correct.
It was organised by the Tribal Secretary and not by Ministry of Tourism. Hence statement (2) is incorrect.
The 2019 edition of the Aadi Mahotsav held at Leh in Ladakh.

29. Which of the following statements regarding construction of Rohtang Tunnel is not correct?
(a) It is located at an altitude of 5,000 feet
(b) It will provide all-year connectivity to Lahaul and Spiti Valley
(c) The tunnel is being built by the Border Roads Organisation
(d) It will reduce the length of the Leh-Manali Highway by approximately 50 km

(*a*) Option (a) is not correct because the Rohtang Tunnel is located at an altitude of 3,100 metres (10,171 ft). It is being built by the Border Roads Organisation to provide all year connectivity to Lahaul and Spiti Valley. It also aims at reducing the distance of Leh-Manali Highway by approx 50 km.

30. Who among the following recently became the first woman pilot in Indian Navy?
(a) Astha Segal
(b) Roopa A
(c) Sakthi Maya S
(d) Shubhangi Swaroop

(*d*) Shubhangi Swaroop has made history by becoming the first woman to be inducted into the Indian Navy as a pilot. Shubhangi Swaroop is belonged to Bareilly in Uttar Pradesh.

31. Who among the following Indians did not hold the title of Miss World?
(a) Reita Faria (b) Sushmita Sen
(c) Diana Hayden (d) Yukta Mookhey

(*b*) Sushmita Sen is the winner of the Miss Universe Pageant of 1994. She was the first Indian woman to won the crown. All the other options are of former Miss World.

32. Which one of the following countries has failed to qualify for the first time in 60 years for the FIFA World Cup to be held in Russia in the year 2018 ?
(a) Mexico (b) Iran
(c) Saudi Arabia (d) Italy

(*d*) Italy has failed to qualify for the first FIFA World Cup in 60 years after play-off defeat to Sweden.

33. The Defence Technology and Trade Initiative (DTTI) is a forum for dialogue on defence partnership between India and
(a) Russia
(b) United States of America
(c) Israel
(d) France

(*b*) Defence Technology and Trade Initiative (DTTI), originally called the 'Carter Initiative' after the name of US Deputy Secretary of Defence Ashton Carter.

Its main objective is to strengthen defence cooperation by facilitating the Indian companies to collaborate with US partners in defence co-production, where the US provides technology and guidance for building modern weapon systems.

34. As per the policy applicable in 2017, how much Foreign Direct Investment (FDI) is permitted in the defence sector in India?
(a) 49 percent through the automatic route
(b) 26 percent through the government route
(c) 26 percent through the automatic route and beyond that up to 49 percent through the government route
(d) 75 percent through the automatic route

(*a*) In the defence sector, Foreign Direct Investment (FDI) up to 49% is permitted under the automatic route. Foreign Direct Investment beyond 49% and upto 100% is permitted through government approval, wherever it is likely to result in access to modern technology or for other reasons to be recorded.

35. Which one of the following countries did NOT participate in the 21st Edition of Exercise Malabar?
(a) United States of America
(b) Japan
(c) India (d) Australia

(*d*) The 2017 Malabar Exercise was the 21st Edition of the Exercise and conducted from 10th to 17th July, 2017. This edition involved navies from India, USA and Japan. The exercise included a harbor phase at Chennai. Hence, Australia was not a participant of this exercise.

Malabar Exercise, 2019 (23rd edition) was held in Japan from 26th September to 4th October, 2019.

36. Justice Dalveer Bhandari of India was recently re-elected to the International Court of Justice after Christopher Greenwood pulled out before 12th round of voting. Christopher Greenwood was a nominee of
(a) Canada (b) Russia
(c) Britain (d) USA

(*c*) Sir Christopher John Greenwood is a British judge at the International Court of Justice. He was elected on 6th November, 2008. He pulled out before the 12th round of voting.

37. In order to review the Income Tax Act, 1961 and to draft a new Direct Tax Law in consonance with economic needs of the country, the Government of India in November, 2017 has constituted a Task Force. Who among the following is made the convenor of it?
(a) Shri Arvind Subramanian
(b) Shri Arbind Modi
(c) Shri Amitabh Kant
(d) Dr. Bibek Debroy

(*b*) The Union Government has constituted a Task Force to review the Income Tax Act, 1961 and draft a new Direct Tax Law in consonance with current economic needs of the country. Central Board of Direct Taxes (CBDT) member Arbind Modi will be the convenor of task force while Chief Economic Adviser Dr. Arvind Subramanian will be a permanent Special Invitee. The task force will draft an appropriate direct tax legislation keeping in view the direct tax system in various countries, international best practices and economic needs of the country. It will submit its report within six months.

2017 (II)

38. Which one of the following is the theme of the 'World Health Day', 2017 celebrated by the World Health Organisation?
(a) Diabetes
(b) Food safety
(c) Depression : Let's Talk
(d) Ageing and Health

(*c*) The theme of 2017 World Health Day campaign was Depression: Let's Talk. 'World Health Day', celebrated on 7th April every year to mark the anniversary of the founding of WHO. It provides a unique opportunity to mobilise action around a specific health topic of concern to people all over the world. The theme of World Health Day, 2019 is Universal Health Coverage:Everyone, Everywhere.

39. Which one of the following ministries has launched a new programme on Interdisciplinary Cyber Physical Systems (ICPS) to foster and promote R and D?

(a) Ministry of Earth Sciences
(b) Ministry of Science and Technology
(c) Ministry of Information and Broadcasting
(d) Ministry of New and Renewable Energy

(*b*) Ministry of Science and Technology has launched a new programme 'Interdisciplinary Cyber Physical Systems' (ICPS) to foster and promote R&D in this emerging field of research. A Cyber Physical System (CPS) is a mechanism controlled or monitored by computer-based algorithms.

40. In April, 2017, India celebrated 100 years of Mahatma Gandhi's
(a) Satyagraha in Kheda
(b) Dandi March
(c) Satyagraha in Champaran
(d) Return from South Africa

(*c*) India celebrated 100 years of Gandhiji's Satyagraha in Champaran in April, 2017. The Champaran Satyagraha of 1917 was the first Satyagraha Movement inspired by Gandhi and a major revolt in the Indian Independence Movement. It was farmer's uprising that took place in Champaran.

41. Which one of the following is the theme of the International Day for Monuments and Sites (World Heritage Day), 2017?
(a) The Heritage of Sport
(b) Cultural Heritage and Sustainable Tourism
(c) Past and Present Heritage
(d) Heritage and Science

(*b*) On 18th April, 2017, whole world celebrated 'World Heritage Day'. Its theme was 'Cultural Heritage and Sustainable Tourism'. The World Heritage is observed every year on 18th April to spread awareness of the importance of protecting and preserving various sites around the world that has achieved world heritage status. The theme of World Heritage Day, 2019 was Rural Landscapes.

42. In April, 2017, the USA dropped MOAB (Massive Ordinance Air Blast popularly known as the Mother of All Bombs in the suspected hideouts of militants in which one of the following countries?
(a) Iran
(b) Syria
(c) Afghanistan
(d) Somalia

(*c*) The USA dropped the 'Mother of All Bombs', the most powerful conventional bomb in the American arsenal, on an Islamic State cave complex in Afghanistan. The strike was the first combat use of what is formally named the GBU-43/B Massive Ordinance Air Blast.

43. Who among the following is the winner of the Singapore Open Super Series Badminton Men's Singles title, 2017?
(a) Kidambi Srikanth (b) Lin Dan
(c) B Sai Praneeth (d) Kento Momota

(*c*) The 2017 Singapore Open Super Series was the fourth Super Series Tournament of the 2017 BWF Super Series. The tournament takes place in Singapore City, (Singapore) from 11-16th April, 2017. B Sai Praneeth won the Singapore Open Super Series, 2017. Kento Momoto won the Singapore Open Super Series, 2019.

2017 (I)

44. Which one of the following agencies enforces the laws on food security in India?
(a) FDA (b) WHO (c) FSSAI (d) FAO

(*c*) Food Safety and Standards Authority of India (FSSAI) is an autonomous body established under the Ministry of Health and Family Welfare, Government of India. It is responsible for protecting and promoting public health through supervision of food safety.

45. Who among the following personalities is not a Governor of any Indian State?
(a) Najma Heptulla
(b) Ram Nath Kovind
(c) Acharya Devvrat
(d) Arvind Subramanian

(*d*) Except Arvind Subramanian, all other given personalities are or remained governor of any Indian State. Arvind Subramanian is an Indian economist and the former Chief Economic Advisor to the Government of India, who served the position from 16th October, 2014 to 20th June, 2018.

46. Which of the following cricket teams was defeated by India to lift the Women's T-20 Asia Cup, 2016?
(a) Bangladesh (b) Sri Lanka
(c) Pakistan (d) Afghanistan

(*c*) Indian women's squad beat Pakistan by 17 runs in the final match of Women's T-20 Asia Cup, 2016 in Bangkok. Mithali Raj was player of the match as well as player of the series.

In Women's T-20 Asia Cup, 2018, Bangladesh defeated India by 3 wickets.

47. Which one of the following was the venue of second BRICS Youth Summit of the Ministers, Officials and Youth Delegations?
(a) New Delhi (b) Mumbai
(c) Shillong (d) Guwahati

(*d*) The second BRICS Youth Summit was held in Guwahati, Assam with the theme 'Youth as bridge for intra-BRICS exchanges'.

48. Headquarters of the World Meteorological Organisation is located in
(a) Washington (b) Geneva
(c) Moscow (d) London

(b) World Meteorological Organisation (WMO) is an inter-governmental organisation with a membership of 191 member states and territories. Its headquarter is in Geneva. It is originated from the International Meteorological Organisation (IMO). It is a specialised agency of the United Nations for meteorology.

49. Which one among the following countries is the largest trading partner of India in external trade for the year 2015-16?
(a) United States of America
(b) United Kingdom
(c) United Arab Emirates
(d) China

(*d*) China was the largest trade partner of India in its external trade for the financial year 2015-16. The cumulative trading amount was 65 billion USD including 56.95 billioin USD worth imports and exports worth 8.25 billion USD.

For the year 2018-19 also China is the largest trading Partner of India.

50. Which one of the following cities hosted the 3rd World Trauma Congress?
(a) New Delhi (b) Dacca
(c) Singapore (d) Bangkok

(*a*) New Delhi hosted the 3rd World Trauma Congress with the purpose of exchange ideas, experiences and lessons learned about the role of internationally accepted systems and protocols.

5th world Trauma congress (2020) to be held in Australia.

51. Arrange the following countries in ascending order on the basis of the total medals earned by them in Rio Olympic, 2016
1. United States 2. China
3. Great Britain 4. Russia

Select the correct answer using the codes given below.
(a) 1, 2, 3, 4
(b) 1, 3, 2, 4
(c) 4, 3, 2, 1
(d) 4, 3, 1, 2

(**c**) Rio Olympics, 2016 was an international multi-sport event that was held in Rio de Janeiro, Brazil. The United States topped the medal tally by winning highest number of gold (46) and overall medals (121). USA is followed by Great Britain (67), China (70) and Russia (56).

So, on the basis of medals earned the correct ascending order of countries will be–Russia<Great Britain<China<USA. Hence, option (c) is the correct choice.

Rio Olympic, 2020 to be held in Tokyo, Japan.

2016 (II)

52. Which of the following statement(s) regarding recent developments in international affairs is/are correct?

1. NATO's Defence Ministers agreed to boost support for Ukraine with a Comprehensive Package of Assistance, which will help it strengthen defence and to modernise its armed forces.
2. Israel announces plans for construction of dwelling units for Jewish settlers in a Palestinian neighbourhood of East Jerusalem.

Select the correct answer using the codes given below.
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(**c**) NATO's Defence Ministers agreed to support for Ukraine with a Comprehensive Package of Assistance.
The package aims:
- To help Ukraine strengthen its defences by building stronger security structures.
- To help Ukraine establish more effective and efficient defence and security structures, and to strengthen civilian control over them.
- To develop new projects, including in the areas of countering hybrid warfare and explosive devices.

Israel authorities have approved plans for the construction of 800 new housing units for Jewish settlers in a palestinian neighbourhood of East Jerusalem.

Hence both statements are correct.

53. Which country won the Thomas Cup Badminton Championship, 2016?
(a) Indonesia (b) Denmark
(c) China (d) Malaysia

(b) The Thomas Cup, sometimes called the World Men's Team Championship, is an international badminton competition among teams representing member nations of the Badminton World Federation (BWF). Thomas Cup Badminton Championship, 2016 won by Denmark. Thomas Cup after defeating Indonesia in the 2016 final. It was held in China. Thomas Cup 2018 was held in Bangkok (Thailand) and won by China. Thomas Cup 2020, 2022 and 2024 will be hosted by Denmark, Thailand and China respectively.

54. The world's largest biomass cookstove system for cooking in community kitchen has been installed at
(a) Tirupati (b) Amritsar
(c) Shirdi (d) Udupi

(**c**) The world's largest biomass cookstove system for cooking in community kitchen has been installed at Shirdi in Maharashtra to cook for 20,000 people per day and is saving around 60,000 kg of LPG every year. The kitchen is completely based on solar cooking system for community kitchen.

55. The United Nations proclaimed 21st June as
(a) International Mother Language Day
(b) International Widows' Day
(c) International Day of Yoga
(d) International Day against Drug Abuse and Illicit Trafficking

(**c**) International Yoga Day or Yoga Day, is celebrated on 21st June and was declared so by United Nations General Assembly (UNGA) on 11th December, 2014. Yoga is a physical, mental and spiritual practice or discipline that originated in India.

56. Who are Bhawana Kanth, Avani Chaturvedi and Mohana Singh?
(a) Sprinters (b) Rock climbers
(c) Fighter pilots
(d) Mountaineers

(**c**) In a historic event in Telangana for the first time women combat pilots were inducted in the Indian Air Force. The then Defence Minister Manohar Parrikar Commissioned three fighter Pilots Mohana Singh, Bhawana Kanth and Avani Chaturvedi. The trio's commissioned held at an elaborate combined graduation parade ceremony at the Air Force Academy in Dundigal.

57. Consider the following statement(s)

He was an Indian journalist, editor and author. He was associated with the Statesman, the Guardian, the Times of India, etc. He received the Ramnath Goenka Lifetime Achievement Award in 2013. He died in June, 2016.

He person referred to above is
(a) BK Karanjia
(b) KM Mathew
(c) Inder Malhotra
(d) Nanasaheb Parulekar

(**c**) Veteran journalist Inder Malhotra passed away at the age of 86 in Delhi on 11th June, 2016. Malhotra began his career in journalism at United Press of India after graduating from Punjab University. Inder Malhotra has authored several books including 'Indira Gandhi: A Personal and Political Biography (1989)', and 'Dynasties of India and Beyond (2003)'. He was also working on the book 'Indian Security: Past, Present and Future' which remains unfinished.

2016 (I)

58. Who among the following is/was associated with 'Maharashtra Andhashraddha Nirmoolan Samiti'?
(a) Chandi Prasad Bhatt
(b) Narendra Dabholkar
(c) GD Aggarwal
(d) Kailash Satyarthi

(**b**) Narendra Dabholkar, a rationalist, founded the Maharashtra Andhashraddha Nirmoolan Samiti in 1989. He actively campaigned against superstitions and confronted dubious tantriks. Dabholkar was murdered on 20th August, 2013.

59. *Nabhah Sprsam Diptam* is the motto of
(a) Central Industrial Security Force
(b) Indian Air Force
(c) ISRO (d) Indian Navy

(**b**) *Nabhah Sprsam Diptam* (Touch the sky with glory) is the motto of Indian Air Force. This has been taken from eleventh chapter of the Gita, the discourse given by Lord Krishna to Arjuna during the war by Mahabharata.

60. Who among the following is the founder of the 'Bachpan Bachao Andolan'?
(a) Shantha Sinha (b) Kailash Satyarthi
(c) Aruna Roy (d) Anil Agarwal

(*b*) Bachpan Bachao Andolan works in the field of Rights of children. It was started in 1980 by Nobel Laureate Kailash Satyarthi. The focus is on ending bonded labour, child labour and human trafficking. He was awarded with Nobel Peace Prize in 2014. He shared the Nobel Prize with Malala Yousafzai of Pakistan.

61. Arrange the following centres of AIIMS from East of West:

1. Rishikesh 2. New Delhi
3.Patna 4. Bhubaneswar

Select the correct answer using the codes given below.

(a) 4, 3, 1 and 2 (b) 4, 1, 3 and 2
(c) 3, 4, 2 and 1 (d) 3, 2, 1 and 4

(**a**) The correct order of AIIMS centres from East to west is-
Bhubaneswar ® Patna ® Rishikesh ® New Delhi

2015 (II)

62. The first Indian satellite, Aryabhatta, was launched in the year

(a) 1972 (b) 1975
(c) 1976 (d) 1979

*(**b**)* Aryabhatta Satellite was launched on 19th April, 1975, with weight of 360 kg. It was launched by Soviet Intercosmos rocket. The objectives of this project were to indigenously design and fabricate a space-worthy satellite system and evaluate its performance in orbit.

63. Which one of the following sea routes is the shortest from point to point?

(a) Kolkata to Yangoon
(b) Kolkata to Chennai
(c) Chennai to Port Blair
(d) Mumbai to Columbo

*(**c**)* Among the given options, Chennai to Port Blair is the shortest sea route. Distance between Chennai to Port Blair is 794 nautical miles. Distance between Kolkata to Yangon is 975 nautical miles. Distance between Kolkata to Chennai is 890 nautical miles. Distance between Mumbai to Columbo is 943 nautical miles.

64. The SAARC Secretariat is located at

(a) New Delhi (b) Columbo
(c) Kathmandu (d) Karachi

*(**c**)* The SAARC Secretariat is located in Kathmandu, Nepal. It was established in the year 1985. It coordinates and monitors implementation of activities, prepares for and services meetings, and serves as a channel of communication between the Association and its Member States as well as other regional organisations. Member countries of SAARC are India, Bhutan, Bangladesh, Afghanistan, Maldives, Nepal, Pakistan and Sri Lanka.

65. Who among the following is the author of the book 'Pakistan Paradox : Instability and Resilience'?

(a) Christopher Jaffrelot
(b) G Parthasarathy
(c) Imran Khan
(d) Mamnoon Hussain

(**a**) *The Pakistan Paradox* by Christopher Jaffrelot, a redoubtable scholar on South Asia, reveals a country grappling with multiple dichotomies, most of which can be traced to separatist ideals espoused by the country's early leaders.

G Parthasarathy, an Indian economist authored the Book 'Economic reforms and Rural Development in India.'

Imran Khan , ex- cricketer of Pakistan and currently a politician, wrote many books. One of them is 'Warrior Race : A Journey Through the Land of Tribal Pathan'. Imran Khan became 22nd Prime Minister of Pakistan.

Mamnoon Hussain is the 12th (2013-2018) President of Pakistan.

66. Which one of the following was the theme of the 'International Day of United Nations Peacekeepers' for the year 2015?

(a) Peacekeeping is a Global Partnership
(b) UN70 and UN Peacekeeping : Past, Present and Future
(c) Women in Peacekeeping
(d) A Force for the Future

*(**b**)* The theme of International Day of UN Peacekeepers, 2015, was 'UN70 and UN Peacekeeping: Past, Present and Future.' UN Peacekeepers day is celebrated on 29th May each year. It reaffirms our commitment to working together for peace. Since, its inception in 1948, the UN has established 69 peacekeeping operations in Africa, Asia, Europe and Middle East.

67. Indian athlete Vikas Gowda is associated with

(a) wrestling (b) sprint
(c) discus throw (d) archery

*(**c**)* Vikas Gowda is an Indian discus thrower and shot putter. Vikas Gowda won his first gold medal in Asian Championship in 2013 in Pune. His best moment came when he clinched the Gold for India in the 2014 Commonwealth games. He has qualified for the 2016 Olympics in the discus throw.

2015 (I)

68. Which of the following country/countries successfully inserted a spacecraft into Mars' orbit by maiden attempt?

(a) The USA (b) The USA and Russia
(c) India (d) China

*(**c**)* The Mars Orbiter Mission (MOM) also called Mangalyaan, is India's first interplanetary mission and ISRO has become the fourth space agency to reach Mars, after the Soviet Space Programme, NASA and the European Space Agency. It is a maiden attempt of India which is a very successful programme. Around the same time, NASA also send its another Mars Mission 'MAVEN'.

69. The European Space Agency (ESA) successfully landed a spacecraft on a speeding comet that lies 310 million miles away from Earth. As a result, the scientists may find out

(a) the age of the Sun
(b) the age of the Universe
(c) new fundamental atomic particles
(d) the role of the comets in bringing organic matter and water to Earth

*(**d**)* The head of the European Space Agency underlined Europe's pride in having achieved a unique first ahead of its US counterpart, NASA. The scientist may find out the role of the comets in bringing organic matter and water to Earth.

70. Who among the following nobel laureates has recently announced to put his Nobel Medal under auction?

(a) Kailash Satyarthi
(b) JD Watson
(c) Mikhail Gorbachev
(d) Jody Williams

*(**b**)* **James Dewey Watson** announced to put his Nobel Medal under auction in 2015. James Watson is an American molecular biologist, geneticist and zoologist. He is one of the scientists credited with discovering the structure of DNA. He had won the 1962 Nobel Prize in medicine with Francis Crick and Maurice Wilkins.

71. Which one among the following books is centered around 'environment'?

(a) The Late, Great Planet Earth
(b) Silent Spring
(c) Here I Stand
(d) And Then One Day

(b) 'Silent Spring' is an environmental science book written by Rachel Carson and published by Houghton Mifflin on 27th September, 1962.

List of other books and their authors is

Books	Authors
The Late, Great Planet Earth	Hal Lindsey
Here I Stand, (A life of Martin Luther)	Ronald Bainton
And Then One Day	Naseeruddin Shah

72. Who among the following cricket players has not died while playing a match?
(a) Wasim Raja
(b) Phillip Hughes
(c) Raman Lamba
(d) Vijay Hazare

(d) Vijay Hazare didn't die on field while playing a match. Vijay Samuel Hazare captained India in 14 matches between 1951 and 1953.

The other options belong to cricket players who died while playing a game. They died directly from injuries sustained while playing, or died after being taken ill on the ground.
- Wasim Raja was a Pakistani player. He suffered a heart attack on the pitch.
- Raman Lamba was an Indian cricketer who played mainly as a batsman. He died when a ball struck on the head while fielding.
- Phillip Joel Hughes was an Australian cricketer who died when a ball struck on his neck.

73. Which one among the following films was conferred the 'Golden Peacock' Award at the 45th International Film Festival of India, 2014?
(a) Ek Hazarachi Note
(b) A Short Story
(c) Leviathan
(d) Ajeyo

(c) The Russian film 'Leviathan' has been awarded the Golden Peacock Award at the 45th International Film Festival of India (IFFI), 2014. This Russian drama film directed by Andrey Zvyagintsev.

74. Which one of the following books won the Man Booker Prize, 2014?
(a) History of the Rain
(b) The Lives of Others
(c) The Narrow Road to the Deep North
(d) How to be Both

(c) Australian author Richard Flanagan had won the 2014 Man Booker Prize for his book 'The Narrow Road to the Deep North'. Until 2014, only novels written in English and from authors in the Commonwealth, including the UK, the Republic of Ireland and Zimbabwe were eligible for consideration; however from 2014 rules were changed to extend eligibility to any novel written in English.

The 2019 Booker Prize for fiction was awarded jointly to Margret Atwood for the Testaments' and Bernardine Evaristo for Girl, Woman and Other.

75. East zone won the prestigious Deodhar Trophy cricket tournament 2014-15 by defeating
(a) South zone (b) Central zone
(c) West zone (d) North zone

(c) East zone defeat West zone by 24 runs and won the 2014-15 Deodhar Trophy of cricket on 3rd December, 2014.

2014 (II)

76. In the term GIS, 'G' stands for
(a) Global (b) Geographic
(c) Goodness (d) Geological

(b) GIS means Geographic Information System. It is a computer system designed to capture, store, manipulate, analyse, manage and present all types of spatial or geographical data. GIS applications are used in a number of fields.

77. Consider the following statements about Indian Regional Navigation Satellite System (IRNSS)
1. IRNSS is a constellation of five satellites, which were launched by PSLV.
2. It is an independent regional navigation satellite system designed to provide position information in the Indian region.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) Indian Regional Navigation Satellite System (IRNSS) is an independent regional navigation system, designed to provide accurate position information to the users in India as well as the region extending up to 1500 km from its boundary.

The space segment of IRNSS consists of seven satellites with three satellites in geostationary orbit and four satellites in inclined geosynchronous orbit. Hence, only statement(2) is correct.

78. The National Defence Academy has foreign cadets from which of the following countries?
1. Bhutan 2. Bangladesh
3. Nepal 4. Afghanistan

Select the correct answer using the codes given below.
(a) 1, 2, 3 and 4 (b) 1, 3 and 4
(c) 3 and 4 (d) 1 and 2

(b) The National Defence Academy has so far trained more than 700 cadets from about 27 friendly countries including Bhutan, Nepal, Afghanistan, Iran, Malaysia, etc.

Hence, option (b) is the correct answer.

79. Consider the following statements
1. China won both Thomas Cup and Uber Cup badminton tournaments, 2014.
2. The Uber Cup is the World Team Championship for women and the Thomas Cup is for men.

Which of the statement(s) given above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) The Thomas and Uber Cup are the premier badminton team championships for men and women respectively. The 2014 Thomas Cup and the Uber Cup was held on 18th to 25th May, 2014 in New Delhi, India. This was the first time India hosted the two cups.
- The Thomas Cup final featured two teams Japan and Malaysia. Japan came out on top by a score of 3 matches to 2, thus winning their first Thomas Cup.
- The Uber Cup final featured the top two teams, China and Japan. China came out on top by a 3–1 score. It was their 13th title overall for China.

2018 Thomas Cup was won by China while 2018 Uber Cup was won by Japan. The Thomas and Uber Cup in 2020 are scheduled to be held at Aarhus, Denmark.

80. Consider the following statements about *Drishti* system
1. It is a sophisticated instrument to assess the runway visual range.
2. This system is developed by Airports Authority of India.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Drishti system is a sophisticated instrument for assessment of runway visual range, which is a critical parameter for safe landing and take off aircraft in poor visibility. Hence, statement (1) is correct

It is designed and developed by CSIR-NAL. Hence, statement(2) is incorrect.

81. Which of the following about Malavath Poorna is correct?

(a) First Indian swimmer to cross Suez Canal
(b) Youngest woman to scale Everest
(c) First Indian to win FIDE World Junior Championship
(d) Winner of Young Scientist Award, 2014

(b) Malavath Poorna is an Indian mountaineer from Telangana. Poorna, a girl of 13 years and 11 months, scaled the highest peak of Mount Everest and became the youngest girl to have reached the summit of Everest.

First Indian swimmer to cross Suez Canal is Mihir Sen.

First Indian to win FIDE World Junior Championship is Viswanathan Anand.

Winner of Young Scientist Award, 2014 is Sahil Doshi.

82. Which of the following statement(s) about SIMBEX 14 is/are correct?

1. It was a bilateral naval exercise between Indian Navy and Sri Lankan Navy.
2. It was the 21st in the series.
3. It was conducted in the Andaman Sea.

Select the correct answer using the codes given below

(a) 1, 2 and 3 (b) 2 and 3
(c) Only 2 (d) 1 and 3

(b) The 2014 edition of the 'SIMBEX 14' series of annual bilateral naval exercises between the Indian Navy (IN) and the Republic of Singapore Navy (RSN), was conducted in the Andaman Sea from 22th - 28th May, 2014. It was the 21st in the series. Hence option (b) is correct.

The operational interaction between the Indian Navy and Singapore Navy has grown steadily since 1994.

In 1999, this operational interaction was formalised as an annual bilateral exercise and was christened 'SIMBEX'.

83. Consider the following statements

1. *Mobile Seva*, the National Mobile Governance Initiative of Department of Electronics and Information Technology of India, has won the United Nations Public Service Award, 2014.
2. The innovative initiative is aimed at mainstreaming mobile governance in the country.

Which of the statements given above is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) An initiative conceptualised by Indian Government's Department of Electronics and Information Technology to enable delivery of public services electronically through the mobile platform has been awarded a United Nations Public Service Award. This programme was launched in 2011.

84. Consider the following statements about Prithvi-II missile

1. It is a nuclear-capable ballistic missile.
2. It has a maximum range of 100 km.
3. It is India's first native made ballistic missile.

Which of the statement(s) given above is/are correct?

(a) 1, 2 and 3
(b) Only 1
(c) 1 and 3
(d) 2 and 3

(c) Prithvi is a tactical surface-to-surface Short-Range Ballistic Missile (SRBM) developed by DRDO of India under the Integrated Guided Missile Development Program (IGMDP).

Prithvi II class is a single-stage liquid-fueled missile which has been indigenously built. It is also nuclear-capable. Hence statement (1) and (3) are correct.

Prithvi II missile has an extended range of 250 km (Prithvi I had range of 100 km) thus having the maximum range of upto 350 km. Hence, statement (2) is incorrect.

Prithvi III class is a two-stage surface-to-surface missile. The first stage is solid-fueled while the second stage is liquid-fueled.

2014 (I)

85. Which of the following is/are true relating to Nelson Mandela?

1. He was the first President of South Africa.
2. Mandela spent 18 years in prison on Robben island.
3. Mandela became the President in 1990 after his release from the prison.

Select the correct answer using the codes given below.

(a) 1 and 3 (b) 2 and 3
(c) Only 2 (d) 1, 2 and 3

(c) Nelson Mandela was a South African anti-apartheid revolutionary, political leader, and philanthropist.

Pieter Willem Botha was the first President of South Africa during apartheid. Nelson Mandela was the country's first black head of state and the first elected in a fully representative democratic election. He served as President of South Africa from 1994 to 1999. Hence, statement (1) and (3) is incorrect.

Nelson Mandela served 27 years in prison, split between Robben Island, Pollsmoor Prison, and Victor Verster Prison. Mandela was imprisoned at Robben Island for 18 years out of the 27 years of prision. Hence statement (2) is correct.

86. Which of the following books was written by BR Ambedkar?

(a) The Discovery of India
(b) My Life
(c) Buddha or Karl Marx
(d) Hind Swaraj

(c) Buddha or Karl Marx was written by BR Ambedkar. Other books written by BR Ambedkar - 'The Buddha and his Dharma', 'Annihilation of Caste','Who were the Sudras', 'Pakistan or Partition of India', 'Idea of a Nation', 'The Untouchable.'

'The Discovery of India was written by Jawaharlal Nehru'.

'My Life is autobiography of Bill Clinton'. 'Hind Swaraj' was written by Mohandas Karamchand Gandhi.

87. Consider the following statements about 'Sahayog Kaijin'

1. It is a joint exercise of the Coast Guards of India and Japan.

2. Its 13th edition was held recently off Chennai Port.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› *(**a**)* The 1st edition of bilateral exercise by the Indian Coast Guard and Japanese counterpart titled 'Sahayog Kaijin'.

The 13th edition of Sahayog Kaijin held at Kochi. Hence, only statement(1) is correct.

88. Consider the following statements about grand slam tennis tournaments

1. There are four grand slam annual tennis tournaments.
2. The first grand slam of a year is the US Open.
3. The Australian and the US tournaments are played on grass court.

Which of the statement(s) given above is/are correct?

(a) 1, 2 and 3 (b) 1 and 2
(c) Only 1 (d) 2 and 3

› *(**c**)* There are four Grand Slams in a year- Australian Open, French Open, Wimbledon and US Open. Hence, statement (1) is correct.

Australian Open is the first Grand Slam of a year. The Grand Slam itinerary consists of the Australian Open in mid-January, the French Open in May and June, Wimbledon in July, and the US Open in August and September. Hence statement (2) is incorrect.

Australian and US Open are played on hard court, French Open on clay court and Wimbledon on grass court. Hence, statement (3) is incorrect.

89. EV Ramaswami Naicker was associated with

1. reforming Brahminism
2. the Self-Respect Movement
3. the Low Caste Movement
4. the Communist Movement

Select the correct answer using the codes given below.

(a) 1, 2, 3 and 4 (b) 1 and 3
(c) 1 and 4 (d) 2 and 3

› *(**d**)* EV Ramaswami Naicker was associated with anti-brahminism. He worked for the lower castes. He was also influenced by the idea of communism. His greatest contribution is though the Self-Respect Movement. Self-Respect Movement was started by EV Ramaswami (popularly known as Periyar) in Tamil Nadu in 1925.

90. Consider the following statements about 'Param Vir Chakra'

1. It is the highest peacetime gallantry award.
2. It is the replacement of British Victoria Cross.
3. It can be awarded to civilians.

Which of the statement(s) given above is/are correct?

(a) Only 1 (b) Only 2
(c) 2 and 3 (d) 1, 2 and 3

› *(**b**)* Param Vir Chakra is the replacement of British Victoria Cross. It cannot be awarded to civilians and is not a peacetime gallantry award. Ashok Chakra is the peacetime award.

Param Vir Chakra is India's highest military decoration awarded for the highest degree of valour or self-sacrifice in the presence of the enemy. It is a war time gallantry award established on 26th January, 1950. It is lower to Bharat Ratna and higher to Ashoka Chakra.

91. What was the code name for the Indian Air Force operations during the Kargil conflict of 1999?

(a) Operation Vijay
(b) Operation Meghdoot
(c) Operation Indradhanush
(d) Operation Safed Sagar

› *(**d**)* **Operation Safed Sagar** was the code name of Indian Air Force during Kargil War of 1999, that was aimed to flush out regular and irregular troops of the Pakistani Army from vacated Indian positions in the Kargil sector along the line of control.

Operation Vijay Name of Indian Army operation to clear Kargil sector in 1991.

(1961 'Operation Vijay' - Operation by military of India that led to capture of Goa, Daman and Diu and Anjediva Island).

Operation Meghdoot Code name for the Indian Armed force operation to capture the Siachen glacier in Kashmir, launched on 13th April, 1984. This military operation was unique as the 1st assault launched in the world's highest battle launched in the world's highest battlefield.

Operation Indradhanush 4 of India's fleet of Russian designed S4-30 MK1 Flanker Fighter Aircraft were pitted against RAF's Typhoon FGR4 fighter planes in Lincolnshire in this operation.

92. The Indian National Army was organised to fight against

(a) Germany and Japan
(b) British in India
(c) Japan
(d) Russia

› *(**b**)* The Indian National Army was organised to fight against the British in India. The Indian National Army (INA) fought for Japan in the Burma Theater 1942-45. It consisted of soldiers from Britain's then colony India who had been captured by the Japanese and convinced to fight against British and Commonwealth forces.

The Indian National Army was an armed force formed by Indian nationalists in 1942 in South-East Asia during World War II. Its aim was to secure Indian independence from British rule. Its formed an alliance with Imperial Japan in the latter's campaign in the South-East Asian theatre of WWII.

93. Consider the following statements

1. 'SAMARTH 2014', a national festival for showcasing abilities of persons with disabilities, was organised recently in New Delhi by the Ministry of Culture
2. India is a party to the United Nations Convention on the Rights of Persons with Disabilities.

Which of the statement given above is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

› *(**b**)* SAMARTH 2014, a national festival for show casing abilities of persons and disability was inaugurated in New Delhi. It was organised by Department of Disability Affairs, Ministry of Social Justice and Empowerment. India is a party to the United National Convention on the Rights of Persons with Disabilities. Hence, only statement (2) is correct.

94. Which one among the following is not true about Bal Gangadhar Tilak?

(a) He started the New English School in Poona
(b) He founded the Shri Shivaji Fund Committee for celebration of Shiv Jayanti
(c) He preached the cult of self-reliance
(d) He transformed the household worshipping of Ganesha into a public event

*(**c**)* Bal Gangadhar Tilak started the new English school in Poona (Pune) and also founded the Shri Shivaji fund Committee. He transformed the household worshipping of Ganesha into a public event whose energy and momentum was used in the National Movement.
Cult of self-reliance was preached by Rabindranath Tagore.
Hence option (c) is incorrect.